Adding a horizontal gauge symmetry to the Weinberg-Salam model: An eight-quark model

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An eight-quark model is constructed. It incorporates a horizontal SU(2) gauge symmetry in addition to the $SU(2)_L \times U(1)$ gauge symmetry of Weinberg and Salam. Cabibbo universality is reproduced. *CP* violation can also be obtained. Experimental data on the K_L-K_S mass difference and neutral-current experiments are used to estimate the lower bounds on the masses of the horizontal SU(2) gauge bosons; the values turn out to be less than $\alpha^{-1/2}M_W$. No mixing exists between the horizontal gauge fields and the $SU(2)_L \times U(1)$ gauge fields; the Weinberg-Salam model of weak and electromagnetic interactions is preserved perfectly.

I. INTRODUCTION: CABIBBO UNIVERSALITY AS AN INDICATION OF HORIZONTAL STRUCTURE

The recent determination[1] of the u-quark and d-quark content of the hadronic weak neutral current using data from four types (i.e., elastic, inclusive deep-inelastic, exclusive pion production, and inclusive pion production) of neutrino-nucleon scattering experiments, and the observation[2] of a helicity asymmetry of magnitude $-9.6\times10^{-5}q^2$ in the polarized-electron–nucleon deep-inelastic scattering experiments have provided remarkably strong evidence for the Weinberg-Salam model[3] of weak and electromagnetic interactions. In particular, the polarized-electron–nucleon deep-inelastic scattering experiments have effectively dismissed the skepticism rooted in the parity-violation atomic experiments.[4] There is now a consensus that the Weinberg-Salam model is at least a good approximation to the ultimate theory of weak and electromagnetic interactions.

The description of the weak and electromagnetic interactions in terms of the Weinberg-Salam model and that of the strong interactions in terms of quantum chromodynamics[5] naturally lead to the speculation that the three interactions may be unified further into a grand unified gauge field theory. This grand unified theory, if it exists, would probably contain more than the three interactions, just as the Weinberg-Salam model contains more than electromagnetic interactions and the charged weak interactions. A flavor dynamics which includes the Weinberg-Salam model as a proper subset is presumably a first step before the goal of grand unification is reached.

It is an essential property of gauge field theories that the existence and nature of the gauge fields are completely dictated by the local gauge symmetries of the theory. The gauge symmetries of the theory also demand that the matter fields have a definite representation content. Gauge symmetries, especially non-Abelian gauge symmetries, thus provide a unifying principle not only for the interactions but also for the matter fields, though in differing degrees. In spontaneously broken gauge field theories, the gauge symmetries of the Lagrangian are distorted by the nonsymmetric vacuum, which causes mixing among the gauge fields and among the matter fields, and changes also the mass spectrum of the theory. If the number of the Higgs scalars in the theory is not unlimited, then there could be a disparity[6] between the degree of gauge-symmetry breaking in the sector of gauge fields and that in the sector of matter fields. This observation provides a basis for the possibility of attributing certain regularities in the mass spectrum and mixings of the fermion fields to some broken gauge symmetries of the theory, and vice versa (i.e., to decoding broken gauge symmetries from the mass spectrum and mixings of the fermions).

In the Glashow-Iliopoulos-Maiani (GIM) model[7] (which is the four-quark version of the Weinberg-Salam model), there are the $SU(2)_L\times U(1)$ gauge fields, the four quarks u, d, s, and c, with their left-handed components forming two $SU(2)_L$ doublets and their right-handed components forming four $SU(2)_L$ singlets. The masses for the gauge bosons and quarks are generated by the vacuum expectation values of a $SU(2)_L$-doublet Higgs scalar field and its conjugate. In addition to its other merits, the GIM model also provides a natural explanation for the Cabibbo universality which is so far the most impressive mixing pattern observed in the fermion sector.

The discovery[8] of the heavy lepton τ(1.78 GeV) and its neutrino ν_τ (<250 MeV) with dominant $(V-A)$ weak interaction implies, through the lepton-quark symmetry in general and the anomaly cancellation[9] in particular, that there are at least two more heavy quarks, one (t) with electric charge $+\frac{2}{3}$, another (b) with electric charge $-\frac{1}{3}$, to be discovered. The observation[10] of Υ(9.4 GeV) and Υ'(10.0 GeV) is usually[11] taken as evidence for the b quark. With six quarks, the GIM model is smoothly generalized to the Kobayashi-Maskawa (KM) model[12] in which the left-handed quarks form three $SU(2)_L$ doublets and the right-handed quarks form six $SU(2)_L$ singlets. The Cabibbo-GIM mixing between the d quark and the s quark is now replaced by a four-parameter mixing among d, s, and b quarks. The naturalness of the Cabibbo universality is lost.[13] The advantage of the KM model is that the complex phase appearing in the general four-parameter mixing provides room for accommodating CP violation.[14]

Admittedly the GIM → KM type of generalization is the simplest form we have for six quarks. But is it the correct one? Further GIM → KM generalization to the eight-quark case (four u-like quarks and four d-like quarks) means that the Cabibbo-GIM mixing is replaced by a nine-parameter mixing, and the observation of Cabibbo universality is obviously even more unexpected. The mixing pattern suggested by the GIM → KM type of generalization is, however, based on the assumption that all the left-handed doublets are identical except for differences in masses. Following this line of reasoning, we may now interpret the observation of Cabibbo universality as evidence for the existence of some horizontal structure by which I mean some quality other than masses that may distinguish two left-handed doublets or two right-handed singlets with equal electric charges.

