O(4) ⊗ U(1) gauge model

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We propose an O(4) ⊗ U(1) gauge model for weak and electromagnetic interactions, in which the Cabibbo angle is expressed as a free mixing angle between two charged vector bosons without rotating quarks. Four left-handed leptons (ν_e, e, ν_μ, μ) and quarks (u, d, c, s) are, respectively, assumed to belong to quartets of O(4) whereas all the right-handed ones are assumed to belong to singlets. We use the representation $(\underline{2},\underline{1})+(\underline{1},\underline{2})$ of $O(4)\simeq SU(2)\otimes SU(2)$ for our quartets, different from that used by Pais, $(\underline{2},\underline{2})$ of $SU(2)\otimes SU(2)$. Our model naturally ensures the mechanism and respects quark-lepton universality. *CP* violation can be incorporated in the model to reproduce effectively results of the superweak theory by a small rotation of d and s quarks. The electric dipole moment D_n of a neutron is shown to be small, $|D/e|\lesssim 10^{-29}$ cm. The parity-violating asymmetry in the inelastic scattering of longitudinally polarized electrons from the deuterium recently measured at SLAC is reproduced in our model. A strangeness-changing neutral current also appears but is suppressed to the order $G_F\alpha$.

I. INTRODUCTION

The bulk of weak-interaction phenomena is successfully explained by a SU(2) ⊗ U(1) gauge model of weak and electromagnetic interactions. This theory was originated by Weinberg and Salam[1] and was later extended to include four quarks and four leptons, which satisfy the so-called Glashow-Iliopoulos-Maiani (GIM) mechanism.[2] Many variations of a gauge model SU(2) ⊗ U(1) have been proposed, based on various motivations. In constructing gauge models some criteria demanded by experimental data are also used, such as no flavor-changing neutral current,[3] e- or μ-number conservation, and so on.

Let us now pay attention to the Cabibbo angle, which enters into the charged current for quarks. This angle is usually considered to be generated by diagonalizing the quark mass matrix, and recent study is mainly devoted to the theoretical calculation of it by assuming a discrete symmetry.[4] Instead of rotating quarks, it is possible to interpret the Cabibbo angle as a ratio of masses of two charged vector bosons, which couple to strangeness-changing and -nonchanging currents, respectively.[5] This interpretation is easily seen in Fig. 1; that is, the ratio among the amplitudes of the three diagrams is 1: cosθ:sinθ. Then the Cabibbo angle θ is defined by $\tan\theta = m_{W_1}{}^2/m_{W_2}{}^2$. The first realization of this idea in a gauge model was achieved by Pais,[6] who adopted O(4) ⊗ g, the same group as will be used in this paper. He and others[7] succeeded in explaining *CP* violation in *K* decays, nonleptonic decays, etc. There seem, however, to be some defects in his model, e.g. the large contribution of heavy neutral leptons to the anomalous magnetic moment of μ,[8] though some improvements can be made.[9]

Now consider what will happen when one wants to construct a gauge model which realizes the above idea within only four quarks, u, d, c, and s, and four leptons, ν_e, e, ν_μ, and μ. One will soon encounter a serious problem, namely that nonleptonic decays do not occur in tree diagrams. In order to avoid this problem one must abandon the idea of expressing the Cabibbo angle as the mass ratio of two charged vector bosons.

Though one must give up the above idea, it is possible to construct a gauge model with four quarks and four leptons and to relate the Cabibbo angle with gauge bosons in some sense. That is, rotate two charged vector bosons instead of quarks; then, a resultant mixing angle effectively becomes the Cabibbo angle. Moreover, when one requires that the charged current be the GIM current, a relevant gauge group is uniquely determined to be O(4) ⊗ U(1). Let us illustrate how one will be led to

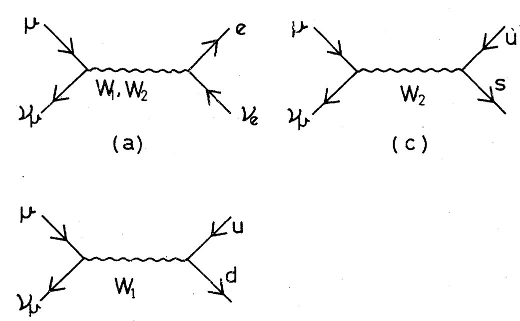

FIG. 1. The amplitudes which give the ratio 1: cosθ: sinθ with θ the Cabibbo angle.

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a gauge model of O(4) ⊗ U(1).

The GIM charged current is given by

$$J=(\bar{u}d+\bar{c}s)\cos\theta+(\bar{u}s-\bar{c}d)\sin\theta\,, \quad (1.1)$$

where the γ matrices have been omitted. First, there must be two charged vector bosons which couple to $\bar{u}d+\bar{c}s$ and $\bar{u}s-\bar{c}d$, respectively. The Cabibbo angle is a mixing angle of these gauge bosons. Then the relevant generators of some group, which we want to have, are given by

$$\bar{u}d+\bar{c}s=\bar{q}\begin{pmatrix}\tau_+ & 0\\ 0 & \tau_+\end{pmatrix}q \quad (1.2)$$

and

$$\bar{u}s-\bar{c}d=\bar{q}\begin{pmatrix}0 & \tau_+\\ -\tau_+ & 0\end{pmatrix}q\,, \quad (1.3)$$

with $q=(u,\ d,\ c,\ s)^t$ and $\tau_+=\frac{1}{2}(\tau_1+i\tau_2)$, where τ_i are the Pauli matrices. One more generator may be extracted from the charge operator Q. If quarks have fractional charges, Q is given by

$$\begin{aligned}\text{diag}Q&=(\tfrac{2}{3},-\tfrac{1}{3},\tfrac{2}{3},-\tfrac{1}{3})\\ &=\tfrac{1}{2}(1,-1,1,-1)+\tfrac{1}{6}(1,1,1,1)\,,\end{aligned} \quad (1.4)$$

where only the diagonal elements are written and the other elements are zero. The first matrix of the right-hand side of Eq. (1.4) is considered to be one generator of some group, while the second one is the U(1) charge. Next calculate commutators of the generators (1.2)–(1.4) and continue to make commutators of resultant generators until the same ones appear. Fortunately enough, the algebra closes to become a spinor representation of SU(2) ⊗ SU(2) = O(4). Thus we obtain a gauge group O(4) ⊗ U(1), which is coincidentally the same group as that adopted by Pais,[6] though his fermions are assigned to a vector representation of O(4).

Finally we briefly comment on the idea that the Cabibbo angle may be expressed as a mixing angle of gauge bosons. This idea was already realized by Gupta and Mani,[10] but with three flavors, u, d, and s. They used a gauge model of SU(3) ⊗ U(1).

