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Particle solutions in a unified field theory

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A numerical solution is obtained for the static electric spherically symmetric field in Bonnor's unified field theory. This solution can be interpreted as a distributed particle with a finite self-energy. Important constants of the theory k and p are evaluated by assuming the particle carries the electron's charge and mass.

I. INTRODUCTION

In the thirty years that has elapsed since Einstein proposed his nonsymmetric unified field theory[1] progress in the area has been slow. Closed-form solutions for Einstein's equations were obtained by Papapetrou,[2] Wyman,[3] and Bonnor.[4] These solutions were not, however, in accord with nature. It appeared that Einstein's equations did not lead to the correct Lorentz equations of motion for electromagnetic charges.[5] This problem was again examined in 1971 when Johnson developed a fast-motion approximation which showed that in the lowest order the proper equations of motion are obtained.[6] In higher-order correction terms, however, long-range forces appear that have not yet been observed in nature.[7]

Because of the problems associated with Einstein's theory two modifications of it soon appeared. The first was published in 1952 by Kursunoğlu[8]; the other was published in 1954 by Bonnor.[9] The form used by Bonnor was previously considered and rejected by Einstein for lack of a compelling reason to include it.[10] In this paper we will be concerned only with Bonnor's theory. Closed-form magnetic-monopole solutions for Bonnor's equations were obtained by Pant in 1975.[11] Pant's interpretation of his result is based on Einstein's original identification of the dual of the electromagnetic tensor with the asymmetric part of the metric tensor. An identical solution was obtained by Boal and Moffat but they chose to interpret it as an electric monopole.[12] This is, of course, in apparent accord with nature. Pant comments, in fact, that his result seems to allow magnetic monopoles which are not observed, but forbids electric monopoles which are observed.

In this paper we prefer to stay with Einstein's interpretation.[13] Using it, a numerical solution is obtained for the static, electric, spherically symmetric field in Bonnor's theory. Particle-type solutions are obtained which demonstrate a short-range attraction that stabilizes the charge distribution with a finite self-energy. The Bonnor term in the field equations may be interpreted as a generalized energy-momentum density. The charge and mass of the electron is used to determine the constants introduced by Bonnor.

II. FIELD EQUATIONS

Bonnor's modification of Einstein's nonsymmetric theory may be derived from the Hamiltonian

$$\mathcal{H} = \sqrt{-g}\,g^{\mu\nu}(R_{\mu\nu} - \tfrac{1}{2}p^2 g_{[\mu\nu]}), \tag{2.1}$$

where

$$R_{\mu\nu} = \partial_\alpha \Gamma^\alpha_{\mu\nu} - \partial_\nu \Gamma^\alpha_{\mu\alpha} - \Gamma^\alpha_{\mu\sigma}\Gamma^\sigma_{\alpha\nu} + \Gamma^\alpha_{\mu\nu}\Gamma^\sigma_{\alpha\sigma} \tag{2.2}$$

is the contracted curvature tensor, $g^{\mu\nu}$ is the contravariant nonsymmetric metric tensor, and $\Gamma^\alpha_{\mu\nu}$ is the nonsymmetric affine connection. The field equations are obtained from

$$\delta \int \mathcal{H}\, d\tau = 0, \tag{2.3}$$

where $g^{\mu\nu}$ and $\Gamma^\alpha_{\mu\nu}$ are varied independently subject to the constraint

$$\Gamma^\nu_{[\mu\nu]} = 0. \tag{2.4}$$

The resulting field equations are

$$\partial_\alpha g_{\mu\nu} - g_{\sigma\nu}\Gamma^\sigma_{\mu\alpha} - g_{\mu\sigma}\Gamma^\sigma_{\alpha\nu} = 0, \tag{2.5}$$

$$R_{(\mu\nu)} + I_{(\mu\nu)} = 0, \tag{2.6}$$

$${}_{[\lambda}R_{[\mu\nu]]} + {}_{[\lambda}I_{[\mu\nu]]} = 0, \tag{2.7}$$

$$\Gamma^\nu_{[\mu\nu]} = 0, \tag{2.8}$$

where

$$I_{\mu\nu} = -\tfrac{1}{2}p^2(g_{\mu\sigma}g^{[\sigma\rho]}g_{\rho\nu} + \tfrac{1}{2}g_{\mu\nu}g_{\sigma\rho}g^{[\sigma\rho]} + g_{[\mu\nu]}) \tag{2.9}$$

