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$\eta'(958) \rightarrow 4\pi$ decay in a broken-$SU_6 \times O_3$ quark model

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An attempt is made to calculate the decay rate of the rare four-body decay mode, $\eta'(958)\rightarrow 4\pi$, within the general framework of a broken-$SU_6 \times O_3$ quark model of relativistically invariant coupling structures. The calculation is based on the assumption that the 4π final state is realized from the dominance of the decay by the $\rho\rho$ intermediate state, viz., $\eta' \rightarrow \rho\rho \rightarrow (2\pi)(2\pi)$.

I. INTRODUCTION

The $\eta'(958)$ meson has enjoyed considerable attention ever since its existence was confirmed by Kalbfleisch *et al.*[1] and independently by Goldberg *et al.*[2] from observations in the reactions $K^-p \rightarrow \Lambda$ + neutrals, $\Lambda\pi^+\pi^-$ + neutrals, and $\Lambda\pi^+\pi^+\pi^-\pi^-\pi^0$, where the effective mass of the particles recoiling against the Λ exhibited an enhancement in the 960-MeV region. In spite of the confirmation of its existence, the situation regarding its properties has been rather unclear, especially in relation to the quantum-number assignment and the decay processes. The apparent absence of the $\pi^+\pi^-\pi^0$ decay mode and the studies analyzing the Dalitz plots of the $\eta\pi^+\pi^-$ and $\pi^+\pi^-\gamma$ decay modes, in conjunction with the observation of the $\eta' \rightarrow 2\gamma$ mode suggest the quantum-number assignment $I^G(J^P)=0^+(0^-)$. Though these analyses favor the pseudoscalar ($J^P=0^-$) assignment, they are not sufficiently definitive to rule out the possibility $J^P=2^-$. The spin-parity $J^P=0^-$ assignment is strongly favored since the indication of any anisotropy in the decay of the very forward produced η' has not yet been established. Additional support for the pseudoscalar assignment is furnished by several theoretical considerations,[3] wherein the calculcated two- and three-body strong and electromagnetic decay rates of the η', viz., $\eta' \rightarrow (2\gamma,\ \rho\gamma,\ \omega\gamma,\ \eta\pi\pi,\ \pi\pi\gamma,\ \pi\gamma\gamma$, etc.), were found to be consistent with the available experimental evidence.

The purpose of the present investigation is to continue the program initiated earlier and calculate the very sparse four-body decay process $\eta' \rightarrow 4\pi$ within the general theoretical formulation of a broken-$SU_6 \times O_3$ quark model of relativistically invariant hadron coupling structures.[4] The investigation into the multibody final states is motivated by the fact that these decay channels are expected to be an important source of information on the interaction of unstable particles. At present, however, the experimental data on the absolute decay rate $\eta' \rightarrow 4\pi$ are not adequate enough so as to be important for the purposes of comparing the estimated rate; nevertheless the immense temptation offered by a simple $SU_6 \times O_3$ quark model bestowed with impressive success to try its application to somewhat less conventional decay modes such as $\eta' \rightarrow 4\pi$ is difficult to resist.

In Sec. II, we briefly outline the relativistically invariant meson couplings evaluated within a broken-$SU_6 \times O_3$ quark-model formulation, appropriate for the evaluation of the $\eta' \rightarrow 4\pi$ decay rate. The process $\eta' \rightarrow 4\pi$ is assumed to be dominated by a $\rho\rho$-intermediate state such that $\eta' \rightarrow \rho\rho \rightarrow (2\pi)(2\pi)$. As enumerated above the η' is taken to belong to the pseudoscalar nonet ($J^P=0^-$). The couplings involved in this two-step process correspond to the three-point vertices VVP and VPP, the explicit forms for which are readily available in the present model. With the help of these couplings we compute the $\eta' \rightarrow 4\pi$ decay rate. A discussion of the result is given in Sec. III.

II. MESON COUPLINGS AND CALCULATION OF $\eta' \rightarrow 4\pi$ RATE

The relativistically invariant meson coupling structures necessary for the calculation of the four-body decay process $\eta' \rightarrow 4\pi$ are constructed with the help of a quark-model formulation embodying the broken-$SU_6 \times O_3$ group structure. The version of the model considered here is based on the framework in which the hadron couplings result from the corresponding basic $\bar{q}qP$ vertex. The description of the $\bar{q}qP$ vertex is facilitated by a single quark (q) transition with the emission of a pseudoscalar meson (P), which in the present approach is treated as a radiation quantum. These couplings are then supplemented through the introduction of a suitable phenomenological form factor, so designed as to possess the requisite amount of symmetry breaking in terms of the physical masses of the hadrons corresponding to the interaction vertices. Since a detailed formulation of these couplings is given elsewhere in our earlier work we shall briefly outline only the main steps for the purpose of estimating the $\eta' \rightarrow 4\pi$ decay rate.

