PHYSICAL REVIEW D VOLUME 19, NUMBER 1 1 JANUARY 1979

Study of ω and ϕ effects on the pion form factor

B. Costa De Beauregard, B. Pire, and Tran N. Truong
Centre de Physique Théorique, École Polytechnique, 91120 Palaiseau, France
(Received 13 June 1978)

The resonant G-parity-violation effect due to ω and ϕ on the pion form factor is studied using the solution of the Muskhelishvili-Omnes integral equation with inelastic unitarity. The relative phases of the so-called ω-ρ and ϕ-ρ interference effects are accurately determined by analyticity and time-reversal invariance. For the case of the ω, the results are shown to be identical with those of the two-coupled-channel problem and also with those obtained from the modified propagator (or mass-matrix) approach. Similar results but with somewhat less accuracy are also valid for the ϕ effect on the pion form factor.

I. INTRODUCTION

The problem of ω-ρ mixing, due to a large G-parity-violating amplitude in $\omega \to 2\pi$ transition, was first suggested by Glashow[1] and was subsequently observed by experiments. There have been many theoretical studies of this problem. They can be classified into two categories. The first one[2] is similar to the Wigner-Weisskopf treatment of resonances,[3] which has previously been used for the $K\bar{K}$ phenomena, and is also suitable for the treatment of the time development of the system. The second one[4] is based on the propagator approach which leads directly to scattering amplitudes. Using any of these methods, and using the fact that the ρ width is much larger than the ω width, the phase of the ω-ρ interference is shown to be essentially that of the ρ propagator function evaluated at the ω mass. This method will not work in the hypothetical case where both ω and ρ were equally narrow and in the practical case of the ϕ-ρ interference, owing to the approximation used in the calculation (see Sec. III).

The purpose of this paper is to give a more general treatment of this problem with the help of analyticity, unitarity, and time-reversal-invariance properties of the pion form factor. Three methods are presented. The first one is based on the solution of the Muskhelishvili-Omnès integral equation with inelastic unitarity.[5,6] A theorem is established for the phase of the inelastic spectral function due to G-parity-violation mixings, which in turn enables us to calculate the ω-ρ interference phase with precision. The second methods consists in constructing a two-coupled-channel problem,[7] $\pi\pi \to \pi\pi$ via ρ and $3\pi \to 3\pi$ via ω, with G-parity-violation mixing. The solutions of the coupled integral equations for the pion form factor and the 3π form factor are then explicitly given. These two methods give identical results. The last method is a modification of the usual propagator approach[4] and is shown to be equivalent to the first two methods. All three methods are valid for the calculation of the phase of the ω-ρ interference, and, with less accuracy, the ϕ-ρ interference. They also work for the case of two overlapping resonances of comparable widths. The only restriction is that the inelastic factor η of the P-wave $\pi\pi$ amplitude $(\eta e^{2i\delta} - 1)/2i$ is approximately unity.

Before solving this problem we would like to make two comments on its experimental status and methods of analysis:

(i) Recent experimental results at Orsay[8] give the branching ratio of 2π in ω decay as

$$B(\omega \to 2\pi) = (2.1 \pm 0.9)\%, \tag{1.1a}$$

to be compared with the world-average value[9] which comes mainly from photoproduction experiments,[10]

$$B(\omega \to \pi\pi) = (1.02 \pm 0.19)\%, \tag{1.1b}$$

which is apparently more accurate than the Orsay results. We would like to point out that the value of the branching ratio and also of the phase obtained from photoproduction (hadronic processes) requires more assumptions which must be verified experimentally, namely the equality of ρ and ω photoproduction amplitudes (modulus and phase). This assumption is not needed in e^+e^- experiments. We therefore advocate that e^+e^- results on the branching ratio and phase should be considered as direct measurements that can be used to deduce the relative ρ and ω photoproduction amplitude.

(ii) The pion form factor in the ω region can experimentally be parametrized as[8,11]

$$F(s) = \frac{M_\rho^2(1+\beta)}{M_\rho^2 - s - iM_\rho\Gamma_\rho} + Ae^{i\lambda}\frac{M_\omega^2}{M_\omega^2 - s - iM_\omega\Gamma_\omega}, \tag{1.2}$$

with $A = (1.4 \pm 0.4) \times 10^{-2}$ and $\lambda = 102° \pm 13°$. While this form is satisfactory for most purposes a more adequate form, as will be shown, is

$$F(s) = \frac{M_\rho^2(1+\beta)}{M_\rho^2 - s - iM_\rho\Gamma_\rho}\left(1 + Ne^{i\theta}\frac{M_\omega^2}{M_\omega^2 - s - iM_\omega\Gamma_\omega}\right). \tag{1.3}$$

From the last Orsay results,[8] $\lambda = 102^\circ \pm 13^\circ$, $M_\rho = 777$ MeV, $\sigma_\rho = 159$ MeV, we deduce $\theta = 8^\circ \pm 13^\circ$. We shall show below that $\theta = 0$ modulo π. Equation (1.3) is preferred because the phase θ does not depend on the ρ mass and width which are experimentally determined with much uncertainty.

The plan of this paper is organized as follows: In Sec. II the phase of the ρ-ω interference is calculated by using the solution of the Muskelishvili-Omnès integral equation[5] with G-parity-violating contributions in the inhomogeneous term σ. It is shown that the corresponding part of σ has the phase $e^{-i\delta}$ and this enables us to calculate the phase of the ω-ρ interference. In Sec. III a coupled-channel problem[7] for the ω-ρ mixing is formulated in terms of the coupled $2\pi \to 2\pi$ and $3\pi \to 3\pi$ amplitudes with a $2\pi \to 3\pi$ mixing. Unitary coupled partial-wave amplitudes are constructed which have both resonances in 2π (ρ) and 3π (ω) channels. The 2π and 3π form factors are calculated in terms of the constructed amplitudes. It will be shown that the results obtained here are equivalent to those in Sec. II. The inelastic effect on the 2π channel due to the other $I = 1$ channels is parametrized in a simple manner. In Sec. IV the usual mass-matrix formulation[4] of the problem is reexamined, and it will be modified to show the equivalence of this method and those given in previous sections. In Sec. V we discuss the $\phi \to \pi\pi$ effect in the pion form factor with the help of the methods of the second section. It is shown that there is essentially no change in the result except that the $\phi \to 2\pi$ amplitude is much smaller.

II. SOLUTION OF THE MUSKHELISHVILI-OMNÈS INTEGRAL EQUATION

Let us recall that the pion form factor $F(s)$ is an analytic function in the cut plane with its discontinuity across the cut purely imaginary, if time-reversal invariance is assumed. The unitarity condition may be written in the form

$$\mathrm{Im}F(s) = f^*(s)F(s) + \sigma(s)\,, \tag{2.1}$$

where $f(s) = (\eta f^{2i\delta} - 1)/2i$. δ is the P-wave $\pi\pi$ phase shift and η the inelasticity factor of the P-wave $\pi\pi$ channel. The first term in the right-hand side of Eq. (2.1) is the contribution to the absorptive part of $F(s)$ of the $\pi\pi$ intermediate state. $\sigma(s)$ is the inelastic spectral function which sums up the remaining contributions. Assuming a once-subtracted dispersion relation for $F(s)$ we have a linear integral equation,

$$F(s) = 1 + \frac{s}{\pi}\int \frac{f^*(s')F(s') + \sigma(s')}{s'(s' - s - i\epsilon)}\, ds'\,, \tag{2.2}$$

whose solution can be written[6]

$$F(s) = e^{u(s)}\left[1 + \frac{s}{\pi}\int \frac{2}{1+\eta}\, e^{-Pu(s')}\mathrm{Re}(\sigma e^{i\delta}) \times \frac{ds'}{s'(s' - s - i\epsilon)}\right], \tag{2.3}$$

where P stands for the principal-part integration and where

$$u(s) = \frac{s}{\pi}\int \frac{\delta(s')ds'}{s'(s' - s - i\epsilon)}\,. \tag{2.4}$$