and p is a constant to be determined experimentally. The notation used here is

$${}_{[\lambda}R_{\mu\nu]} \equiv \partial_\lambda R_{\mu\nu} + \partial_\mu R_{\nu\lambda} + \partial_\nu R_{\lambda\mu},$$

$$g_{(\mu\nu)} \equiv \tfrac{1}{2}(g_{\mu\nu} + g_{\nu\mu}),$$

$$g_{[\mu\nu]} \equiv \tfrac{1}{2}(g_{\mu\nu} - g_{\nu\mu}).$$

III. SOLUTIONS

A. Analytic results

The 82 simultaneous equations (2.5) to (2.8) are considerably simplified by selecting the spherically symmetric metric corresponding to a radial electric field (this is a magnetic field in Boal and Moffat's paper) to be

$$g_{\mu\nu}=\begin{bmatrix} -\alpha & 0 & 0 & 0\\ 0 & -\beta & f\sin\theta & 0\\ 0 & -f\sin\theta & -\beta\sin^2\theta & 0\\ 0 & 0 & 0 & \gamma \end{bmatrix}. \tag{3.1}$$

The corresponding contravariant $g^{\mu\nu}$ is obtained from

$$g^{\mu\nu}g_{\alpha\nu}=\delta^{\mu}_{\alpha}. \tag{3.2}$$

A universal constant k connects $g_{[\mu\nu]}$ with the dual of the electromagnetic field tensor $F^{\mu\nu}$. The relationship is

$$kF^{\mu\nu}\equiv\tfrac{1}{2}e^{\mu\nu\alpha\beta}g_{[\alpha\beta]},$$

where $e^{\mu\nu\alpha\beta}$ is the Levi-Civita tensor density.

Using (3.1) in (2.5) we obtain 64 linear simultaneous algebraic equations. They determine the affine connections in terms of the $g_{\mu\nu}$ and its derivatives. It is easy to see that these $\Gamma^{\mu}_{\nu\alpha}$ satisfy (2.8). The task of solving 64 simultaneous equations may seem formidable, but in practice the use of (3.1) results in considerable simplification. The 64 equations break into independent groups each of which includes only a fraction of the original number of variables. The solution of these equations has been obtained previously by Papapetrou.[14] We simply quote his results:

$$\begin{aligned}
&\Gamma^1_{11}=\frac{\alpha'}{2\alpha},\\
&\Gamma^1_{22}=\csc^2\theta\,\Gamma^1_{33}=\frac{fB-\beta A}{2\alpha},\\
&\Gamma^1_{23}=-\Gamma^1_{32}=\frac{fA+\beta B}{2\alpha}\sin\theta,\\
&\Gamma^1_{44}=\frac{\gamma'}{2\alpha},\\
&\Gamma^2_{12}=\Gamma^2_{21}=\tfrac{1}{2}A,\\
&\Gamma^2_{13}=-\Gamma^2_{31}=\tfrac{1}{2}B\sin\theta,\\
&\Gamma^2_{33}=-\sin\theta\cos\theta,\\
&\Gamma^3_{13}=\Gamma^3_{31}=\tfrac{1}{2}A,\\
&\Gamma^3_{21}=-\Gamma^3_{12}=\tfrac{1}{2}B\csc\theta,\\
&\Gamma^3_{23}=\Gamma^3_{32}=\cot\theta,\\
&\Gamma^4_{14}=\Gamma^4_{41}=\frac{\gamma'}{2\gamma},
\end{aligned} \tag{3.3}$$

where

$$\begin{aligned}
A&=\frac{ff'+\beta\beta'}{f^2+\beta^2},\\
B&=\frac{f\beta'-\beta f'}{f^2+\beta^2}.
\end{aligned} \tag{3.4}$$

The primes denote differentiation with respect to r.

Using the affine connections (3.3) in Eqs. (2.6) we obtain six equations which are not identically zero. They are

$$R_{11}+I_{11}=-A'-\tfrac{1}{2}(A^2+B^2)+A\frac{\alpha'}{2\alpha}+\frac{\gamma'}{2\gamma}\left(\frac{\alpha'}{2\alpha}-\frac{\gamma'}{2\gamma}\right)-\left(\frac{\gamma'}{2\gamma}\right)'-\frac{p^2}{2}\frac{\alpha f^2}{f^2+\beta^2}=0, \tag{3.5}$$

