Leptonic decay of heavy leptons

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The energy-angle distribution of a light charged lepton l from the decay of an arbitrary polarized heavy charged lepton L, $L^- \to \nu_L + \bar{\nu}_l + l^-$, is derived assuming that (1) the mass of ν_L is not zero and (2) the coupling of $WL\nu_L$ is some combination of V and A. The case when ν_L and $\bar{\nu}_l$ are identical is also discussed.

When the heavy lepton τ was discovered[1] it was necessary to determine the following properties:

(1) whether τ^- has its own leptonic number or has the same leptonic number as that of either e^+, e^-, μ^+, or μ^-;

(2) whether the neutrino associated with τ is massless or massive;

(3) the form of coupling between W and $\tau\nu_\tau$.

The formulas to be given below were derived at various times[1,2] for the experimentalists at SLAC in order to answer the questions listed above. The purpose of this paper is to record these expressions for future reference. Since our formulas can be used for any lepton, we shall refer to the heavy lepton as $L^\pm$ and the associated neutrinos as $\bar{\nu}_L$ and ν_L, and to any charged leptons lighter than L as $l^\pm$ and the associated neutrinos as $\bar{\nu}_l$ and ν_l. The processes under consideration are $L^+ \to \bar{\nu}_L + \nu_l + l^+$ and $L^- \to \nu_L + \bar{\nu}_l + l^-$. Assuming CP invariance, the energy-angle distribution of the decay of L^+ is related[3] to that of L^- by changing the sign of polarization vector of L^-.

I. STANDARD MODEL

We shall refer to the special case satisfying the following specifications as the standard model:

(a) L^- and ν_L have their own lepton number different from that of l^- and ν_l and also from that of l^+ and $\bar{\nu}_l$.

(b) ν_L is massless.

(c) $V-A$ coupling between W and $L\nu_L$.

All the known leptons (e^-, ν_e), (μ^-, ν_μ), and (τ^-, ν_τ) seem[1,2] to share this property. This case was considered in my previous paper.[3] For convenience of comparison, we summarize the results here from that paper.

The energy-angle distribution of $l^\mp$ from the decay of an arbitrary polarized heavy lepton $L^\mp$ in the rest frame of L satisfying the criteria mentioned above is

$$\Gamma\begin{pmatrix} L^- \to \nu_L + \nu_l + l^- \\ L^+ \to \bar{\nu}_L + \nu_l + l^+ \end{pmatrix} = \frac{G^2M^5}{3\times 2^7\pi^4}\,\frac{8}{M^4}\int_0^{p_{\max}} p^2\,dp \int d\Omega \left[3M - 4E - \frac{2m^2}{E} + \frac{3m^2}{M} \mp (\vec{\mathrm{w}}\cdot\hat{p})\frac{p}{E}\left(4E - M - \frac{3m^2}{M}\right)\right], \tag{1}$$

where $G = 1.02\times 10^{-5}/M_p{}^2$, m, E, and p are respectively mass, energy, and momentum of l in the rest frame of L, w is the polarization vector of L, and $p_{\max} = (M^2 - m^2)/(2M)$. The integration with respect to p and the solid angle $d\Omega$ can be carried out analytically, and the result[4] is

$$\Gamma\begin{pmatrix} L^- \to \nu_L + \bar{\nu}_l + l^- \\ L^+ \to \bar{\nu}_L + \nu_l + l^+ \end{pmatrix} = \frac{G^2M^5}{3\times 2^6\pi^3}(1 - 8y + 8y^3 - y^4 - 12y^2\ln y), \tag{2}$$

where

$$y = m^2/M^2 .$$

If we ignore the mass of l, Eq. (1) can be written as $(x = p/p_{\max})$

$$\Gamma\begin{pmatrix} L^- \to \nu_L + \nu_l + l^- \\ L^+ \to \nu_L + \nu_l + l^+ \end{pmatrix}_{m=0} = \frac{\Gamma_0}{4\pi}\int d\Omega \int_0^1 x^2[6 - 4x \mp (\vec{\mathrm{w}}\cdot\hat{p}_l)(4x-2)]\,dx, \tag{3}$$

where Γ_0 is the width given by Eq. (2) with the mass of l ignored $(y=0)$:

$$\Gamma_0 = \frac{G^2M^5}{3\times 2^6\pi^3}. \tag{4}$$

The expressions of partial width such as (2) and (4) are useful in calculating the branching ratios. However, in the actual experiments[1,2] the detectors accept only certain regions of energy and angle and the expressions such as (1) and (3) are necessary to correct the effects due to the finite acceptance. The masses of electron and muon are negligible in the τ decay. However, if leptons heavier than τ exist then they will decay into τ via $L\rightarrow\nu_L+\nu_\tau+\tau$, where the mass of τ is not negligible. Equations (1) and (2) will be useful in such a process.

II. ν_L AND $\bar{\nu}_l$ ARE IDENTICAL PARTICLES (L IS A PARALEPTON)

In the reaction $L^-\rightarrow\nu_L+\bar{\nu}_l+l^-$, if ν_L and $\bar{\nu}_l$ were two identical particles, then the effect due to the Pauli principle would be to double the decay rate compared with the standard case. This was first pointed out by Bjorken and Llewellyn Smith.[5] In practice, because of the incomplete experimental acceptance and the difference in the energy-angle distributions[1] in the two cases, the observed ratio will not be equal to two even if ν_L were identical to $\bar{\nu}_l$. Let us assume that ν_l and $\bar{\nu}_l$ have negative and positive helicities, respectively (true for neutrinos associated with e, μ, and also τ). Since in this section we assume ν_L is identical to $\bar{\nu}_l$ and $\bar{\nu}_L$ is identical to ν_l, the helicities of ν_L and $\bar{\nu}_L$ must be $\frac{1}{2}$ and $-\frac{1}{2}$, respectively. We also have to use the convention that $(L^+,\bar{\nu}_L)$ are fermions and (L^-,ν_L) are antifermions if we use the convention that (l^-,ν_l) are fermions and $(l^+,\bar{\nu}_l)$ are antifermions. With this convention only the $V-A$ current contributes to the decay because the $V+A$ part will project out the neutrinos with the wrong helicity.