The next section (Sec. II) describes the structure of the gauge group O(4) ⊗ U(1), the Higgs mesons, masses of fermions and gauge bosons, the Cabibbo angle, and the Adler-Bell-Jackiw anomalies.[11,12] Using our model, in Sec. III we shall analyze CP-violating phenomena: the neutral-K mass matrix, direct CP violation in K^0 decays, and an electric dipole moment of a neutron. Effects of neutral currents in our model are discussed in Sec. IV. It is shown that with our model one can consistently explain the evidence of neutral currents found so far in many scattering processes, and also the parity-violating asymmetry in the inelastic ed scattering recently measured at SLAC. Finally, a summary and concluding remarks are given in Sec. V.

II. STRUCTURE OF O(4)⊗U(1)

Our gauge model of O(4) ⊗ U(1) gives the covariant derivative

$$D_\mu=\partial_\mu-\tfrac{1}{2}ig(\vec{\mathrm{I}}\cdot\vec{\mathrm{A}}_\mu+\vec{\mathrm{J}}\cdot\vec{\mathrm{B}}_\mu)-\tfrac{1}{2}ig'I_0C_\mu\,, \quad (2.1)$$

where $\vec{\mathrm{A}}_\mu$, $\vec{\mathrm{B}}_\mu$, and C_μ are the seven Hermitian gauge fields and $I_0/2$ is the U(1) charge. The generators $\vec{\mathrm{I}}$ and $\vec{\mathrm{J}}$ satisfy

$$\begin{aligned}&[I_i,I_j]=2i\epsilon_{ijk}I_k\,,\\ &[J_i,J_j]=2i\epsilon_{ijk}J_k\,,\\ &[I_i,J_j]=0\ \text{ for any } i \text{ and } j\,,\end{aligned} \quad (2.2)$$

which are characteristics for O(4) = SU(2) ⊗ SU(2). The group O(4) restricts the covariant derivative D_μ in the form (2.1), which is also invariant under the simultaneous exchange of $\vec{\mathrm{I}}$, $\vec{\mathrm{A}}_\mu$ and $\vec{\mathrm{J}}$, $\vec{\mathrm{B}}_\mu$. Our representation for $\vec{\mathrm{I}}$ and $\vec{\mathrm{J}}$ is

$$\vec{\mathrm{I}}=\tfrac{1}{2}\vec{\tau}\otimes(1+\tau_2),\quad \vec{\mathrm{J}}=\tfrac{1}{2}\vec{\tau}\otimes(1-\tau_2)\,, \quad (2.3)$$

where τ_i are the Pauli matrices. Note that our representation for O(4) = SU(2) ⊗ SU(2) is $(\underline{2},\underline{1})+(\underline{1},\underline{2})$, i.e., a spinor representation different from a vector representation $(\underline{2},\underline{2})$ adopted by Pais.[6] Hence, instead of Eq. (2.2), the products I_iJ_j, and not just the commutators $[I_i,J_j]$ vanish in our representation

$$I_iJ_j=0\ \text{ for any } i \text{ and } j. \quad (2.2')$$

In our spinor representation the operation of the interchange of $\vec{\mathrm{I}}$ and $\vec{\mathrm{J}}$ is realized by

$$R_s=1\otimes\tau_1 \text{ or } 1\otimes\tau_3\,. \quad (2.4)$$

Next introduce the definitions

$$\begin{aligned}&\tilde{W}^\pm_\mu=\tfrac{1}{2}[A^1_\mu+B^1_\mu\mp i(A^2_\mu+B^2_\mu)]\,,\\ &\tilde{X}^\pm_\mu=\mp\tfrac{1}{2}i[A^1_\mu-B^1_\mu\mp i(A^2_\mu-B^2_\mu)]\,,\\ &Z_\mu=\frac{1}{\sqrt{2}}[(A^3_\mu+B^3_\mu)\cos\gamma-\sqrt{2}\,C_\mu\sin\gamma]\,,\\ &A_\mu=\frac{1}{\sqrt{2}}[(A^3_\mu+B^3_\mu)\sin\gamma+\sqrt{2}\,C_\mu\cos\gamma]\,,\\ &Y_\mu=\frac{1}{\sqrt{2}}(A^3_\mu-B^3_\mu)\,,\end{aligned} \quad (2.5)$$

and

$$\tan\gamma=\sqrt{2}\,g'/g\,. \quad (2.6)$$

The D_μ becomes

$$D_\mu = \partial_\mu - ieQA_\mu - \tfrac{1}{2}ie[\cot\gamma(I_3+J_3) - \tan\gamma I_0]Z_\mu - \frac{ig}{2\sqrt{2}}(I_3 - J_3)Y_\mu - \tfrac{1}{2}ig[(I_+ + J_+)\tilde{W}_\mu^+ + i(I_+ - J_+)\tilde{X}_\mu^+ + \text{H.c.}], \quad (2.7)$$

with

$$I_\pm = \tfrac{1}{2}(I_1 \pm iI_2) \text{ and } J_\pm = \tfrac{1}{2}(J_1 \pm iJ_2), \quad (2.8)$$

where the charge operator Q and the charge e are given by

$$Q = \tfrac{1}{2}(I_3 + J_3 + I_0), \quad (2.9)$$

$$e = gg'/(g^2 + 2g'^2)^{1/2} = \frac{1}{\sqrt{2}} g\sin\gamma = g'\cos\gamma. \quad (2.10)$$

Notice that $\tilde{W}_\mu^\pm$ and $\tilde{X}_\mu^\pm$ are not the physical fields. The physical fields which diagonalize the mass terms are given by

$$W_\mu^+ = \tilde{W}_\mu^+\cos\theta + \tilde{X}_\mu^+\sin\theta, \quad X_\mu^+ = -\tilde{W}_\mu^+\sin\theta + \tilde{X}_\mu^+\cos\theta. \quad (2.11)$$

The Higgs scalars which cause such a mixing angle are discussed below, together with the mass generation of leptons and quarks.

The left-handed leptons $l_L' = (\nu_e', e', \nu_\mu', \mu')_L$ and quarks $q_L = (u, d, c, s)_L$ belong to quartets of a spinor representation of O(4), whereas all the right-handed ones belong to singlets. The U(1) charges are

$$I_0 = \begin{cases} -1 & \text{for } l_L', \\ \frac{1}{3} & \text{for } q_L, \\ -2 & \text{for } e_R' \text{ and } \mu_R', \\ \frac{4}{3} & \text{for } u_R \text{ and } c_R, \\ -\frac{2}{3} & \text{for } d_R \text{ and } s_R, \end{cases} \quad (2.12)$$

where the left- and right-handed fields are defined by $\Psi_L = \frac{1}{2}(1+\gamma_5)\Psi$ and $\Psi_R = \frac{1}{2}(1-\gamma_5)\Psi$. Here, since neutrinos are assumed to be massless, they have no right-handed U(1) charge. Looking at the lepton currents, there occurs e- and μ-number nonconservation in two charged currents and one neutral current. Then the physical leptons l, which conserve e or μ number and diagonalize the mass terms, must be defined as

$$l = Ul' \quad (2.13)$$

with

$$l = (\nu_e, e, \nu_\mu, \mu)^t, \quad l' = (\nu_e', e', \nu_\mu', \mu')^t,$$

and

$$U = \frac{1}{\sqrt{2}} 1 \otimes \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} = \frac{1+i}{2\sqrt{2}} 1 \otimes [1 - i(\tau_1 + \tau_2 + \tau_3)]. \quad (2.14)$$

The explicit forms of the currents for quarks and leptons which couple to each vector boson are given in Appendix A. As seen from Eq. (A6), the charged current J_μ^- becomes the GIM current,[2] namely the Cabibbo angle θ is properly located in the charged current, provided our $W_\mu^\pm$ is the same as that in the Weinberg-Salam model.[1] Also, as seen from Eqs. (A9)–(A14), all the currents conserve e or μ number.