As stated in Sec. I, the four-body decay $\eta' \rightarrow 4\pi$

is assumed to proceed as a two-step process via an intermediate $\rho\rho$ state, viz., $\eta'\to\rho\rho\to(2\pi)(2\pi)$. The vertices involved are of the *VVP* and *VPP* type whose explicit forms can be readily obtained from the present model. In particular, the $\eta'\rho\rho$ vertex is described in terms of the coupling

$$\eta'\rho\rho:\ \ im^{-1}\epsilon_{\lambda\mu\beta\nu}(\partial_\lambda\rho^1_\mu\rho^2_\beta)(\partial_\nu\eta')c_{\eta'\rho\rho}\ , \tag{2.1}$$

where $\rho^{1,2}$ and η' represent the fields of the corresponding mesons. The coupling for the $\rho\pi\pi$ vertex can be written down in the form

$$\rho\pi\pi:\ \ i\epsilon_{\alpha\beta\gamma}\rho^\alpha_\mu\pi^\beta(\partial_\mu\pi^\gamma)c_{\rho\pi\pi}\ , \tag{2.2}$$

where, as before, the particle symbols are used to denote the corresponding fields. $c_{\eta'\rho\rho}$ and $c_{\rho\pi\pi}$ are the appropriate SU_3 coefficients.

In the present scheme, the η', ρ, and π mesons participating in the interaction are assigned to the $L=0$ supermultiplet. Therefore, the $\eta'\rho\rho$ and $\rho\pi\pi$ vertices are characterized by transitions within the same multiplet (i.e., $L=0$ to $L=0$). In this case the relevant form factor acquires the form

$$f_0=g_0\mu^{-1}(\mu/m)^{1/2}(2m)\ , \tag{2.3}$$

where m and μ refer to the masses of the decaying and the emitted (pseudoscalar) mesons, respectively, and g_0 is a dimensionless coupling constant describing the entire supermultiplet transition. The value of g_0 has already been adjusted[3,4] at the value $g_0{}^2/4\pi=0.03$. The dressed couplings are then obtained by multiplying expressions (2.1) and (2.2) by the form factor (2.3).

These prescriptions are now applied to the calculation of the decay rate for $\eta'\to\rho\rho\to 4\pi$. The invariant T-matrix element squared for this four-body strong decay process can be cast in the form

$$|T|^2=K^2\frac{m_\pi{}^4}{8m_\rho{}^2}\frac{-4m_{\eta'}{}^2s_2+(s_1+u_1+4m_{\eta'}{}^2+2m_\pi{}^2)^2}{[(-s_2+s_1+u_1+3m_{\eta'}{}^2+2m_\pi{}^2+m_\rho{}^2)^2+m_\rho{}^2\Gamma_\rho{}^2][(-s_2+m_\rho{}^2)^2+m_\rho{}^2\Gamma_\rho{}^2]}\ , \tag{2.4}$$

where $K=c_{\eta'\rho\rho}f_{\eta'\rho\rho}c_{\rho\pi\pi}{}^2f_{\rho\pi\pi}{}^2$, and s_1, s_2, and u_1 are Mandelstam-type variables defined in terms of the four momenta of the η' and the π mesons in the final state. Using the standard covariant phase-space calculations of Kumar[5] for a four-particle final state, the resulting expression for the decay width $\Gamma(\eta'\to 4\pi)$ can be written in the form

$$\Gamma(\eta'\to 4\pi)=\frac{\pi^3}{4m_{\eta'}{}^3}\int_{9m_\pi{}^2}^{(m_{\eta'}-m_\pi)^2}ds_1\int_{4m_\pi{}^2}^{(\sqrt{s_1}-m_\pi)^2}ds_2\int_{u_{1-}}^{u_{1+}}du_1\,\frac{1}{s_2}\,\lambda^{1/2}(s_2,m_\pi{}^2,m_\pi{}^2)|T|^2\ , \tag{2.5}$$

where

$$u_{1\pm}=m_{\eta'}{}^2+m_\pi{}^2-\tfrac{1}{2}(s_1+m_{\eta'}{}^2-m_\pi{}^2)\pm\frac{1}{2s_1}\,\lambda^{1/2}(s_1,m_\pi{}^2,m_\pi{}^2)\lambda^{1/2}(m_{\eta'}{}^2,s_1,m_\pi{}^2)$$

and $\lambda(x,y,z)=x^2+y^2+z^2-2xy-2yz-2zx$.

Substituting for $|T|^2$ from (2.4) into (2.5), we can write

$$\Gamma(\eta'\to 4\pi)=K^2\frac{m_\pi{}^4\pi^3}{32m_\rho{}^2m_{\eta'}{}^3}\,I_p(\eta'\to 4\pi)\ , \tag{2.6}$$

where $I_p(\eta'\to 4\pi)$ is a phase-space integral having the explicit form

$$I_p(\eta'\to 4\pi)=\int_{9m_\pi{}^2}^{(m_{\eta'}-m_\pi)^2}ds_1\int_{4m_\pi{}^2}^{(\sqrt{s_1}-m_\pi)^2}\frac{ds_2}{s_2}\,\frac{\lambda^{1/2}(s_2,m_\pi{}^2,m_\pi{}^2)}{(-s_2+m_\rho{}^2)^2+m_\rho{}^2\Gamma_\rho{}^2}$$
$$\times\int_{u_{1-}}^{u_{1+}}du_1\,\frac{-4m_{\eta'}{}^2s_2+(s_1+u_1+4m_{\eta'}{}^2+2m_\pi{}^2)^2}{(-s_2+s_1+u_1+3m_{\eta'}{}^2+2m_\pi{}^2+m_\rho{}^2)^2+m_\rho{}^2\Gamma_\rho{}^2}\ . \tag{2.7}$$

