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Generalization of the Glashow-Iliopoulos-Maiani mechanism: Horizontal and vertical flavor mixing

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For theories with more than four quarks we generalize the Glashow-Iliopoulos-Maiani mechanism for flavor-change suppression in the gauge sector and the Glashow-Weinberg theorems for the corresponding suppression in the Higgs sector. We note that there is a class of theories in which many but not all flavor changes are naturally suppressed. $d \leftrightarrow s$ and $u \leftrightarrow c$ are in the naturally suppressed category. Such theories have distinctive experimental signatures. We discuss the simplest example where the new mechanism applies: $SU(3) \times U(1)$ with six quarks, two with charge 2/3, four with charge $-1/3$. We also indicate how this six-quark system can be incorporated in a full-fledged gauge theory.

I. INTRODUCTION: PHYSICS AFTER τ AND Υ

In a world of four quarks (u, d, s, c) and four leptons $(\nu_e, e^-, \mu^-, \nu_\mu)$ with weak and electromagnetic interactions described by the standard $SU(2) \times U(1)$ gauge model,[1] the Glashow-Iliopoulos-Maini (GIM) mechanism[2] is the virtually unique answer to the question: "Why are there no flavor-changing neutral-current (FCNC) effects in order G or αG?" Alas, this beautiful theory does not seem to apply to our world, since there are apparently at least five quarks (assuming Υ to be a $b\bar{b}$ state, where b has charge[3] $-\frac{1}{3}$) and at least six leptons (counting τ^- and an associated neutrino ν_τ).[4] We must generalize our answer to the question of suppression of FCNC's by generalizing the GIM mechanism. While we are at it, we should also consider generalizing the question.

If we take the question to mean: "How can we naturally suppress *all* flavor-changing neutral-current effects?" then it has been answered by Glashow and Weinberg.[5] We recall the answer for the case of the $SU(2) \times U(1)$ theory. In order to ensure in a natural way that there are no FCNC effects in order G or αG, one must put all fields with the same conserved quantum numbers and chirality in equivalent positions within equivalent representations of the gauge group. In general we encounter particle mixtures at these equivalent positions (particle states being defined as eigenstates of the quark mass matrix). We call such mixings horizontal mixings. In such a theory all mixings and Cabibbo-type angles connect fields with the same gauge properties. If the b quark has charge $-\frac{1}{3}$ in such a horizontal mixing scheme, its left-handed chiral component (like those of the d and s quarks) must be the $T_3 = -\frac{1}{2}$ component of a weak-SU(2) doublet. Thus there are at least three quark doublets, and there must exist a t quark with charge $\frac{2}{3}$ whose left-handed chiral component (like those of the u and c quarks) is the $T_3 = \frac{1}{2}$ component of a weak SU(2) doublet.

The gauge couplings of the six-quark, three-doublet, horizontal mixing scheme are described by three mixing angles and a phase.[6] The angles and phase must be small if this scheme is to be consistent with experimental bounds on the violation of Cabibbo universality. The occurrence of extra parameters in this model (as compared with the single Cabibbo angle in the four-quark model) would make it unattractive except that it is possible by enlarging the gauge group to relate them naturally to the quark masses in a simple way.[7] This leads to a set of specific predictions for the decay modes of b and t quarks and even to a prediction for the t-quark mass.[8]

It is straightforward to generalize the horizontal mixing defined above to gauge groups larger than $SU(2) \times U(1)$ in such a way that the natural suppression of *all* FCNC's is again guaranteed. Numerous generalizations of this type are found in the literature.

We stress that we shall not argue against this attractive class of theories, but we wish to point out that there is another way of generalizing the GIM mechanism to systems of more than four quarks and leptons. In our generalization, we expand the quark and lepton systems in the "vertical" direction by enlarging the weak gauge group. In this type of generalization, horizontal mixings do not give FCNC's, as in the pure horizontal mixing theories. But vertical mixings are associated with FCNC effects. At present, there is experimental evidence that FCNC effects are smaller than $O(\alpha G_F)$ in the s-d system and the c-u system, but we do not know whether the same is true in the s-b or d-b systems.

We will illustrate our vertical mixing scheme in an SU(3) × U(1) gauge model with two triplets. In this model there is no t quark. The sixth quark has charge $-\frac{1}{3}$ (we call it l). Nevertheless there is a simple system of Higgs mesons which ensures that FCNC's in the d-s and u-c systems are naturally suppressed (and the same is true in the d-l, s-b, and b-l systems), but there are FCNC effects of $O(G_F)$ in the d-b and s-l systems. The key idea is that the mass matrix of the charge $-\frac{1}{3}$ quarks naturally takes the form of a tensor product of a 2×2 matrix in the horizontal space with a 2×2 matrix in the vertical space. This idea can be generalized to larger matrices in both the horizontal and vertical spaces and to larger gauge groups. As we will see in detail in Sec. II, the tensor-product mass matrix can be diagonalized with unitary transformations which are likewise tensor products of horizontal and vertical transformations, and this translates easily into our generalization of the GIM mechanism.