In the neighborhood of the ω resonance, we can further write $\sigma(s)$ as

$$\sigma(s) = \sigma_1(s) + \sigma_2(s)\,, \tag{2.5}$$

where $\sigma_2(s)$ is the contribution of the G-parity-violating intermediate states, whatever their origin, which have an ω resonantlike behavior and σ_1 is the remaining contribution which may also include the nonresonant G-parity-violating amplitudes besides the G-parity-conserving ones. Because the effect of σ_2 is localized in the neighborhood of the ω mass and is large, a special method must be developed to handle this situation. The modulus of σ_2 is given by

$$|\sigma_2|^2 = \frac{s^2}{16\pi^2\alpha^2}\left(\frac{s}{s - 4m_\pi^2}\right)^{1/2} \times \sigma(e^+e^- \to \text{“}\omega\text{”})\sigma(\pi\pi \to \text{“}\omega\text{”})\,. \tag{2.6}$$

We shall later use the Breit-Wigner approximation for the relevant cross sections. If we separate the contributions of σ_1 and σ_2 to σ in Eq. (2.3) we can write

$$F(s) = e^{u(s)}[g_1(s) + g_2(s)]\,, \tag{2.7}$$

where $g_1(s)$ and $g_2(s)$, with $g_1(0) = 1$ and $g_2(0) = 0$, are, respecitvely, the contribution of σ_1 and σ_2. g_1 can be shown to be nearly real in the ρ-ω region. In fact $\mathrm{Re}g_1$ is of the order of unity, while from Eq. (2.3) $\mathrm{Im}g_1$ is bounded by

$$|\mathrm{Im}g_1| \leq \frac{2}{1+\eta}\left|\frac{\sigma_1}{e^u}\right|\,. \tag{2.8}$$

If we assume ρ dominance, the contributions of the $\pi^0\gamma$, $\eta\gamma$, $\pi^+\pi^+\pi^-\pi^-$, and $\pi^+\pi^-\pi^0\pi^0$ channels to σ_1 are proportional to the corresponding branching ratios of ρ. $\Gamma(\rho \to \pi^0\gamma)$ and $\Gamma(\rho \to \eta\gamma)$ are less than 100 keV.[9] $\Gamma(\rho \to 4\pi)$ is unknown; however, comparison with $B(\rho^+ \to 4\pi)$ shows[9] that it must be less than 1 MeV. Then $|\sigma_1| < 0.04$, and $|\mathrm{Im}g_1| < 0.008$, since $|e^u| \simeq 5$ at $s \simeq s_\omega$. Hence, the phase of g_1 is less than 0.5° at $s = s_\omega$. In the following we shall set g_1 real. We shall show below that $|\sigma_2(s = s_\omega)| \simeq 1$, so that σ_1 is negligible as compared to σ_2 at the ω mass.

We now want to show that as $(1-\eta)_{s=s_\omega} \ll 1$ the phase of σ_2 is $e^{-i\delta}$. This is a straightforward consequence of the final-state theorem[12] in the special case where all channels are mutually orthogonal under strong interactions. The transitions between different channels are due to G-parity or isospin violations. In this case, the final state theorem states that to the first order in isospin-breaking amplitudes the corresponding form factors must have the phase $e^{i\delta_i}$, where δ_i is the eigenphase of the ith channel, and that the T_{1i} amplitudes must have the phase $e^{i(\delta_1+\delta_i)}$. This result can be generalized to the present situation where channel $1=\pi\pi$ is weakly coupled to a set of channels which have large phase space, and strongly coupled to channels which have little phase space.

Theorem.[13] Let K be a set of inelastic channels k (e.g., odd G parity) which are orthogonal to $\pi\pi$ and to all other inelastic channels j (e.g., even G parity), but are not necessarily mutually orthogonal. If time-reversal invariance holds for all matrix elements T_{1j} and T_{1k}, the contribution σ_K of K to the inelastic spectral function of the pion form factor has the phase-δ, i.e., $\sigma_K = \pm|\sigma_K|e^{-i\delta}$, to a precision better than $[(1-\eta)/2]^{1/2}$.

Proof of the theorem. The channels $k=3\pi$, etc. (k running from 2 to K) are weakly coupled to 1, since they have opposite G parity. σ_K denotes the sum

$$\sigma_K = \sum_{k=2}^{K} F_k\rho_k T_{k1}^* , \tag{2.9}$$

where F_k is the k-channel form factor, ρ_k its phase-space factor, and T_{1k} the matrix element of the transition $\pi\pi\to k$. The intermediate states k are eigenstates of the isospin. The G-parity-violation effect is described by the transition amplitudes. Let us write the inelastic spectral function, defined in Eq. (2.1), under the form

$$\sigma = \sigma_K + \sum_{j>K} F_j\rho_j T_{j1}^* , \tag{2.10}$$

where $j=4\pi$, etc. Assuming time-reversal invariance for all T_{1i} transition amplitudes, we have

$$\mathrm{Im}\,T_{1i} = T_{11}\rho_1 T_{1i}^* + \sum_{k=2}^{K} T_{1k}\rho_k T_{ki}^* + \sum_{j>K} T_{1j}\rho_j T_{j1}^* , \tag{2.11}$$

hence,

$$\mathrm{Im}(\sigma_K e^{i\delta}) = \frac{1-\eta}{2i}\,\sigma_K e^{i\delta} + F_1\rho_1 \sum_{k=2}^{K} |T_{1k}|^2\rho_k e^{-i\delta} + \sum_{\substack{k=2,K\\ j>K}} F_j\rho_j T_{jk}^*\rho_k T_{k1} e^{-i\delta} - \sum_{\substack{k=2,K\\ j>K}} F_k\rho_k T_{kj}^*\rho_j T_{j1} e^{i\delta} . \tag{2.12}$$

The second and third terms of Eq. (2.12) are of second order in G-parity violation and should be negligible, as compared to the first and fourth ones. Generally speaking, assuming that the matrix elements T_{kj} are of the order of T_{k1}, we find from $|\rho_j T_{j1}| \le [(1-\eta^2)\rho_j/4\rho_1]^{1/2}$ that the fourth term is less than $|\sigma_K|[(1-\eta^2)/4]^{1/2}\sup(\rho_j/\rho_1)^{1/2}$. Hence the theorem is proved. We can translate these conditions in terms of physical quantities as follows: In the ω-ρ interference problem this theorem is valid as long as $\Gamma(\omega\to 4\pi)/\Gamma(\omega\to 2\pi)$ is of the order of unity. This is a very weak assumption since this ratio should be much smaller than 10^{-2} owing to phase-space arguments; in this case $\sigma_2 = |\sigma_2|e^{-i\delta}$ to a very good precision [better than $(1-\eta)/2$]. In the case of the ϕ effect the corresponding ratio, $\Gamma[\phi\to K\bar{K}(I=1)]/\Gamma(\phi\to\pi\pi)$, should be of the order of unity because the ratio of the phase-space factors cancels the Zweig-rule violation factor [of course $\Gamma(\phi\to 4\pi)/\Gamma(\phi\to 2\pi)$ is expected to be much smaller than one because of the phase-space factors]. Hence for the ϕ effect we expect $\sigma_2 \simeq |\sigma_2|e^{-i\delta}$ to a precision of about $[(1-\eta)/2]^{1/2}$.