Assuming ν_L, ν_l, and l to be massless, we obtain for this case

$$\Gamma\begin{pmatrix}L^-\rightarrow\nu_L+\bar{\nu}_l+l^-\\ L^+\rightarrow\nu_L+\nu_l+l^+\end{pmatrix}=\frac{2\Gamma_0}{4\pi}\int d\Omega\int_0^1 x^2[9-8x\pm(\vec{w}\cdot\hat{p})(5-4x)]dx\,, \tag{5}$$

where Γ_0 is given by Eq. (4). After integration with respect to $x=p/p_{max}$ and the solid angle, the right-hand side of Eq. (5) is $2\Gamma_0$ as mentioned before.

III. SEQUENTIAL $L, M_\nu\neq 0$, AND ARBITRARY VA COMBINATIONS

Let us suppose that L and ν_L have their lepton number different from those of e and μ. Let us write the leptonic current involving L and ν_L as

$$J_\mu=\bar{\nu}_L\gamma_\mu[g_L(1-\gamma_5)+g_R(1+\gamma_5)]L\,, \tag{6}$$

where g_L and g_R are normalized such that $g_L=1$ and $g_R=0$ in the standard case. The energy-angle distribution in the decay $L^-\rightarrow\nu_L+\bar{\nu}_l+l^-$ and $L^+\rightarrow\bar{\nu}_L+\nu_l+l^+$ can then be written as

$$\Gamma\begin{pmatrix}L^-\rightarrow\nu_L+\bar{\nu}_l+l^-\\ L^+\rightarrow\bar{\nu}_L+\nu_l+l^+\end{pmatrix}=g_R{}^2\Gamma_R^{\mp}+g_L{}^2\Gamma_L^{\mp}+g_Rg_L\Gamma_{RL}^{\mp}\,, \tag{7}$$

where

$$\Gamma_R^{\mp}=\frac{4G^2M}{(2\pi)^4}\int d^3p(E_{max}-E)(1\mp\vec{w}\cdot\hat{p})f\,, \tag{8}$$

$$\Gamma_L^{\mp}=\frac{4G^2M}{(2\pi)^4}\int d^3p\,\{(E_{max}-E)f+(2E-E_{max})\tfrac{1}{2}f^2-\tfrac{1}{3}Ef^3 \pm(\vec{w}\cdot\hat{p})[(E_{max}-E)f-\tfrac{1}{2}f^2(M+E_{max}-2E)+\tfrac{1}{3}f^3(M-E)]\}\,, \tag{9}$$

$$\Gamma_{RL}^{\mp}=\frac{-2G^2MM_\nu}{(2\pi)^4}\int d^3p f^2(1\mp\vec{w}\cdot\hat{p})\,, \tag{10}$$

where $f=1-M_\nu{}^2/M(M-2E)$, $E^{max}=(M^2-M_\nu{}^2)/(2M)$, and M_ν is the mass of ν_L. The interference term $\Gamma_{RL}^{\pm}$ becomes zero because when $M_\nu=0$, $(1-\gamma_5)$ and $(1+\gamma_5)$ project out different helicity eigenstates of the neutrino ν_L. Equation (9) reduces to Eq. (3) if we ignore the mass of ν_L, i.e., $f=1$. In the limit $M=0$, Eq. (8), which is usually referred to as the $V+A$ case, can be written as

$$\Gamma_R^{\mp}=\frac{\Gamma_0}{4\pi}\int d\Omega\int_0^1 x^2[12(1-x)(1\mp\hat{p}\cdot\vec{w})]dx\,. \tag{11}$$

Many years ago, Kinoshita and Sirlin[6] derived the general expression of the energy-angle distribution containing all possible non-parity-conserving couplings (S,P,T,V,A) assuming that both ν_L and l are massless. The expression contains three parameters ρ, ξ, and δ:

$$\Gamma=\frac{\Gamma}{4\pi}\int d\Omega\int_0^1 4x^2\left\{3(1-x)+2\rho\left(\frac{4x}{3}-1\right)+\xi\hat{p}\cdot\vec{w}\left[(1-x)+2\delta\left(\frac{4x}{3}-1\right)\right]\right\}dx\,. \tag{12}$$

Our Eqs. (3), (5), and (11) also have this form. The three parameters corresponding to these three cases are

Eq. (3) (sequential L, $V-A$, $m=0$, $M_\nu=0$): $\rho=\frac{3}{4}$, $\delta=\frac{3}{4}$, and $\xi_\mp=\mp 1$.

Eq. (11) (sequential L, $V+A$, $m=0$, $M_\nu=0$): $\rho=0$, $\delta=0$, and $\xi_\mp=\mp 3$.

Eq. (5) (para L, $V-A$, $m=0$, $M_\nu=0$): $\rho=\frac{3}{8}$, $\delta=\frac{3}{16}$, and $\xi_\mp=\mp 2$.

ξ_- and ξ_+ refer to the values of ξ in Eq. (12) for the decay of L^- and L^+, respectively.

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