In Sec. II, we discuss the quark sector of our SU(3) × U(1) example. There are six quarks in the model, two with charge $\frac{2}{3}$ (u and c) and four with charge $-\frac{1}{3}$ (d, s, b, and l). The left-handed quark fields make up two triplets under the weak SU(3) gauge group. The right-handed quark fields are all weak SU(3) singlets.

We break the SU(3) × U(1) gauge symmetry in two steps: SU(3)×U(1) → SU(2)×U(1) → U(1). The breakdown in the first step is taken to be superstrong. This is achieved by assigning a large vacuum expectation value (VEV) to a Higgs meson which does not couple to quarks. With some simple discrete symmetries, we ensure that the Higgs fields which do couple to quarks give a quark mass matrix of the tensor product form. We show that FCNC effects in the u-c and d-s (and some other) systems are suppressed not only in the $W^\pm$ and Z couplings but in Higgs-boson couplings as well. In fact, the model shares many of the features of the four-quark SU(2) × U(1) model. For example, we show that all phases can be removed from the couplings of the $W^\pm$ and Z^0 to the quarks, so that any CP violation must come from the exchange of Higgs bosons or superheavy vector bosons.

We incorporate the vertical-mixing idea in a full-fledged gauge model by including leptons. One such scheme is described in Sec. III and another in an appendix. We discuss some experimental signatures for the vertical-mixing scheme. In Sec. IV we summarize our main conclusions.

II. VERTICAL FLAVOR MIXING

To illustrate the idea of vertical mixing, we consider an SU(3) × U(1) gauge model of weak and electromagnetic interactions. The quark fields are color-SU(3) triplets, but we suppress the color index. They belong to the following representations of the weak SU(3) gauge group: The left-handed fields are two triplets,

$$\Psi_{j_L}=\begin{pmatrix} U_j \\ D_j^1 \\ D_j^2 \end{pmatrix}_L , \quad j=1 \text{ or } 2; \tag{2.1}$$

the right-handed fields are singlets,

$$U_{j_R}, D^x_{j_R}, \quad j=1 \text{ or } 2, \quad x=1 \text{ or } 2. \tag{2.2}$$

The SU(3) generators are denoted by T^a, $a=1$ to 8, and the U(1) generator by S. The gauge-covariant derivative is

$$\partial^\mu + i\,\frac{e}{\sin\theta}T^a W_a^\mu + ie\left(\frac{3\sin\theta}{\sin 3\theta}\right)^{1/2} S X^\mu , \tag{2.3}$$

where W_a^μ and X^μ are the gauge fields. The couplings have been normalized to correspond to the standard SU(2) × U(1) couplings in the sense that θ is the usual weak mixing angle. The gauge properties of the quark fields are given by

$$\begin{aligned} &T^a\Psi_{j_L}=\tfrac{1}{2}\lambda^a\Psi_{j_L}, \quad S\Psi_{j_L}=0,\\ &T^aU_{j_R}=T^aD^x_{j_R}=0,\\ &SU_{j_R}=\tfrac{2}{3}U_{j_R}, \quad SD^x_{j_R}=-\tfrac{1}{3}D^x_{j_R},\end{aligned} \tag{2.4}$$

where λ^a are the standard SU(3) matrices. We take the electric charge operator to be

$$Q=T_3+\frac{1}{\sqrt{3}}T_8+S. \tag{2.5}$$

Then the U_j fields describe two charge $\frac{2}{3}$ quarks and the D_j^x fields describe four charge $-\frac{1}{3}$ quarks. The U_j will be linear combinations of the mass-eigenstate fields u and c describing u and c quarks. The D_j^x are linear combinations of d, s, d, and l quark fields.

In the charge $-\frac{1}{3}$ sector, described by the fields D_j^x, the subscript labels the position in what we call the horizontal space while the superscript labels the position in the vertical space. This notation will simplify the discussion of the quark mass matrix.

We include in our model four triplets of Higgs mesons ϕ, ϕ_U, and ϕ_D^x, $x=1,2$. All the ϕ's satisfy $T_a\phi=\lambda_a\phi/2$, while $S=-\frac{2}{3}$ for ϕ_U and $+\frac{1}{3}$ for ϕ and ϕ_D^x. The Higgs VEV's are

$$\langle\phi\rangle=\begin{pmatrix}0\\0\\\lambda\end{pmatrix},\quad \langle\phi_U\rangle=\begin{pmatrix}\alpha\\0\\0\end{pmatrix},$$
$$\langle\phi_D^x\rangle=\begin{pmatrix}0\\\beta^{1x}\\\beta^{2x}\end{pmatrix}. \tag{2.6}$$

