Quark mass differences, magnetic moments, and radiative decay widths

Lambodar P. Singh
Physics Department, Utkal University, Vani Vihar, Bhubaneswar-751004, India
(Received 24 April 1978)

Magnetic moments and transition moments of charmed hadrons and radiative decay widths of ordinary and charmed vector mesons have been calculated using quark masses consistent with the gauge model of weak, electromagnetic, and strong interactions of De Rújula, Georgi, and Glashow.

Symmetry breaking in static and dynamic properties of hadrons will certainly have contributions from the inequality of the masses of the various kinds of quarks which are believed to be the ultimate constituents of matter. To start with, a small mass difference between neturon and proton is suggestive of a splitting in the mass degeneracy of up and down quarks,[1] where the latter is the heavier of the two. In this connection, it is interesting to note that a comparison with the accurately determined magnetic moments of neutron and proton using the assumptions that the entire contribution to these moments comes from the intrinsic quark moments and that quark moments, in turn, are proportional to their charge-to-mass ratio determines the up quark to be heavier than the down quark.[2] This result may, however, be erroneous to some measure because of the neglect of relativistic effects and orbital angular momentum in the wave functions. But whatever the exact nature of the mass splitting, the static properties of neutron and proton lead to a small mass difference between up and down quarks. However, since a small mass difference affects our results negligibly we prefer to work with $m_u = m_d$. Similarly attempts have been made to describe large mass splittings between strange and nonstrange baryons and mesons, in the first place, on the basis of the mass difference between nonstrange and strange quarks.[3] Recently while analyzing the implications of the standard gauge model of weak, electromagnetic, and strong interactions for hadron spectroscopy, De Rújula, Georgi, and Glashow used $m_u = m_d = 336$ MeV and $m_s = 540$ MeV, where $m_{u,d,s}$ stands for the masses of up, down, and strange quarks, to obtain rather good agreement with experimental numbers for baryon magnetic moments. Finally the interpretation of the narrow resonances ψ and ψ' as states of charmonium inevitably leads to a rather large mass for the charmed quarks. De Rújula, Georgi, and Glashow obtain $m_c = 1660$ MeV.

In this paper we calculate the magnetic moments of charmed hadrons and the radiative decay widths of ordinary and charmed vector mesons using the quark mass differences, the principle of quark additivity, and the stated assumption that quark magmetic moments are proportional to their charge-to-mass ratio. We use, to repeat, $m_u = m_d = 336$ MeV and $m_s = 540$ MeV and $m_c = 1660$ MeV, or alternatively $x = m_u/m_d = 1$, $y = m_u/m_s = 0.62$, and $z = m_u/m_c = 0.2$. It may be remarked that Lichtenberg[5] has calculated the magnetic moments of charmed baryons in the above mechanism using $x = 1$, $y = 0.62$, $z = 0.2$. However, we also calculate magnetic moments of charmed mesons and estimate the radiative decay widths of ordinary and charmed vector mesons. We follow the notations and method of calculation of Ref. 6.

The magnetic moments of charmed baryons which carry the effect of up and down quark mass difference are (in units of proton magnetic moment)

$$\mu_{C_1^+} = \tfrac{2}{9}(2 - x - z), \quad \mu_{C_1^0} = -\tfrac{2}{9}(2x + z)$$

$$\mu_{X_d^+} = \tfrac{1}{9}(x + 8z), \quad \mu_{S^0} = -\tfrac{2}{9}(x + y + z)$$

$$\mu_{C_1^{*+}} = \tfrac{1}{3}(-x + 2 + 2z), \quad \mu_{C_1^{*0}} = \tfrac{2}{3}(-x + z)$$

$$\mu_{S^{*0}} = \tfrac{1}{3}(-x - y + 2z), \quad \mu_{X_d^{*+}} = \tfrac{1}{3}(-x + 4z).$$

