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Where and what are the scalar mesons?

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A self-consistent analysis of the $J^{PC} = 0^{++}$ partial waves obtained from high-statistics $K\pi$, $K\bar{K}$, and $\pi\pi$ data has been performed to verify the existence and to determine the resonance parameters of the $I = 1/2$ and $I = 0$ scalar mesons. A mass-dependent parametrization of the $K\pi$ partial wave yields one and only one strange scalar meson, the $\kappa(1510)$, with a width of several hundred MeV. Isoscalar 0^{++} mesons found in simultaneous fits to $K\bar{K}$ and $\pi\pi$ data are the $\epsilon(800)$, $S^*(1005)$, and $\epsilon'(1540)$ with widths of about 1000, 8, and 200 MeV, respectively. No simple interpretation of all these states and the $\delta(980)$ and possible $\delta'(1300)$ as $q\bar{q}$ mesons, $q\bar{q}q\bar{q}$ bound states, or gluon-gluon bound states exists. Possible candidates for an SU(3) nonet of $q\bar{q}$ states are found to be (a) $\epsilon(800)$, $\epsilon'(1540)$, $\sigma(980)$, and $\kappa(1540)$ or (b) $S^*(1005)$, $\epsilon'(1540)$, $\sigma'(1300)$, and $\kappa(1540)$.

I. INTRODUCTION

The scalar mesons ($J^{PC} = 0^{++}$) are supposed, in a simple $q\bar{q}$ interpretation of the mesons, to be associated with the lowest-lying nonets of tensor and axial-vector mesons. Of the uncharmed mesons, only the 2^{++} nonet is well established; the 1^{++} mesons suffer from problems of separating Deck from resonance effects, while the 0^{++} mesons are well masked by the existence of higher-angular-momentum states at about the same masses. In fact, the only clean example of 2^{++}, 1^{++}, and 0^{++} states is the $c\bar{c}$ χ mesons. It is important to establish the existence of the uncharmed scalar mesons if one wishes to examine the spin-dependent quark-quark forces.

Theoretically, one also expects in the MIT bag model[1] and, presumably, in almost any model in which the quarks are bound, to have mesons which are $q\bar{q}q\bar{q}$ bound states. In Ref. 1, the lowest lying of such states were found to be scalar mesons with masses around 1 GeV. The situation is further complicated by the possible existence of flavorless 0^{++} gluon-gluon bound states.[2] The determination of the masses and couplings of the scalar mesons is clearly necessary before one can attempt to interpret them as $q\bar{q}$, $q\bar{q}q\bar{q}$, or gg states.

Experimentally, the study of the scalar mesons is also difficult. Although they can decay to two pseudoscalars, the 0^{++} mesons are masked by the leading peripheral 2^{++} mesons so that they do not generally appear as bumps in cross sections.

Moreover, the existence of large S-wave background and/or overlapping resonances means they may even appear as dips rather than bumps in the S-wave cross section.[3] The determination of resonance parameters, therefore, requires careful, preferably coupled-channel, fits to the 0^{++} partial-wave amplitudes. Since the work of Ref. 4, which concluded that there was indeed an SU(3) nonet of scalar mesons, new high-quality data and partial-wave analyses of $K\pi$ (Ref. 5) and $K\bar{K}$ (Refs. 6–8) data have appeared.

Here we present, in Sec. II, a discussion of the candidates for the noncharmed scalar mesons and, where possible and necessary, a determination of resonance pole positions and couplings. Since the SU(3) properties of $q\bar{q}$, $q\bar{q}q\bar{q}$, and gg states are different, we investigate, in Sect. III, the SU(3) properties of the scalar mesons of Sec. II. Section IV contains a summary of results and our conclusions.

II. THE SCALAR MESONS

It is convenient for discussion purposes to classify the scalar mesons according to isopin. We therefore discuss the evidence for $I=1$ scalar mesons in Sec. II A. In Sec. II B, we determine the resonance parameters of the strange ($I=\frac{1}{2}$) scalar mesons by fitting the mass dependence of the $K\pi$ S wave. In Sec. II C, we combine $\pi\pi$ and $K\bar{K}$ data to determine the masses and couplings of the $I=0$ scalar mesons.