$$\begin{aligned}
R_{22}+I_{22}&=[(fB-\beta A)/(2\alpha)]'+(fB-\beta A)\frac{[\ln(\alpha\gamma)]'}{4\alpha}+B(fA+\beta B)/(2\alpha)+1+\frac{p^2}{2}\frac{\beta f^2}{f^2+\beta^2}\\
&=0=\csc^2\theta\,(R_{33}+I_{33}),
\end{aligned} \tag{3.6}$$

$$R_{44}+I_{44}=\left(\frac{\gamma'}{2\alpha}\right)'+\frac{\gamma'}{2\alpha}\left(\frac{\alpha'}{2\alpha}-\frac{\gamma'}{2\gamma}+A\right)+\frac{p^2}{2}\frac{\gamma f^2}{f^2+\beta^2}=0, \tag{3.7}$$

$$\begin{aligned}
\csc\theta\,(R_{23}+I_{23})&=[(fA+\beta B)/(2\alpha)]'-B(fB-\beta A)/(2\alpha)+(fA+\beta B)\left(\frac{\alpha'}{2\alpha}+\frac{\gamma'}{2\gamma}\right)\Big/(2\alpha)+\frac{p^2}{2}f\left(\frac{\beta^2}{f^2+\beta^2}+1\right)\\
&=+iQ=-\csc\theta\,(R_{32}+I_{32}).
\end{aligned} \tag{3.8}$$

In Eq. (3.8), iQ is a constant of integration. Its meaning will be clarified later.

Now, let

$$\begin{aligned}
\beta&=R\cosh\psi,\\
f&=iR\sinh\psi;
\end{aligned} \tag{3.9}$$

then

$$\begin{aligned}
A&=R'/R,\\
B&=-i\psi'.
\end{aligned} \tag{3.10}$$

Taking the linear combination

$$\gamma(R_{11}+I_{11})+\alpha(R_{44}+I_{44})=0, \qquad (3.11)$$

we obtain

$$A'+\tfrac{1}{2}(A^2+B^2)-\tfrac{1}{2}A[\ln(\alpha\gamma)]'=0, \qquad (3.12)$$

which becomes

$$\left(\frac{R'}{R}\right)'+\frac{1}{2}\left[\left(\frac{R'}{R}\right)^2-(\psi')^2\right]-\frac{1}{2}\frac{R'}{R}[\ln(\alpha\gamma)]'=0. \qquad (3.13)$$

This may be written as

$$\left[\frac{R'}{\sqrt{R\alpha\gamma}}\right]'-\frac{1}{2}\left[\frac{R'}{\sqrt{R\alpha\gamma}}\right]\frac{R}{R'}(\psi')^2=0, \qquad (3.14)$$

which has the formal solution

$$\sqrt{\alpha\gamma}=\frac{C}{2}\frac{R'}{\sqrt{R}}e^{\Phi}, \qquad (3.15)$$

where

$$\Phi'=\frac{R}{2R'}(\psi')^2$$

and C is a constant.

Now proceed to $R_{22}+I_{22}=0$, which we simplify by the use of

$$\xi=\beta A-fB \text{ and } i\eta=fA+\beta B \qquad (3.16)$$

and (3.9). They give

$$\left(\frac{-\xi}{2\alpha}\right)'-\frac{\xi}{4\alpha}[\ln(\alpha\gamma)]'+\psi'\eta/(2\alpha)+1-\tfrac{1}{2}p^2R\cosh\psi\sinh^2\psi=0, \qquad (3.17)$$

which may be written as

$$\left(\xi\frac{\sqrt{\alpha\gamma}}{2\alpha}e^{\Theta}\right)'-\sqrt{\alpha\gamma}\,e^{\Theta}(1-\tfrac{1}{2}p^2R\cosh\psi\sinh^2\psi)=0, \qquad (3.18)$$

where

$$\Theta'=-\frac{\eta}{\xi}\psi'.$$

Integrating and solving for α we obtain

$$\alpha=\frac{\tfrac{1}{2}\sqrt{\alpha\gamma}\,\xi\,e^{\Theta}}{C_1+\int_0^r e^{\Theta}\sqrt{\alpha\gamma}\,(1-\tfrac{1}{2}p^2R\cosh\psi\sinh^2\psi)dr}. \qquad (3.19)$$

The lower limit is taken as zero and C_1 set to zero to exclude a singularity at the origin.