The horizontal structure may take the form of some discrete symmetry, a continuous global symmetry, or a gauge symmetry.[15] Discrete symmetries are presumably the simplest ones. Continuous global symmetries will result in massless Goldstone bosons after spontaneous symmetry breaking and gauge symmetries are necessarily accompanied by the corresponding gauge fields. The recent attempts[16] to express the Cabibbo angle in terms of quark masses are concrete examples demonstrating a probable consequence of discrete symmetries. Unfortunately, good formulas relating Cabibbo angle and quark masses are obtained only for the ambidextrous models.[17] It is found[18] difficult to implement such a program in the Weinberg-Salam model.

In this paper we shall study the possibility of introducing a horizontal structure in the form of a gauge symmetry. We construct an eight-quark model with a horizontal SU(2) gauge group, henceforth to be referred to as $SU(2)_{HV}$ (V for vector) in addition to the $SU(2)_L \times U(1)$ gauge group. Both helicity states of quarks transform like $SU(2)_{HV}$ doublets. In this model, Cabibbo universality is reproduced. CP violation also has a natural place. The strangeness-changing neutral current of $\bar{d}s$ type and the charm-changing neutral current of $\bar{u}c$ type are absent from the gauge couplings, while the αG-order corrections are suppressed by a GIM-type mechanism. The details of our model are presented in Sec. II.

In Sec. III, we confront our model with the relevant experimental data in order to estimate the lower bound for the masses of the $SU(2)_{HV}$ gauge bosons. It turns out that the lower bound is below the value $\alpha^{-1/2}M_W$. Our model therefore has the advantage[19] that the tree-graph approximation may still be a reliable tool for studying the self-coupling of Higgs scalars.

Section IV gives a summary of our results.

II. THE $SU(2)_L \times U(1) \times SU(2)_{HV}$ MODEL

The model consists of eight quarks: u_0, c_0, t_0, g_0, with electric charge $+\frac{2}{3}$, and d_0, s_0, b_0, h_0, with electric charge $-\frac{1}{3}$. The subscript "0" is used to denote eigenstates of the flavor interactions mediated by the $SU(2)_L \times U(1) \times SU(2)_{HV}$ gauge fields. The same symbols without the subscript represent the eigenstates of mass. The representation content of the quarks is given in the following:

$$\begin{pmatrix} u_0 \\ d_0 \end{pmatrix}_L \rightleftarrows \begin{pmatrix} t_0 \\ b_0 \end{pmatrix}_L, \quad \begin{pmatrix} c_0 \\ s_0 \end{pmatrix}_L \rightleftarrows \begin{pmatrix} g_0 \\ h_0 \end{pmatrix}_L,$$
$$u_{0R} \rightleftarrows t_{0R}, \quad c_{0R} \rightleftarrows g_{0R}, \tag{1}$$
$$d_{0R} \rightleftarrows b_{0R}, \quad s_{0R} \rightleftarrows h_{0R},$$

where each round bracket forms an $SU(2)_L$ doublet and each arrow combines two quarks to form an $SU(2)_{HV}$ doublet. The electric-charge operator is just the original one used in the Weinberg-Salam model, namely

$$Q = T_{3L} + \tfrac{1}{2} Y, \tag{2}$$

where T_{3L} is the third component of the $SU(2)_L$ isospin and Y is the hypercharge associated with the U(1) gauge group. The hypercharge of left-handed quarks is $+\frac{1}{3}$ and that of right-handed quarks with electric charge $-\frac{1}{3}$ $(+\frac{2}{3})$ is $-\frac{2}{3}$ $(+\frac{4}{3})$. We shall denote the $SU(2)_{HV}$ gauge fields by θ_μ^a, the $SU(2)_{HV}$ isospin by T_{HV}, and the associated gauge coupling constant by g_H.