Let us now calculate $g_2(s)$. From the above theorem $\mathrm{Re}(\sigma_2 e^{i\delta}) = \pm|\sigma_2|$, the modulus of σ_2 can be calculated from Eq. (2.6) in the Breit-Wigner approximation. One gets

$$|\sigma_2| = B\frac{M_\omega}{\Gamma_\omega}\frac{M_\omega^2\Gamma_\omega^2}{(s-s_\omega)^2+M_\omega^2\Gamma_\omega^2} , \tag{2.13}$$

with

$$B = \frac{6}{\alpha\beta_\pi^{3/2}}\frac{\Gamma_\omega}{M_\omega}[1-B(\omega\to\pi\pi)] \times [B(\omega\to e^+e^-)B(\omega\to\pi\pi)]^{1/2} , \tag{2.14}$$

where $\beta_\pi = (s-4m_\pi^2)^{1/2}/\sqrt{s}$. This approximation is valid if the resonance is narrow over the region of interest, which is indeed the case considered here, since $\Gamma_\omega = 10$ MeV $\ll \Gamma_\rho = 150$ MeV. It is also useful to modify Eq. (2.13) so that it can be used for a more general situation. This can be done by noticing that $\sigma_2 = T_{1k}^*\rho_k F_k$ and k in this case can be approximated by the 3π channel (in the form of the ω resonance). T_{1k} has the phase of $\delta_1+\delta_k$. Using the analytic property of T_{1k} we can write

$$T_{1k} = T_{1k}(0)\,\exp\left(\frac{s}{\pi}\int\frac{ds'}{s'(s'-s-i\epsilon)}(\delta_1+\delta_k)\right).$$

This requires that Eq. (2.13) is modified to

$$|\sigma_2| = B\frac{M_\omega}{\Gamma_\omega}\,|e^{u(s)-u(s_\omega)}|\,\frac{M_\omega^2\Gamma_\omega^2}{(M_\omega^2-s)^2+M_\omega^2\Gamma_\omega^2} . \tag{2.15}$$

Putting Eq. (2.15) into the integrand of Eq. (2.3) the function e^{-Ru} cancels with $|e^{u(s)}|$ and the pion form factor is given by

$$F(s)=e^{u(s)}\left[g_1(s)+\frac{2B}{1+\eta(s_\omega)}e^{-Pu(s_\omega)}\frac{s}{s_\omega-s-iM_\omega\Gamma_\omega}\right], \tag{2.16}$$

with no condition on the ratio $\Gamma_\omega/\Gamma_\rho$. If T_{12} has a zero near the ω mass the above calculation is not modified: If σ_2 is multiplied by a polynomial, the result of the integration remains unchanged except that the final expression is multiplied by the same polynomial. Hence if σ_2 has a zero near the ω mass, g_2 will have a zero at the same energy. Present experimental evidence does not show such a zero exists.

The phase of the ω-ρ interference is given by the relative phase of the two terms in the square brackets of the right-hand side of Eq. (2.16). Since the phase of $g_1(s)$ is less than 0.5° the interference effect is given by the ω propagator. Let us now rewrite Eq. (2.16) as

$$F(s)=e^{u(s)}g_1(s)\left[1+\frac{2B}{1+\eta(s_\omega)}(g_1(s)e^{Pu(s_\omega)})^{-1}\frac{s}{s_\omega-s-iM_\omega\Gamma_\omega}\right]. \tag{2.17}$$

The modulus of the product $|e^{u(s)}g_1(s)|$ is an experimentally measurable quantity and hence independent of dynamics. The factor B defined in Eq. (2.14) can be deduced directly from the experimental measurements of the magnitude of the interference, and hence the branching ratio of ω decay into the $\pi\pi$ channel is determined independently of the dynamics of the pion form factor.

We now turn to the analysis of the last experimental results. If we neglect the G-parity-violation effect on the e^u factor, we can put in a first approximation:

$$e^{u(s)}=\frac{(1+\delta)M_\rho^2}{M_\rho^2-s-iM_\rho\Gamma_\rho},$$

where $1+\delta=1.09$ is the finite-width correction factor.[14] We rewrite Eq. (2.17) in the form of Eq. (1.3):

$$F(s)=\frac{M_\rho^2(1+\beta)}{M_\rho^2-s-iM_\rho\Gamma_\rho}\left(1+Ne^{i\theta}\frac{s}{s_\omega-s-iM_\omega\Gamma_\omega}\right), \tag{2.18}$$

with $1+\beta=(1+\delta)g_1(s)$ and

$$N=\frac{2B}{1+\eta(s_\omega)}\frac{|M_\rho^2-M_\omega^2-iM_\rho\Gamma_\rho|}{M_\rho^2(1+\beta)}. \tag{2.19}$$

Experimental results give $1+\beta\simeq1.2$, as also expected by theoretical calculations.[15] Equation (2.17) shows that $\theta=0°$ and this is our theoretical prediction.

Until now experimental results have been analyzed with $F(s)$ written under the form[8,11]

$$F(s)=\frac{(1+\beta)M_\rho^2}{M_\rho^2-s-iM_\rho\Gamma_\rho}+Ae^{i\lambda}\frac{s}{s_\omega-s-iM_\omega\Gamma_\omega}. \tag{2.20}$$

From Eqs. (2.16) and (2.14) we have

$$A=|e^{u(s)-u(s_\omega)}|\frac{2}{1+\eta(s_\omega)}[1-B(\omega\to\pi\pi)]\times[B(\omega\to ee)B(\omega\to\pi\pi)]^{1/2}\frac{6}{\alpha\beta_\pi^{3/2}}\frac{\Gamma_\omega}{M_\omega}. \tag{2.21}$$

Neglecting $|e^{u(s)-u(s_\omega)}|$, and $2[1-B(\omega\to2\pi)]/[1+\eta(s_\omega)]$, since in the Breit-Wigner approximation,

$$\frac{1-\eta^2}{4}\simeq[1-B(\omega\to2\pi)]B(\omega\to2\pi)\times\frac{M_\omega^2\Gamma_\omega^2}{(M_\omega^2-s)^2+M_\omega^2\Gamma_\omega^2}, \tag{2.22}$$

we find for $F(s)$ the same expression as in vector-dominance-model calculations, except that now the phase λ is predicted to be equal to δ. We can now compare the results of our formalism to the latest Orsay experimental results[8]: $\lambda=102°\pm13°$, with $M_\rho=777$ MeV, $M_\omega=783.5$ MeV, and $\Gamma_\rho=159$ MeV. With these values one finds $\delta(s_\omega)=94°$; hence we predict $\lambda=94°$. The agreement is indeed very good. As will be shown later in a simple model, the electromagnetic G-parity-violation effect on the $e^{u(s)}$ factor can give rise to a correction on the experimental value of λ of about 6°: This is only a matter of definition. It also affects the measurement of the magnitude of the interference effect and hence of $B(\omega\to2\pi)$ by a correction of the order of 1-cos6° which is less than 1%. Lastly let us recall that the most recent Orsay results[8] is $B(\omega\to2\pi)=(2.1\pm0.9)\%$. This justifies, *a posteriori*, $|\sigma_1|\ll|\sigma_2|$ since Eq. (2.14) gives $|\sigma_2(s_\omega)|\simeq1.0\pm0.3$ to be compared to $|\sigma_1|<0.04$, and $\frac14(1-\eta^2)\ll1$ since Eq. (2.22) gives $\frac14[1-\eta^2(s_\omega)]\simeq2\%$.