Turning now to the equation for R_{23}, we can write it as

$$\left(\frac{\eta}{2\alpha}\right)'-\frac{\xi}{2\alpha}\psi'+\frac{\eta}{2\alpha}[\ln(\sqrt{\alpha\gamma})]'-Q+\tfrac{1}{2}p^2R\sinh\psi(1+\cosh^2\psi)=0. \qquad (3.20)$$

This can be integrated to give the expression

$$\alpha=\frac{\tfrac{1}{2}\eta\sqrt{\alpha\gamma}\,e^{\Lambda}}{C_2+\int_0^r e^{\Lambda}\sqrt{\alpha\gamma}[Q-\tfrac{1}{2}p^2R\sinh\psi(1+\cosh^2\psi)]dr}, \qquad (3.21)$$

where $\Lambda'=(-\xi/\eta)\psi'$.

Now we have two apparently different expressions for α. In order to cause the two expressions to be the same, we use the method of Procrustes and require

$$\tfrac{1}{2}\sqrt{\alpha\gamma}\,\xi e^{\Theta}\left[C_2+\int_0^r e^{\Lambda}\sqrt{\alpha\gamma}[Q-\tfrac{1}{2}p^2R\sinh\psi(1+\cosh^2\psi)]dr\right]=\tfrac{1}{2}\sqrt{\alpha\gamma}\,\eta e^{\Lambda}\left[C_1+\int_0^r e^{\Theta}\sqrt{\alpha\gamma}\,(1-\tfrac{1}{2}p^2R\cosh\psi\sinh^2\psi)dr\right]. \qquad (3.22)$$

This is an integrodifferential equation with dependent variable ψ and independent variable r. Equation (3.16) can be used to eliminate $\sqrt{\alpha\gamma}$. We have used only three of the four equations to arrive at our result, but Kursunoğlu has shown that the four equations are not independent. The fourth equation is satisfied identically by a solution of the other three. Also, the Bianchi relation requires that the choice of the radial coordinate be arbitrary. Choosing $r^2=R$ we can eliminate R; this leaves us with an ordinary nonlinear integrodifferential equation in ψ and r. Before proceeding to its solution we will examine its asymptotic forms. They will allow us to interpret the constants of integration. C_1 and C_2 are dropped.

B. Asymptotic forms

We can write Eq. (3.22) as

$$\frac{\xi e^{\Theta}}{\eta e^{\Lambda}}=\frac{\int_0^r e^{\Theta}\sqrt{\alpha\gamma}(1-\tfrac{1}{2}p^2R\cosh\psi\sinh^2\psi)dr}{\int_0^r e^{\Lambda}\sqrt{\alpha\gamma}[Q-\tfrac{1}{2}p^2R\sinh\psi(2+\sinh^2\psi)]dr}. \qquad (3.23)$$

Consider the case where $p=0$. Rewriting Eq. (3.16) as

$$\xi = R' \cosh\psi + R\psi' \sinh\psi, \qquad \eta = R' \sinh\psi - R\psi' \cosh\psi, \tag{3.24}$$

we try $\psi = \text{constant}$ ($\psi' = 0$). With this $\Theta = \Lambda = 0$ and

$$\xi/\eta = \coth\psi_0 = \text{constant}. \tag{3.25}$$

Thus, we have from Eq. (3.23)

$$e^{\Lambda}\sqrt{\alpha\gamma} = e^{\Theta}\sqrt{\alpha\gamma}\, Q^{-1} \tanh\psi_0, \tag{3.26}$$

and therefore,

$$Q^{-1} = \coth\psi_0. \tag{3.27}$$

The integrodifferential equation (3.23) is therefore satisfied when $p^2 = 0$ by

$$\psi \equiv \psi_0 = \text{constant},$$

$$\beta = R \cosh\psi_0,$$

$$f = iR \sinh\psi_0,$$

and

$$\sqrt{\alpha\gamma} = \frac{C}{2}\frac{R'}{\sqrt{R}}\ . \tag{3.28}$$

The choice $\beta \equiv r^2$ makes the solution identical to Papapetrou's solution[14] for the homogeneous Einstein set of equations.

When this solution is interpreted as representing a distributed charge and energy density, we find the integrals do not converge in an asymptotically flat space. We regard this solution as an asymptotic solution describing the charge distribution near the center of the particle.