The first requirement for building a realistic model is to spontaneously break the $SU(2)_L \times U(1) \times SU(2)_{HV}$ gauge symmetry down to a U(1) gauge symmetry corresponding to the electromagnetic interaction. This requirement alone can be fulfilled[20] by introducing two complex Higgs multiplets: one (ϕ) transforms as a $[T_L = \frac{1}{2},\ Y = 1,\ T_{HV} = 0]$ representation, another (χ) as a $[T_L = 0,\ Y = 0,\ T_{HV} = \frac{1}{2}]$ representation. Nonvanishing vacuum expectation values of ϕ and χ generate the following masses for the gauge fields:

$$M^2(W_\mu^{\pm}) = \left|\frac{1}{\sqrt{2}}\, g\,\langle\phi\rangle_0\right|^2 ,$$
$$M^2(Z_\mu^0) = \left|\frac{1}{\sqrt{2}}\, g\,\langle\phi\rangle_0 \sec\theta_W\right|^2 , \qquad (3)$$
$$M^2(A_\mu) = 0 ,$$

and

$$M^2(\theta_\mu^a) = \left|\frac{1}{\sqrt{2}}\, g_H \langle\chi\rangle_0\right|^2 .$$

There is no mixing between the $SU(2)_{HV}$ gauge fields and the $SU(2)_L \times U(1)$ gauge fields. The $W_\mu^{\pm}$, Z_μ^0, and A_μ are just those appearing in the usual Weinberg-Salam model. Note that the relation $M(W_\mu^{\pm}) = M(Z_\mu^0)\cos\theta_W$ is preserved in Eq. (3).

The Higgs scalars ϕ and χ are, however, not enough to generate a realistic mass spectrum for the quarks. Since the left-handed quarks belong to ($T_L = \frac{1}{2}$, $Y = \frac{1}{3}$, $T_{HV} = \frac{1}{2}$) representations and the right-handed quarks belong to ($T_L = 0$, $Y = -\frac{2}{3}$, $T_{HV} = \frac{1}{2}$) and ($T_L = 0$, $Y = \frac{4}{3}$, $T_{HV} = \frac{1}{2}$) representations, the Higgs scalar χ does not couple to the quarks through Yukawa coupling. The mass spectrum of the quarks resulting from the Yukawa coupling between ϕ and quarks remains $SU(2)_{HV}$ symmetric after the spontaneous symmetry breaking. To generate different masses for $SU(2)_{HV}$-isospin-up quarks and $SU(2)_{HV}$-isospin-down quarks we need a Higgs scalar (η) transforming as a ($T_L = \frac{1}{2}$, $Y = 1$, $T_{HV} = 1$) representation. Therefore the final list of the Higgs scalars (all are complex) in our model is χ, ϕ, and η. The conjugate representations $\tilde{\phi}$ and $\tilde{\eta}$ are useful in constructing Yukawa couplings (ϕ and η generate masses for negatively charged quarks while their conjugates, $\tilde{\phi}$ and $\tilde{\eta}$, generate masses for positively charged quarks) but add no new degrees of freedom.

While the symmetry-breaking effects of χ and ϕ are relatively simple, that of η is a little complicated and deserves some explanation. The patterns of symmetry breaking in gauge theories have been studied by Li[20] with Higgs scalars in various representations of the gauge groups. He shows that a Higgs scalar forming an adjoint representation of a $SU(n)$ gauge group would break the $SU(n)$ gauge symmetry in one of the following two ways: (1) $SU(n) \to SU(l) \times SU(n-l) \times U(1)$, (2) $SU(n) \to SU(n-1)$, where $l = n/2$ for even n, and $(n+1)/2$ for odd n. The way in which the symmetry breaking will proceed depends on which part of the parameter-space the parameters in the Higgs scalar self-interaction potential belong. We shall now take inspiration from Li's result and assume[21] in our model that the η, which is an adjoint representation of $SU(2)_{HV}$, breaks the $SU(2)_{HV}$ down to a $U(1)_{HV}$ symmetry in some finite region of the parameter space. The $SU(2)_{HV}$ symmetry of the Lagrangian before spontaneous symmetry breaking leaves us further freedom to identify this $U(1)_{HV}$ as the one associated with the third generator (T_{3HV}) of the $SU(2)_{HV}$ group. [The $U(1)_{HV}$ is eventually broken by χ.] This identification is realized by demanding that only the $T_{3HV} = 0$ component of η develops a nonvanishing vacuum expectation value. Consequently the $SU(2)_{HV}$ symmetry of the Yukawa couplings is only broken partly; the T_{3HV} charge is conserved by the Yukawa couplings.

