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Light-quark spectroscopy and radial excitations

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We discuss the mass spectrum and radiative decays of the low-lying pseudoscalar and vector mesons in a nonrelativistic potential model based on quantum chromodynamics. We are able to account successfully for the smallness of the pion mass.

The quark model has progressed from a useful mnemonic to a dynamically viable scheme for describing hadrons. Much work has been done over the years in attempts to account for the mass spectra and transitions of mesons and baryons, taken to be composites of quarks and antiquarks.[1] Even with heuristic guesses for the interquark forces the model has reproduced many of the qualitative features of the hadronic spectra. Now, however, a plausible candidate for a theory of strong interactions, quantum chromodynamics (QCD), exists.[2] With the specification in QCD of at least part of the short-range force between quarks, one can hope to investigate the quark model more quantitatively.

A number of QCD studies of this nature have indeed been undertaken[3-8] and the results have been encouraging,[3-8] not only for the relative positions of the lowest-lying mesons and baryons, but for their masses. One of the major problems with this work has been the general failure to achieve a reasonable pion mass.[3-5] This failure has often been explained away by alluding to the unique dynamical situation of the pseudoscalar octet, and especially of the pion, as would-be Goldstone bosons. According to this philosophy the pseudoscalar mesons would have to be treated differently from other hadrons. In this paper we wish to suggest a solution to the pion-mass problem in which no special status is accorded the pseudoscalar mesons.

In addition to the $S, P, D, \ldots$ orbital excitations of the meson spectrum, the quark model predicts radial excitations, characterized by a principal quantum number, $n = 1, 2, 3 \ldots$ In a departure from most previous approaches, we take into account mixing between the lowest radially excited levels. (This idea was studied originally in connection with charmonium[8]). The masses of the pseudoscalar and vector-meson nonets and their first radial excitations are accounted for quite adequately in this scheme. As a check on the reasonableness of the eigenvectors obtained in the analysis the vector- and pseudoscalar-meson radiative decay widths and vector-meson e^+e^- decay widths are calculated and found to be in rather good agreement with experiment.

Our approach is based on the following usual assumptions:

(a) The quark constituents of the low-lying (non-charm-carrying) hadrons have small effective masses.

(b) The quark motion inside hadrons can be described by a nonrelativistic wave equation.

(c) The constituent-quark Hamiltonian H has the form

$$H = H_0 + V_c + V_{ex} + V_s + \cdots , \tag{1}$$

where H_0 contains the quark kinetic energy and mass terms, V_c is the (long-range) confining potential, V_{ex} arises from (short-range) single-gluon exchange, and V_s accounts for lowest-order spin-dependent and other singular terms. Higher-order effects, represented by the dots, include $q\bar{q}$ annihilation which will be discussed below.

Assumption (a) is perhaps the weakest of the three. There is considerable support for light effective masses both from deep-inelastic leptoproduction and from previous QCD analyses of the hadronic mass spectra, with whose results we substantially agree.

The use of a nonrelativistic wave equation to describe quarks inside hadrons [assumption (b)] is probably not correct (indeed, in the present and most models the quarks have $v/c \sim 1$). However, there is reason to believe[4] that relativistic corrections would not substantially change the energy levels found in the present model, although the interpretation of effective quark masses and other parameters may be less obvious.

There are several ambiguities and uncertainties associated with assumption (c). In the first place one can only guess at the form of V_c in Eq. (1). QCD arguments[9] suggest that $V_c(r) \sim ar + b$. However, other forms certainly can not be ruled out. For calculational use we have chosen instead to work with a harmonic-oscillator potential. The respective energy levels and eigenfunctions associated with these two choices should not be too different.[10]

At short distances $V_{ex}(r) \sim 1/r$, with a strength given by $\alpha_s(Q)$, the running QCD coupling, which has a logarithmic dependence on the momentum transfer Q^2. The Q^2 dependence of α_s slightly modulates the $1/r$ behavior of V_{ex}, but we will ignore this effect here by taking an average value of α_s to describe the lowest-lying hadronic states.