The integration over u_1 can be performed analytically,[6] thereby reducing (2.7) to the expressions

$$I_p(\eta'\to 4\pi)=\int_{9m_\pi{}^2}^{(m_{\eta'}-m_\pi)^2}ds_1\int_{4m_\pi{}^2}^{(\sqrt{s_1}-m_\pi)^2}\frac{ds_2}{s_2}\,\frac{\lambda^{1/2}(s_2,m_\pi{}^2,m_\pi{}^2)}{(-s_2+m_\rho{}^2)^2+m_\rho{}^2\Gamma_\rho{}^2}\,f(s_1,s_2) \tag{2.8}$$

with

$$f(s_1,s_2)=\frac{1}{s_1}\,\lambda^{1/2}(s_1,m_\pi{}^2,m_\pi{}^2)\lambda^{1/2}(m_{\eta'}{}^2,s_1,m_\pi{}^2)-\xi\ln\left(\frac{u_{1+}{}^2+4\beta u_{1+}+\delta}{u_{1-}{}^2+4\beta u_{1-}+\delta}\right)$$
$$-\alpha\left[\arctan\left(\frac{u_{1+}+\beta}{\gamma}\right)-\arctan\left(\frac{u_{1-}+\beta}{\gamma}\right)\right]\ , \tag{2.9}$$

where

$$\gamma = m_\rho \Gamma_\rho, \quad \xi = m_\rho{}^2 - m_{\eta'}{}^2 - s_2, \quad \beta = -s_2 + s_1 + 3m_{\eta'}{}^2 + 2m_\pi{}^2 + m_\rho{}^2 ,$$
$$\delta = \beta^2 + \gamma^2, \quad \alpha = \gamma^{-1}(\gamma^2 - \xi^2 + 4m_{\eta'}{}^2 s_2) . \tag{2.10}$$

The width of the decay process $\eta' \to 4\pi$ can now be readily estimated from expressions (2.6) through (2.10), performing numerical integration over the variables s_1 and s_2. In computing these expressions we have used[7] $m_{\eta'} = 958$ MeV, $m_\rho = 770$ MeV, $\Gamma_\rho = 150$ MeV, and $m_\pi = 140$ MeV. The resulting decay rate turns out to be

$$\Gamma(\eta' \to 4\pi) = 2.4 \times 10^{-4} \text{ MeV} . \tag{2.11}$$

This value is roughly 2 orders of magnitude below the observed experimental upper bound[7] which is about 10^{-2} MeV.

III. DISCUSSION

We have attempted to study the rare four-body strong decay mode, $\eta' \to 4\pi$, treating it as a two-step process through ρ dominance, viz., $\eta' \to \rho\rho \to 4\pi$, using the broken-$SU_6 \times O_3$ quark model with phenomenological hadron couplings. The description is based on the plausible assumption that the η' belongs to the pseudoscalar nonet ($J^P = 0^-$). We have taken η' to be pure SU_3 singlet with the quark contents $\eta' = (1/3)^{1/2}(\overline{u}u + \overline{d}d + \overline{s}s)$, thus neglecting the effect of any mixing[8] between η and η'. The resulting decay width is obtained to be 2.4×10^{-4} MeV, which is about 2 orders of magnitude short of the observed experimental upper bound.

It is tantalizing, however, to note that a serious comparison of the theoretical prediction can hardly be made particularly when the currently available experimental information on the absolute decay rate of $\eta' \to 4\pi$ is not sufficiently definitive. Indeed there exists only the experimental upper limit on this rate and any model yielding a value below this limit is naturally acceptable. In this sense, the model used here does not seem to be inconsistent when confronted with experimental evidence. We look forward to comparing the predictions of this model with more precise data in the future. Mention should also be made that contribution to the $\eta' \to 4\pi$ decay can come from other decay mechanisms, viz., $\eta' \to (A_1, A_2)\pi \to (\rho\pi)\pi \to 4\pi$, which, strictly speaking, ought to be considered in the evaluation of the decay rate. We have, however, not explored these possibilities. In any case, given the nature of the upper bound where minor violations are not very critical, the contribution due to these decay channels may be ignored at least as a first approximation.

The main advantage of the present model lies in its mathematical simplicity and ability to provide a unified treatment of both baryons and mesons. Its performance has been credited with splendid success in accounting for a tremendous proliferation of decay patterns of hadrons in terms of a single dimensionless coupling constant governing the entire supermultiplet transition. The present calculation demonstrates the internal consistency and the relative ease with which the $SU_6 \times O_3$ quark model is capable of describing rather complicated decay processes.

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