With the three complex Higgs scalars χ, ϕ, and η, the mass spectrum for the gauge bosons becomes

$$M^2(W_\mu^{\pm}) = \tfrac{1}{2} g^2 (|\langle\phi\rangle_0|^2 + |\langle\eta_0\rangle_0|^2) ,$$
$$M^2(Z_\mu^0) = \tfrac{1}{2} g^2 (|\langle\phi\rangle_0|^2 + |\langle\eta_0\rangle_0|^2) \sec^2\theta_W ,$$
$$M^2(A_\mu) = 0 , \qquad (4)$$
$$M^2(\theta_{\pm\mu}) = g_H^2 (2|\langle\eta_0\rangle_0|^2 + \tfrac{1}{2}|\langle\chi\rangle_0|^2) ,$$

and

$$M^2(\theta_{3\mu}) = \tfrac{1}{2} g_H^2 |\langle\chi\rangle_0|^2 ,$$

where η_0 is the $T_{3HV} = 0$ component of η. Note that the "±" attached to θ_μ denotes T_{3HV} charge instead of the electric charge as in the case of $W_\mu^{\pm}$. As promised above $|\langle\eta_0\rangle_0|$ does not contribute to $M^2(\theta_{3\mu})$. As a bonus, there is no mixing between the $SU(2)_{HV}$ gauge fields and the $SU(2)_L \times U(1)$ gauge fields. The Weinberg-Salam model of weak and electromagnetic interactions is thus preserved perfectly in our model. Generally the mass scale of θ_μ gauge bosons can be made very high by adjusting the vacuum expectation value $\langle\chi\rangle_0$ without affecting the masses of other gauge bosons and quarks.

Let us now consider the consequences of the Yukawa coupling between the Higgs scalars and the quarks. The quark mass term in the Lagrangian after the spontaneous symmetry breaking takes the form

$$L_m = -(\bar{b}_0, \bar{h}_0, \bar{d}_0, \bar{s}_0)_L M_- \begin{pmatrix} b_0 \\ h_0 \\ d_0 \\ s_0 \end{pmatrix}_R - (\bar{t}_0, \bar{g}_0, \bar{u}_0, \bar{c}_0)_L M_+ \begin{pmatrix} t_0 \\ g_0 \\ u_0 \\ c_0 \end{pmatrix}_R + \text{Hermitian conjugates} . \qquad (5)$$

The T_{3HV} symmetry is not spoiled in the Yukawa couplings during spontaneous symmetry breaking; the mass matrices M_- and M_+ consequently take the following block forms:

$$M_- = \begin{bmatrix} A & B & & \\ C & D & & 0 \\ & & E & F \\ 0 & & G & H \end{bmatrix}, \tag{6}$$

$$M_+ = \begin{bmatrix} A' & B' & & \\ C' & D' & & 0 \\ & & E' & F' \\ 0 & & G' & H' \end{bmatrix}.$$

The matrix elements are products of Yukawa coupling constants and the vacuum expectation values of ϕ, $\tilde{\phi}$, η, and $\tilde{\eta}$. They are in general complex, and the mass matrices may not be Hermitian matrices. The mass matrices can be diagonalized by biunitary transformation:

$$\begin{pmatrix} b_0 \\ h_0 \end{pmatrix}_{L/R} \to \begin{pmatrix} b \\ h \end{pmatrix}_{L/R} = U_{bh}^{L/R} \begin{pmatrix} b_0 \\ h_0 \end{pmatrix}_{L/R},$$
$$\begin{pmatrix} d_0 \\ s_0 \end{pmatrix}_{L/R} \to \begin{pmatrix} d \\ s \end{pmatrix}_{L/R} = U_{ds}^{L/R} \begin{pmatrix} d_0 \\ s_0 \end{pmatrix}_{L/R}, \tag{7}$$
$$\begin{pmatrix} t_0 \\ g_0 \end{pmatrix}_{L/R} \to \begin{pmatrix} t \\ g \end{pmatrix}_{L/R} = U_{tg}^{L/R} \begin{pmatrix} t_0 \\ g_0 \end{pmatrix}_{L/R},$$

and

$$\begin{pmatrix} u_0 \\ c_0 \end{pmatrix}_{L/R} \to \begin{pmatrix} u \\ c \end{pmatrix}_{L/R} = U_{uc}^{L/R} \begin{pmatrix} u_0 \\ c_0 \end{pmatrix}_{L/R},$$

where each equation stands for two unitary transformations: one (U^L) for the left-handed (L) quarks, another (U^R) for the right-handed (R) quarks.

The currents coupled to $W_\mu^\pm$ gauge bosons have the quark content

$$(\bar{t}_0, \bar{g}_0, \bar{u}_0, \bar{c}_0)_L \begin{bmatrix} b_0 \\ h_0 \\ d_0 \\ s_0 \end{bmatrix}_L = (\bar{t}, \bar{g}, \bar{u}, \bar{c})_L \begin{pmatrix} U_{tg}^L U_{bh}^{L-1} & 0 \\ 0 & U_{uc}^L U_{ds}^{L-1} \end{pmatrix} \begin{bmatrix} b \\ h \\ d \\ s \end{bmatrix}. \tag{8}$$