Note that if one substitutes $x = 1$ and $x = y = z = 1$ in the above expressions one recovers the results of Lichtenberg (Ref. 5) and Singh (Ref. 6), respectively. These expressions are also recovered if one substitutes $\mu_u = \frac{2}{3}$, $\mu_d = -\frac{1}{3}x$, $\mu_s = -\frac{1}{3}y$, and $\mu_c = \frac{2}{3}z$ in the expressions obtained by Johnson and Shah-Jahan.[7] The magnetic moments of charmed vector mesons and transition moments, in the units of proton magnetic moment, are

$$\mu_{D^{*+}} = -\mu_{D^{*-}} = \tfrac{1}{3}(x + 2z), \quad \mu_{D^{*0}} = -\mu_{\bar{D}^{*0}} = -\tfrac{2}{3}(1 - z),$$

$$\mu_{F^{*+}} = -\mu_{F^{*-}} = +\tfrac{1}{3}(y + 2z), \quad \mu_{\phi_c} = 0,$$

$$\mu_{\rho^+\pi^+} = \mu_{\rho^0\pi^0} = -\mu_{\rho^-\pi^-} = \tfrac{1}{3}(x - 2), \quad \mu_{\omega\pi^0} = -\tfrac{1}{3}(x + 2),$$

$$\mu_{\phi\pi^0} = -\frac{0.08}{3}(x + 2), \quad \mu_{\omega\eta} = \frac{1}{3\sqrt{2}}(2 - x),$$

$$\mu_{\phi\eta} = [1.98y - 0.08(2 - x)]/3\sqrt{2}, \quad \mu_{\rho^0\eta} = \frac{1}{3\sqrt{2}}(2 + x),$$

$$\mu_{K^{*\pm}K^{\pm}} = \tfrac{1}{3}(y-2)\,, \quad \mu_{K^{*0}K^0} = \tfrac{1}{3}(y+x)$$
$$\mu_{D^{*+}D^+} = \mu_{D^{*-}D^-} = \tfrac{1}{3}(x-2z)\,,$$
$$\mu_{D^{*0}D^0} = \mu_{\bar{D}^{*0}\bar{D}^0} = -\tfrac{2}{3}(1+z)$$
$$\mu_{F^{*+}F^+} = \mu_{F^{*-}F^-} = \tfrac{1}{3}(y-2z)\,, \quad \mu_{\phi_c\eta_c} = 2/\sqrt{3}\,z\,.$$

In obtaining these relations we have used

$$|\eta\rangle = \cos\theta_P\,|\bar{s}s\rangle - \frac{\sin\theta_P}{\sqrt{2}}\,|\bar{u}u + \bar{d}d\rangle$$

and

$$|\phi\rangle \cos\theta_V\,|\bar{s}s\rangle + \frac{\sin\theta_V}{\sqrt{2}}\,|\bar{u}u + \bar{d}d\rangle\,.$$

These states in SU(6) notation are,

$$|\eta\rangle = \frac{\cos\theta_P}{\sqrt{2}}\,|\bar{s}{\uparrow}s{\downarrow} - \bar{s}{\downarrow}s{\uparrow}\rangle - \frac{\sin\theta_P}{2}\,|\bar{u}{\uparrow}u{\downarrow} - \bar{u}{\downarrow}u{\uparrow} + \bar{d}{\uparrow}d{\downarrow} - \bar{d}{\downarrow}d{\uparrow}\rangle$$

and

$$|\phi\rangle = \frac{\cos\theta_V}{\sqrt{2}}\,|\bar{s}{\uparrow}s{\downarrow} + \bar{s}{\downarrow}s{\uparrow}\rangle + \frac{\sin\theta_V}{2}\,|\bar{u}{\uparrow}u{\downarrow} + \bar{u}{\downarrow}u{\uparrow} + \bar{d}{\uparrow}d{\downarrow} + \bar{d}{\downarrow}d{\uparrow}\rangle\,.$$

We have taken $\theta_V = 5°$ and $\theta_P = 45°$ as has been suggested by the quadratic mass formula for mesons.[8]

Having obtained the transition moments, the calculation of the radiative decay width is straightforward since[9]

$$\Gamma = \langle\mu\rangle^2 k^3/3\pi\,,$$

where Γ is the decay width, μ is the transition magnetic moment, and k, the photon momentum, is given by $(m_V{}^2 - m_P{}^2)/2m_V$ in the rest frame of the decaying vector meson of mass m_V. Here m_P is the mass of the pseudoscalar meson. The results for the decay widths are shown in the Table I. The experimental values are obtained from Ref. 10.