As a further check for the validity of the method presented above we shall make again the calculation in a slightly different way. With the condition of time-reversal invariance it is possible to write an integral equation for g in terms of $|\sigma|$ only,

$$(\mathrm{Im}g)^2=\frac{|\sigma|^2}{\eta|e^u|^2}-\frac{(1-\eta)^2}{4}|g|^2. \tag{2.23}$$

As $\frac14(1-\eta^2)\leq2\%$ and $|\sigma_2|\gg|\sigma_1|$ we shall neglect the second term in the right-hand side of Eq. (2.23) and take $|\sigma|\simeq|\sigma_2|$. Then Eq. (2.23) reduces to

$$|\mathrm{Im}g| = \frac{|\sigma_2|}{\eta\,|e^u|} \tag{2.24}$$

and assuming a once-subtracted dispersion relation for g we find

$$F = e^u\left(1 + \frac{1}{[\eta(s_\omega)]^{1/2}} B e^{-Pu(s_\omega)} \frac{s}{s_\omega - s - iM_\omega\Gamma_\omega}\right). \tag{2.25}$$

Equations (2.25) and (2.16) differ from each other only by the change of $2/(1+\eta)$ into $1/\sqrt{\eta}$, a difference of the second order in $\frac{1}{2}(1-\eta)$; in addition g_1 is replaced by unity. The result of Eq. (2.25) can be improved by iterations of Eq. (2.23); The first one gives a correction of 0.1%, which is negligible. This short discussion shows the precision of the method which is developed in this paragraph; this precision is mainly due to the smallness of $\frac{1}{2}(1-\eta)$.

III. MULTICHANNEL APPROACH TO THE ω-ρ MIXING

In this part we use the $1/t$ construction[7] to study the ω-ρ mixing problem. Actually we shall consider three channels for completeness: "1" and "2" denote the $J=1$ $\pi\pi$ and 3π states, which are, respectively, coupled to the ρ and ω resonances, and "3" represents the 4π $I=1$ channels. However, we consider only the violation of G-parity from the $2\pi\to3\pi$ transitions and neglect those from the $4\pi\to3\pi$ ones. As a review let us consider the elastic single-channel $\pi\pi$ case:

$$T_{11} = \frac{1}{\rho_1} e^{i\delta_1}\sin\delta_1 , \tag{3.1}$$

where $\rho_1 = (s-4M_\pi^2)^{3/2}s^{-1/2}$ is the $J=1$ $\pi\pi$ state phase-space factor. The inverse amplitude $1/T_{11}$ has, as well as T_{11}, both right- and left-hand cuts. Along the right-hand cut,

$$\frac{1}{T_{11}} = H(s) - i\rho_1(s) . \tag{3.2}$$

At $s=s_\rho$ T_{11} goes through a resonance of width Γ_ρ. Hence $1/T_{11}$ may be written

$$\frac{1}{T_{11}} = \frac{1}{K_1}[M_\rho^2 - s - iM_\rho\Gamma_{\rho\to\pi\pi}(s)] + h(s) , \tag{3.3}$$

with $K_1\,\rho(s) = M_\rho\Gamma_{\rho\to\pi\pi}(s)$, and[14,17] $h(M_\rho^2)=0$ and

$$\frac{d}{ds}h(s)\bigg|_{s=M_\rho^2} = 0 .$$

Since we are only interested in the ω-ρ energy region we shall, from now on, neglect $h(s)$. Let us define

$$t_{11} = \frac{\eta_1 e^{2i\delta_1}-1}{2i}\,\frac{1}{M_\rho\Gamma_{\rho\to\pi\pi}(s)}$$

and (3.4)

$$t_{22} = \frac{\eta_2 e^{2i\delta_2}-1}{2i}\,\frac{1}{M_\omega\Gamma_{\omega\to3\pi}(s)} .$$

For simplicity we set $\Gamma_\rho(s) = \Gamma_{\rho\to2\pi}(s)$ and $\Gamma_\omega(s) = \Gamma_{\omega\to3\pi}(s)$. With these definitions the $1/t$ matrix can be written as

$$\frac{1}{t} = \begin{pmatrix} M_\rho^2 - s - iM_\rho\Gamma_\rho(s) & \Lambda(s) & \epsilon(s) \\ \Lambda(s) & M_\omega^2 - s - iM_\omega\Gamma_\omega(s) & 0 \\ \epsilon(s) & 0 & a^{-1}(s) \end{pmatrix}, \tag{3.5}$$

where $\Lambda(s)$, $\epsilon(s)$, and $a(s)$ are slowly varying functions. From unitarity $\Lambda(s)$ and $\epsilon(s)$ are real. Since $2\pi\to4\pi$ transitions are small we shall work only to linear order in $\epsilon(s)$. It is straightforward to show

$$\begin{aligned} t_{11}(s) &= \frac{1}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s) - \Lambda^2(s)/[M_\omega^2 - s - iM_\omega\Gamma_\omega(s)]} , \\ t_{12}(s) &= \frac{-\Lambda(s)}{[M_\rho^2 - s - iM_\rho\Gamma_\rho(s)][M_\omega^2 - s - iM_\omega\Gamma_\omega(s)] - \Lambda^2(s)} , \\ t_{13}(s) &= \frac{-\epsilon(s)a(s)}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s) - \Lambda^2(s)/[M_\omega^2 - s - iM_\omega\Gamma_\omega(s)]} , \\ t_{33}(s) &= a(s) . \end{aligned} \tag{3.6}$$

Owing to the lack of phase space of the 4π state we set $t_{33}(s)$ real.

The integral equation for the form factors F_i

$$\mathrm{Im}F_i = \sum_j T_{ij}^* \rho_j F_j , \tag{3.7}$$

has the solution

$$F_i(s) = \sum_j T_{ij}(s) C_j(s) , \tag{3.8}$$

where the $C_j(s)$ have left-hand cuts that cancel those of the $T_{ij}(s)$. Since we neglect the left-hand cuts in the neighborhood of the ρ and ω resonances we can take the C_i as constants.[7,16] The pion form factor can be written

$$F_1 = \frac{C_1K_1}{M_\rho^2 - s - iM_\rho\Gamma_\rho - \Lambda^2/(M_\omega^2 - s - iM_\omega\Gamma_\omega)} - \frac{C_2(K_1K_2)^{1/2}\Lambda(s)}{(M_\rho^2 - s - iM_\rho\Gamma_\rho)(M_\omega^2 - s - iM_\omega\Gamma_\omega) - \Lambda^2}$$
$$- \frac{C_3 a(s)\epsilon(s)}{M_\rho^2 - s - iM_\rho\Gamma_\rho - \Lambda^2/(M_\omega^2 - s - iM_\omega\Gamma_\omega)}, \tag{3.9}$$

where K_2 is defined by $K_2\rho_{3\pi}(s) = M_\omega\Gamma_\omega(s)$. Similar forms are valid for F_2 and F_3. We can recombine the first and third terms in the right-hand side of Eq. (3.9) to get

$$F_1(s) = \frac{C_1K_1}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s) - \Lambda^2(s)/[M_\omega^2 - s - iM_\omega\Gamma_\omega(s)]}\left[1 - \frac{C_3a(s)\epsilon(s)}{C_1K_1} - \frac{C_2}{C_1}\left(\frac{K_2}{K_1}\right)^{1/2}\frac{\Lambda(s)}{M_\omega^2 - s - iM_\omega\Gamma_\omega(s)}\right]. \tag{3.10}$$