The currents coupled to $\theta_{\pm\mu}$ have the quark content

$$(\bar{t}_0, \bar{g}_0, \bar{b}_0, \bar{h}_0)_{L/R} \begin{bmatrix} u_0 \\ c_0 \\ d_0 \\ s_0 \end{bmatrix}_{L/R} = (\bar{t}, \bar{g}, \bar{b}, \bar{h})_{L/R} \begin{pmatrix} U_{tg}^{L/R} U_{uc}^{L/R-1} & 0 \\ 0 & U_{bh}^{L/R} U_{ds}^{L/R-1} \end{pmatrix} \begin{bmatrix} u \\ c \\ d \\ s \end{bmatrix}_{L/R}. \tag{9}$$

The currents coupled to Z_μ^0, A_μ, and $\theta_{3\mu}$ are invariant with respect to the biunitary transformation and remain flavor-conserving. The currents of the type $(\bar{d}s)$ and $(\bar{u}c)$ disappear in the lowest order. Making use of the freedom of redefining the phases of quark fields we can remove all complex phases from the mixing matrix of Eq. (8), and parametrize the matrix by two real angles θ_1 and θ_2, i.e., the matrix becomes

$$\begin{bmatrix} \cos\theta_1 & \sin\theta_1 & & \\ -\sin\theta_1 & \cos\theta_1 & & 0 \\ & & \cos\theta_2 & \sin\theta_2 \\ 0 & & -\sin\theta_2 & \cos\theta_2 \end{bmatrix}. \tag{10}$$

The angle θ_2 can be identified as the Cabibbo angle. The Cabibbo universality is thus reproduced in the present model. The matrices of Eq. (9) are parametrized simultaneously as

$$\begin{bmatrix} \cos\psi_{L/R} e^{i\sigma_{L/R}} & \sin\psi_{L/R} e^{i\rho_{L/R}} & & 0 \\ -\sin\psi_{L/R} e^{i\delta_{L/R}} & \cos\psi_{L/R} e^{i(-\sigma+\rho+\delta)_{L/R}} & & \\ & 0 & \cos\phi_{L/R} e^{i\gamma_{L/R}} & \sin\phi_{L/R} e^{i\alpha_{L/R}} \\ & & -\sin\phi_{L/R} e^{i\beta_{L/R}} & \cos\phi_{L/R} e^{i(-\gamma+\alpha+\beta)_{L/R}} \end{bmatrix} \tag{11}$$

and one of the sixteen phase factors can be replaced by unity.

Thus CP violation can be attributed to the $\theta_{\pm\mu}$ interactions in the present model. We see from Eq. (11) that the left-handed current and the right-handed current coupled to $\theta_{\pm\mu}$ would generally have different mixing angles and phases unless the mass matrices given by Eq. (6) are Hermitian. There are 17 mixing angles and phases in the present model; this number reduces to 9 if the mass matrices, Eq. (6), are Hermitian. For comparison, we mention that the number is 9 in the GIM – KM type eight-quark model, and 25 in the general ambidextrous model.

The present model has the feature that four Higgs multiplets (ϕ, η, $\tilde{\phi}$, and $\tilde{\eta}$) are involved in the Yukawa couplings and so the Yukawa coupling constant matrix may not be diagonal in the basis formed by the quark mass eigenstates. Vertices of the form $\bar{d}$–s–Higgs-scalar and $\bar{u}$–c–Higgs-scalar are not absent in general. Therefore the present model demands heavy massess ($\sim 10 M_W$) for the Higgs scalars.[22]

III. LOWER BOUNDS ON THE MASSES OF THE SU(2)$_{HV}$ GAUGE BOSONS

In the preceding section we have seen that the GIM model is preserved exactly for the SU(2)$_L$ ×U(1) interactions of the u, d, s, and c quarks. An important new feature resulting from the introduction of the SU(2)$_{HV}$ gauge symmetry is that the u, d, s, and c quarks are now involved also in the new interactions mediated by the SU(2)$_{HV}$ gauge fields. The $\theta_{3\mu}$ interaction conserves flavor while the $\theta_{\pm\mu}$ interaction couples u, d, s, and c quarks to t, b, g, and h quarks. In this section we shall first study the K_L-K_S mass difference. This is an observed phenomenon involving no leptons and yet possessing contributions from the $\theta_{\pm\mu}$ interaction at the one-loop level. Through this phenomenon we wish to derive a rough estimate of the lower bound for the mass of $\theta_{\pm\mu}$ gauge boson. Following this we give a trivial extension of the model to include eight leptons which allows an estimate of the lower bound for the mass of the $\theta_{3\mu}$-gauge boson from the present neutral-current experiments. Obviously the masses of the SU(2)$_{HV}$ gauge bosons should not be lighter than M_W and M_Z if the gauge coupling constants g and g_H are comparable. What we are concerned with is whether M_θ are 10 times or 100 times as heavy as $M_{W,Z}$.

A. K_L - K_S mass difference

The K_L-K_S mass difference can be regarded as a consequence of the $\Delta S=2$ transition: $K^0 \to \bar{K}^0$. The contribution of the SU(2)$_{HV}$ gauge interactions to the $\Delta S=2$ transition is given by the two-$\theta_{\pm\mu}$-exchange box diagrams at lowest order (Fig. 1).