To find the appropriate asymptotic solution in the region of large r we assume $f \simeq iF = \text{constant}$. Again choosing $\beta = r^2$ and neglecting terms where $F^2 \ll p^2 r^4$, we find

$$\cosh\psi \simeq 1, \quad \sinh\psi \simeq F/r^2, \quad \xi \simeq 2r, \quad \eta \simeq 4F/r. \tag{3.29}$$

Using them in (3.15) and the right-hand side of (3.19) we obtain

$$\sqrt{\alpha\gamma} \simeq C,$$

$$e^{\Theta} \simeq 1,$$

and

$$\alpha^{-1} \simeq r^{-1}\left(\int_0^r dr - \int_0^\infty \tfrac{1}{2}Rp^2 \cosh\psi \sinh^2\psi\, dr + \int_r^\infty \frac{F^2p^2}{2}\frac{dr}{r^2}\right). \tag{3.30}$$

If now we let

$$2M \equiv \int_0^\infty \tfrac{1}{2}R^2p^2 \cosh\psi \sinh^2\psi\, dr, \tag{3.31}$$

we can write (3.30) as

$$\alpha^{-1} = 1 - \frac{2M}{r} + \frac{F^2p^2}{2}\frac{1}{r^2}\ . \tag{3.32}$$

This is in agreement with the Reissner-Nordström[15] solution if we identify the mass and charge by

$$2M = \frac{2m\kappa}{c^2} \tag{3.33}$$

and

$$\frac{F^2p^2}{2} = \frac{2\kappa e^2}{c^4}\ . \tag{3.34}$$

Turning now to the expression on the right-hand side of Eq. (3.21) and using $F = \text{constant}$ as above, we find that

$$e^{\Lambda} \simeq r,$$

and this in turn gives

$$\alpha^{-1} \simeq (2F)^{-1}\int_0^r r[Q - \tfrac{1}{2}p^2F(2 + \sinh^2\psi)]dr = (2F)^{-1}\int_0^r r(Q - p^2F)dr - \tfrac{1}{4}p^2\int_0^r r\sinh^2\psi\, dr. \tag{3.35}$$

This is obviously not the previous asymptotic form. To accomplish this we put $Q = Fp^2$; this eliminates the first term in (3.35).

We conclude, in agreement with Pant, that only the trivial solution exists in the region where the charge density vanishes ($F' = 0$). In order to find the solution we seek we must relax that assumption and allow the space to be modified by the presence of a nonzero charge density. Since we seek a solution free from singularities and discontinuous derivative, we must expect a charge density that is not identically zero at any finite distance.

By choosing $f \simeq F + \sum_n A_n/r^n$ we may find A_n such that the Reissner-Nordström solution is obtained as an asymptotic form.[16]

C. Numerical integration

Standard finite-difference methods were used to obtain numerical solutions to Eq. (3.23). It can be written as

$$(2\sinh\psi + r\psi'\cosh\psi)\, e^{\Lambda(r)} \int_0^r e^{\Theta(\gamma)+\Phi(\gamma)}(1-\tfrac{1}{2}p^2\gamma^2\cosh\phi\sinh^2\phi)d\gamma$$
$$= (2\cosh\psi - r\psi'\sinh\psi)e^{\Theta(r)}\int_0^r e^{\Lambda(\gamma)+\Phi(\gamma)}[\tanh\psi_0 - \tfrac{1}{2}p^2\gamma^2\sinh\phi(1+\cosh^2\phi)]d\gamma, \quad (3.36)$$

where

$$\Lambda(r) = -\int_0^r \frac{\xi}{\eta}\frac{d\phi}{d\gamma}d\gamma = -\int_{\psi_0}^{\psi(r)} \frac{\xi}{\eta}d\phi, \quad (3.37)$$

$$\Theta(r) = -\int_0^r \frac{\eta}{\xi}\frac{d\phi}{d\gamma}d\gamma = -\int_{\psi_0}^{\psi(r)} \frac{\eta}{\xi}d\phi, \quad (3.38)$$

$$\Phi(r) = \frac{1}{4}\int_0^r \gamma(\phi')^2 d\gamma = \frac{1}{4}\int_{\psi_0}^{\psi(r)} \gamma\frac{d\phi}{d\gamma}d\phi. \quad (3.39)$$

ϕ and γ are dummy variables standing for ψ and r.

The radial variable may be rescaled to absorb the constant p, so we need only specify the starting values ψ_0 and $\psi'(0)$ to determine the solution. When $\psi'(0) \geq 0$, the solution rapidly diverges for $\psi_0 > 0$. When $\psi'(0) < 0$ the solution generally diverges. However, the divergence is found to be in the positive direction when $\psi_0 > \lambda$ and in the negative direction when $\psi_0 < \lambda$ where λ is a number that depends only upon $\psi'(0)$.

