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Finite renormalization, flavor mixing, and weak decays

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We discuss the treatment of flavor-changing self-energies, both for leptons and for quarks in bound states. It is shown that finite self-energy effects can contribute to $\Delta S = 1$ hadronic weak decays.

I. INTRODUCTION

Our present understanding of the observed hadrons and their interactions is that they are manifestations of an underlying theory which is best expressed in terms of quarks and gauge bosons. So far the approximations to that theory are crude, but are steadily becoming more sophisticated. One problem which has not yet received adequate treatment is the renormalization of quarks in bound states. Infinite ultraviolet divergences are due to the short-distanct behavior of the fields and can be handled directly by renormalizing the underlying Lagrangian. However, infrared effects (whether infinite or finite) are not easily handled in this way, and are best described by phenomenological hadronic wave functions or structure functions. This paper considers one small aspect of the general problem: the renormalization and effects of self-energies that change the quark flavor.

One motivation for this work arises from weak nonleptonic processes. In the decays of hyperons and kaons, the change in strangeness and parity is the same as would occur if the strange quark were simply mixed with the down quark by the weak interaction. Of course this does occur, but the mixing is generally removed by a redefinition of the quarks.[1,2] It has therefore been assumed that the off-diagonal self-energy is unimportant in weak processes. However, it is possible that the mixing of the quarks would be different in mesons and baryons, or would have some momentum dependence. The standard redefinition of the fields is then not possible, and an alternate procedure must be found. We examine the situation somewhat more generally than in the past and find that differences in self-energies can in fact lead to weak decays.

In Sec. II, we describe the renormalization of off-diagonal self-energies, first for leptons, and then for bound states of quarks.[3] The emphasis is on finite effects, with the renormalization of infinities assumed to have been taken care of in the usual way. Then, in Sec. III, the s-d mixing is studied in relation to weak decays. Although reliable calculations are beyond us, we show how the self-energy may generate nonleptonic transitions.

II. RENORMALIZATION

One of the attractive features of the combination of quantum chromodynamics (QCD) and weak gauge theories is the simple way that flavor-changing infinities are removed from the theory.[2] The infinities all occur in dimension-three and -four operators, corresponding to a change in the Lagrangian similar to

$$\Delta\mathcal{L} = \bar{d}[i\not{D}(\alpha L + \beta R) + \gamma L + \delta R]s + \text{H.c.}, \tag{1}$$

where $L = (1+\gamma_5)/2$ and $R = (1-\gamma_5)/2$ are the left and right projection operators. However, by performing separate transformations (in general, not unitary) on the left and right components of the s and d fields, one can bring the full Lagrangian to diagonal form

$$\mathcal{L}_{\text{QCD}} + \Delta\mathcal{L} \to \mathcal{L}'_{\text{QCD}}, \tag{2}$$

where the prime means with renormalized masses and vacuum angle. This is a very powerful result, as it implies that even in the presence of $\Delta\mathcal{L}$ we have a conserved quantum number which can be called strangeness. $\Delta\mathcal{L}$ will not induce strangeness-changing decays.

However, once infinities have been removed, this procedure is not definite enough for a final finite renormalization. In general $\alpha \to \delta$ will depend on momentum or, in a theory with confinement, will depend on the environment in which they are calculated. "Environment" is used to mean "in a pion," "in a kaon," "in a proton." Diagonalization of the fields in one environment will not be equivalent in another. In a theory where particles are free and do not mix, as in QED, the renormalization prescription is known: One adjusts the parameters such that bare plus radiative mass equals the experimentally measured mass, and such that the full propagator has the standard form, when $p^2 \approx m_{\text{exp}}^2$. This is the experimental data which determine the renormalized parameters. With flavor-changing processes we need similar input.