The factor which multiplies the square brackets in this equation has, by definition, the phase ϕ of $f = (\eta e^{2i\delta} - 1)/2i$. However, the relation

$$tg(\phi - s) = \frac{1-\eta}{1+\eta}\frac{1}{\tan\delta}$$

shows that δ and ϕ do not differ significantly since $1 - \eta/2 \lesssim 2\%$ and $\delta \simeq 90°$. Hence, it is interesting to compare Eqs. (3.10) and (2.16). $g_1(s)$ can be identified with $1 - C_3 a\epsilon/C_1K_1$ and $2Bse^{-Pu(s_\omega)}/(1+\eta)$ with $-\Lambda(s)C_2\sqrt{K_2}/C_1\sqrt{K_1}$. If we neglect G-parity violation we find

$$C_2\sqrt{K_2}/C_1\sqrt{K_1} \simeq [\Gamma(\omega \to e^+e^-)/\Gamma(\rho \to e^+e^-)]^{1/2}$$

which is equal to about $\frac{1}{3}$, and $B(\omega \to 2\pi) \simeq 0.02$ gives $\Lambda \simeq 0.15 M_\rho(\Gamma_\rho\Gamma_\omega)^{1/2}$.

To the first order in Λ the phase of the interference, as defined in Eq. (1.2), is $\delta(s_\omega)$. However, if we develop Eq. (3.10) to second order in Λ we find

$$\lambda = \delta(s_\omega) + 6° .$$

As we have already said those 6° are not significant from an experimental point of view. However, this shows how sensitive is the prediction and the measurement of λ in Eq. (1.2) to the parametrization of $e^{u(s)}$, i.e., of δ. A difference of 0.1° between the phases of $M_\rho^2 - s - iM_\rho\Gamma_\rho$ and

$$M_\rho^2 - s - iM_\rho\Gamma_\rho - \Lambda^2/(M_\omega^2 - s - iM_\omega\Gamma_\omega)$$

at $s = s_\omega$ leads to a difference of several degrees in the final result on λ. The consequence of this uncertainty is only of academic interest as previously discussed in Sec. II.

IV. METHOD OF THE PROPAGATOR OR MASS MATRIX

We now present a short review of the method of the propagator to show its equivalence with the method of the $1/t$ matrix of Sec. III, and, hence, that no approximation is needed to get the result on the phase of the ω-ρ interference[18]; in particular we want to show that this result is general and not due to the smallness of the ω width as compared with the ρ one.

In this method one considers the 2×2 mass matrix $\mathfrak{M}$ of the ρ and ω resonances. Its eigenvectors are the physical intermediate states, the propagators of which are defined by the corresponding eigenvalues. Let us set

$$\mathfrak{M} = \begin{pmatrix} s_{\rho^0} & M^2 \\ M^2 & s_{\omega^0} \end{pmatrix} \tag{4.1}$$

in the basis of the pure isospin states $|\rho^0\rangle$ and $|\omega^0\rangle$; s_{ρ^0}, s_{ω^0}, and M^2 are complex and their imaginary parts are proportional to the on-mass-shell part of the corresponding self-energy operators.

It is necessary to know the value of $\mathrm{Re}M^2$ to calculate $B(\omega \to 2\pi)$, but we shall show below that the phase of the interference is independent of $\mathrm{Re}M^2$. To simplify our discussion we consider 2π and 3π channels only. Then

$$\begin{aligned} \mathrm{Im}s_{\rho^0} &= -\langle\rho_0|2\pi\rangle^2\rho_{2\pi} - \langle\rho_0|3\pi\rangle^2\rho_{3\pi}, \\ \mathrm{Im}s_{\omega^0} &= -\langle\omega^0|2\pi\rangle^2\rho_{2\pi} - \langle\omega_0|3\pi\rangle^2\rho_{3\pi}, \\ \mathrm{Im}M^2 &= -\langle\omega^0|2\pi\rangle\langle\rho^0|2\pi\rangle\rho_{2\pi} - \langle\omega^0|3\pi\rangle\langle\rho^0|3\pi\rangle\rho_{3\pi}, \end{aligned} \tag{4.2}$$

where the matrix elements $\langle\rho^0|2\pi\rangle$, $\langle\rho^0|3\pi\rangle$, $\langle\omega^0|2\pi\rangle$, and $\langle\omega^0|3\pi\rangle$ are real, and where the phase-space factors are denoted by $\rho_{2\pi}$ and $\rho_{3\pi}$ for the 2π and 3π channels. One obtains the eigenvectors $|\rho\rangle$ and $|\omega\rangle$ from $|\rho_0\rangle$ and $|\omega_0\rangle$ by a complex nonunitary transformation

$$\begin{aligned} |\rho\rangle &= \tau|\rho^0\rangle - \eta|\omega^0\rangle, \\ |\omega\rangle &= \tau|\omega^0\rangle + \eta|\rho^0\rangle, \end{aligned} \tag{4.3}$$

with

$$\tau^2 + \eta^2 = 1, \tag{4.4}$$

and

$$\frac{\eta\tau}{\tau^2 - \eta^2} = \frac{M^2}{s_{\omega^0} - s_{\rho^0}}. \tag{4.5}$$

τ and η are complex, and because the G-parity violation is small, $|\tau| \simeq 1$ and $|\eta| \ll 1$.

The corresponding eigenvalues of $\mathfrak{M}$ are s_ρ and s_ω,

$$s_\rho = \frac{\tau^2 s_{\rho 0} - \eta^2 s_{\omega 0}}{\tau^2 - \eta^2}, \quad s_\omega = \frac{\tau^2 s_{\omega 0} - \eta^2 s_{\rho 0}}{\tau^2 - \eta^2}. \tag{4.6}$$

This is equivalent to

$$s_{\rho 0} = \tau^2 s_\rho + \eta^2 s_\omega, \quad s_{\omega 0} = \tau^2 s_\omega + \eta^2 s_\rho. \tag{4.7}$$

With these relations it is straightforward to calculate the pion form factor

$$F_1 = \frac{\langle\gamma|\rho\rangle\langle\tilde\rho|2\pi\rangle}{s_\rho - s} + \frac{\langle\gamma|\omega\rangle\langle\tilde\omega|2\pi\rangle}{s_\omega - s}, \tag{4.8}$$

where the states $\langle\tilde\rho|$ and $\langle\tilde\omega|$ are dual to $|\rho\rangle$ and $|\omega\rangle$.