Note that these are the most general VEV's consistent with electromagnetic gauge invariance. We have used the gauge freedom to choose $\langle\phi\rangle$ to have a simple form. In fact, we can use the gauge invariance to choose λ and α real. We will assume that $\lambda\gg\alpha, |\beta^{xy}|$. Then the VEV of ϕ breaks the symmetry down to an $SU(2)\times U(1)$ generated by T^1, T^2, T^3 which couple to $(e/\sin\theta)W_a^\mu$, $a=1$ to 3, and by S' which couples to $(e/\cos\theta)X'^\mu$. Here

$$S'=S+\frac{1}{\sqrt{3}}T_8,$$
$$X'^\mu=\frac{1}{\sqrt{3}}\tan\theta\, W_8^\mu+\frac{1}{\cos\theta}\left(\frac{\sin 3\theta}{3\sin\theta}\right)^{1/2}X^\mu. \tag{2.7}$$

All other vector bosons are very heavy, and we will ignore them in analyzing the phenomenology of the weak interactions. Thus, as far as the gauge couplings are concerned, we can treat the model as an effective $SU(2)\times U(1)$ model. The top two components of ψ_{j_L}, U_{j_L} and $D^1_{j_L}$, form SU(2) doublets, and all other fields are singlets. The $(W_1\pm iW_2)/\sqrt{2}$ are the standard $W^\pm$ fields which couple only to the SU(2) doublets. The Z boson is the linear combination

$$Z^\mu=\cos\theta W_3^\mu-\sin\theta X'^\mu. \tag{2.8}$$

It couples to both doublets and singlets with strength proportional to $T_3-\sin^2\theta Q$.

Clearly this is very different from the standard generalization of the GIM mechanism. Two of the left-handed charge $-\frac{1}{3}$ quark fields are parts of SU(2) doublets ($D^1_{j_L}$) while the other two are singlets ($D^2_{j_L}$).

Even though the effective gauge theory at low momentum transfers is an $SU(2)\times U(1)$ theory, the SU(3) gauge symmetry of the original Lagrangian still constrains the Yukawa couplings which give rise to the quark masses. To further constrain the quark mass matrix, we impose the following discrete symmetries on the dimension-four couplings in the Lagrangian.

$$\begin{aligned}&(a)\ \phi\to-\phi,\\&(b)\ D^1_{j_R}\to-D^1_{j_R},\quad \phi^1_D\to-\phi^1_D,\\&(c)\ D^1_{j_R}\leftrightarrow D^2_{j_R},\quad \phi^1_D\leftrightarrow\phi^2_D.\end{aligned} \tag{2.9}$$

Symmetry (a) ensures that ϕ does not couple to quarks. Symmetries (b) and (c) together with the gauge symmetry, Eq. (2.4), restrict the Yukawa couplings to have the following form[9]:

$$\sum_{j,k}A_{jk}\overline{\Psi}_{j_L}\phi_U U_{k_R}+\sum_{\substack{j,k\\y}}B_{jk}\overline{\Psi}_{j_L}\phi_D^y D^y_{k_R}+\text{H.c}. \tag{2.10}$$

Combining Eq. (2.10) with the VEV's, Eq. (2.6), we find the following quark mass matrix:

$$\begin{aligned}&\overline{U}_{j_L}\alpha A_{jk}U_{k_R}+\text{H.c.},\\&+\overline{D}^x_{j_L}\beta^{xy}B_{jk}D^y_{k_R}+\text{H.c}.\end{aligned} \tag{2.11}$$

The mass matrix of the charge $\frac{2}{3}$ quarks is just an arbitrary 2×2 matrix, but the mass matrix of the charge $-\frac{1}{3}$ quark is very special: It is a tensor product of an arbitrary 2×2 matrix B_{jk} in the horizontal space times an arbitrary 2×2 matrix β^{xy} in the vertical space.

To display the composition of the gauge interactions in terms of mass eigenstate fields, we must diagonalize the mass matrices in Eq. (2.11). Because the charge $-\frac{1}{3}$ quark mass matrix is a tensor product, it can be diagonalized with unitary matrices which are themselves tensor products of unitary matrices in the horizontal and vertical spaces. Given a 2×2 matrix X, we can find unitary matrices $V_L(X)$ and $V_R(X)$ such that

$$V_L(X)^\dagger XV_R(X) \tag{2.12}$$

is diagonal and positive with the larger eigenvalue in the lower right-hand corner. Then

$$\alpha V_L(A)^\dagger AV_R(A)=\begin{pmatrix}m_u&0\\0&m_c\end{pmatrix}, \tag{2.13a}$$

$$V_L(B)^\dagger BV_R(B)[V_L(\beta)^\dagger\beta V_R(\beta)]_{11}=\begin{pmatrix}m_d&0\\0&m_s\end{pmatrix}, \tag{2.13b}$$

$$V_L(B)^\dagger BV_R(B)[V_L(\beta)^\dagger\beta V_R(\beta)]_{22}=\begin{pmatrix}m_b&0\\0&m_l\end{pmatrix}. \tag{2.13c}$$