A. The isovector scalar mesons

A recent analysis[9] of $\delta(980)$ production in the reactions $K^-p \to \eta\pi^-\Sigma(1385)^+$ and $K^-p \to K^0K^-\Sigma(1385)^+$ at 4.2 GeV/c (Ref. 10) finds $m_\delta = 979 \pm 5$ MeV, $\Gamma_{\eta\pi} = 51 \pm 4$ MeV, and $g^2_{\delta K\bar{K}}/g^2_{\delta\eta\pi}$ consistent with the SU(3) value of $\frac{3}{2}$. A large ($\Gamma_\delta \sim 300$ MeV width for the δ, as suggested in Ref. 11, appears to be ruled out by the behavior of the K^0K^- mass spectrum above 1060 MeV.[10,12]

Partial-wave analyses of the reactions $\pi^-p \to K_SK_Sn$ (Ref. 6) and $\pi^-p \to K_SK^-p$ (Ref. 7) show a bump in the $K\bar{K}$ S-wave magnitude near 1.3 GeV. The interpretation of this bump as a resonance requires corroboration from $\pi^-p \to K^-K^+n$ and π^+n

$\to K^- K^+ p$ data,[8] as well as from $\eta\pi$ data. A reliable determination of the resonance parameters of the $\delta(980)$ and $\delta'(1270)$ would require joint fits to $K\bar{K}$ and $\eta\pi$ data.

B. Strange scalar mesons

Previous SU(3) analyses of the scalar mesons[4,6] assumed the existence of a strange scalar meson, the κ, with a mass of about 1.1 GeV. However, recent high-statistics $K\pi$ data[5] eliminate the possibility of such a low mass for the κ, and suggest, instead, that κ has a mass near 1.45 GeV. The $I=\frac{1}{2}$ $K\pi$ S-wave phase, shown in Fig. 1, is a slowly increasing function of $K\pi$ mass and is well described by an effective-range parametrization for $K\pi$ masses from 0.7 to about 1.3 GeV where the phase finally reaches 90°. The S-wave magnitude, after its steady rise from threshold to about 1.3 GeV, rapidly decreases with increasing $K\pi$ mass starting at 1.4 GeV. Associated with this behavior is a reasonably rapid phase motion. Although there are discrete ambiguities in the determination of the $K\pi$ partial waves from the data for $K\pi$ masses greater than about 1.45 GeV, this resonancelike behavior of the S wave occurs in all four possible solutions, as illustrated by the Argand diagrams of Fig. 2. In order to update and examine the SU(3) structure of the 0^+ nonet, it is important to determine the possible range of κ

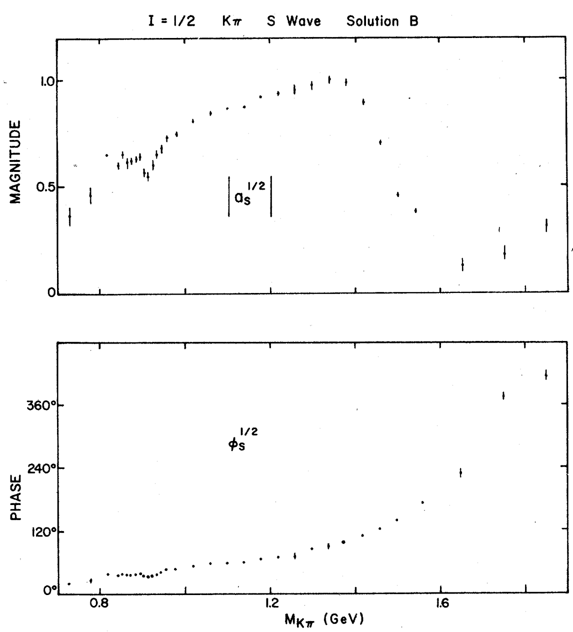

FIG. 1. The magnitude and phase of the $J^P=0^+$, $I=\frac{1}{2}$ $K\pi$ partial wave for solution B of Ref. 5. The unitarity limit for the magnitude is unity.