A possibly more serious problem concerns V_s. A wide range of choices have been made for this contribution, reflecting the uncertainty in its detailed structure. The most straightforward guess for V_s is the Fermi-Breit interaction, but it is not clear whether the derivatives ocurring in it should act on V_{ex} or on $V_{ex}+V_c$.[3,5] We have used the former prescription.[3]

It is probably equally valid, at this stage in our understanding, to employ (as other authors have done[8]) only the most singular terms in the Fermi Breit interaction. Thus, for the sake of comparison, we have also carried out our analysis with $V_{ex}=0$ and

$$V_s(\vec{r}) = \text{const} \times \delta(\vec{r})\vec{S}_1 \cdot \vec{S}_2 , \quad (2)$$

where $\vec{S}_i$ refer to quark spins.

In the following we shall deal with the S wave, n^3S_1 and n^1S_0, mesons, which are taken to be bound states of a u, d, or s quark and antiquark. The mass matrices are constructed by taking matrix elements of Eq. (1) in the $n=1,2,\ldots$ harmonic oscillator basis. To the $I=1$, $\frac{1}{2}$, and 0 sectors correspond submatrices whose eigenvalues are the masses of the particles

$$I=1: \; \pi, \pi', \ldots ; \; \rho, \rho', \ldots ,$$

$$I=\tfrac{1}{2}: \; K, K', \ldots ; \; K^*, K^{*\prime}, \ldots ,$$

$$I=0: \; \eta, \eta', \ldots ; \; \omega, \omega', \ldots ,$$

$$X, X', \ldots ; \; \phi, \phi', \ldots .$$

Within each sector the difference between the pseudoscalar ($S=0$) and vector ($S=1$) meson mass matrices arises from hyperfine splitting generated by a term in H of the form of Eq. (2).

In addition to the quark masses m_u ($=m_d$) and m_s there are four other parameters in the most economical version of the model of De Rújula *et al.*[3] These are the SU(3)-symmetric harmonic-oscillator strength κ, the quark-gluon coupling α_s, and two constants δ_p and δ_v which represent $q\bar{q}$ annihilation into gluons in the $I=0$ sector for $S=0$ and 1 states, respectively.[11]

In terms of the oscillator strength κ, the spacing between *unmixed* $n=1$ and $n=2$ $q_1\bar{q}_2$ levels would be $2\omega_{12}$ with $\omega_{12}=(\kappa/\mu_{12})^{1/2}$ where μ_{12} is the reduced mass of q_1 and q_2.

As mentioned above, we take an effective value of α_s in our analysis, ignoring its momentum-

TABLE I. Masses of lowest-lying pseudoscalar and vector mesons and their first radial excitations as predicted in radially mixed versions of the quark model. The experimental values are taken from Ref. 12 except where otherwise indicated.

Particle	Model I	Mass (GeV) Model II	Experiment
$I=1$: π'	1.22	1.29	•••
π	0.16	0.22	0.135
ρ'	1.34	1.74	1.60
ρ	0.74	0.71	0.773
$I=\frac{1}{2}$: K'	1.24	1.38	1.4 [a]
K	0.48	0.35	0.496
$K^{*\prime}$	1.39	1.84	•••
K^*	0.89	0.89	0.892
$I=0$: η	0.48	0.44	0.549
η'	1.89	1.39	•••
X	1.11	0.99	0.957
X'	2.61	1.61	•••
ω	0.76	0.74	0.783
ω'	1.19	1.77	••• [b]
ϕ	0.98	1.07	1.02
ϕ'	1.34	1.89	••• [b]

[a]Reference 13.
[b]See Refs. 14 and 15 for possible new states.

TABLE II. Vector- and pseudoscalar-meson radiative decay widths as predicted in radially mixed versions of the quark model.