From (2.13b) and (2.13c) it follows that

$$m_l=m_bm_s/m_d, \tag{2.14}$$

so that the l quark is very heavy and is not likely to be observed anytime soon. The SU(3) triplets can now be expressed in terms of mass-eigenstate fields. The result is more transparent if we redefine the fields in the horizontal space as follows:

$$\Psi'_{j_L} = V_L^\dagger(B)_{jk}\Psi_{k_L}\,,$$
$$D^{x\prime}_{j_R} = V_R^\dagger(B)_{jk}D^x_{k_R}\,. \tag{2.15}$$

This is equivalent to choosing Yukawa couplings in which the B_{jk} matrix is diagonal. Then with a suitable choice of the phases of the mass-eigenstate fields, the effective SU(2) doublets are the following:

$$\begin{pmatrix} \cos\theta_1 u - \sin\theta_1 c \\ \cos\theta_2 d + \sin\theta_2 b \end{pmatrix}_L ,$$
$$\begin{pmatrix} \cos\theta_1 c + \sin\theta_1 u \\ \cos\theta_2 s + \sin\theta_2 l \end{pmatrix}_L . \tag{2.16}$$

This simple form makes the nature of our generalization of the GIM mechanism clear. In the u-c system, there is horizontal mixing only and the GIM mechanism operates as usual. In the charge $-\frac{1}{3}$ system, we have eliminated the horizontal mixing by our choice of the doublets (which is to say that the GIM mechanism again applies to the horizontal mixing), but the vertical mixing remains, described by the angle θ_2.

Note that all phases have been removed from the doublets in Eq. (2.16). This is possible because the structure of the Yukawa couplings in the horizontal space is precisely the same as in the four-quark theory. The phases are removed by the same manipulations which remove them in the four-quark model. The difference now is simply that the fields which play the role of the d and s fields in the four-quark model have been replaced by the vertically mixed fields $\cos\theta_2 d + \sin\theta_2 b$ and $\cos\theta_2 s + \sin\theta_2 l$. Since only these fixed combinations take part in the charged-current weak interactions, their phases can be chosen arbitrarily.

The fact that the horizontal structure of the theory is like that of the four-quark theory has another important consequence. Namely, the neutral Higgs bosons do not mediate FCNC interactions, except in the d-b and s-l systems. In the u-c system this is obvious because only one (complex) neutral scalar Higgs boson couples, so it must couple in proportion to the mass matrix. There are four (complex) neutral Higgs bosons which couple to the charge $-\frac{1}{3}$ quarks, but they all share a common horizontal coupling B_{jk}, which is the horizontal part of the quark mass matrix. Thus these Higgs bosons do not mediate horizontal FCNC effects.

Before going on to discuss the leptons, we review what we have learned about the quark sector of our theory. Our two triplets of Eq. (2.1) may be rewritten as

$$\begin{pmatrix} u \\ c_1(c_2 d + s_2 b) + s_1(c_2 s + s_2 l) \\ c_1(c_2 b - s_2 d) + s_1(c_2 l - s_2 s) \end{pmatrix}_L ,$$
$$\begin{pmatrix} c \\ c_1(c_2 s + s_2 l) - s_1(c_2 d + s_2 b) \\ c_1(c_2 l - s_2 s) - s_1(c_2 b - s_2 d) \end{pmatrix}_L , \tag{2.17}$$

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$. Here we have rotated the doublets of Eq. (2.16) so that the mass eigenstate u and c fields appear unmixed. We have also included the effective SU(2) singlet fields in the third row. Clearly, θ_1 is the Cabibbo angle as defined by the ratio of the semileptonic decays of d and s quarks. Of the seven possible flavor changes in the couplings to Z, only $b \leftrightarrow d$ and $l \leftrightarrow s$ occur, the others are naturally absent. The theory contains the following b-quark transitions:

$$b \to u + W^- \quad (c_1 s_2)\,, \tag{2.18}$$
$$b \to c + W^- \quad (s_1 s_2)\,, \tag{2.19}$$
$$b \to d + Z \quad (c_2 s_2)\,. \tag{2.20}$$

It is at once evident from Eqs. (2.18)–(2.20) that B mesons (of type $\bar{b}d$ and $\bar{b}u$ and their conjugates) decay preferably to noncharmed channels. The Z-boson coupling, Eq. (2.20), leads to strong mixing for the neutral states $\bar{b}d \leftrightarrow \bar{b}d$ and to the peculiar neutral-current decays $b \to d + \mu^+ + \mu^-$ or $b \to d + e^+ + e^-$.[10] We shall discuss more detailed phenomenological properties in Sec. III where we incorporate our quark couplings into a full-fledged gauge model.