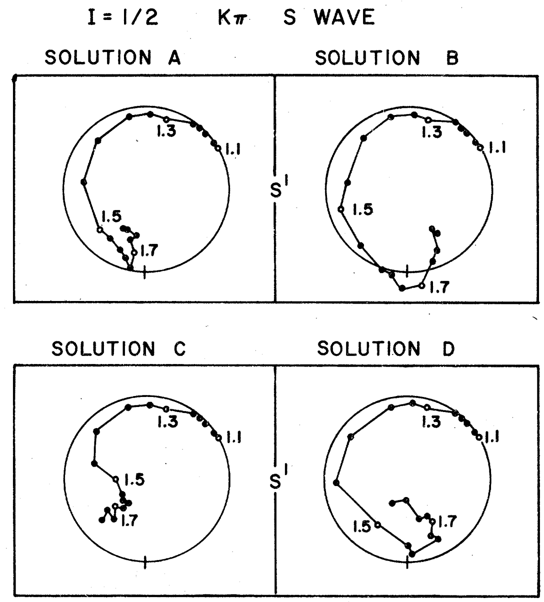

FIG. 2. Argand diagrams for the $I=\frac{1}{2}$ $K\pi$ S waves of Ref. 5 for all four partial-wave solutions.

resonance parameters. However, the determination of resonance parameters from the 0^+ partial wave of any one solution is complicated by (i) the large elastic $K\pi$ background or, possibly, extremely broad resonance, and (ii) the nonelasticity of the partial waves above about 1.3 GeV. It is therefore essential to allow for the possibility of inelastic channels and to be able to handle overlapping resonances properly if one is to extract meaningful resonance parameters from the data.

Since there is no sign of inelasticity in the $K\pi$ S wave below 1.3 GeV, we choose to consider $K\eta'$ as the only important inelastic channel. This choice is further motivated by the SU(3) prediction $\Gamma(\kappa\to K\eta)<\frac{1}{10}\Gamma(\kappa\to K\pi)$. Unfortunately, there are no data available on either the $K\eta$ or the $K\eta'$ S wave. Least-squares fits to the $I=\frac{1}{2}$ $K\pi$ S-wave magnitude and phase as functions of $K\pi$ mass were performed in order to determine the number, masses, and couplings of κ resonances. In order to investigate the sensitivity of the results to the form of the parametrization of the mass dependence, we tried several different forms. First we parametrized the M-matrix[13] elements, M_{ij},

$$\begin{aligned} M_{11} &= a_{11}+\tfrac{1}{2}rq_1{}^2 , \\ M_{12} &= a_{12} , \\ M_{22} &= a_{22} , \end{aligned} \tag{1}$$

where q_1 is the K momentum in the $K\pi$ rest frame, the subscripts 1 and 2 refer to $K\pi$ and $K\eta'$ channels, respectively, and r and the a_{ij} are param-

TABLE I. Position of the second-sheet pole for the $J^P = 0^+ K\pi$ resonance.

Partial-wave solution	Re M_κ (GeV)	$-$Im M_κ (GeV)	$g_{K\eta'}/g_{K\pi}$
A	1.51 ± 0.02	0.13 ± 0.03	1.0 ± 0.2
B	1.49 ± 0.02	0.20 ± 0.05	0.6 ± 0.2
C	1.57 ± 0.02	0.06 ± 0.02	1.0 ± 0.2
D	1.48 ± 0.02	0.14 ± 0.02	0.6 ± 0.2

eters to be determined by the fit. Recall that the relation between the T and M matrices is given by

$$\underline{T} = \underline{Q}^{1/2}(\underline{M} - i\underline{Q})^{-1}\underline{Q}^{1/2} \tag{2}$$

with

$$\underline{Q} \equiv \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix}. \tag{3}$$