Now we discuss the Higgs scalars which cause boson and/or lepton mixings such as Eqs. (2.11) and (2.13). In order to cause a lepton mixing such as Eq. (2.13) and to generate quark masses, there must be at least two Higgs scalars in a spinor representation of O(4), ϕ_1 and ϕ_2 with $I_0 = 1$ which have the vacuum expectation values (VEV's)

$$\langle\phi_1\rangle = \begin{bmatrix} 0 \\ d \\ 0 \\ 0 \end{bmatrix} \text{ and } \langle\phi_2\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ d \end{bmatrix} \quad (2.15)$$

with a real parameter d, respectively. The interaction Lagrangian between leptons and the Higgs scalars is given by

$$G\bar{l}_L'(\phi_1 e_R' + \phi_2 \mu_R') + G'\bar{l}_L'(\phi_2 e_R' - \phi_1 \mu_R') + \text{H.c.}, \quad (2.16)$$

with

$$G = (m_e + m_\mu)/2d, \quad G' = i(m_e - m_\mu)/2d, \quad (2.17)$$

where m_e and m_μ are masses of e and μ. As is seen from the above equation, e-μ universality is broken. As for the quark masses, we need the G-parity-conjugate Higgs scalars which are defined by

$$\phi^G \equiv \exp[\tfrac{1}{2}i\pi(I_2 + J_2)]\phi^*, \quad (2.18)$$

where the definition should be used only in a spinor representation of O(4). Using this definition, the VEV's of the G-parity-conjugate Higgs scalars ϕ_1^G and ϕ_2^G are given by

$$\langle\phi_1^G\rangle = \begin{bmatrix} d \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \langle\phi_2^G\rangle = \begin{bmatrix} 0 \\ 0 \\ d \\ 0 \end{bmatrix}. \quad (2.19)$$

Four VEV's of the Higgs scalars in Eqs. (2.15) and (2.19) are sufficient to generate quark masses and to cause a small rotation of quarks, which will later be necessary in order to bring about the CP violation.

Concerning masses of gauge bosons, we need at least two Higgs scalars in a vector representation of O(4) to produce a charged boson mixing such as Eq. (2.11) and to render the masses of the extra gauge bosons $X_\mu^\pm$ and Y_μ large. The generators of a vector representation are given by

$$\vec{I} = \vec{\tau} \otimes 1, \quad \vec{J} = 1 \otimes \vec{\tau}\,, \tag{2.20}$$

where the same notations for the generators as in a spinor representation are used and the same vector representation as that used by Pais[6] is adopted. Instead of the relation (2.2′), the generators (2.20) satisfy

$$[I_i, J_j] = 0\,. \tag{2.21}$$

However, the other commutation relations of (2.2) are satisfied by these generators and the definition of the charge operator Q is the same as Eq. (2.9). In a vector representation, the Higgs scalar is denoted as $H^{(I_0)} = (H_1, H_2, H_3, H_4)$ with I_0 being the U(1) charge. Quartet members are ordered such that the respective eigenvalue pairs are $(I_3, J_3) = (1,1)$, $(-1,1)$, $(1,-1)$, and $(-1,-1)$, corresponding to the respective charges $(+,0,0,-)$ when $I_0 = 0$. In our vector representation (2.20), $H^{(0)}$ is expressed as

$$H^{(0)} = \frac{1}{\sqrt{2}}(-\xi + i\eta,\ \zeta - i\chi,\ \zeta + i\chi,\ \xi + i\eta)\,, \tag{2.22}$$

where ξ, η, ζ, and χ are real scalar fields. Since H_2 and H_3 with $I_0 = 0$ are neutral components, ζ and/or χ may have the VEV by assuming an appropriate Higgs potential. The contribution of the Higgs scalar $H^{(I_0)}$ to the mass terms of gauge bosons is calculated from $|D_\mu H^{(I_0)}|^2$ with D_μ given by Eq. (2.7) and is given by

$$\tfrac{1}{2}g^2\tilde{W}_\mu^+\tilde{W}^{-\mu}(H_2+H_3)^\dagger(H_2+H_3) + \tfrac{1}{2}g^2\tilde{X}_\mu^+\tilde{X}^{-\mu}(H_2-H_3)^\dagger(H_2-H_3) + \tfrac{1}{2}ig^2(\tilde{W}_\mu^+\tilde{X}^{-\mu} + \tilde{W}_\mu^-\tilde{X}^{+\mu})(H_3^\dagger H_2 - H_2^\dagger H_3) + \tfrac{1}{2}g^2(Y_\mu)^2(H_2^\dagger H_2 + H_3^\dagger H_3)\,. \tag{2.23}$$

In a vector representation, a "parity" operator which exchanges $\vec{I}$ and $\vec{J}$ is defined as

$$R_v = \tfrac{1}{2}(1 + \vec{I}\cdot\vec{J})\,. \tag{2.24}$$

Operating it on $H^{(0)}$ in Eq. (2.22) it is easily seen that R_v behaves as a parity operator of O(4):

$$R_v(H_1, H_2, H_3, H_4) = (H_1, H_3, H_2, H_4)\,. \tag{2.25}$$

With reference to Eq. (2.22), the exchange of H_2 and H_3 is to change the sign of χ.[6] The contribution of the Higgs scalar $\phi = (\phi_1, \phi_2, \phi_3, \phi_4)$ with $I_0 = 1$ in the spinor representation is also calculated from $|D_\mu\phi|^2$ and is given (in this case the neutral components are ϕ_2 and ϕ_4) by

$$\tfrac{1}{4}g^2(\tilde{W}_\mu^+\tilde{W}^{-\mu} + \tilde{X}_\mu^+\tilde{X}^{-\mu})(\phi_2^\dagger\phi_2 + \phi_4^\dagger\phi_4) + \tfrac{1}{4}g^2(\tilde{W}_\mu^+\tilde{X}^{-\mu} - \tilde{W}_\mu^-\tilde{X}^{+\mu})(\phi_4^\dagger\phi_2 - \phi_2^\dagger\phi_4) + \left[\tfrac{1}{8}g^2(Y_\mu)^2 + \frac{e^2}{\sin^2 2\gamma}(Z_\mu)^2\right](\phi_2^\dagger\phi_2 + \phi_4^\dagger\phi_4)\,. \tag{2.26}$$