For leptons we require a generalization of the standard prescription. We can write the inverse propagator in matrix form as

$$S^{-1}(p)=\not p[A_l(p^2)L+A_R(p^2)R]-B_L(p^2)L-B_R(p^2)R\,, \tag{3}$$

where A_L, A_R, B_L, and B_R are matrix functions of p^2 in lepton flavor space. We require that near the ith pole the propagator be in its usual form

$$S(p)=A_i\frac{\not p+m_i}{p^2-m_i^2+i\epsilon}+R(p)\,, \tag{4}$$

where A_i is a matrix (which may contain γ_5) with no momentum dependence and $R(p)$ is regular at the pole. From $S^{-1}S=SS^{-1}=1$ we can write

$$S(p)=\alpha_i\otimes\frac{\not p+m_i}{p^2-m_i^2+i\epsilon}\alpha_i^\dagger+R(p)\,, \tag{5}$$

where the condition that $S^{-1}S$ be well behaved at the pole requires that the column vector α_i satisfy

$$S^{-1}(p)\alpha_i(\not p+m_i)=0 \tag{6}$$

at $p^2=m_i^2$. Decomposing α_i into left and right portions,

$$\alpha_i=\alpha_{iL}L+\alpha_{iR}R\,, \tag{7}$$

we find that

$$\begin{aligned}&[m_i^2A_L(m_i^2)-B_R(m_i^2)A_R^{-1}(m_i^2)B_L(m_i^2)]\alpha_{iL}=0\,,\\&[m_i^2A_R(m_i^2)-B_L(m_i^2)A_L^{-1}(m_i^2)B_R(m_i^2)]\alpha_{iR}=0\,.\end{aligned} \tag{8}$$

Both can be satisfied provided that

$$\det[m_i^2-B_R(m_i^2)A_R^{-1}(m_i^2)B_L(m_i^2)A_L^{-1}(m_i^2)]=0\,. \tag{9}$$

This is the equation which locates the position of the poles. The resulting eigenvectors α_{iL} and α_{iR} tell us which combinations of left- and right-handed bare fields make up the physical particle whose pole this is. The physical field (denoted hereafter by a tilde) is the one with definite propagation behavior and is given by

$$\tilde\psi_i=\sum_j(\alpha_{iL}^j\psi_{jL}+\alpha_{iR}^j\psi_{jR})\,, \tag{10}$$

where α_{iL}^j is the jth component of α_{iL}. Note that in general $\alpha_i^j\neq-\alpha_j^i$ because of the momentum dependence in the equations for the eigenvectors. If there were no momentum dependence the above procedure would reduce to the standard diagonalization of dimension-three and -four operators that was mentioned bedore. At this stage if we wish we can rewrite the Lagrangian in terms of the physical fields by a transformation:

$$\begin{aligned}&\psi=S\tilde\psi\,,\\&\tilde{\mathcal{L}}=S^\dagger\mathcal{L}S\,.\end{aligned} \tag{11}$$

With quarks the above method will not be applicable. We have no direct information about quarks, and self-energies of isolated quarks are not calculable because of infrared difficulties. The experimental input that we do have involves only bound states of quarks. This is of course a disadvantage as we are only able to construct approximations to the true bound states. In practice, however, this will be acceptable because the phenomena that we wish to discuss (e.g., particle decay) also involve the composite states.

As an example, consider the scalar and pseudoscalar bosons. One can in principle calculate the self-energy in this basis, and obtain the inverse propagator for the bosons,

$$\Delta^{-1}(p)=p^2-M_0^2-\Sigma_s(p)-\Sigma_w(p)\,, \tag{12}$$

where M_0^2 is the bare mass matrix for the mesons calculated with $\mathcal{L}_{\rm QCD}$ (in some approximation). $\Sigma_s(p)$ is the strong self-energy which results when the strong states are taken off-shell, and we choose it to be diagonal and to vanish at the strong-interaction pole. $\Sigma_w(p)$ is the matrix self-energy due to the weak interactions, and will couple together states of different strangeness and parity. In this basis, the procedure is similar to that given above. We require that, near a pole, the propagator have the form

