Comment on polarized fragmentation functions

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The spin observables of quark fragmentation in parity-conserving reactions are discussed. Simple predictions are given, and possible spin correlations are mentioned.

Recently, considerable effort has been devoted to the study of how an energetic quark develops into a jet of hadrons.[1] The properties of the hadrons are described by a phenomenological set of fragmentation functions. The standard analysis concentrates on the energy distribution and flavor dependence of the fragmentation function for pseudoscalar and vector mesons. However, when vector mesons are studied, an extra degree of freedom is available, i.e., spin. A previous paper by the author introduced spin-dependent fragmentation functions and used their parity-odd components as a probe of the quark helicity in processes which violate parity (i.e., neutrino scattering).[2] There is also information contained in the parity-even components. The purpose of this comment is to discuss the latter for use in electroproduction or e^+e^- experiments. The example throughout will be a ρ^0, but other particles may be used instead.

Let us consider a ρ^0 in the jet produced by a left-handed quark (helicity $-\frac{1}{2}$). The fragmentation function can be given as a product of helicity-independent and helicity-dependent portions (λ,λ' labels the vector-meson helicity)

$$D^{\rho}_{\lambda\lambda'}(z)=D^{\rho}(z)\rho_{\lambda\lambda'}(z)\,, \tag{1}$$

where we normalize the helicity density matrix $\rho_{\lambda\lambda'}(z)$ by

$$\sum_{\lambda}\rho_{\lambda\lambda}(z)=1\,. \tag{2}$$

$D^{\rho}(z)$ is the usual spin-averaged fragmentation function ($z=E_\rho/E_{\text{quark}}$). We can measure $\rho_{\lambda\lambda'}(z)$ by studying the angular distribution of the π^+ in the decay $\rho^0\to\pi^+\pi^-$. In the ρ^0 rest frame

$$W(\theta,\phi)=W_{\text{even}}(\theta,\phi)+W_{\text{odd}}(\theta,\phi)\,, \tag{3}$$

$$\begin{aligned}W_{\text{even}}(\theta,\phi)=\frac{1}{4\pi}[1&-\tfrac{1}{2}(2\rho_{00}-\rho_{11}-\rho_{-1-1})(1-3\cos^2\theta)\\&-3\sqrt{2}\sin\theta\cos\theta\sin\phi\,\text{Re}(\rho_{10}-\rho_{-10})\\&-3\sin^2\theta\cos 2\phi\,\text{Re}\rho_{1-1}]\,,\end{aligned} \tag{4}$$

$$\begin{aligned}W_{\text{odd}}(\theta,\phi)=\frac{1}{4\pi}[&3\sqrt{2}\sin\theta\cos\theta\sin\phi\,\text{Im}(\rho_{10}-\rho_{-10})\\&+3\sin^2\theta\sin 2\phi\,\text{Im}\rho_{1-1}]\,.\end{aligned} \tag{5}$$

Here the z axis is given by the direction of motion of the ρ^0. The choice of x axis will be discussed later. The subscripts even and odd refer to the parity behavior of the density matrix elements. For a right-handed quark $W_{\text{odd}}(\theta,\phi)$ changes sign, but the magnitude of all the elements is unchanged. In neutrino scattering all five terms may be measured. In electromagnetic processes, with equal amounts of left- and right-handed quarks, $\rho_{10}=-\rho_{-10}$, and ρ_{1-1} is real, so that $W_{\text{odd}}(\theta,\phi)$ vanishes leaving $W_{\text{even}}(\theta,\phi)$ with three spin components of interest.

The spin component which is the easiest to measure is the *alignment* η,

$$\eta=\tfrac{1}{2}(2\rho_{00}-\rho_{11}-\rho_{1-1})\,, \tag{6}$$

as it requires only the identification of the z axis. If we neglect the transverse momentum in the fragmentation process, there is a simple interpretation which may be given to the alignment. There are two amplitudes for the forward fragmentation into a vector meson, depending on whether the quark helicity flips or remains unchanged, as is illustrated in Fig. 1. The alignment measures the relative amounts of these two amplitudes:

$$\eta=\frac{1}{2}\,\frac{2|a_{++}|^2-|a_{+-}|^2}{|a_{++}|^2+|a_{+-}|^2}\,. \tag{7}$$

This leads to bounds on η

$$-\tfrac{1}{2}\leqslant\eta\leqslant 1\,. \tag{8}$$

In addition there is a simple assumption which leads to a prediction for η. It is plausible that the ρ will couple to quarks like a vector current. This is a quark-level application of the old ideas of vector dominance and the current-field identity.[3] However, a vector current does not change the quark helicity, leading to $\eta=+1$. The prediction holds no matter at which step in the fragmentation chain the ρ^0 is produced. (Note also that in

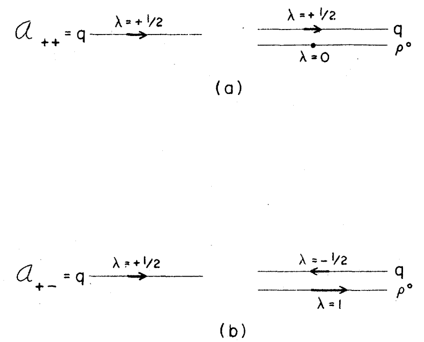

FIG. 1. (a) Helicity-conserving fragmentation amplitude. (b) Helicity-flip amplitude. Helicities are denoted by λ.