Here we employ two $H^{(0)}$'s which have the VEV's

$$\langle H_1^{(0)}\rangle = (0, a-ib, a+ib, 0) \tag{2.27}$$

and

$$\langle H_2^{(0)}\rangle = (0, -ic, ic, 0)\,, \tag{2.28}$$

respectively. The existence of such two $H^{(0)}$'s is warranted in general by assuming a relevant interacting Higgs potential $V(H_1^{(0)}, H_2^{(0)})$. Together with Eq. (2.15), the gauge boson masses are given by

$$m_Z^2 = \frac{2e^2}{\sin^2 2\gamma}d^2\,, \qquad m_Y^2 = g^2(a^2+b^2+c^2+\tfrac{1}{4}d^2)\,. \tag{2.29}$$

The mass matrix for the charged bosons is given by

$$\begin{array}{c} \\ \tilde{W}^+ \\ \tilde{X}^+ \end{array}\begin{array}{c} \begin{array}{cc} \quad\tilde{W}^- & \qquad\quad\tilde{X}^- \end{array} \\ \begin{bmatrix} g^2(a^2+\tfrac{1}{2}d^2) & g^2ab \\ g^2ab & g^2(b^2+c^2+\tfrac{1}{2}d^2) \end{bmatrix} \end{array}, \tag{2.30}$$

which is diagonalized by Eq. (2.11). Then the masses of $W_\mu^\pm$ and $X_\mu^\pm$ and the angle θ are given by

$$\begin{aligned} m_W^2 &= \tfrac{1}{2}g^2(a^2+b^2+c^2+d^2) - \tfrac{1}{2}g^2[(a^2-b^2-c^2)^2+4a^2b^2]^{1/2}\,, \\ m_X^2 &= \tfrac{1}{2}g^2(a^2+b^2+c^2+d^2) + \tfrac{1}{2}g^2[(a^2-b^2-c^2)^2+4a^2b^2]^{1/2}\,, \end{aligned} \tag{2.31}$$

and

$$\tan 2\theta = 2ab/(a^2+b^2+c^2+d^2)\,. \tag{2.32}$$

The parameters a, b, c, and d, introduced above, are restricted from the suppression of a strangeness-changing neutral current to order of $G_F\alpha$, and from the value of the Cabibbo angle. That is,

$$\frac{g^2}{16m_Y^2}=\frac{1}{16(a^2+b^2+c^2+\frac{1}{4}d^2)} \ll G_F\alpha \sim 8.34\times 10^{-8}\ \text{GeV}^{-2}, \qquad (2.33)$$

$$\tan 2\theta = 2ab/(a^2+b^2+c^2+d^2)=0.501 \qquad (2.34)$$

for $\sin\theta = 0.230$.

Finally, we must consider the Adler-Bell-Jackiw triangle anomalies.[11,12] Owing to the spinor representation of O(4) for leptons and quarks and to their charge structure, there comes about the cancellation of anomalies between lepton and quark triangle loops. Here the color degree of freedom $SU(3)_c$ for quarks is taken into account.

III. *CP* VIOLATION

Our model may reproduce the results of the superweak theory for the *CP* violation in the *K*-meson system, and can also give a restriction for an electric dipole moment of a neutron. To generate the *CP*-violating phase in coupling constants between gauge bosons and quarks, we must rotate *d* and *s* quarks a little by a general unitary matrix *U* with one mixing angle θ' and three phase parameters:

$$U=e^{i\alpha_1}e^{i\alpha_2\tau_3}\begin{bmatrix}\cos\theta' & \sin\theta' \\ -\sin\theta' & \cos\theta'\end{bmatrix}e^{i\alpha_3\tau_3}. \qquad (3.1)$$

After absorbing as many phases into quarks as possible, there remains one phase parameter, α. The phases occur in the currents $J_\mu^{\pm}$, $J_\mu^{\pm\prime}$, and $J_\mu^{\pi\prime}$, or alternatively the phases occur only in the currents $J_\mu^{\pm\prime}$ and $J_\mu^{\pi\prime}$. Either expression for the *CP*-violating phase must give the same physical quantities. There explicit forms are given in Appendix B. There is no change in the other currents, which are given by Eqs. (A2) and (A4). Before proceeding with the calculations, let us check whether there may occur *CP* violation without a quark rotation. The only possibility to cause *CP* violation is through the gauge boson Y_μ since the current which couples to Y_μ has a pure imaginary coupling constant given in Eq. (A5). The lowest diagrams are illustrated in Fig. 2. The explicit calculation, however, shows that the sum of these diagrams vanishes in the external-momentum-zero limit. The same cancellation as above also happens when the *CP*-violating phase is introduced by a quark rotation.

Now let us analyze *CP* violation when rotating *d* and *s* quarks by Eq. (3.1). Our analyses are mainly based on Refs. 13–17.

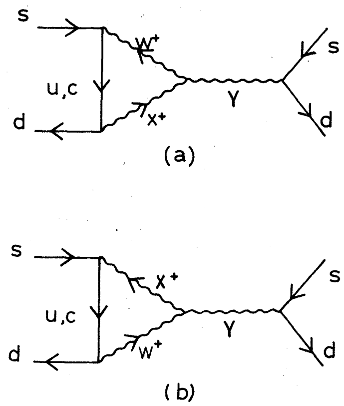

FIG. 2. The possible *CP*-violating diagrams without rotating quarks.

A. *CP* violation in the neutral-*K* mass matrix

The box diagrams shown in Fig. 3 are customarily considered to give an estimate for the $\Delta S=2$ transition. Evaluating these diagrams by the approximation of the vanishing external momentum limit,[13] and using the phase convention (ii) in Appendix B, only the diagrams of Figs. 3(c) and 3(d) contribute to the imaginary part of the mass matrix. The real part is approximated with Fig. 3(a), assuming $m_W^2 \ll m_X^2$, and its effective Lagrangian is given by

$$L_{\text{eff}}=-\frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi}\kappa(\cos\phi\,\sin\phi)^2[\bar{d}\gamma_\mu\tfrac{1}{2}(1+\gamma_5)s]^2, \qquad (3.2)$$

where the Fermi coupling constant G_F is given by

$$\frac{1}{\sqrt{2}}G_F=g^2/16m_W^2, \qquad (3.3)$$

and

$$\kappa=(m_c^2-m_u^2)^2/(m_c^2m_W^2\sin^2\gamma)\simeq m_c^2/(m_W^2\sin^2\gamma) \qquad (3.4)$$

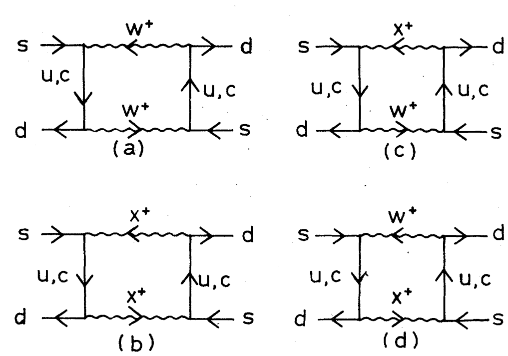

FIG. 3. The diagrams which contribute to the off-diagonal parts of the neutral-*K* mass matrix.