Finally, we note that the Yukawa couplings of (d, s, b, l) to the four neutral Higgs fields contained in ϕ^x_D can be written as

$$\begin{aligned} &(\lambda \bar{d}_R d_L + \mu \bar{s}_R s_L)(\rho + c_2\Delta_1^1 - s_2\Delta_2^1) \\ &\quad + (\lambda \bar{d}_R b_L + \mu \bar{s}_R l_L)(s_2\Delta_1^1 + c_2\Delta_2^1) \\ &\quad + (\lambda \bar{b}_R b_L + \mu \bar{l}_R l_L)(\sigma + s_2\Delta_1^2 + c_2\Delta_2^2) \\ &\quad + (\lambda \bar{b}_R d_L + \mu \bar{b}_R s_L)(c_2\Delta_1^2 - s_2\Delta_2^2) + \text{H.c.}\,, \end{aligned} \tag{2.21}$$

where $\lambda\rho = m_d$, $\mu\rho = m_s$, $\lambda\sigma = m_b$, $\mu\sigma = m_l$. The Δ^i_j are the electrically neutral fields contained in $\phi^x_D - \langle\phi^x_D\rangle$. Equation (2.21) shows explicitly that we have not only generalized the GIM mechanism but also the Glashow-Weinberg results on flavor change induced by the exchange of physical Higgs bosons. Not only in the gauge sector but also in the Higgs sector all flavor change is suppressed except for $d \leftrightarrow b$, $s \leftrightarrow l$. None of these flavor changes affect the K-$\bar{K}$ system since the *chains* $d \to b \to s$ and $d \to l \to s$ are forbidden.

III. LEPTONS AND PHENOMENOLOGY

There are many ways of including leptons in a theory with vertical flavor mixing. In this section, we will describe the most economical lepton scheme in the sense that it cancels the anomalies associated with the quark representations and has a natural embedding into a superunified theory [SU(6)] which requires four charged lepton fields. We will discuss the phenomenology of the gauge theory for this specific lepton scheme. We will describe the embedding of this model into SU(6) in Appendix A. In Appendix B, we discuss an alternative lepton scheme.

The lepton fields are the following:

$$L^x_{j_R} = \begin{pmatrix} l^x_j \\ \nu^x_j \\ \nu'^x_j \end{pmatrix}_R \qquad (3.1)$$

are four triplets of right-handed fields. They satisfy $T^a L^x_{j_R} = \frac{1}{2}\lambda^a L^x_{j_R}$, $S L^x_{j_R} = \frac{1}{3} L^x_{j_R}$.

$$\Lambda_{j_L} = (M^0_j, l^2_j, -l^1_j)_L \qquad (3.2)$$

are two antitriplets of left-handed fields. They satisfy $T^a \Lambda_{j_L} = -\Lambda_{j_L}\lambda^a/2$, $S\Lambda_{j_L} = \frac{2}{3}\Lambda_{j_L}$. Finally, there are two singlest $T^a M^0_{j_R} = S M^0_{j_R} = 0$. The l^x_j fields are charge +1 antilepton fields. The M^0_j, ν^x_j, and ν'^x_j are massive neutral lepton fields and neutrino fields. We introduce no new Higgs fields.

All of the fermion fields we have introduced, leptons and quarks, carry the horizontal index $j=1$ or 2. Thus there are two "families" of fermions with the same gauge structure. Each family consists of the fields listed in Table I. Ignoring the horizontal mixing (the Cabibbo angle), one family consists of the u, d, and b quarks and, as we shall see, e and τ leptons and various neutral leptons. The other consists of c, s, and l quarks, μ and T leptons, and neutral leptons. The b and τ in this scheme are not mere copies of the s and μ. They are new and different objects. This should be compared with the standard six-quark, three-charged-lepton model in which there are three families with the same gauge structure. As in the original four-quark GIM model, we need only double (not triple) the basic family structure.

TABLE I. A family of fermion fields.

(1) a weak-SU(3) $\underline{3}$ of left-handed quark fields, Ψ_L;

(2) a weak-SU(3) singlet of $Q=\frac{2}{3}$ right-handed quark fields, U_R;

(3) two weak-SU(3) singlets of $Q=-\frac{1}{3}$ right-handed quark fields, in a vertical pair ($x=1$ and 2), D^x_R;

(4) two weak-SU(3) $\underline{3}$'s of right-handed antilepton fields, each with charges (+1, 0, 0) in a vertical pair L^x_R;

(5) a weak-SU(3) $\underline{\bar{3}}$ of left-handed antilepton fields with charges (0, +1, +1), Λ_L;

(6) a neutral SU(3) singlet right-handed antilepton field, M^0_R.

It is easy to see that each family is anomaly-free.[11] In fact, as we will see in Appendix A, a single family (except for the neutral singlet lepton) can be embedded in the smallest anomaly-free but complex representation of the superunifying group SU(6).