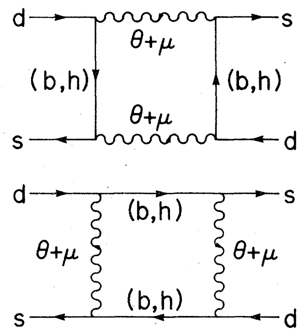

FIG. 1. Two-$\theta_{\pm\mu}$-exchange diagrams contributing to $K^0 \longleftrightarrow \bar{K}^0$ transition.

Similar diagrams involving two-$W_\mu^\pm$ exchange are shown in Fig. 2. The latter have been calculated in great detail by Gaillard and Lee,[23] which leads to the effective Lagrangian

$$L_{\rm eff}^W = -\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi}\,\epsilon_0 \cos^2\theta_C \sin^2\theta_C (\bar{s}_L \gamma_\mu d_L)^2 + \text{H.c.}, \tag{12}$$

where

$$\epsilon_0 = (m_c^{\,2} - m_u^{\,2})^2/[(m_c^{\,2} + m_u^{\,2}) M_W^{\,2} \sin^2\theta_W].$$

To estimate the magnitude of the $K^0 \to \bar{K}^0$ transition induced by the effective Lagrangian $L_{\rm eff}^W$, they insert the vacuum state between the two currents and use PCAC (partial conservation of axial-vector current):

$$\langle \bar{K}^0 | -L_{\rm eff}^W | K^0 \rangle \simeq \frac{G_F}{\sqrt{2}} f_K^{\,2} m_K^{\,2}\,\frac{\alpha}{4\pi}\,\epsilon_0 \cos^2\theta_C \sin^2\theta_C . \tag{13}$$

The experimental value of the mass difference m_L-m_S implies that

$$\epsilon_0 \simeq 1.4 \times 10^{-3} . \tag{14}$$

Repeating similar computation for the two-$\theta_{\pm\mu}$-

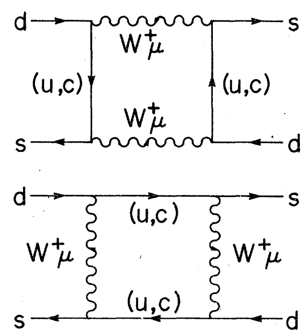

FIG. 2. Two-$W_\mu^\pm$-exchange diagrams contributing to $K^0 \longleftrightarrow \bar{K}^0$ transition.

exchange diagrams leads to

$$\langle \bar{K}^0 | -L_{\rm eff}^{\theta_\pm} | K^0 \rangle \simeq \frac{G_F}{\sqrt{2}} f_K^2 m_K^2 \times \frac{\alpha}{4\pi} \epsilon' f(\cos\phi_{L/R}, \sin\phi_{L/R}, e^{i(\gamma-\alpha)_{L/R}}), \tag{15}$$

where $f(\cos\phi_{L/R}, \ldots)$ is a polynomial in its arguments resulting from the mixing matrix, Eq. (11), in the $\theta_{\pm\mu}$ interaction, and

$$\epsilon' = \frac{(m_h^2 - m_b^2)^2}{(m_h^2 + m_b^2) M_{\theta_\pm}^2} \left(\frac{g_H^2 M_W}{e g M_{\theta_\pm}} \right)^2 . \tag{16}$$

The phases $e^{i(\gamma-\alpha)_{L/R}}$ in $\langle \bar{K}^0 | -L_{\rm eff}^{\theta_\pm} | K^0 \rangle$, Eq. (15), allow for CP violation in $K_L^0 \to 2\pi$ decay.[24]

The values of $\phi_{L/R}$ and $(\gamma-\alpha)_{L/R}$ are not determined *a priori* in our model and so should be regarded as parameters to be determined experimentally. For a rough estimation, which is the only thing we can do for the moment, we assume that

$$\mathrm{Re} f(\cos\phi_{L/R}, \ldots) \approx \cos^2\theta_C \sin^2\theta_C \, . \tag{17}$$

[The imaginary part of $f(\cos\phi_{L/R}, \ldots)$ should of course be small to account for the small CP-violation parameter $|\epsilon| \simeq (1.95 \pm 0.25) \times 10^{-3}$.] Then

$$\epsilon' \lesssim 1.4 \times 10^{-3} , \tag{18}$$

which implies that

$$\frac{g_H^4 M_W^2 (m_h^2 - m_b^2)^2}{e^2 g^2 (m_h^2 + m_b^2) M_{\theta_\pm}^4} \lesssim 1.4 \times 10^{-3} . \tag{19}$$