$$\Delta(p)=\frac{\alpha_i\otimes\alpha_i^\dagger}{p^2-m_i^2+i\epsilon}+R(p)\,, \tag{13}$$

where α_i is an eigenvector of $\Delta^{-1}(p^2=m_i^2)$ with eigenvalue zero:

$$\Delta^{-1}(p^2=m_i^2)\alpha_i=0\,. \tag{14}$$

For this to be possible we must have

$$\det[p^2-M_0^2-\Sigma_s(p)-\Sigma_w(p)]=0\,, \tag{15}$$

which will determine the location of the poles. In principle this feeds back to determine the parameters in the Lagrangian. This is in fact just a generalization of the standard quark model practice of fitting the parameters by calculating bound-state masses. For baryons, the prescription is the same as for leptons, except the basis states include the ground-state baryons and all the baryon resonances.

For weak mixing the effect of this analysis is simple. The shift in the pole is second order in the off-diagonal perturbation, so we will ignore it and all other second-order corrections. The physical meson fields are expressible in terms of the bare fields by

$$\tilde{\phi}_i = \phi_i + \sum_{j\neq i} \alpha_i^j \phi_j , \tag{16}$$

where

$$\alpha_i^j = \frac{\langle M_j(p^2=M_i^2)\,|\mathcal{H}_w|\,M_i(p^2=M_i^2)\rangle}{M_i^2 - M_j^2 - \Sigma_{jj}^s(M_i^2)} . \tag{17}$$

The possible momentum dependence in the weak matrix elements may mean that the mixing is more complicated than the usual treatment of, say, ω-ϕ mixing, but this presents no real difficulty. In a quark model it means that we have mixed the s and d quarks differently in different particles, and the result cannot be described as a simple redefinition of fields at the Lagrangian level. It should be emphasized that, although the focus of much of this paper is on quark or lepton self-energies, for the mixing of the bound states one must use the full Hamiltonian in Eq. (17).

The effects of the flavor mixing on semileptonic processes is negligible, being an $O(G_F)$ correction. However, in nonleptonic decays it would appear that the effects would be more important since the states which are mixed in at order G_F could decay through the strong interactions at $O(1)$ yielding a result of the same magnitude as the standard contribution.

It is the physical states $\tilde{\phi}$ which have the definite propagation behavior, and we would therefore like to study their matrix elements. This could be difficult, as it would require the calculation of the mixing angles. However, Feinberg, Kabir, and Weinberg have shown that one obtains the same results if one calculates amplitudes using the bare, unmixed states when all particles are on their mass shell (up to a renormalization constant of order of G_F^2). The proof uses the Lehmann-Symanzik-Zimmermann reduction on the bare states, e.g.,

$$_{\rm out}\langle \pi_i \pi_j | K_k \rangle_{\rm in} = -i\int d^4x\, d^4y\, d^4z\, e^{iq_i\cdot x}(-q_i^2+m_\pi^2)e^{iq_j\cdot y}(-q_j^2+M_\pi^2)\Gamma_{ijk}(-k^2+m_k^2)e^{-ik\cdot z} , \tag{18}$$

where we have integrated by parts twice, and Γ is defined by

$$\Gamma_{ijk} = \langle 0|T(\phi_i(x)\phi_j(y)\phi_k(z))|0\rangle . \tag{19}$$

Written in terms of the physical fields,

$$\Gamma_{ijk} = \langle 0|T(\tilde{\phi}_i(x)\tilde{\phi}_j(y)\tilde{\phi}_k(z))|0\rangle - \sum_l [\alpha_i^l\langle 0|T(\tilde{\phi}_l(x)\tilde{\phi}_j(y)\tilde{\phi}_k(z))|0\rangle + \alpha_j^l\langle 0|T(\tilde{\phi}_i(x)\tilde{\phi}_l(y)\tilde{\phi}_k(z))|0\rangle + \alpha_k^l\langle 0|T(\tilde{\phi}_i(x)\tilde{\phi}_j(y)\tilde{\phi}_l(z))|0\rangle] . \tag{20}$$