We have considerable latitude in writing down Yukawa couplings which give mass to the charged leptons. For simplicity, we will assume that there is a global symmetry which prevents the transitions μ or $T \leftrightarrow e$ or τ but not (see below) the transitions $\tau \leftrightarrow e$, $T \leftrightarrow \mu$. Then the Yukawa couplings are the following:

$$\sum_{j,x} C_j \epsilon \bar{\Lambda}_{j_L} \phi^x_D L^x_{j_R} + \text{H.c.} \qquad (3.3a)$$

$$+ \sum_j E_j \phi^\dagger_U \bar{\Lambda}_{j_L} M^0_{j_R} + \text{H.c.} \qquad (3.3b)$$

In Eq. (3.3a), the ϵ symbol stands for the three-index completely antisymmetric tensor in the weak-SU(3) space.

The Yukawa couplings [Eq. (3.3)] together with the VEV's [Eq. (2.6)] determine the lepton masses. Because of the global symmetry, the charged-lepton mass matrix splits into two unconnected pieces, one describing the e^+ and τ^+ and the other describing the μ^+ and an as yet unobserved lepton T^+. Because of the tensor product form of the mass matrix and the fact that the same ϕ^x_D Higgs multiplets have been used for quarks and leptons, we obtain the mass relations

$$\frac{m_\tau}{m_e} = \frac{m_T}{m_\mu} = \frac{m_b}{m_d} = \frac{m_l}{m_s}. \qquad (3.4)$$

The relation between masses of already "observed" quarks and leptons, $m_\tau/m_e = m_b/m_d$, is a relation which is also obtained in a wide class of superunified theories. Equation (3.4) implies $m_d \simeq 1.3$ MeV while current algebra suggests $m_d \simeq 7.5$ MeV. Some have argued[13] that the very small current-algebra mass of the d quark (compared to typical hadron masses) makes the comparison of Eq. (3.4) with current algebra chancy. We choose to regard Eq. (3.4) as a minor flaw. It can always be remedied, if necessary, by using different Higgs fields for the leptons from those used for the quarks. For simplicity, we will continue to describe the theory with the same set of Higgs fields for both quarks and leptons.

To further simplify the result, we will assume that

$$E_j \alpha \gg C_j \beta^{xy}. \tag{3.5}$$

This allows us to neglect the mixing of the massless antineutrino fields in $L^x_{j_R}$ with the massive lepton fields $M^0_{j_R}$. It also implies that the massive neutral leptons are much heavier than the τ. In this approximation, we can write the effective weak-SU(2) doublets of the theory in terms of mass-eigenstate fields as follows: The top two components of the $L^x_{j_R}$ fields can be taken to be doublets

$$\begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix}_R, \begin{pmatrix} \tau^+ \\ \bar{\nu}_\tau \end{pmatrix}_R, \begin{pmatrix} \mu^+ \\ \bar{\nu}_\mu \end{pmatrix}_R; \begin{pmatrix} T^+ \\ \bar{\nu}_T \end{pmatrix}_R. \tag{3.6a}$$

The first two components of Λ_{j_L} can be written in the following doublet form:

$$\begin{pmatrix} \cos\theta_2 \tau^+ - \sin\theta_2 e^+ \\ M^0_1 \end{pmatrix}_L, \quad \begin{pmatrix} \cos\theta_2 T^+ - \sin\theta_2 \mu^+ \\ M^0_2 \end{pmatrix}_L. \tag{3.6b}$$

All other lepton fields are singlets under the effective SU(2) gauge group.

The gauge structure of Eq. (3.6) completes our gauge model, and we can discuss the specific phenomonology it describes. The charged-current structure is simple. The b quark decays primarily into a u quark by ordinary charged-current weak interactions, but the decay rate is suppressed by a factor of $\sin^2\theta_2$. The decay of a b quark into a charmed quark is suppressed by $\sin^2\theta_1 \sin^2\theta_2$.

In addition to the charged-current decays of the b quark, it has neutral-current decays into d quarks. These are also suppressed by $\sin^2\theta_2$. The primary decays of the b quark in this model are shown in Table II. We estimate the b-quark lifetime to be 10^{-15} sec$/s_2^2$.

There is a rare decay mode of the $b\bar{d}$ bound state which is very interesting. The neutral-current interactions cause the exclusive decay $b\bar{d} \to \mu^+\mu^-$, which can show up as a peak in $\mu^+\mu^-$ invariant mass in hadron-hadron scattering experiments. The branching ratio, unfortunately, is very small, about 10^{-6}.

Because the θ_2 mixing appears in the left-handed quark doublets [Eq. (2.16)] but not in the right-handed antilepton doublets [Eq. (3.6a)], Cabibbo universality is not exact if $\theta_2 \neq 0$. The success of universality puts the following limit on $\sin^2\theta_2$:[14]

TABLE II. b-quark decay modes.