Decay	Model I	Width (keV) Model II	Experiment
$\rho\to\pi\gamma$	90	84	35 ±10 [a]
$\rho\to\eta\gamma$	66	78	50 ±13 [b]
$K^{*+}\to K^+\gamma$	116	87	<80 [c]
$K^{*0}\to K^0\gamma$	119	125	75 ±35 [d]
$\omega\to\pi\gamma$	841	782	880 ±61 [c]
$\omega\to\eta\gamma$	6.9	8.7	3 ±2.5 [b]
$\phi\to\pi\gamma$	16	6.4	5.9 ±2.1 [c]
			55 ±12 [b]
$\phi\to\eta\gamma$	38	41	82 ±17 [c]
$X\to\rho\gamma$	7.3	13	<304 [c]
$X\to\omega\gamma$	0.3	1.9	<44 [c]
$\frac{\Gamma(X\to\rho\gamma)}{\Gamma(X\to\omega\gamma)}$	24	6.6	9 ±2 [c]
$\rho\to e^+e^-$	6.75	6.08	6.54±0.76 [c]
$\omega\to e^+e^-$	0.95	0.81	0.76±0.17 [c]
$\phi\to e^+e^-$	0.62	1.04	1.31±0.08 [c]
$\rho'\to e^+e^-$	1.16	2.89	
$\omega'\to e^+e^-$	0.13	0.52	
$\phi'\to e^+e^-$	1.39	0.59	

[a]Reference 19.
[b]Reference 20.
[c]Reference 12.
[d]Reference 21.

transfer dependence over the low-lying mesons.

Our model consists of diagonalizing the $n=1,2$ submatrix in each sector, neglecting to a first approximation the mixing with higher radial excitations. (Note that for the $I=0$ sector this results in the diagonalization of 4×4 matrices). It is important to realize that this gives quite different results from the usual approach, which is based on first-order perturbation theory in V_{ex} and V_s.

Table I gives the results for the mass spectrum. Model I refers to Eq. (1) with the assumptions of Ref. 3 for H_0, V_{ex}, and V_s. In model II, $H_0=m_1+m_2$, $V_{ex}=0$, and V_s are given by Eq. (2).[8] For both models the harmonic-oscillator potential is used for V_c.

It can be seen from Table I that both models give a reasonably good description of the P, P', V, and V' mass spectra. Variations of the predicted masses between the models is typical of fitting procedures. The important point is that *a low-pion mass is achieved in both models without seriously distorting the remaining mass spectra*. This feature should thus arise in any model with a basic QCD structure. There is no longer simple octet-singlet mixing relations for the $\eta, X, \ldots$ or $\omega, \phi, \ldots$ in either model. It is in determining the correct eigenfunction that the radiative decays $V\to P+\gamma$ and $P\to V+\gamma$ and the leptonic decays $V\to e^+e^-$ play an important role.

There have been indications[16,17] that the inclusion of higher-mass vector mesons may help account for the "problem"[18] of the radiative decays. In our approach the eigenvectors are explicitly known and we can use an exactly calculated form of the overlap integral. As in conventional quark-model approaches there will be the usual problem of the nonrelativistic nature of the calculation.[18] The radiative decay widths, as predicted in models I and II, are shown in Table II (Note that we have assumed that the quark magnetic moment $\mu_q \sim 1/m_q$).

The e^+e^- widths of ρ, ω, and ϕ and their higher-mass counterparts have been calculated as well and these results are also presented in Table II. These are determined from knowledge of the vector-meson wave function at the origin and may be sensitive to the approximation scheme, since it is assumed in the calculation that only the $q\bar{q}$ bound-state wave function is to be used.

As may be appreciated from Table II there is fairly good agreement overall between theory and experiment. However, the inclusion of radially excited states does not remove the discrepancy in $\Gamma(\rho^-\to\pi^-\gamma)/\Gamma(\omega\to\pi\gamma)$.