with γ given by Eq. (2.6) and ϕ given by Eq. (B6). Here and hereafter it is assumed that the masses of $X_\mu^\pm$ and Y_μ are sufficiently large so that the Fermi coupling constant is effectively given by Eq. (3.3). The effective Lagrangian which contributes to the CP-violating parameter ϵ is calculated from Figs. 3(c) and 3(d), and is given by

$$L'_{\text{eff}} = i\,\frac{G_F}{\sqrt{2}}\,\frac{\alpha}{2\pi}\,\kappa'\cos 2\phi\,\sin(\beta_1-\beta_2)\times[\bar{d}\gamma_\mu\tfrac{1}{2}(1+\gamma_5)s]^2\,, \tag{3.5}$$

where

$$\kappa' = 4m_c^{\,2}/(m_X^{\,2}\sin^2\gamma) \tag{3.6}$$

and β_1 and β_2 are given by Eqs. (B7). Neglecting the contribution from the Y_μ gauge boson, the ratio of the imaginary to the real part of the off-diagonal mass matrix is, from Eqs. (3.2) and (3.5), given by

$$\xi_1 \equiv \left|\frac{\langle K^0|-L'_{\text{eff}}|\bar{K}^0\rangle}{\langle K^0|-L_{\text{eff}}|\bar{K}^0\rangle}\right| \simeq 16\left(\frac{m_W}{m_X}\right)^2\frac{\cos 2\phi}{\sin^2 2\phi}\sin(\beta_2-\beta_1)\,. \tag{3.7}$$

Approximating $\phi\simeq\theta$ [see Appendix B (ii)] and taking account of the experimental results,[18] $\xi_1\sim 6.5\times10^{-3}$ and $\sin\theta=0.230$, the following restriction on our parameters is obtained:

$$\left(\frac{m_W}{m_X}\right)^2\sin(\beta_2-\beta_1)\simeq 9.10\times10^{-5}\,. \tag{3.8}$$

B. Direct CP violation in K^0 decay

In the following we shall estimate the deviation from the superweak theory.[19] In the conventional analysis of CP violation the following quantities are used[20]:

$$\eta_{+-}=\frac{A(K_L^0\to\pi^+\pi^-)}{A(K_S^0\to\pi^+\pi^-)}\,,\quad \eta_{00}=\frac{A(K_L^0\to\pi^0\pi^0)}{A(K_S^0\to\pi^0\pi^0)}\,,$$
$$\epsilon=\frac{A(K_L^0\to2\pi(I=0))}{A(K_S^0\to2\pi(I=0))}\,,\quad \epsilon'=ie^{i(\delta_2-\delta_0)}\text{Im}\left(\frac{A_2}{A_0}\right)\,, \tag{3.9}$$

where $A_ne^{i\delta_n}=A(K^0\to2\pi(I=n))$ $(n=0,2)$ and δ_n are the associated strong phases. The phase convention $\text{Im}A_0=0$ is adopted here. Assuming the $\Delta I=\frac{1}{2}$ rule for CP-conserving decays, we have[20]

$$\eta_{+-}\simeq\epsilon+\frac{\epsilon'}{\sqrt{2}}\,,\quad \eta_{00}\simeq\epsilon-\sqrt{2}\,\epsilon'\,. \tag{3.10}$$

In our model, the phase difference between A_2 and A_0 arises only from the diagram shown in Fig. 4(b), if we use the phase convention (ii) in Appendix B. The diagrams Figs. 4(a) and 4(c) do not contribute to CP violation, though the latter diagram Fig. 4(c) is usually considered as the dominant contribution over Figs. 4(a) and 4(b) in

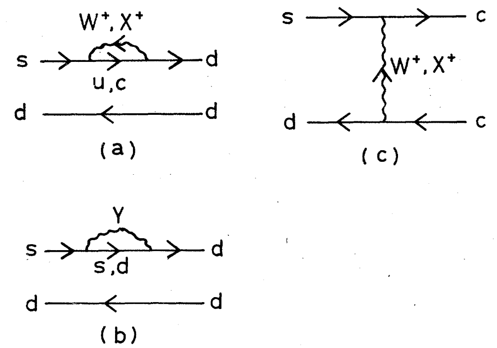

FIG. 4. The diagrams which may contribute to the direct CP violation in K^0 decay.

the literature, Refs. 14–16. The contribution of Fig. 4(b) to the CP-violating phase, i.e., a ratio of the imaginary part of Fig. 4(b) to the ordinary CP-conserving amplitude, is given by

$$\xi_2\equiv\frac{1}{4\pi^2}\left(\frac{m_W}{m_Y}\right)^2\cos^3\theta'\sin\theta'\sin(\beta_1+\beta_2)/\cos\phi\,\sin\phi\,, \tag{3.11}$$

where θ' is a mixing angle of d and s quarks, and β_1, β_2, and ϕ are given by Eq. (B7). Comparing the quantities ξ_1 and ξ_2 given by Eqs. (3.7) and (3.11), we find that ξ_2 is multiplied by a further suppression factor $\sin\theta'$. By taking into account Eqs. (2.33) and (3.8), masses of $X_\mu^\pm$ and Y_μ are expected to be of the same order. Then ξ_2 is considered to be smaller than ξ_1. Their ratio is the order of

$$\xi_2/\xi_1\simeq1.2\times10^{-2}\left(\frac{m_X}{m_Y}\right)^2 \tag{3.12}$$

for $\sin\theta'=0.1$. It is convenient to introduce the following expression for ϵ[18,20]:

$$\epsilon\simeq\frac{\frac{1}{2}\text{Im}\,\Gamma_{12}+i\,\text{Im}M_{12}}{\frac{1}{2}i(\Gamma_s-\Gamma_L)-\Delta m}\,, \tag{3.13}$$

where $\Gamma_{12}=2\pi\Sigma\rho_F\langle\bar{K}^0|H_W|F\rangle\langle F|H_W|K^0\rangle$, $\Gamma_s-\Gamma_L$ is the difference between the K_s^0 and K_L^0 widths, and $\text{Im}M_{12}/\Delta m=\frac{1}{2}\xi_1$. This relation holds under the $\Delta I=\frac{1}{2}$ rule. We estimate

$$|\epsilon|\simeq\frac{1}{2\sqrt{2}}\,\xi_1\,,$$
$$|\epsilon'|\simeq\frac{1}{\sqrt{2}}\,\xi_2|A_2/A_0|\simeq\frac{1}{20\sqrt{2}}\,\xi_2\,, \tag{3.14}$$
$$|\phi_D|\simeq\frac{\xi_2}{2}\left[\frac{\Gamma(K^0\to2\pi(I=2))+\Gamma(K^0\to3\pi)}{\Gamma(K^0\to2\pi(I=0))}\right]\Big/\left|\frac{\text{Im}M_{12}}{\Delta m}\right|\simeq2\times10^{-3}\xi_2\Big/\left|\frac{\text{Im}M_{12}}{\Delta m}\right|\,,$$

where

$$\phi_D = -\tan^{-1}(\mathrm{Im}\,\Gamma_{12}/2\,\mathrm{Im}M_{12}) \tag{3.15}$$

and the $\Delta I = \frac{1}{2}$ rule, $|A_2/A_0| \sim 0.05$, has been used. In the superweak theory,[19] CP violation occurs only through the K^0-$\bar{K}^0$ mixing, and $\epsilon' = \phi_D = 0$.