If we look at the last term, where $\tilde{\phi}_l$ replaces ϕ_k, we note that vertices involving $\tilde{\phi}_l$ have a pole at m_l but not at the kaon mass, so we may let $k^2 \to m_K^2$ in the numerator and obtain a vanishing result for an on-shell kaon. The other terms are disposed of similarly, leaving

$$_{\rm out}\langle \pi_i \pi_j | K_k\rangle_{\rm in} = {}_{\rm out}\langle \tilde{\pi}_i \tilde{\pi}_j | \tilde{K}_k\rangle_{\rm in} . \tag{21}$$

The mixed in fields do not contribute to transitions when all the external fields are on their mass shell. This justifies the usual procedure of calculating with the bare states.

When used with the diagonalization theorem for dimension-three and -four operators [Eq. (2)] this becomes even more useful. Matrix elements of a Hamiltonian given by Eq. (1) must vanish when calculated between bare states (i.e., those states formed *before* the rotation which diagonalizes the fields). This resolves the problem of how to handle self-energies: One simply proceeds to calculate them using the bare states. Any term which are removable by a redefinition of the fields [i.e., equivalent to Eq. (1)] will disappear, if the calculation is done correctly, and any effect which remains constitutes a physical transition. As an example of this in leptons, one can easily verify that if all the Feynman diagrams for $\mu \to e\gamma$ are calculated using an interaction analogous to Eq. (1), the sum is zero.[1] In addition it is well known that when one uses a Hamiltonian consisting of a scalar density, the parity-conserving hyperon decay amplitudes vanish.[4] We will return to this problem in the next section.

There is one situation where calculations with bare states are not correct, and the flavor mixing should be accounted for. This occurs when one of the particles is taken off its mass shell, as is done with the pion in the standard current-algebra manipulations of nonleptonic amplitudes.[5] Here

$$\begin{aligned} _{\rm out}\langle B\tilde{\pi}_i(q)|A\rangle_{\rm in} &= i\int d^4x\, e^{iq\cdot x}(-q^2+m_\pi^2)\langle B|\tilde{\phi}_i(x)|A\rangle \\ &= i\int d^4x\, e^{iq\cdot x}(-q^2+m_\pi^2) \\ &\quad\times \langle B|\phi_i(x) + \sum_j \alpha_i^j\phi_j(x)|A\rangle . \end{aligned} \tag{22}$$

The new terms can be accounted for by multiplying and dividing by q^2-m_j^2 to obtain

$$
{}_{\rm out}\langle B\tilde{\pi}_i(q)\,|A\rangle_{\rm in} = {}_{\rm out}\langle B\tilde{\pi}^i(q)\,|A\rangle_{\rm in} + \sum_j \frac{q^2 - m_\pi^2}{q^2 - m_j^2}\,\alpha_i^j \times {}_{\rm out}\langle BM_j(q)\,|A\rangle_{\rm in}\,. \qquad (23)
$$

The last series of terms is the off-shell scattering amplitudes for processes that take place via the strong interactions. With $\alpha_j^i \sim O(G_F)$ they are the same order of magnitude as the first term. When one takes the $q_\mu \rightarrow 0$ limit one may use the soft-pion techniques on the first piece, but one should also include the new terms. These are of the order of m_π^2/m_j^2, and in many cases could be neglected. However, care should be taken to ascertain that this is the case.

III. WEAK DECAYS

In this section we will study the strangeness-changing self-energy and see that it may lead to observable transitions. Previously we have seen that if the self-energy behaves as an operator of dimension three or four, its effect must vanish when calculated for on-shell states. However, environmental dependences of the self-energy may cause deviations from this simple behavior, and may result in nonzero effects. The archetype for this program is the Lamb shift, which may be thought of as the difference between the self-energy of a free electron and that of one bound in an atomic S state.