Decay mode	Branching ratio (%)
$ud\bar{u}$	33
$u\nu e^-$, $u\nu\mu^-$	12 each
$us\bar{c}$	11
$dd\bar{d}$	10
$ds\bar{s}$	7
$d\nu\bar{\nu}$	7
$d\mu\bar{\mu}$, $de\bar{e}$	2 each
$us\bar{u}$	2

$$s_2^2 \leq 0.014, \quad \theta_2 \lesssim 6.7^0. \tag{3.7}$$

We can also obtain an upper bound on θ_2 by noting that D^0-$\bar{D}^0$ mixing is not observed. This requires that the $\Delta C = 2$ mass mixing be smaller than the decay rate. But, in this theory, the $\Delta C = 2$ amplitude (from the box diagram) gets a contribution from the virtual heavy l and b quarks of the order of $s_1^2 s_2^4 m_l^2 G_F^2$. Thus we must have

$$s_2^2 < \frac{m_c}{s_1 m_l} = \frac{1}{s_1}\frac{m_c}{m_b}\frac{m_d}{m_s}. \tag{3.8}$$

This bound is already satisfied because of Eq. (3.7).

A lower bound on s_2^2 can be obtained from our b-lifetime estimate and recent experiments which suggest[15]

$$\tau_b < 5 \times 10^{-8} \text{ sec}. \tag{3.9}$$

Thus we must have

$$s_2^2 > 2 \times 10^{-8}. \tag{3.10}$$

The peculiar neutral-current structure of the model has interesting consequences in the lepton sector. The τ and the e appear mixed in a weak SU(2) doublet in Eq. (3.6b) so there are neutral-current decays of the τ. These decays are rare, but if they can be seen they provide a nice test of the idea of vertical flavor mixing. For example, the decay $\tau^- \to e^- e^- e^+$ has a branching ratio of roughly

$$s_2^2[\sin^4\theta + 2(-\tfrac{1}{2} + \sin^2\theta)^2]/5. \tag{3.11}$$

With Eq. (3.7) and $\sin^2\theta = 0.2$, this branching ratio is less than 6×10^{-4}.

IV. CONCLUDING REMARKS

We summarize the main points of this paper.

(A) There exist nontrivial generalizations of the GIM and of the Glashow-Weinberg mechanisms in which $d \leftrightarrow s$ and $u \leftrightarrow c$ mixings are naturally suppressed as of old, but in which certain other flavor mixings may appear to order G. $SU(3) \times U(1)$ is the simplest example of the application of our mechanism. Our main purpose in reporting our results at this time is to stress that there exist hitherto unforeseen options for model building.

(B) The particular model we have described has the following characteristics:

(0) There is no t quark;
(1) $b \to u$ dominates over $b \to c$;
(2) $b\bar{d}$ and $d\bar{b}$ mix strongly;
(3) neutral-current decays $b \to d + \mu^+ + \mu^-$ and $b \to d + e^+ + e^-$;
(4) $\Delta S = 2$, $\Delta C = 2$ and strangeness-changing neutral-current effects are naturally suppressed;
(5) the b quark is relatively long-lived, with a lifetime $> 10^{-13}$ sec;
(6) lepton and quark masses are related [see Eq. (3.4)];
(7) $\tau \to 3e$ with small branching ratio.

The first six properties, (0)–(5), are inevitable consequences of our generalization of the GIM mechanism. The last two, (6) and (7), depend on the particular lepton content of the model of Sec. III. In Appendix B we exhibit an alternative lepton scheme which does not lead to properties (6) and (7).

Of course, experiments to date do not make clear whether any nontrivial generalization of the standard mechanism is needed at all. It may well be that the next stage of the developments will consist of the discoveries of more sequential weak left-handed doublets of quarks and leptons. But it may also be that heavy leptons and new quarks will bring us welcome new surprises and will give us new ideas about the nature of economy in model building.

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APPENDIX A: EMBEDDING IN SU(6)

We describe the fields of a single family (in the notation of Table I) in terms of only left-handed fields, specifying their transformation properties under $SU(3)_{\text{weak}} \times SU(3)_{\text{color}} \times U(1)$.

$$\begin{aligned}
&(1)\text{ a } (3,3)_{\bar{Q}=0} \text{ of quarks (where the subscript indicates the average charge of the multiplet, } \bar{Q} \equiv S),\\
&(2)\text{ a } (1,\bar{3})_{\bar{Q}=-2/3} \text{ of antiquarks},\\
&(3)\text{ two } (1,\bar{3})_{\bar{Q}=1/3} \text{ of antiquarks},\\
&(4)\text{ two } (\bar{3},1)_{\bar{Q}=-1/3} \text{ of leptons},\\
&(5)\text{ a } (\bar{3},1)_{\bar{Q}=2/3} \text{ of antileptons},\\
&(6)\text{ a } (1,1)_{\bar{Q}=0} \text{ neutral lepton}.
\end{aligned} \tag{A1}$$