The parameters corresponding to the two models have the following values and interpretation. Apart from the dimensionless parameter α_s all parameters listed below are in units of GeV. In model I there is an overall mass eigenvalue $M_0(1.1)$ of the Hamiltonian H_0. The harmonic-oscillator frequencies are[22] $\omega_{uu}(0.39)$, $\omega_{us}(0.39)$, and $\omega_{ss}(0.31)$. The annihilation terms[11] are given by $\delta_p=0.55$, $\delta_p'=0.43$, $\delta_v=0.015$, and $\delta_v'=0$. Finally, the running coupling constant $\alpha_s=0.8$ and the quark masses are $m_u=0.243$, $m_s=0.478$.

For model II, there is no parameter M_0. The hyperfine interaction is described by a δ function [Eq. (2)] which gives rise to three parameters[8] $A_{uu}=0.094$, $A_{us}=0.1$, $A_{ss}=0.068$. The harmonic-oscillator frequencies are found to be $\omega_{uu}=0.48$, $\omega_{us}=0.43$, and $\omega_{ss}=0.38$. The quark masses are $m_u=0.314$ and $m_s=0.496$. The annihilation parameters are $\delta_p=0.23$, $\delta_p'=0$, $\delta_v=\delta_v'=0.015$.

The value of α_s in model I is an effective value in which we have not taken the variation with mass into account.

Among the new effects predicted by our model is that there should exist a radially excited π meson, the π' with a mass of ~1 GeV. We note that, in contrast to the charmonium and Υ cases, radially excited states of the low-lying mesons have been notoriously difficult to detect. We might expect therefore that it will be no less difficult to see a $\pi'(1.2)$. Presumably evidence for such a particle will come from phase-shift analyses of multipion production. The existence of such a particle may have interesting consequences for the analyses of the decays of the heavy lepton τ.[23]

One immediate and open question is the effect of higher radial excited states on the mixing between 1S and 2S states. This problem is compounded with the possibility of nonorthogonal mixing coming, perhaps, from gluon states.[24] Within the computational prescription given here it is difficult to estimate fully these effects since the harmonic-oscillator approximation becomes less reliable for the higher radial states. We can expect, however, that the 1S results will be rather insensitive to the effect of 3S mixing.

To summarize, we have shown that it is possible, within a nonrelativistic potential model of quarks based on QCD ideas and in which $m_u \sim 0.3$ GeV, $m_s \sim 0.5$ GeV, to account successfully for the mass spectrum of the low-lying pseudoscalar and vector nonets. This result uses the idea of radial mixing which has been applied elsewhere to charmonium,[5,8] to predicting properties of the Υ (Ref. 25) and to baryons.[26] We predict the existence of (among others) a new excited state π' with a mass of around 1 GeV. The spectrum of these new radially excited states may be affected, however, by mixing with 3S states which we have not considered here. The known radiative decays are fitted about as well as by the naive quark-model or vector-dominance approximations,[18] but this new mixing scheme does not resolve the descrepancy in the ratio $\Gamma(\rho\to\pi\gamma)/\Gamma(\omega\to\pi\gamma)$. From Tables I and II we see that model II, which emphasizes the most singular term in the potential, does just as well in describing the low-lying meson spectra and their radiative and leptonic decays. We have noted previously that there are many ambiguities in interpreting QCD in a nonrelativistic potential-model form. The success of the second, simplified, model suggests that for the light-mass meson system the quark-confining potential and the hyperfine interaction are the important ingredients.[27]

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[11]The isoscalar mass matrix has a contribution from the annihilation terms (ignoring radial excitations) of the form

$$\begin{pmatrix} 2\delta & \sqrt{2}\delta \\ \sqrt{2}\delta & \delta \end{pmatrix}.$$

We assume for simplicity, that there is no annihilation contribution between the 1S and 2S states.

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