For strangeness-changing effects, we will use the weak Hamiltonian of the Weinberg-Salam model.[6] Some contributions to the quark self-energy are given in Fig. 1. The dashed lines are to remind us that these diagrams are to be evaluated in a specific bound state (either a meson or a baryon). The bound-state wave functions and propagators appropriate to that state must be used. Because of the short range of the W propagator, only the short-distance piece of the fermion propagator will be relevant for Fig. 1(a). Thus, this diagram will not be sensitive to the large-scale properties of the bound state and any environmental dependences to the self-energy will be suppressed by powers of M_W. However, in Figs. 1(b) and 1(c) the light quarks and gluon propagators and wave functions are drastically affected by confinement, and will depend on bound-state properties (such as the hadronic radius). Thus we would expect significant differences if these latter diagrams were calculated in a baryon or in a meson. We will give below an explicit calculation of the experimental effects connected with these diagrams.

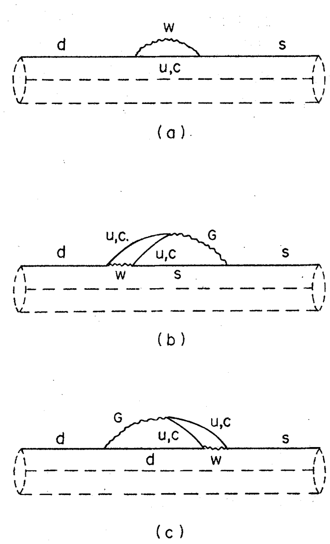

FIG. 1. Examples of quark self-energy diagrams. In (b) and (c), G is a gluon. The dotted lines indicate that these processes take place inside of a hadron, in the presence of other quarks.

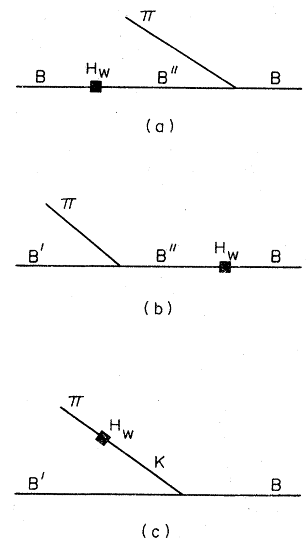

FIG. 2. Pole diagrams for $B \rightarrow B'\pi$. (a) and (b) are baryon poles, and (c) is a kaon pole.

Because the bound-state parameters are determined by the collective action of all the quarks, it could be argued that the effects described in this section are not true self-energies effects but rather are a class of nonperturbative interactions with other quarks. The difference is only semantic, but in fact these diagrams have been neglected in past studies because of a feeling that self-energies should not be included. We will see that they should not be overlooked.

The most straightforward example is the parity-conserving (pc) amplitude in hyperon decay. We will treat this in a pole model and only include the lowest baryon and meson octets. The appropriate diagrams are given in Fig. 2. The weak matrix elements which are needed are of the form

$$\langle B' |\mathcal{H}_w|B\rangle\,,\quad \langle \pi |\mathcal{H}_w|K\rangle\,,$$