From Table I it is clear that by the introduction of quark mass splittings the radiative decay widths of $K^{*\pm}$ to $K^\pm$ alone are disturbed off the experiment by a substantial amount. This mechanism obviously also provides no answer to the decay processes $\rho\to\pi\gamma$, $\omega\to\pi\gamma$, and $\phi\to\pi\gamma$, particularly the first one. On the other hand the agreement of the decay widths of $\omega\to\eta\gamma$, $\phi\to\eta\gamma$, and $\rho^0\to\eta\gamma$ with experiment is excellent. Particularly the agreement in case of $\phi\to\eta\gamma$ is a positive result of this analysis since it has not been possible to reconcile[11] theory with experiment for this case in the framework of SU(3) or naive quark model of vector-meson dominance. The decay widths of K^{*0} to $K^0\gamma$ and $\phi_c\to\eta_c\gamma$ are also considerably improved though the values are still off the experi-

TABLE I. Comparison of calculated radiative decay widths of ordinary and charmed vector mesons with experiment.

Process	Radiative decay widths in keV $x=y=z=1$	$x=1, y=0.62, z=2$	Experiment
$\rho \to \pi\gamma$	1254	1254	35 ±10
$\omega \to \pi^0\gamma$	1173	1173	880 ±60
$\omega \to \eta\gamma$	9.49	9.49	$3.0^{+2.5}_{-1.8}$
			or 29 ± 7
$\rho^0 \to \eta\gamma$	76.8	76.8	50 ±13
			or 76 ±15
$\phi \to \eta\gamma$	203.9	74.4	64 ±10
$\phi \to \pi^0\gamma$	17.1	17.1	5.7± 2.1
$K^{*\pm} \to K^\pm\gamma$	68.3	130.2	<80
$K^{*0} \to K^0\gamma$	273.5	179.4	74 ±35
$D^{*\pm} \to D^\pm\gamma$	5.19	1.87	···
$D^{*0} \to D^0\gamma$	100	36.1	···
$F^{*\pm} \to F^\pm\gamma$	2.9	0.14	···
$\phi_c \to \eta_c\gamma$	46.8	18.7	<3.5

ment.

Thus in recognition of the simple symmetry-breaking mechanism employed in this analysis, the rather good general agreement of the theoretical and experimental values for various radiative decay widths is certainly interesting.

ACKNOWLEDGMENT

We are thankful to Dr. J. Maharana for encouragement and L. Maharana for useful discussions. Computation facilities of Utkal University Computer Center is also gratefully acknowledged.

[1]R. G. Moorhouse, in *High Energy Physics*, proceedings of the European Physical Society International Conference, Palermo, 1975, edited by A. Zichichi (Editrice Compositori, Bologna, 1976), p. 1203.

[2]J. Franklin, Phys. Rev. 172, 1807 (1968).

[3]See, for example, J. J. J. Kokkedee, *The Quark model* (Benjamin, New York, 1969), p. 34.

[4]A. De Rújula, H. Georgi, and S. I. Glashow, Phys. Rev. D 12, 147 (1975).

[5]D. B. Lichtenberg, Phys. Rev. D 15, 345 (1977).

[6]L. P. Singh, Phys. Rev. D 16, 158 (1977).

[7]R. J. Johnson and M. Shah-Jahan, Phys. Rev. D 15, 1400 (1977), see pp. 1400, and 1401.

[8]R. G. Moorhouse, in *High Energy Particle Interactions*, proceedings of the Triangle Conference, Sonolenice, Czechoslovakia, 1975, edited by D. Krupta and J. Pisut (Veda, Bratislava, 1976), pp. 172 and 173.

[9]See for example, W. Thirring, Phys. Lett. 16, 335 (1965); R. Van Royen and V. F. Wiesskopf, Nuovo Cimento 50, 617 (1967).

[10]B. J. Edwards and A. N. Kamal, Phys. Rev. Lett. 39, 66 (1977).

[11]M. Greco, in *Deep Scattering and Hadronic Structure*, proceedings of the XII Recontre de Moriond, Flaine, 1977, edited by J. Trân Thanh Vân (Editions Frontiéres, Paris, 1977), and references therein.