The source of the divergence is apparently related to the divergence noted when the terms in Eq. (3.36) fail to cancel. This case may be related to the cosmological solutions discussed by Synge.[17]

The limitations of the numerical methods preclude integration to the point at infinity. This sort of problem is common when a boundary condition at a distant point is to be satisfied by choosing an initial value and initial slope. The approximate results obtained from the series expansion about ∞ indicate that the Reissner-Nordström solution is approached asymptotically as r increases. Thus we need only press the numerical procedure to obtain results that approach the Reissner-Nordström solution to the desired degree of accuracy (Fig. 1).

D. The fundamental constant

The numerical solution of the integrodifferential equation provides the gravitational field which gives the appearance of a charged spherically symmetric particle with a mass derived from its electrical energy density. When we identify this solution with an existing particle of known charge and mass, we can determine the fundamental constant p as well as the constant relating the electric field to the metric. The value of ψ_0 corresponding to the electron provides such a small change in α that the numerical method is expected to fail through lack of significant figures. The solution for ψ, however, approaches an asymptotic form, and the physical results are obtained by extrapolation from the region where the computations are more readily accomplished.

Comparison with the Reissner-Nordström equation gives the results

$$2m\kappa/c^2 = 2M = p^2\int_0^\infty \tfrac{1}{2}\gamma^4\cosh\psi\sinh^2\psi\, d\gamma \quad (3.40)$$

and

$$\tfrac{1}{2}p^2\tanh\psi_0 = 2\kappa e^2/c^4\,. \quad (3.41)$$

The integral is evaluated as the numerical solution proceeds and then is augmented by an amount determined analytically for the region where the numerical solution has become unusable. For example, in the case where $\psi_0 = 10^{-4}$

$$p2M = 0.5295\times10^{-8} + \tanh^2\psi_0\int_{5.6568}^\infty \frac{dr}{r^2}$$
$$= 0.7062\times10^{-8}, \quad (3.42)$$

where $r = 5.6568$ is about as far as the numerical procedure produces reasonable results. The last term is the remaining field energy approximated by the inverse-square field (Fig. 2).

The shape of the curve seems to change imperceptibly for smaller ψ, so we deduce

$$2M = \frac{2\kappa m}{c^2} = \frac{0.7062\tanh^2\psi_0}{2p} \quad (3.43)$$

for small ψ_0.

Also

$$\frac{\tanh^2\psi_0}{2p^2} = \frac{2\kappa e^2}{c^4}\,. \quad (3.44)$$

This gives

$$p \simeq \sqrt{2}/a = 0.501\times10^{13}\ \text{cm}^{-1}, \quad (3.45)$$

where a is the classical electron radius, $a = e^2/mc^2$, and

$$\tanh\psi_0 = \left(\frac{8\kappa m^2}{e^2}\right)^{1/2} \equiv \frac{m}{e}\sqrt{8\kappa} = 1.385\times10^{-21}, \quad (3.46)$$

$$k = \frac{i\tanh\psi_0}{p^2 e} = \frac{ia}{c^2}\sqrt{2\kappa}$$
$$= i1.149\times10^{-37}\ \text{sec(cm/g)}^{1/2}. \quad (3.47)$$