which we will denote by

$$M_{AB} = \langle B |\mathcal{H}_w|A\rangle\,. \tag{24}$$

The pc amplitudes are then

$$\begin{aligned}
B(\Lambda^0_-) &= \frac{g_{\pi^- pn} M_{\Lambda n}}{m_\Lambda - m_N} - \frac{g_{\pi\Lambda\Sigma} M_{\Sigma^+ p}}{m_\Sigma - m_N} - \frac{g_{\Lambda pK} M_{K^-\pi^-}}{m_K^2 - m_\pi^2}\,,\\
B(\Sigma^-_-) &= \frac{-g_{\pi^-\Sigma\Lambda} M_{\Lambda n}}{m_\Lambda - m_N} - \frac{g_{\pi^-\Sigma^-\Sigma^0} M_{\Sigma^0 n}}{m_\Sigma - m_N} - \frac{g_{\Sigma^- nK} M_{K^-\pi^-}}{m_K^2 - m_\pi^2}\,,\\
B(\Sigma^+_+) &= \frac{g_{\pi^+ np} M_{\Sigma^+ p}}{m_\Sigma - m_N} - \frac{g_{\pi^+\Lambda\Sigma^+} M_{\Lambda n}}{m_\Lambda - m_N} - \frac{g_{\pi^+\Sigma^0\Sigma^+} M_{\Sigma^0 n}}{m_\Sigma - m_N}\,,\\
B(\Xi^-_-) &= \frac{g_{\pi^-\Lambda\Sigma} M_{\Xi^-\Sigma^-}}{m_\Xi - m_\Sigma} - \frac{g_{\pi^-\Xi^-\Xi^0} M_{\Xi^0\Lambda}}{m_\Xi - m_\Lambda} - \frac{g_{\Xi\Lambda K} M_{K^-\pi^-}}{m_K^2 - m_\pi^2}\,.
\end{aligned} \tag{25}$$

The remaining amplitudes are given by the $\Delta I = \frac{1}{2}$ sum rules.

The above is general and could be used with any Hamiltonian, but we wish to specialize to the study of self-energies. It is known that if the weak Hamiltonian is a scalar density all the amplitudes vanish.[4] This is a demonstration of the results of the previous section, and we may use it to simplify the amplitudes. We will assume, as is true in the following calculation, that the self-energy has the same SU(3) transformation as a scalar density, but a *different* magnitude for mesons and baryons. Thus

$$\begin{aligned}
B(\Lambda^0_-) &= \frac{-g}{\Delta m_B\sqrt{3}}(1+2f)C\,,\\
B(\Sigma^-_-) &= \frac{g\sqrt{2}}{\Delta m_B}(2f-1)C\,,\\
B(\Sigma^+_+) &= 0\,,\\
B(\Xi^-_-) &= \frac{g}{\Delta m_B\sqrt{3}}(4f-1)C\,,
\end{aligned} \tag{26}$$

where g and f describe the strong vertex ($g^2/4\pi = 14.6$, $f \cong \frac{2}{5}$) and

$$C = A_{\Sigma^+ p} - \frac{\langle p |\bar{d}s|\Sigma^+\rangle}{\langle \pi^- |\bar{d}s|K^-\rangle} A_{K^-\pi^-}\,. \tag{27}$$

Even at this stage we can see that this mechanism cannot be the complete description of the P-wave processes. The amplitude for $\Sigma^+ \to N\pi^+$ has no meson pole and hence vanishes by our assumptions, contrary to experiment. However, it is still important to determine the role of the self-energy even if it is not the unique contribution.

A calculation of the self-energy in hadrons within the MIT bag model can be extracted from the quark-sea work of the author and Golowich.[7] The gluon propagator in the bag is known,[7] and the propagators for the virtual quarks are given by summing over all possible intermediate-state modes. The coupling constant is that found in the MIT fit to hadron masses,[8] and for simplicity all light-quark masses are taken to be zero. If we neglect charm for the moment, a simple and general feature can be extracted. The only dimensional parameter which enters the calculation is the hadronic radius R, so that by dimensional analysis alone the self-energy must be of the form

$$\text{self-energy} = \frac{KG_F}{R^3}\,. \tag{28}$$

This vanishes in the absence of confinement ($R \to \infty$) as must be the case for massless quarks. Since the radius of baryons ($R \approx 5$ GeV^{-1}) and mesons ($R \approx 3.3$ GeV^{-1}) is different in the bag model[8] this portion of the self-energy will be different in these two cases by a large amount. (The scalar density is independent of R in the massless limit.) For charmed quarks in the intermediate state, the self-energy contains another dimensional parameter m_c, so that

$$(\text{self-energy})_{\text{charm}} = \frac{KG_F}{R^3} f(m_c R)\,, \tag{29}$$

with $f(0) = 1$. The charm contribution tends to cancel that of the light quarks, leaving

$$C = M_{\Sigma^+ p}\left[1 - \frac{R_p^3[1 - f(m_c R_\pi)]}{R_\pi^3[1 - f(m_c R_p)]}\right]. \tag{30}$$