From Eqs. (3.12) and (3.14) we have

$$|\epsilon'| \simeq 1.2\times 10^{-3}\left(\frac{m_X}{m_Y}\right)^2 |\epsilon|\,,$$
$$|\phi_D| \simeq 4.8\times 10^{-5}\left(\frac{m_X}{m_Y}\right)^2 . \tag{3.16}$$

Even the ratio $m_X/m_Y = 1$ is sufficient to make Eqs. (3.16) consistent with experimental results[18]:

$$|\epsilon'| \lesssim \tfrac{1}{50}|\epsilon|, \quad |\phi_D| \lesssim \tfrac{1}{20} . \tag{3.17}$$

We have thus succeeded in approximately reproducing superweak results,[19] which are due to the small phase and/or mixing parameters, the large masses of the extra gauge bosons, and the $\Delta I = \frac{1}{2}$ enhancement in Eq. (3.14).

C. Electric dipole moment of the neutron

The violation of P and T induces the effective $\bar{q}q\gamma$ vertex of the form

$$e f_D(k^2)\bar{\Psi}_2\sigma^{\mu\nu}\gamma_5\Psi_1 k_\nu A_\mu(k^2) \tag{3.18}$$

with $\sigma_{\mu\nu} = \frac{1}{2}i[\gamma_\mu, \gamma_\nu]$. Hermiticity of the current requires that $f_D(k^2)$ be purely imaginary. The strength of the electric dipole moment D is defined as

$$|D| = e|f_D(0)| . \tag{3.19}$$

The origin of the name "electric dipole moment" may be easily seen when we rewrite Eq. (3.18) in the nonrelativistic form

$$e f_D(k^2) w_2^*(\vec{\mathrm{E}}\cdot\vec{\sigma}) w_1 , \tag{3.20}$$

where w_i are the usual two-component spinors and $\vec{\mathrm{E}}$ is the electric strength. The diagrams which effectively induce Eq. (3.18) arise from perturbation of the fourth order in the weak coupling constant in our model. The electric dipole moment of a neutron D_n is considered to come from d and u quarks whose contributions are calculated from the diagrams shown in Fig. 5.

The estimate of D_n in the two-loop diagrams is quite complicated, as discussed in Ref. 15. Moreover, its value D_n estimated from electric dipole moments of quarks is not warranted in general. From this point of view, we here give a crude estimate of the upper bound on D_n. As an example, to consider the angle factors which contribute to D_n, we take Fig. 5(a). This diagram gives as the angle factors

$$\tfrac{1}{4}\sin 2\theta \sin 2\theta' \cos^2\theta' \sin\alpha \sim \sin\theta \sin\theta' \sin\alpha , \tag{3.21}$$

where we have used the phase convention (i) in

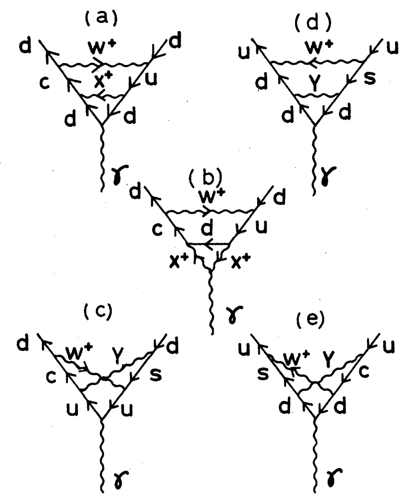

FIG. 5. The diagrams which contribute to the electric dipole moment of a neutron.

Appendix B. Finally we estimate D_n as

$$|D_n/e| \lesssim (G\alpha/\pi^3)\sin\theta \sin\theta' \sin\alpha (m_W/m_X)^2 m_q (m_c/m_W)^2$$
$$\lesssim 10^{-29}\ \mathrm{cm} \tag{3.22}$$

for $m_c/m_X \sim 10^{-2}$, $m_q/m_{\mathrm{neutron}} \sim 0.5$, and $\sin\theta' = \sin\alpha \sim 0.1$. The numerical factors in Eq. (3.22), except for the angle factors, have been taken from Ref. 15. This value of D_n is well below the present experimental upper limit of $|D_n/e| < 5\times 10^{-24}$ cm.[18]

IV. EFFECTS OF NEUTRAL CURRENTS

In this section we discuss the effects of neutral currents in our model.[21,22] We consider only the neutral currents coupled to Z_μ, Eqs. (A4) and (A11) in Appendix A, but not those coupled to Y_μ, since the mass of the gauge boson Y_μ is very large [see Eq. (2.33) in Sec. II].

One can easily see from Eqs. (A4) and (A11) that the structure of the neutral currents coupled to Z_μ is the same as those of the Weinberg-Salam model.[1] In our model, however, there is no relation between the masses of the gauge bosons $W_\mu^\pm$ and Z_μ [see Eqs. (2.29) and (2.30)], contrary to the Weinberg-Salam model[1] where there is a relation ($m_W = m_Z \cos\theta_W$). Therefore, in our model we can choose the parameters so as to explain any phenomena due to neutral currents which can be explained in the Weinberg-Salam model. At the same time, we cannot explain the absence of parity violation in heavy atoms[23] as in the Weinberg-Salam model. This experiment, however, may not be a decisive one to select out of many gauge models because of the complexity of heavy atoms.

Here we adopt two kinds of experiments on neutral currents in order to restrict the values of the Z-boson mass m_Z and the mixing angle $\sin\gamma$. One experiment is to measure the parity-violating asymmetry in the inelastic scattering of longitudinally polarized electrons from the deuterium, which has been recently carried out at SLAC,[24] and the data now rule out almost all models proposed so far, except for the Weinberg-Salam model. Another experiment is to measure the cross section for the process $\nu_\mu + e^- \to \nu_\mu + e^-$.[25] Although this experiment is not so accurate, we adopt it since this process occurs only through the Z boson in the lowest-order diagram and the data on this experiment have now accumulated.