Assuming further that $g_H^2 \approx eg$, $m_b \approx 5$ GeV, and $m_h \approx 25$ GeV, then Eq. (19) results in

$$M_{\theta_\pm} \gtrsim 7 M_W \, . \tag{20}$$

Note also the following appealing possibility: If the mass matrices of Eq. (6) are Hermitian then $\phi_L = \phi_R$, $\alpha_L = \alpha_R$, $\beta_L = \beta_R$, and the gauge coupling between quarks and $\theta_{\pm\mu}$ is purely vectorial. Consequently the effective Lagrangian $L_{\rm eff}^{\theta_\pm}$ would represent a V-V interaction which makes no contribution to the $K^0 \leftrightarrow \bar{K}^0$ transition in the vacuum-intermediate-state approximation used above. One then has to proceed to the approximation scheme in which other intermediate states, e.g., the one-pion state, are inserted between the two currents. This may provide additional suppression, and a lower bound less than $7 M_W$ may be possible for $M_{\theta_\pm}$.

B. Neutral-current experiments

We shall assume further that leptons also form definite representations for the $\mathrm{SU}(2)_{HV}$ gauge group and are involved in the interactions mediated by the $\mathrm{SU}(2)_{HV}$ gauge fields. The representation content of the leptons is given by the following:

$$\begin{pmatrix} \nu_{e0} \\ e_0 \end{pmatrix}_L \to \begin{pmatrix} \nu_{\tau 0} \\ \tau_0 \end{pmatrix}_L , \quad \begin{pmatrix} \nu_{\mu 0} \\ \mu_0 \end{pmatrix}_L \to \begin{pmatrix} \nu_{\lambda 0} \\ \lambda_0 \end{pmatrix}_L , \quad e_{0R} \to \tau_{0R} , \quad \mu_{0R} \to \lambda_{0R} \, . \tag{21}$$

Here eight leptons are assumed, among which all four neutrinos are massless. The round brackets and the arrows have the same meaning as in Eq. (1).

Having extended the $\mathrm{SU}(2)_{HV}$ gauge interactions to the leptons, the neutral-current experiments such as neutrino-nucleon scattering, neutrino-electron scattering, and electron-nucleon scattering should now be perceived as processes involving not only the $Z_\mu{}^0$ boson but also $\theta_{3\mu}$ boson (the effect of the $\theta_{\pm\mu}$ boson is suppressed relative to that of the $\theta_{3\mu}$ boson). Consider the neutrino-quark scattering (Fig. 3). There are two relevant tree graphs in lowest-order perturbation theory: One consists of one-$Z_\mu{}^0$ exchange, another one-$\theta_{3\mu}$ exchange. The effective Lagrangian resulting from the one-$Z_\mu{}^0$ exchange diagram is

$$L_{\rm eff}^{Z} = \frac{G}{\sqrt{2}} \bar{\nu}\gamma_\mu (1-\gamma_5)\nu [(T_{3L} - Q\sin^2\theta_W)\bar{q}\gamma^\mu(1-\gamma_5)q - Q\sin^2\theta_W \bar{q}\gamma^\mu(1+\gamma_5)q] \, . \tag{22}$$

The effective Lagrangian resulting from the one-$\theta_{3\mu}$-exchange diagram is

$$L_{\rm eff}^{\theta_3} = \frac{G}{\sqrt{2}} \left(\frac{g_H M_W}{g M_{\theta_3}} \right)^2 (-2T_{3HV}) [\bar{\nu}\gamma_\mu (1-\gamma_5)\nu] [\bar{q}\gamma^\mu q] \, . \tag{23}$$

Our purpose here is to show that the introduction of the $\theta_{3\mu}$ interaction into neutrino-quark scattering can be made consistent with the excellent fit of the data by the original Weinberg-Salam model. To be specific, we shall make use of the analysis done by Abbott and Barnett.[1] They analyze the neutrino-nucleon scattering by the neutrino-quark effective Lagrangian

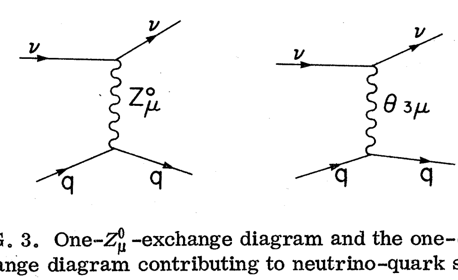

FIG. 3. One-Z_μ^0-exchange diagram and the one-$\theta_{3\mu}$-exchange diagram contributing to neutrino-quark scattering.