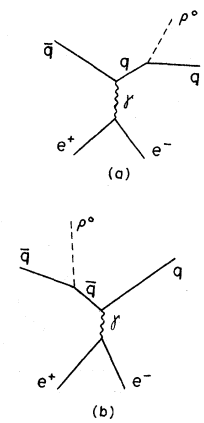

FIG. 2. Diagrams for $e^+e^- \to q\bar{q}\rho^0$.

a recent paper Suaya and Townsend present a calculation that suggests that $\eta = 1$ at large transverse momentum relative to the jet axis.[4]

If we include transverse momentum in the quark jet, the other spin components may be nonzero. However, now we must specify our coordinate system in more detail. The simplest choice would be to put the x axis in the plane formed by the ρ^0 direction of motion and the beam axis. However, the parton model tells us that the quark jet develops without retaining any knowledge about the production plane. With this choice of axes we should find ρ_{01} and ρ_{1-1} to be either zero or falling as a power of Q^2. Nonzero correlations should exist in the scaling limit only when we refer to the internal properties of the jet. Therefore a more appropriate choice of the x-z plane is that plane formed by the ρ^0 direction ($\hat{z}$) and the jet axis, with $\hat{x}$ being directed towards the jet axis. In this frame the off-diagonal tensor polarization may be nonzero.

In order to understand better the generation of these elements, and their dependence on $\rho_\perp$, it is instructive to perform a model calculation for $e^+e^- \to q\bar{q}\rho^0$. The diagrams involved are given in Fig. 2, and the quark-meson coupling is assumed to be vectorial:

$$H = g\bar{q}\gamma_\mu \tau_3 q \rho^{0\mu} \, . \qquad (9)$$

We neglect the interference of the two diagrams, as this is forbidden by standard parton-model assumptions. The spin density matrix is then calculated. We find that correlations with the beam axis do fall like M_ρ^2/Q^2. The alignment deviates from $\eta = 1$ as the transverse momentum increases, while ρ_{01} rises approximately linearly with $\rho_\perp$, and ρ_{1-1} rises quadratically. The dependence on the relative transverse momentum of the quark and ρ^0 at $z = 0.5$ is given in Fig. 3. While this calculation need not be taken seriously as an accurate description of the fragmentation process, it does suggest an ordering of the spin components which may be somewhat more general. This ordering has η somewhat less than unity, with ρ_{01} smaller and ρ_{1-1} the smallest. The ordering could be strengthened by transverse-momentum cuts relative to the jet axis.

A contrasting class of models which yield predictions about the alignment are generated by various statistical assumptions.[5,6,7] These proceed by assuming that in the fragmentation process extra quark pairs are produced with both helicities of quarks equally likely. The initial quark combines with the produced antiquark to form either a vector meson or a pion. If the helicities of the two quarks line up, the state formed must be a vector meson with $J_z = \pm 1$. If the quark helicities are antiparallel, it is as-

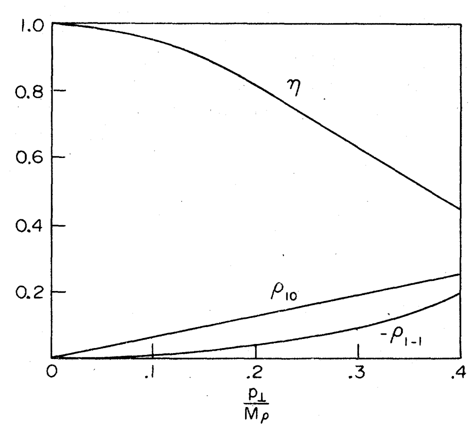

FIG. 3. Density matrix elements for $e^+e^- \to q\bar{q}\rho^0$ at $z = 0.5$.

sumed to form a pion a fraction f of the time, and a vector meson the remaining $1-f$. Popular choices are $f=1$,[1,6] and $f=\frac{1}{2}$.[7] Simple counting arguments then yield the alignment

$$\eta=\frac{1-2f}{4-2f}\,. \tag{10}$$

These models then favor either zero or negative alignment. They do not readily yield the off-diagonal elements, as the above model did. However, they do contain a strong prediction about the relative amounts of primary vector mesons and pions. This yields a ratio

$$R_{\rho/\pi}\equiv\frac{N_\rho}{N_\pi}=\frac{2-f}{f}\,. \tag{11}$$

The alignment can be given in terms of this ratio, and the relationship can be tested experimentally,

$$\eta=\frac{1}{4}\left(1-\frac{3}{R_{\rho/\pi}}\right). \tag{12}$$

The models only make sense for $R_{\rho/\pi}\geq 1$. The predictions of the two types of models are different because of differing physical assumptions about the fragmentation process. Experiment should easily distinguish between them.

Finally we wish to comment on some interesting spin correlations which can occur if one is able to observe a ρ^0 in each of the two jets in e^+e^- annihilation. The helicities of the quarks which form the two jets are correlated because they are produced by a virtual photon. A left-handed quark always occurs with a right-handed antiquark and vice versa. The angular distribution for the two decays is

$$W(\theta_1,\phi_1,\theta_2,\phi_2)=W_{\text{even}}(\theta_1,\phi_1)W_{\text{even}}(\theta_2,\phi_2) - W_{\text{odd}}(\theta_1\phi_1)W_{\text{odd}}(\theta_2,\phi_2)\,, \tag{13}$$

where all the angles refer to the internal properties of the respective quark jet. It is the odd-odd correlation which is new here. For its measurement, the asymmetries defined in Ref. 2 will be useful. Thus the parity-odd density matrix elements which can be measured in neutrino scattering may also be observed in spin-correlation experiments in e^+e^- reactions.

Note added. G. N. Khachatryan and Yu G. Shakhazaryan [Yad. Fiz. 26, 1258 (1977) (Sov. J. Nucl. Phys. 26, 664 (1977))] have also considered the formalism for studying vector-meson polarization in $e^+e^-\to VX$. However, in contrast to the present work, they define their coordinate system by the V direction of motion and the beam direction rather than referring to the internal properties of the quark jet.

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