The asymmetry parameter of the inelastic ed scattering is given by[26,27]

$$A(x,y) = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} = -\frac{9GQ^2}{20\sqrt{2}\,\pi\alpha}\left[(1 - \tfrac{20}{9}\sin^2\gamma) + (1 - 4\sin^2\gamma)\frac{y(2-y)}{2-2y+y^2}\right], \tag{4.1}$$

with

$$m_Z{}^2 = \frac{2\sqrt{2}\,\pi\alpha}{G\sin^2 2\gamma} = \frac{8\pi\alpha}{\sin^2 2\gamma}\,d^2\,, \tag{4.2}$$

where Q^2 is a minus of the momentum transfer squared, x and y the usual scaling variables, and α the fine-structure constant. Equation (4.2) is a definition of the constant G [see also Eq. (2.29)]. In the Weinberg-Salam model, the constant G becomes the Fermi coupling constant G_F, which is defined in our model by Eq. (3.3). The differential cross section for the process $\nu_\mu + e^- \to \nu_\mu + e^-$ in our model reads[21]

$$\frac{1}{E_\nu}\frac{d\sigma}{dy} = \frac{G^2 m_e}{2\pi}\left[(-1+2\sin^2\gamma)^2 + 4y^2\sin^4\gamma\right], \tag{4.3}$$

which gives the cross section

$$\sigma = E_\nu \frac{G^2 m_e}{2\pi}\,(1 - 4\sin^2\gamma + \tfrac{16}{3}\sin^4\gamma)\,, \tag{4.4}$$

where we have neglected the experimental cutoff on the out-going electron energy E_e.

The experimental value for Eq. (4.1) is given by[24]

$$A/Q^2 = (-9.5 \pm 1.6)\times 10^{-5}\ \text{GeV}^{-2} \tag{4.5}$$

for the average value of $y = 0.21$. The experimental value for Eq. (4.4) is given by[25]

$$\sigma = [(1.8 \pm 0.8)\times 10^{-42}\ \text{cm}^2/\text{GeV}]\,E_\nu\,. \tag{4.6}$$

Combining Eqs. (4.1) and (4.4)–(4.6), we can plot Fig. 6 in the G and $\sin^2\gamma$ plane. The intersection of these results shows the domain allowed. The

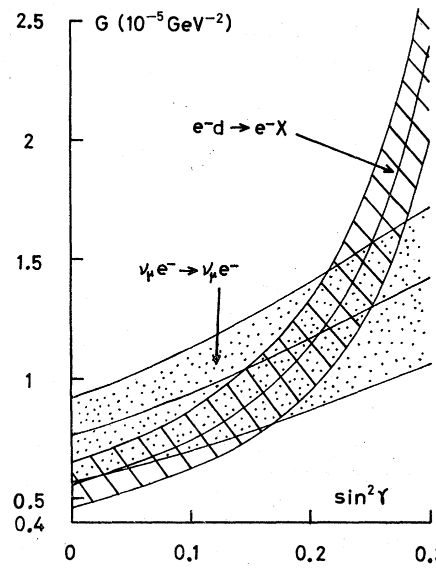

FIG. 6. Limits on G amd $\sin^2\gamma$ plane from the inelastic ed scattering and $\nu_\mu e$ reactions.

most plausible values are given by

$$\sin^2\gamma = 0.21 \quad\text{and}\quad G = 1.2\times 10^{-5}\ \text{GeV}^{-2}\,, \tag{4.7}$$

which correspond to

$$m_Z = 90.2\ \text{GeV} \quad\text{and}\quad m_W = 82.0\ \text{GeV}\,. \tag{4.8}$$

V. SUMMARY AND CONCLUDING REMARKS

Provided that the charged current for four quarks is the GIM current and that the Cabibbo angle is expressed as a mixing angle between two charged vector bosons, we have been led to a gauge model of $O(4)\otimes U(1)$. Each quartet of quarks in our model has the common Cabibbo angle. Using this gauge model, we have succeeded in reproducing the superweak results for CP-violating phenomena, the deviations from which are also calculated to satisfy the experimental upper bounds. The electric dipole moment of a neutron calculated from our model is sufficiently lower than the experimental limit. We have also succeeded in explaining almost all the data on the neutral currents, including the parity-violating asymmetry of the inelastic ed scattering recently measured at SLAC.

Although we are satisfied with the above-mentioned successes, there are still some defects in our model: the appearance of a strangeness-changing neutral current (which may be suppressed by making its associated gauge boson's mass heavy, though), breakdown of e-μ universality in the couplings to the Higgs scalars, and some complication in our generation of the Cabibbo angle.

Finally we want to point out the possibility that the Cabibbo angle may be determined by the hierarchy of spontaneous symmetry breakdown from a larger group as in the Georgi-Glashow SU(5) gauge model,[28] if one succeeds in expressing the Cabibbo angle as a ratio between two coupling constants similarly to the Weinberg angle. Although aiming at such an expression for the Cabibbo angle in the first stage of this work, we have not yet succeeded in it.

Note added. After the completion of our work, we become aware of two papers: N. G. Deshpande, R. C. Hwa, and P. D. Mannheim, Phys. Rev. D 19, 2686 (1979); M. Singer, ICTP Report No. IC/78/68, 1978 (unpublished). They have also expressed the Cabibbo angle as a free mixing angle of the charged gauge bosons (the same idea as ours), using the gauge model of SU(4)⊗U(1) (different from our model), and assigning four leptons (ν_e, e, ν_μ, μ) and four quarks (u, d, c, s) to the quartets of the fundamental representation of SU(4). Deshpande *et al.* have adopted four leptons ($\nu_e\cos\theta+\nu_\mu\sin\theta, e, -\nu_e\sin\theta+\nu_\mu\cos\theta, \mu$) as the quartet members of SU(4) with the Cabibbo angle θ. Under the assumption that the neutrinos are massless, their theory has no Cabibbo rotation in the charged lepton current. Application of the same argument to our gauge model can recover μ-e universality which is broken in our model as is seen from Eqs. (2.16) and (2.17).

It should be stressed that the purpose of this paper is to show that the *minimal* gauge group is O(4)⊗U(1) in order to express the Cabibbo angle as a gauge-boson mixing angle.