$$L=\frac{G}{\sqrt{2}}\,\bar{\nu}\,\gamma^{\mu}(1-\gamma_5)\nu[u_L\bar{u}\gamma_{\mu}(1-\gamma_5)\,u+u_R\bar{u}\gamma_{\mu}(1+\gamma_5)u+d_L\bar{d}\gamma_{\mu}(1-\gamma_5)d+d_R\bar{d}\gamma_{\mu}(1+\gamma_5)d]\;; \tag{24}$$

here u_L, u_R, d_L, and d_R stand for coupling constants, not to be taken as different helicity states of u and d quarks as we did previously. Their result is that

$$\begin{aligned} &u_L=0.35\pm0.07\,,\quad u_R=0.19\pm0.06\,,\\ &d_L=-0.40\pm0.07\,,\quad d_R=0.0\pm0.11\,, \end{aligned} \tag{25}$$

where the errors shown are 90% confidence limits. An overall convention $u_L \geqslant 0$ has been used in Eq. (25). They compare the results, Eq. (25), with the prediction from the Weinberg-Salam model, i.e., Eq. (22), with $\sin^2\theta_W = 0.25$. The differences are

$$\begin{aligned} &u_L-u_L^{WS}=\begin{cases}0.09\\-0.05\end{cases},\quad u_R-u_L^{WS}=\begin{cases}0.04\\-0.08\end{cases},\\ &d_L-d_L^{WS}=\begin{cases}0.09\\-0.05\end{cases},\quad d_R-d_R^{WS}=\begin{cases}0.03\\-0.19\end{cases}. \end{aligned} \tag{26}$$

So within this specific analysis, the effect of $L_{\text{eff}}^{\theta_3}$ in the neutrino-nucleon scattering is within the errors if

$$\frac{1}{2}\left(\frac{g_H M_W}{g M_{\theta_3}}\right)^2 \lesssim 0.03\,. \tag{27}$$

Setting $g_H \approx g$, we get the estimate

$$M_{\theta_3} \gtrsim 4M_W\,. \tag{28}$$

Similarly one can show readily that the electron couplings determined by Abbott and Barnett using data from the neutrino-electron scattering experiment and the polarized-electron–nucleon scattering experiment are consistent with $\sin^2\theta_W = 0.25$ and $M_{\theta_3} \gtrsim 3M_W$.

Since the Weinberg-Salam model is embedded perfectly in our model the above discussion can be interpreted as meaning that our model fits the data of the neutral-current experiments with $\sin^2\theta_W = 0.25$ and $M_{\theta_3} \gtrsim 4M_W$.

Note further that the $\theta_{\pm\mu}$ is also expected to participate in the heavy-lepton decay $\tau \to e\bar{\nu}_e\nu_\tau$ via $\tau \to e\theta_+ \to e\bar{\nu}_e\nu_\tau$. However, given the lower bound $M_{\theta_\pm} \gtrsim 7M_W$, Eq. (20), the effect is within the accuracy of the available data.

IV. CONCLUSION

In this paper we have demonstrated, by constructing a specific eight-quark model, that a horizontal gauge symmetry $SU(2)_{HV}$ may be added to the Weinberg-Salam model without demanding too heavy masses for the new gauge bosons. The lower bounds for the masses are estimated to be $\simeq 7\,M_W$ for $\theta_{\pm\mu}$ gauge bosons and $\simeq 4\,M_W$ for $\theta_{3\mu}$ gauge bosons. These estimated values are within the limit $(\alpha^{-1/2}M_W)$ suggested by Gildener.[19]

The basic idea of the zeroth-order mass relation,[6] that there could be differences between the degrees of gauge symmetry breaking in different sectors of the Lagrangian, has been an important guideline during the construction of our model. The Cabibbo universality is reproduced as a natural consequence of the fact that although the $SU(2)_{HV}$ gauge symmetry of the Lagrangian is completely broken (all θ_μ^a gauge bosons have become massive), the Yukawa couplings between Higgs scalars and quarks preserve a U(1) part of the $SU(2)_{HV}$ gauge symmetry.

An outstanding feature of our model is that the Weinberg-Salam model of weak and electromagnetic interactions is preserved perfectly. There is no mixing between the $SU(2)_L \times U(1)$ gauge fields and the $SU(2)_{HV}$ gauge fields. *CP* violation is also accommodated in our model.[24]

In our model the $\bar{d}s$-type and $\bar{u}c$-type neutral currents are absent from the gauge couplings, but Yukawa couplings of the types $\bar{d}$–s–Higgs scalar and $\bar{u}$–c–Higgs scalar may exist. In order to suppress the transitions $\bar{d}s \to$ Higgs scalar $\to \bar{s}d$ and $\bar{u}c \to$ Higgs scalar $\to \bar{c}u$, one requires the relevant Higgs scalars to be massive ($\sim 10M_W$).[22] This, however, may not be a real drawback for after all no reliable upper bound has been set by experiments and our theoretical understanding of the Higgs scalars is not really deep.

Finally, we note that the $SU(2)_{HV}$ gauge fields couple to both the left-handed and the right-handed quarks (and leptons). Therefore our model may be used as a specific case for a detailed study of the conjecture[25] that the masses of the electron, u and d quark are the results of the radiative corrections to the zeroth-order mass relation $m_u = m_d = m_e = 0$. In order to produce the zeroth-order mass relation, one has to introduce additional symmetry into our model. This possibility will be further studied.

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