The approximation used by Gourdin *et al.*[4] consists in setting $|\eta| \ll 1$ and $|\tau| \simeq 1$. Equation (4.5) becomes

$$\eta \simeq \frac{M^2}{s_\omega - s_\rho}. \tag{4.9}$$

We have

$$F_1 = \frac{\langle\gamma|\rho_0\rangle\langle\rho_0|2\pi\rangle - \eta\langle\gamma|\omega^0\rangle\langle\rho_0|2\pi\rangle}{s_\rho - s} + \frac{\langle\gamma|\omega_0\rangle\langle\omega^0|2\pi\rangle + \eta\langle\gamma|\omega_0\rangle\langle\rho^0|2\pi\rangle}{s_\omega - s}. \tag{4.10}$$

The second set of terms on the right-hand side is small compared to the first one. Since

$$\mathrm{Im}M^2 = -M_\rho\Gamma_\rho\frac{\langle\omega^0|2\pi\rangle}{\langle\rho^0|2\pi\rangle} - M_\omega\Gamma_\omega\frac{\langle\rho^0|3\pi\rangle}{\langle\omega^0|3\pi\rangle}, \tag{4.11}$$

the two terms multiplying with $1/(s_\omega - s)$ are equal to

$$\langle\gamma|\omega^0\rangle\langle\omega^0|2\pi\rangle + \left(\mathrm{Re}M^2 - iM_\rho\Gamma_\rho\frac{\langle\omega^0|2\pi\rangle}{\langle\rho^0|2\pi\rangle} - iM_\omega\Gamma_\omega\frac{\langle\rho_0|3\pi\rangle}{\langle\omega^0|3\pi\rangle}\right)\frac{\langle\gamma|\omega^0\rangle\langle\rho^0|2\pi\rangle}{M_\omega{}^2 - M_\rho{}^2 - iM_\omega\Gamma_\omega + iM_\rho\Gamma_\rho}. \tag{4.12}$$

This gives rise to a cancellation, up to the order of $M_\omega\Gamma_\omega/M_\rho\Gamma_\rho$. The term which is proportional to $\langle\rho^0|3\pi\rangle$ can be argued to be small. Then the factor of $1/(s_\omega - s)$ in Eq. (4.10) reduces to

$$\frac{-i\,\mathrm{Re}M^2}{M_\rho\Gamma_\rho}. \tag{4.13}$$

This is the result of Gourdin *et al.* This cancellation is only possible because $\Gamma_\rho \gg \Gamma_\omega$.

However, one can transform Eq. (4.8) so that to write F_1 under the form

$$F_1 = \frac{H}{s_\rho - s}\left(1 + \frac{X}{s_\omega - s}\right), \tag{4.14}$$

where $H \simeq \langle\gamma|\rho^0\rangle\langle\rho^0|2\pi\rangle$ is a real constant, and X is a real slowly varying linear function of s. If X does not have its zero near the ω mass, this result is the same as in Sec. II and III, and the phase λ, as defined in Eq. (1.2), is found to be equal to δ. This result is obtained without any approximation, unlike that established by Gourdin *et al.*[4]

Let us now show that in detail. We begin first by computing the T matrix in the propagator approach and show that it is unitary. By definition,

$$T(a \to b) = \frac{\langle b|\rho\rangle\langle\tilde\rho|a\rangle}{s_\rho - s} + \frac{\langle b|\omega\rangle\langle\tilde\omega|a\rangle}{s_\omega - s}. \tag{4.15}$$

Let us define t by

$$T = \begin{pmatrix}\langle\rho_0|2\pi\rangle & 0\\ 0 & \langle\omega^0|3\pi\rangle\end{pmatrix} t \begin{pmatrix}\langle\rho^0|2\pi\rangle & 0\\ 0 & \langle\omega^0|3\pi\rangle\end{pmatrix}, \tag{4.16}$$

then we have

$$t_{11} = (\tau\alpha_{2\pi} + \eta)^2\frac{1}{s_\omega - s} + (\tau - \eta\alpha_{2\pi})^2\frac{1}{s_\rho - s},$$
$$t_{22} = (\tau\alpha_{3\pi} - \eta)^2\frac{1}{s_\rho - s} + (\tau + \eta\alpha_{3\pi})^2\frac{1}{s_\omega - s}, \tag{4.17}$$
$$t_{12} = t_{21} = (\tau - \eta\alpha_{2\pi})(\tau\alpha_{3\pi} - \eta)\frac{1}{s_\rho - s} + (\tau\alpha_{2\pi} + \eta)(\tau + \eta\alpha_{3\pi})\frac{1}{s_\omega - s},$$

where

$$\alpha_{2\pi} = \frac{\langle\omega^0|2\pi\rangle}{\langle\rho^0|2\pi\rangle}$$

and

$$\alpha_{3\pi} = \frac{\langle\rho^0|3\pi\rangle}{\langle\omega^0|3\pi\rangle}.$$

By straightforward calculations one finds

$$\left(\frac{1}{t}\right)_{11}=\frac{1}{(1-\alpha_{2\pi}\alpha_{3\pi})^2}\left[s_{\rho^0}+\alpha_{3\pi}{}^2 s_{\omega^0}-2M^2\alpha_{3\pi}-s(1+\alpha_{3\pi}{}^2)\right],$$
$$\left(\frac{1}{t}\right)_{22}=\frac{1}{(1-\alpha_{2\pi}\alpha_{3\pi})^2}\left[s_{\omega^0}+\alpha_{2\pi}{}^2 s_{\rho^0}-2M^2\alpha_{2\pi}-s(1+\alpha_{2\pi}{}^2)\right],\tag{4.18}$$
$$\left(\frac{1}{t}\right)_{12}=\left(\frac{1}{t}\right)_{21}=\frac{1}{(1-\alpha_{2\pi}\alpha_{3\pi})^2}\left[M^2(1+\alpha_{2\pi}\alpha_{3\pi})-s_{\omega^0}\alpha_{3\pi}-s_{\rho^0}\alpha_{2\pi}+s(\alpha_{2\pi}+\alpha_{3\pi})\right].$$

Using Eq. (4.2) we have

$$\mathrm{Im}\left(\frac{1}{t}\right)_{11}=-\langle\rho^0\,|\,2\pi\rangle^2\rho_{2\pi},$$
$$\mathrm{Im}\left(\frac{1}{t}\right)_{22}=-\langle\omega^0\,|\,3\pi\rangle^2\rho_{3\pi},\tag{4.19}$$
$$\mathrm{Im}\left(\frac{1}{t}\right)_{12}=\mathrm{Im}\left(\frac{1}{t}\right)_{21}=0.$$

This is equivalent to the unitarity relations in the previous sections. However, notice that the diagonals $(1/t)_{ii}$ differ from Sec. III by terms of the second order in G-parity-violation parameters. The off-diagonal terms are of the first order as expected. We see that, by construction, the zeros of $\det(1/t)$ are s_ρ and s_ω, the poles of F_1 in Eq. (4.8). From Eq. (4.5), which defines the ω-ρ mixing parameters, s_ω and s_ρ are the roots of $(s_{\omega^0}-s)(s_{\rho^0}-s)-M^4$. In Sec. III the poles of F_1 are the roots of

$$(M_\rho{}^2-s-iM_\rho\Gamma_\rho)(M_\omega{}^2-s-iM_\omega\Gamma_\omega)-\Lambda^2,$$

in the notations of that section. This shows the correspondence between s_{ρ^0}, s_{ω^0}, and M^2 and the parameters of Sec. III, $M_\rho{}^2-iM_\rho\Gamma_\rho$, $M_\omega{}^2-iM_\omega\Gamma_\omega$, and Λ, whatever the values of $\alpha_{2\pi}$ and $\alpha_{3\pi}$ are; this is evidence for the equivalence between the mixing models of Sec. III and IV. For the sake of simplicity, from now on we shall keep only terms of the first order. Then $(1/t)_{11}=s_\rho-s$, $(1/t)_{22}=s_\omega-s$, and $\Lambda=M^2-s_\omega\alpha_{3\pi}-s_\rho\alpha_{2\pi}+s(\alpha_{2\pi}+\alpha_{3\pi})$. We are now in a position to calculate F_1, which we shall write as in Sec. III,