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APPENDIX A

By taking account of Eqs. (2.6), (2.10), and (2.11), the currents which couple to each vector boson are given as follows:

(*i*) *Quarks*. Define the interaction parts between quarks and gauge bosons as

$$A^\mu J_\mu^{\text{em}}+Z^\mu J_\mu^n+Y^\mu J_\mu^{n'}+W^{+\mu}J_\mu^-+X^{+\mu}J_\mu^{-'}+\text{H.c.}, \tag{A1}$$

where the currents are given by

$$J_\mu^{\text{em}}=e\bar{q}\gamma_\mu Qq \tag{A2}$$

with

$$q=(u,\ d,\ c,\ s)^t,\quad Q=\text{diag}(\tfrac{2}{3},-\tfrac{1}{3},\tfrac{2}{3},-\tfrac{1}{3}), \tag{A3}$$

$$J_\mu^n=\tfrac{1}{6}e(3\cot2\gamma-\tan\gamma)(\bar{u}\gamma_\mu u+\bar{c}\gamma_\mu c)-\tfrac{1}{6}e(3\cot2\gamma+\tan\gamma)(\bar{d}\gamma_\mu d+\bar{s}\gamma_\mu s)$$
$$+\frac{e}{2\sin2\gamma}[(\bar{u}\gamma_\mu\gamma_5u+\bar{c}\gamma_\mu\gamma_5c)-(\bar{d}\gamma_\mu\gamma_5d+\bar{s}\gamma_\mu\gamma_5s)], \tag{A4}$$

$$J^{n'}=\frac{ig}{2\sqrt{2}}[(\bar{c}u)-(\bar{u}c)+(\bar{d}s)-(\bar{s}d)], \tag{A5}$$

$$J^-=\tfrac{1}{2}g\{[(\bar{u}d)+(\bar{c}s)]\cos\theta+[(\bar{u}s)-(\bar{c}d)]\sin\theta\}, \tag{A6}$$

$$J^{-'}=\tfrac{1}{2}g\{-[(\bar{u}d)+(\bar{c}s)]\sin\theta+[(\bar{u}s)-(\bar{c}d)]\cos\theta\}, \tag{A7}$$

and $J_\mu^+=J_\mu^{-\dagger}$ and $J_\mu^{+'}=J_\mu^{-'\dagger}$, where we have defined

$$(\bar{q}'q)\equiv\bar{q}'_L\gamma_\mu q_L \tag{A8}$$

(*ii*) *Leptons*. The same notations as in the case (i) are used:

$$J_\mu^{\text{em}}=e\bar{l}\gamma_\mu Ql \tag{A9}$$

with

$$l=(\nu_e,\ e,\ \nu_\mu,\ \mu)^t,\quad Q=\text{diag}(0,-1,0,-1), \tag{A10}$$

$$J_\mu^n = \tfrac{1}{2}e(\tan\gamma - \cot 2\gamma)(\bar{e}\gamma_\mu e + \bar{\mu}\gamma_\mu \mu) + \frac{e}{2\sin 2\gamma}[\bar{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e + \bar{\nu}_\mu\gamma_\mu(1+\gamma_5)\nu_\mu - (\bar{e}\gamma_\mu\gamma_5 e + \bar{\mu}\gamma_\mu\gamma_5\mu)], \quad (A11)$$

$$J^{n'} = \frac{g}{2\sqrt{2}}[(\bar{\nu}_e\nu_e) - (\bar{\nu}_\mu\nu_\mu) - (\bar{e}e) + (\bar{\mu}\mu)], \quad (A12)$$

$$J^- = \tfrac{1}{2}g[(\bar{\nu}_e e)e^{i\theta} - (\bar{\nu}_\mu\mu)e^{-i\theta}], \quad (A13)$$

$$J^{-\prime} = \tfrac{1}{2}ig[(\bar{\nu}_e e)e^{-i\theta} - (\bar{\nu}_\mu\mu)e^{i\theta}], \quad (A14)$$

$J_\mu^+ = J_\mu^{-\dagger}$, and $J_\mu^{+\prime} = J_\mu^{-\prime\dagger}$. The phases which appear in the charged currents above may be eliminated by simultaneous redefinition of the left-handed leptons, which does not affect the mass terms.

APPENDIX B

The currents which have the CP-violating phase are as follows:

$$J^- = \tfrac{1}{2}g[(\bar{u}d)a_1^* + (\bar{c}s)a_1 + (\bar{u}s)a_2 - (\bar{c}d)a_2^*], \quad (B1)$$

which couples to W_μ^+, where

$$a_1 = cc' - e^{-i\alpha}ss', \quad a_2 = sc' + e^{-i\alpha}cs' \quad (B2)$$

with

$$c = \cos\theta, \quad s = \sin\theta, \quad c' = \cos\theta', \quad s' = \sin\theta'. \quad (B3)$$

Note that $|a_1|^2 + |a_2|^2 = 1$:

$$J^{-\prime} = -\tfrac{1}{2}g[(\bar{u}d)a_2^* + (\bar{c}s)a_2 - (\bar{u}s)a_1 + (\bar{c}d)a_1^*], \quad (B4)$$

which couples to X_μ^+:

$$J^{n'} = \frac{ig}{2\sqrt{2}}\{i[(\bar{s}s) - (\bar{d}d)]2c's'\sin\alpha + (\bar{d}s)(c'^2 + e^{-2i\alpha}s'^2) - (\bar{s}d)(c'^2 + e^{2i\alpha}s'^2) + (\bar{c}u) - (\bar{u}c)\}, \quad (B5)$$

which couples to Y_μ. The other currents are not affected by a rotation of d and s quarks and are given by Eqs. (A2) and (A4).

The CP-violating phase may be eliminated from either of the curents (B1) or (B4). We eliminate the phase from (B1). Define

$$a_1 \equiv e^{i\beta_1}\cos\phi, \quad a_2 \equiv e^{i\beta_2}\sin\phi. \quad (B6)$$

The relation between the parameters θ, θ', and α in (B2) and ϕ, β_1, and β_2 in (B6) is given by

$$\begin{aligned}\tan\beta_1 &= ss's''/(cc' - ss'c'') \simeq tt's'', \\ \tan\beta_2 &= cs's''/(sc' - cs'c'') \simeq t's''/t, \\ \tan^2\phi &= [(cs')^2 + (sc')^2 - 2csc's'c'']/[(ss')^2 + (cc')^2 - 2csc's'c''] \simeq t^2\end{aligned} \quad (B7)$$

with

$$c'' = \cos\alpha, \quad s'' = \sin\alpha, \quad t = \tan\theta, \text{ etc.} \quad (B8)$$

for $s' \ll 1$. From (B7), $\phi \simeq \theta$. After eliminating phases from (B1), the respective currents are given by

$$J^- = \tfrac{1}{2}g\{[(\bar{u}d) + (\bar{c}s)]\cos\phi + [(\bar{u}s) - (\bar{c}d)]\sin\phi\},$$

$$J^{-\prime} = \tfrac{1}{2}g\{-[(\bar{u}d)e^{i(\beta_1-\beta_2)} + (\bar{c}s)e^{i(\beta_2-\beta_1)}]\sin\phi + [(\bar{u}s)e^{i(\beta_1-\beta_2)} - (\bar{c}d)e^{i(\beta_2-\beta_1)}]\cos\phi\},$$

and

$$\begin{aligned}J^{n'} = \frac{ig}{2\sqrt{2}}\{&i[(\bar{s}s) - (\bar{d}d)]2c's's'' + (\bar{d}s)e^{-i(\beta_1+\beta_2)}(c'^2 + e^{-2i\alpha}s'^2) \\ &- (\bar{s}d)e^{i(\beta_1+\beta_2)}(c'^2 + e^{2i\alpha}s'^2) + (\bar{c}u)e^{i(\beta_2-\beta_1)} - (\bar{u}c)e^{i(\beta_1-\beta_2)}\}.\end{aligned} \quad (B9)$$

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