This demonstrates how the desired effect can occur. To obtain the magnitude we complete the calculation using bag-model parameters. We find

$$\begin{aligned}
M_{\Sigma^+ p} &= 6\times 10^{-9}\left(\frac{C_+ + C_-}{2}\right)m_p\,,\\
f(m_c R_p) &= 0.11\,,\\
f(m_c R_\pi) &= +0.20\,,\\
C &= -9\times 10^{-9}\left(\frac{C_+ + C_-}{2}\right)m_p\,.
\end{aligned} \tag{31}$$

Here C_+ and C_- are the QCD correction factors due to the short-distance behavior of the Hamiltonian.[6] Because the SU(3) structure of the amplitudes is incorrectly given, there is no unique way to compare this with experiment. However, it contributes a significant fraction to most decays, the largest being (using $C_- = 2$, $C_+ = 1/\sqrt{2}$)

$$B(\Lambda^0_-)_{\text{SE}} = -7.3\times 10^{-7} \tag{32}$$

compared to the experimental value

$$B(\Lambda^0_-)_{\text{expt.}} = 22.5 \pm 0.5 \times 10^{-7}. \tag{33}$$

The sign is opposite that of the usual comparison of P-wave amplitudes extracted from S-wave amplitudes in the soft-pion limit. We see that self-energy diagrams can be a significant contribution to weak decays.

Other decays can be treated less satisfactorily with present techniques. There are two reasons for this. For parity-violating processes, such as S-wave hyperon decay and $K \to 2\pi$, the self-energy mixing is between the ground-state particles and odd-parity excited states. In the case of the parity-violating S wave, the pole-model formalism would apply, with the poles being negative-parity N^*'s and the strange scalar meson, states which are on dubious footing in quark models.[9] The phenomenology of this[10] [and also of $K \to 2\pi$ (Ref. 11)] is worked out, but it would be difficult to perform a reasonable self-energy calculation beyond the simple dependence on the radius quoted above.

The other problem occurs in kaon decay. There we no longer have the interplay of baryon and meson amplitudes. For example, a pole model for $K \to 3\pi$ involves a pion pole (with $M_{K\pi}$ evaluated at $p^2 = m_K^2$) and kaon poles (with $M_{K\pi}$ at $p^2 = m_\pi^2$). The $K \leftrightarrow \pi$ mixing due to a scalar density in chiral $SU(3) \times SU(3)$ is assumed to contain no momentum dependence. Therefore any correct evaluation of the $K\pi$ self-energy which does not involve momentum dependence will lead to the same $K \to 3\pi$ amplitude as the scalar density (namely zero). It is possible for the self-energy to generate a nonzero amplitude proportional to $[M_{K\pi}(p^2 = m_K^2) - M_{K\pi}(p^2 - m_\pi^2)]$, but present techniques are inadequate to handle this. We must hope for improved technologies if we wish to understand better the role of the self-energies in hadronic decays.

IV. CONCLUSION

To discuss the finite renormalization of flavor-changing self-energies, one must specify the experimental data which determine the parameters of the theory. These are found by studying the particles which propagate freely. For quark bound states, the prescription reduces to a generalization of the quark-model procedure of determining Lagrangian parameters by calculating bound-state effects. However, it is not necessary to calculate the weak mixing between hadrons. When using the bare, unmixed states any effects which are removable by a redefinition of the quark fields will cancel (for on-mass-shell states). In Sec. II we used this to show how "environmental" dependences of the self-energy can lead to transition amplitudes in strangeness-changing processes.

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