The lack of spin in this model makes the identi-

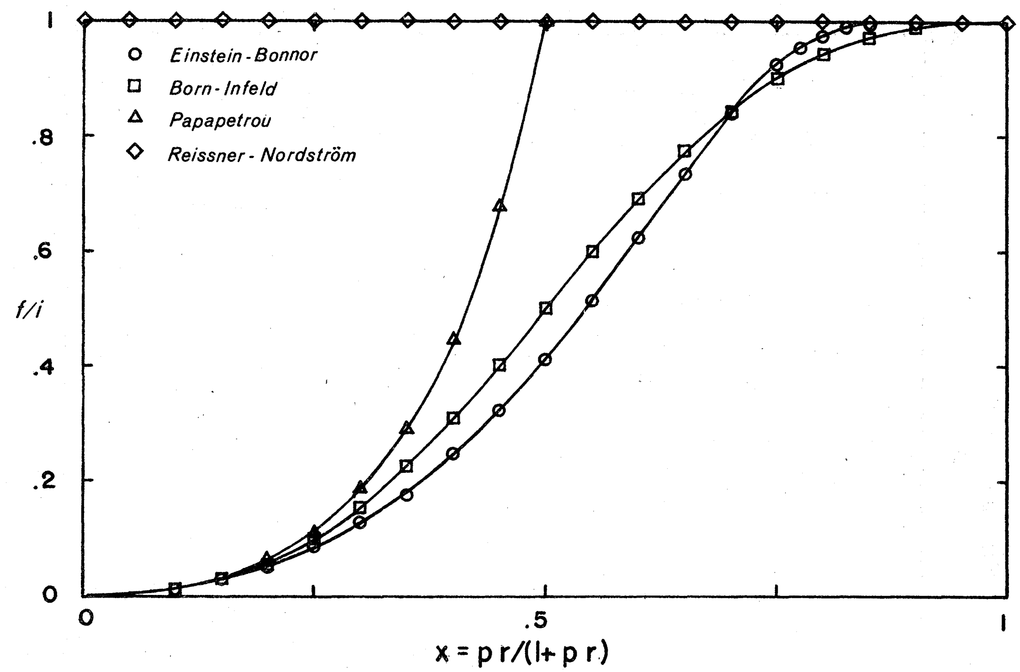

FIG. 1. A comparison of normalized field functions. The Einstein-Bonnor theory approaches the Reissner-Nordström solution for large r. In the region of small r, the Einstein-Bonnor theory approaches Papapetrou's solution. The Born-Infeld theory has a field function similar to the Einstein-Bonnor theory but lies in a Minkowski space.

fication with the electron and subsequent evaluation of the constants somewhat capricious.

E. The Born-Infeld model

An analytic solution of a closely related equation may be found. If ψ' is ignored only in the form

$$\frac{\eta}{\xi} \equiv \frac{\sinh\psi - \frac{1}{2}r\psi' \cosh\psi}{\cosh\psi + \frac{1}{2}r\psi' \sinh\psi} \simeq \tanh\psi, \tag{3.48}$$

then the integrodifferential equation reduces to

$$\tanh\psi\, e^{\Lambda} \int_0^r e^{\overline{\Theta}+\Phi}(1 - \tfrac{1}{2}p^2\gamma^2 \cosh\phi \sinh^2\phi)d\gamma$$
$$= e^{\overline{\Theta}} \int_0^r e^{\overline{\Lambda}+\Phi}[\tanh\psi_0 - \tfrac{1}{2}p^2\gamma^2 \sinh\phi(1 + \cosh^2\phi)]d\gamma, \tag{3.49}$$

where

$$\overline{\Lambda} \equiv -\int_0^r \coth\phi\,\phi'\,d\gamma = -\ln\left|\frac{\sinh\psi}{\sinh\psi_0}\right| , \tag{3.50}$$

$$\overline{\Theta} \equiv -\int_0^r \tanh\phi\,\phi'\,d\gamma = -\ln\left(\frac{\cosh\psi}{\cosh\psi_0}\right). \tag{3.51}$$

Then we have

$$\int_0^r \frac{e^{\Phi}}{\cosh\phi}(1 - \tfrac{1}{2}p^2\gamma^2 \cosh\phi \sinh^2\phi)d\gamma$$
$$= \int_0^r \frac{e^{\Phi}}{\sinh\phi}[\tanh\psi_0 - \tfrac{1}{2}p^2\gamma^2 \sinh\phi(1 + \cosh^2\phi)]d\gamma. \tag{3.52}$$

Therefore, the integrands must be equal and

$$\tanh\psi(1 - \tfrac{1}{2}p^2r^2 \cosh\psi \sinh^2\psi) = \tanh\psi_0 - \tfrac{1}{2}p^2r^2 \sinh\psi(2 + \sinh^2\psi) \tag{3.53}$$

or

$$\frac{s}{\beta} - \frac{p^2}{2}\frac{s^3}{\beta^2 - s^2} = \tanh\psi_0 - \frac{p^2}{2}2s - \frac{p^2}{2}\frac{s^3}{\beta^2 - s^2}, \tag{3.54}$$

where

$$s \equiv r^2 \sinh\psi = -if.$$

This may be written as

$$s = \frac{\beta \tanh\psi_0}{1 + p^2\beta} , \tag{3.55}$$

which corresponds to the Born-Infeld solution when the space is taken to be flat.