In our embedding, the $\underline{6}$ representation of SU(6) transforms as follows:

$$\underline{6} = (3,1)_{\bar{Q}=1/3} + (1,3)_{\bar{Q}=-1/3}. \tag{A2}$$

Then the $\underline{15}$, the antisymmetric combination of two $\underline{6}$'s, transforms like

$$\underline{15} = (3,3)_{\bar{Q}=0} + (\bar{3},1)_{\bar{Q}=2/3} + (1,\bar{3})_{\bar{Q}=-2/3}, \tag{A3}$$

and the $\underline{\bar{6}}$ transforms like

$$\underline{\bar{6}} = (\bar{3},1)_{\bar{Q}=-1/3} + (1,\bar{3})_{\bar{Q}=1/3}. \tag{A4}$$

Evidently, the family in Eq. (A1) has the same representation content as one $\underline{15}$, two $\underline{\bar{6}}$'s, and one SU(6) singlet.[16] The anomaly of the $\underline{15}$ is just twice the anomaly of the $\underline{6}$, so the representation of one $\underline{15}$ and two $\underline{\bar{6}}$'s of left-handed fields is free of anomalies.[17] This reducible representation is the smallest complex anomaly-free representation in SU(6).

The Higgs mesons are the following:

$$\begin{aligned}
&\phi_U \text{ is a } (3,1)_{\bar{Q}=-2/3} \text{ in a } \underline{\overline{15}},\\
&\phi_D^x \text{ are } (3,1)_{\bar{Q}=1/3} \text{ in two } \underline{6}\text{'s},\\
&\phi \text{ is a } (3,1)_{\bar{Q}=1/3} \text{ in a } \underline{6}.
\end{aligned} \tag{A5}$$

In addition to these we need one 35-dimensional adjoint representation with a large vacuum expectation value in the U(1) direction to break the symmetry down to $SU(3)_{weak} \times SU(3)_{color} \times U(1)$.

While we could enlarge the gauge group to $SU(6) \times SU(2)$, gauging an SU(2) in the vertical space, we cannot go all the way to E(6). In E(6) the $\underline{15}$ and two $\overline{6}$'s would be combined in a $\underline{27}$, but so would ϕ_U and ϕ_D^x. Then the horizontal structure of the $Q=\frac{2}{3}$ and $Q=-\frac{1}{3}$ quarks would be the same, and θ_1 would be zero.[18]

APPENDIX B: ALTERNATIVE LEPTONS

In this scheme there are three triplets of right-handed fields:

$$L_{a_R} = \begin{pmatrix} l_a \\ \overline{\nu}_a \\ n_a \end{pmatrix}_R, \quad a=1,2,3 \qquad \text{(B1)}$$

which transform under $SU(3) \times U(1)$ in the same way as do the triplets in Eq. (3.1). $l_1 = e^+$, $l_2=\mu^+$, $l_3=\tau^+$, the $\overline{\nu}_a$ are the corresponding antineutrinos, and the n_a are massive neutral leptons. The left-handed leptons are SU(3) singlets with the appropriate U(1) charge. Higgs Yukawa couplings to ϕ_U serve to give mass to the l_a. Couplings to ϕ give mass to the n_a [see Eq. (2.6)]. A discrete symmetry forbids lepton Yukawa couplings to the ϕ_D^x: The neutrinos are massless. The total set of couplings is therefore

$$\sum_a \left(m_a \overline{l}_{a_L} \phi_U^\dagger L_{a_R} + M_a \overline{n}_{a_L} \phi^\dagger L_{a_R} + \text{H.c.}\right). \qquad \text{(B2)}$$

There is no analog to the lepton-quark mass relations (3.4).

The quark content is as follows. First, there are the two triplets given by Eq. (2.1) and their corresponding right-handed singlets. In addition, there is a third triplet (introduced to cancel the anomalies), a $\overline{3}$:

$$\Psi_{3_L} = (D_3, U_3^1, U_3^2)_L, \qquad \text{(B3)}$$

with $T^a \Psi_{3_L} = -\Psi_{3_L} \lambda^a/2$ and $S\Psi_{3_L} = \Psi_{3_L}/3$. The corresponding right-handed fields are singlets. The mixing between Ψ_{1L} and Ψ_{2L} described in the previous section is not affected by the presence of these additional quarks. ϕ_U gives mass to D_3, ϕ_D^x give mass to U_3^1 and U_3^2. We must beware, however, that these additional ϕ_D^x couplings do not spoil the symmetries of Eq. (2.9). We take the couplings to be

$$f \phi_U^\dagger \overline{\Psi}_{3_L} D_{3_R} + g \sum_x \phi_D^{x\dagger} \overline{\Psi}_{3_L} U_{3R}^x + \text{H.c.}, \qquad \text{(B4)}$$

consistent with Eq. (2.9). The new quarks do not mix with the old, so the lightest is stable. b-quark decays are essentially unchanged. There is no $\tau \to 3e$ decay.

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