$$F_1=C_1T_{11}+C_2T_{12}.\tag{4.20}$$

From Eq. (4.3), (4.8), and (4.17) one has

$$\langle\gamma\,|\,\rho_0\rangle=C_1\langle\rho_0\,|\,2\pi\rangle+C_2\langle\rho^0\,|\,3\pi\rangle,$$
$$\langle\gamma\,|\,\omega^0\rangle=C_1\langle\omega^0\,|\,3\pi\rangle+C_2\langle\omega^0\,|\,3\pi\rangle,\tag{4.21}$$

i.e.,

$$C_1=\frac{\langle\gamma\,|\,\rho^0\rangle\langle\omega^0\,|\,3\pi\rangle-\langle\gamma\,|\,\omega^0\rangle\langle\rho_0\,|\,3\pi\rangle}{\langle\rho^0\,|2\pi\rangle\langle\omega^0\,|\,3\pi\rangle-\langle\rho^0\,|\,3\pi\rangle\langle\omega^0\,|\,2\pi\rangle},$$
$$C_2=\frac{\langle\gamma\,|\,\omega^0\rangle\langle\rho^0\,|\,2\pi\rangle-\langle\gamma\,|\,\rho^0\rangle\langle\omega^0\,|\,2\pi\rangle}{\langle\rho^0\,|\,2\pi\rangle\langle\omega^0\,|\,3\pi\rangle-\langle\rho^0\,|\,3\pi\rangle\langle\omega^0\,|\,2\pi\rangle}.\tag{4.22}$$

From Eq. (4.16) we get

$$T_{11}\simeq\langle\rho^0\,|\,2\pi\rangle^2\frac{1}{s_\rho-s},\tag{4.23}$$
$$T_{12}\simeq\langle\rho^0\,|\,2\pi\rangle\langle\omega^0\,|\,3\pi\rangle\times\frac{M^2-s_\omega\alpha_{3\pi}-s_\rho\alpha_{2\pi}+s(\alpha_{2\pi}+\alpha_{3\pi})}{(s-s_\rho)(s-\omega)},$$

hence, F_1 can be written under the form of Eq. (4.14), with

$$H\simeq\langle\gamma\,|\rho^0\rangle\langle\rho^0\,|\,2\pi\rangle\left(1-\frac{\langle\gamma\,|\,\omega^0\rangle}{\langle\gamma\,|\,\rho^0\rangle}\,\frac{\langle\rho^0\,|\,3\pi\rangle}{\langle\omega^0\,|\,3\pi\rangle}\right),\tag{4.24}$$
$$X\simeq\frac{\langle\gamma\,|\,\omega^0\rangle}{\langle\gamma\,|\,\rho^0\rangle}\left(1-\frac{\langle\gamma\,|\,\rho^0\rangle}{\langle\gamma\,|\,\omega^0\rangle}\alpha_{2\pi}+\frac{\langle\gamma\,|\,\omega^0\rangle}{\langle\gamma\,|\,\rho^0\rangle}\alpha_{3\pi}\right)\times\left[M^2-s_\omega\alpha_{3\pi}-s_\rho\alpha_{2\pi}+s(\alpha_{2\pi}+\alpha_{3\pi})\right].$$

It can be shown that similar results are obtained without approximation. Calculations are more tedious but H remains a real constant and X a real linear function of s. We now verify that the zero of X is far from s_ω, so that in this method the phase λ of the ω-ρ interference, as defined in Eq. (1.2), is found again to be equal to δ. To the first order the zero of X is determined by

$$(s-s_\omega)(\alpha_{2\pi}+\alpha_{3\pi})=(s_\rho-s_\omega)\alpha_{2\pi}-M^2.\tag{4.25}$$

Since $(1/t)_{12}$ is real, Eq. (4.25) is also real and the zero of X does not depend on the widths of ρ and ω. From the experimental results[8] $\mathrm{Re}M^2\simeq 0.6M_\rho\Gamma_\omega$, and Eq. (4.25) becomes

$$s-M_\omega{}^2=M_\omega\Gamma_\omega\left(\frac{-2\alpha_{2\pi}+0.6}{\alpha_{2\pi}+\alpha_{3\pi}}\right).$$

The zero of X will be within the range of the ω resonance only if $\alpha_{2\pi}$ and $\alpha_{3\pi}$ take values of the order of unity, i.e., if G-parity-violating processes are comparable with strong interactions, which cannot be.

This shows the relation between the $1/t$ matrix and the mass matrix approaches in the study of the ω-ρ mixing. The result on the phase of the interference is simply a direct consequence of unitarity, and is quite independent of the respective ρ and ω widths, as was discussed in Sec. II.

V. THE ϕ-ρ INTERFERENCE PROBLEM

In this final part we briefly show how to apply the formalism of Sec. II to the ϕ-ρ interference problem in the pion form factor. We start with a remark: Equation (2.1) holds strictly for the form factor $F(s)$, which is by definition equal to the product of the vertex function and the correction due to the photon propagator. The scattering amplitudes that appear in the same equation must contain also electromagnetic correction. However, this effect cannot be measured experimentally. We must use the unitarity Eq. (2.1) for the vertex function with the electromagnetic effect switched off; the complete pion form factor is obtained after it is multiplied by the hadronic correction to the photon propagator. This is of no importance in the ω-ρ case since it induces a very small correction to the amplitude of the interference, i.e., to the measurement of $B(\omega\to\pi\pi)$ which is anyway very imprecise, and none to the phase. In the ϕ-ρ interference problem the ϕ branching ratio into 2π, $B(\phi\to\pi\pi)$, is so small that the hadronic correction to the photon propagator leads to an interference effect of the same order, or even larger than that induced by the G-parity-violating hadronic transitions. To the first order the hadronic correction to the photon propagator is given by

$$\frac{1}{1+\pi(s)}=1-\frac{3}{\alpha}\frac{s}{s_\phi}\frac{M_\phi\Gamma(\phi\to e^+e^-)}{s_\phi-s-iM_\phi\Gamma_\phi}, \tag{5.1}$$

that is, numerically,[9]

$$\frac{1}{1+\pi(s)}\simeq 1-5.3\times10^{-4}\left(\frac{s}{s_\phi-s-iM_\phi\Gamma_\phi}\right). \tag{5.2}$$

We now turn to the G-parity-violating interference effect on the pion form factor. Neglecting, in a first approximation, the electromagnetic mass difference of K^+ and K^0 we divide the K^+K^- and $K^0\bar{K}^0$ channels into pure isospin states, so that the $K\bar{K}$ contribution to the inelastic spectral function of the pion form factor may be written as

$$\sigma_{K\bar{K}}=\sigma_{K\bar{K}}^{I=0}+\sigma_{K\bar{K}}^{I=1}, \tag{5.3}$$

where $\sigma_{K\bar{K}}^{I=0}$ and $\sigma_{K\bar{K}}^{I=1}$ are pure isospin $I=0$ and $I=1$ contributions.