The early choice of a Hermitian metric ($f = is$) now gives us an approximate solution without the singularity. Bonnor has pointed out that ignoring

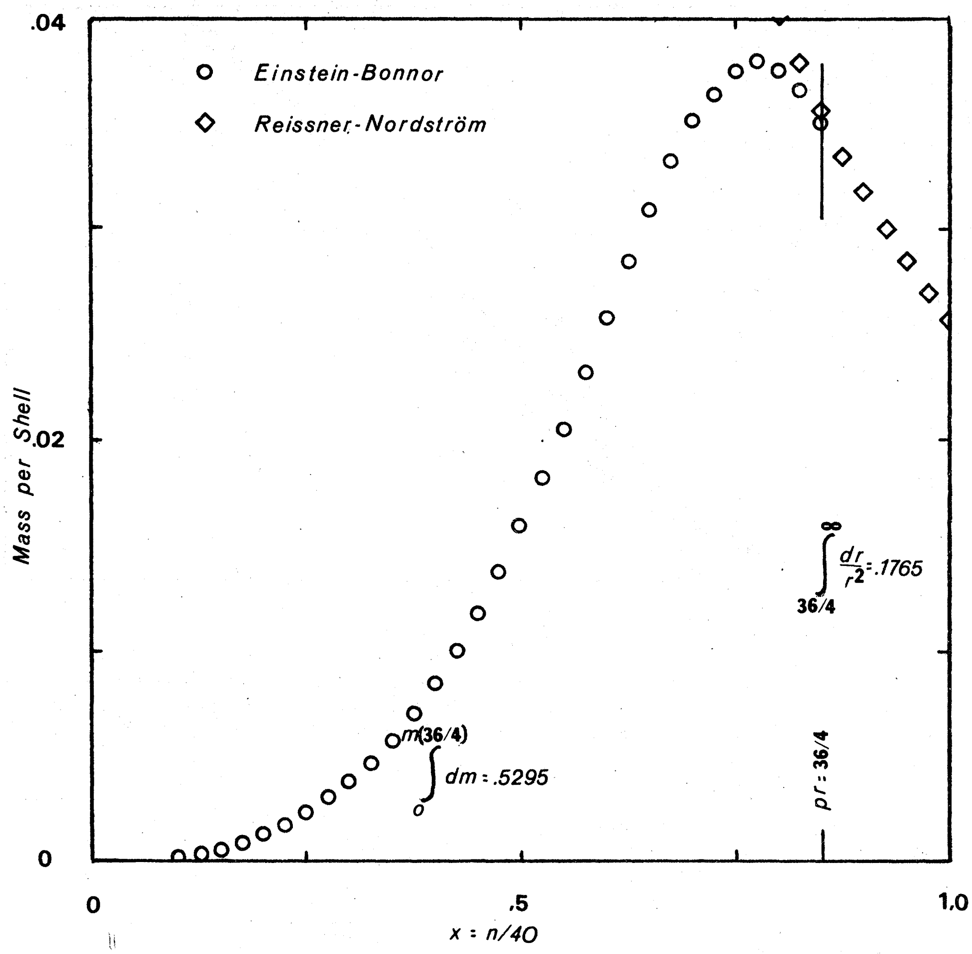

FIG. 2. Mass in nth shell vs x for small charge. The numerical integration fails past $pr = 36/4$, so we use the Reissner-Nordström curve to estimate the remaining area under the curve.

ψ' in (3.48) produces an error of up to a factor of 2 in η/ξ.[18]

IV. CONCLUSIONS

The classical problem of the interaction of a charged particle with its own field has been solved in the Einstein-Bonnor unified field theory. The static model shows that the asymptotic conditions that correspond to a charged particle require a charge distribution that does not vanish at a finite distance, but is sufficiently concentrated near the center to look like a point charge for $r > 10^{-12}$ cm. The particles mass is derived entirely from the generalized energy density. The identification of this model with the electron and subsequent evaluation of p and k is clouded by the electron's spin. The integrodifferential equation and subsequent numerical solution may be readily extended to include interaction terms other than Bonnor's.

Papapetrou's solution, the Born-Infeld solution, and the Reissner-Nordström solution are shown to be closely related to the present solution. Using the charge and mass of the electron, we find that the deviations of α and γ from constants are so slight that they have no observable consequences (other than the existence of this particlelike charge distribution).

The solution presented here has the same properties under space and time inversion and charge conjugation as the current quantum field theories of the electron. It is also noted that the antisymmetric energy tensor is a feature of a spinor field.

If one takes the point of view that the discontinuous derivative located at the center of the particle is a result of a fundamental process not included in the present theory, then we have a theory that predicts a unique charge and charge/mass ratio for the electron. Once p and ψ_0' are specified, the charge and charge/mass ratios are fixed.

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