Again we write for the inelastic spectral function σ,

$$\sigma=\sigma_1+\sigma_2, \tag{5.4}$$

where σ_1 represents the sum of the $I=1$ inelastic channels contributions, including $\sigma_{K\bar{K}}^{I=1}$, and σ_2 that of the electromagnetic and G-parity-violating channels which have a resonant behavior at the ϕ mass, including $\sigma_{K\bar{K}}^{I=0}$. It has been shown in a previous paper[15] that one can calculate the dispersive integral over $\mathrm{Re}(\sigma_1 e^{i\delta})$ in the approximation where the $\omega\pi$ channel dominates the other inelastic $I=1$ channels, in very good agreement with experimental data. At the ϕ mass one finds that g_1, which we recall is defined as in Eqs. (2.3) and (2.7), is about 1.2 and has a phase less than a few degrees. From our theorem in Sec. II we know that $\sigma_2 e^{i\delta}$ is real, and using the Breit-Wigner approximation to evaluate $|\sigma_2|$,

$$|\sigma_2|\simeq\frac{6}{\alpha\beta_\pi^{3/2}}[B(\phi\to e^+e^-)B(\phi\to\pi\pi)]^{1/2}\times\frac{M_\phi^2\Gamma_\phi^2}{(s_\phi-s)^2+M_\phi^2\Gamma_\phi^2}, \tag{5.5}$$

we find, up to the sign ambiguity,

$$g_2=\frac{12}{(1+\eta)\alpha\beta_\pi^{3/2}}[B(\phi\to e^+e^-)B(\phi\to\pi\pi)]^{1/2}\times\frac{\Gamma_\phi}{M_\phi}e^{-Pu(s_\phi)}\frac{s}{s_\phi-s-iM_\phi\Gamma_\phi}, \tag{5.6}$$

i.e., numerically,[9]

$$g_2\simeq\pm4.1\times10^{-2}[B(\phi\to\pi\pi)]^{1/2}\frac{s}{s_\phi-s-iM_\phi\Gamma_\phi}. \tag{5.7}$$

The total pion form factor is given by

$$F(s)=\frac{1}{1+\pi(s)}F^\delta(s)[g_1(s)+g_2(s)] \tag{5.8}$$

and may be approximately written

$$F(s)=F^\delta(s)g_1(s)\left\{1+\frac{3}{\alpha}\frac{\Gamma_\phi}{M_\phi}B(\phi\to e^+e^-)\left[\frac{\pm4e^{-Pu(s_\phi)}}{(1+\eta)g_1B_\pi^{3/2}}\left(\frac{B(\phi\to\pi\pi)}{B(\phi\to e^+e^-)}\right)^{1/2}-1\right]\frac{s}{s_\phi-s-iM_\phi\Gamma_\phi}\right\}. \tag{5.9}$$

Hence if it is parametrized under the usual experimental form,

$$F=F^\delta g_1+Ae^{i\lambda}\frac{s}{s_\phi-s-iM_\phi\Gamma_\phi}, \tag{5.10}$$

with F^δ approximated by a Frazer and Fulco[18] formula or a Gounaris and Sakurai formula[14] we find that the so-called phase of the interference λ is equal to δ, that is, $\lambda\simeq160°$. A is approximately equal to $9.5\times10^{-4}\{\pm65[B(\phi\to\pi\pi)]^{1/2}-1\}$. The formalisms of Sec. III and IV lead to the same result.

VI. CONCLUSION

In this paper we have presented a new method based on the solution of the Muskelishvili-Omnès equation with inelastic unitarity to calculate the

ω-ρ and ϕ-ρ interferences. It is quite general and model independent since it depends only on the assumption of analyticity, unitarity, and time-reversal invariance; it also allows us to treat the inelastic G-parity-violating and G-parity-conserving corrections to the pion form factor on the same footing. Our result is the same as that obtained by other methods which require more assumptions, namely $1/t$ matrix construction for resonant 2π and 3π channels and a mass-matrix formalism for the mixing of resonances.

ACKNOWLEDGMENTS

The authors would like to thank A. Quenzer and G. Sklarz for raising the ω-ρ and ϕ-ρ interference problems which led to this investigation. One of us (B.C.B.) is also indebted to Professor F. M. Renard for helpful conversation and remarks.

[1]S. L. Glashow, Phys. Rev. Lett. 7, 469 (1961); Y. Nambu and J. J. Sakurai, *ibid*. 8, 79 (1962).

[2]J. Bernstein and G. Feinberg, Nuovo Cimento 25, 1343 (1962); D. Horn, Phys. Rev. D 1, 1421 (1970); T. T. Gien, *ibid*. 5, 1773 (1972).

[3]E. P. Wigner and V. F. Weisskopf, Z. Phys. 63, 54 (1930); 67, 18 (1930).

[4]J. Harte and R. G. Sachs, Phys. Rev. 135, B459 (1964); S. Coleman and H. J. Schnitzer, *ibid*. 134, B863 (1964); A. S. Goldhaber, G. C. Fox, and C. Quigg, Phys. Lett. 30B, 249 (1969); M. Gourdin, F. M. Renard, and L. Stodolsky, *ibid*. 30B, 347 (1969); M. Gourdin, in *1969 Boulder Lectures in Physics*, edited by K. T. Mahanthappa and W. E. Brittin (Gordon and Breach, New York, 1970), Vol. XII; R. G. Sachs and J. F. Willemsen, Phys. Rev. D 2, 133 (1970); S. L. Glashow, in *Experimental Meson Spectroscopy*, edited by C. Baltay and A. M. Rosenfeld (Univ. of Pennsylvania, Philadelphia, 1970); F. M. Renard, *Springer Tracts in Modern Physics*, edited by G. Hoehler (Springer-Verlag, Berlin, 1972), Vol. 63, p. 98.

[5]N. I. Muskhelishvili, Tr. Tbilisi Mat. Inst. 10, 1 (1958); in *Singular Integral Equations*, edited by J. Radox (Noordhoff, Groningen, The Netherlands, 1953); R. Omnès, Nuovo Cimento 8, 316 (1958).

[6]V. N. Baier and V. S. Fadin, Zh. Eskp. Teor. Fiz. Pis'ma Red. 15, 219 (1972) [JETP Lett. 15, 151 (1972)]; T. N. Pham and Tran N. Truong, Phys. Rev. D 14, 185 (1976); 16, 896 (1977).

[7]J. D. Bjorken, Phys. Rev. Lett. 4, 473 (1960). See also S. Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1966).

[8]A. Quenzer, thesis, L.A.L. Report No. 1294, 1977 (unpublished).

[9]Particle Data Group, Rev. Mod. Phys. 48, S1 (1976).

[10]P. J. Biggs *et al.*, Phys. Rev. Lett. 24, 1201 (1970); H. Alvensleben *et al.*, *ibid*. 27, 888 (1971).

[11]D. Benaksas *et al.*, Phys. Lett. 39B, 289 (1972).

[12]K. M. Watson, Phys. Rev. 88, 1163 (1952); M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964).

[13]This theorem is a generalization of the result obtained on the total inelastic spectral function σ from the time-reversal invariance constraint in Eq. (2.1):

$$[\mathrm{Im}(\sigma e^{i\delta})]^2=\frac{(1-\eta)^2}{4\eta}\left[\left(\frac{1+\eta}{2}\right)^2|F|^2-|\sigma|^2\right].$$

See B. Costa de Beauregard, T. N. Pham, B. Pire, and Tran N. Truong, Ecole Polytechnique Report No. A272.07.77, 1977 (unpublished).

[14]G. Gounaris and J. Sakurai, Phys. Rev. Lett. 21, 244 (1968).

[15]B. Costa de Beauregard, T. N. Pham, B. Pire, and Tran N. Truong, Phys. Lett. 67B, 213 (1977). (Note that in the caption of Fig. 2 of this paper $|F_\pi|$ should be read as $|F_\pi|^2$, $n=\frac{1}{2}$ and $n=\frac{1}{3}$ as $n=\frac{1}{4}$ and $n=\frac{1}{6}$.)

[16]T. N. Pham and Tran N. Truong, Phys. Rev. D 16, 896 (1977).

[17]W. R. Frazer and J. Fulco, Phys. Rev. Lett. 2, 365 (1959).

[18]In the following discussion we refer more particularly to the papers of M. Gourdin *et al.*, and of F. M. Renard in Ref. 4.