The second type of fit involved a parametrization of the $K\pi$ scattering amplitude (T_{11}) as the sum of an inelastic Breit-Wigner resonance and either an elastic resonance or a background term:

$$T_{11} = T_{\text{bkg}} + T_{\text{res}} e^{i\phi}, \tag{4}$$

$$T_{\text{res}} = \frac{M_R \Gamma_1}{M_R^2 - M_{K\pi}^2 - iM_R(\Gamma_1 + \Gamma_2)} , \tag{5}$$

$$\Gamma_i = q_i \Gamma_i^0 .$$

The "background" term $T_{\text{bkg}} = \sin\delta_B e^{2i\delta_B}$ was parametrized as an elastic resonance ($\Gamma_2^0 = 0$) or by an effective range form

$$q_1 \cot\delta_B = \frac{1}{a} + \frac{1}{2} r q_1^2 . \tag{6}$$

In either case, unitarity requires $\phi = 2\delta_B$.

The third type of fit was based on a parametrization of the K-matrix elements, K_{ij}. Recall the relationship between T and K matrices,

$$\underline{T} = 1 - i\underline{K})^{-1}\underline{K}. \tag{7}$$

The K-matrix elements were parametrized as

$$K_{ij} = (q_i q_j)^{1/2}\left(b_{ij} + \frac{g_i g_j}{S_R - M_{K\pi}^2} + \frac{f_i f_j}{S_{R'} - M_{K\pi}^2}\right) . \tag{8}$$

We found that each of the above prescriptions provided a good qualitative description of the mass dependence of the $K\pi$ S wave. The resonance parameters, M_R, Γ_i of Eq. (5), or $S_R, S_{R'}, g_i, f_i$ of Eq. (8), were very sensitive to the parametrization used. However, we found that the positions and residues of the T_{11} poles did not depend on how the scattering amplitude was parametrized or on the number of resonances included in the T- or K-matrix fits. All the fits resulted in a second sheet pole very near the $K\eta'$ threshold.

The positions and couplings of this pole for the four different discrete solutions for the input $K\pi$ partial waves are listed in Table I. The errors quoted include estimates of the uncertainties due to different parametrizations and the indeterminacy in the choice of overall phase.

Attempts to find a second $K\pi$ resonance resulted either in a high mass (~10 GeV) "resonance" mimicking the background or, if the mass was forced to be below 1.3 GeV, in vanishingly small couplings. We therefore conclude that there is a $K\pi$ S-wave resonance, the $\kappa(1510)$, but no evidence for any lower mass resonance. We defer to Sec. III the discussion of the implications of this statement on the SU(3) properties of the scalar mesons.

C. Isoscalar scalar mesons

The $S^*(993)$ is the only well established scalar meson, although bumps in $\pi\pi$ cross sections have, on occasion, led people to postulate the existence of an $\epsilon(700)$ and/or an ϵ' (~1300). Moreover, a bump in the $I=0$ $K\bar{K}$ cross section has led to the suggestion of an $S^{*\prime}$ (~1300).[6] Estimates of the ϵ' and $S^{*\prime}$ couplings to $\pi\pi$ and $K\bar{K}$ have been obtained by considering the $\pi\pi$ or the $K\bar{K}$ data separately.[6,12] Reliable determinations of the couplings can only come from *simultaneous* fits to both sets of data. In fact, although the S^* mass is well known, estimates of the ratio of its couplings to $K\bar{K}$ and $\pi\pi$ range from 0.9 (Ref. 6) to 2.0.[12] Simultaneous fits to $\pi\pi$ and $K\bar{K}$ data should determine not only the number and positions of resonances, but also their relative $K\bar{K}/\pi\pi$ couplings.

The extraction of partial waves from $\pi\pi$ and $K\bar{K}$ data suffers from the same discrete ambiguity problem as for $K\pi$. Although the $\pi\pi$ S wave must have even isotopic spin, the $K\bar{K}$ S wave is a superposition of isospin zero and one. However, the isoscalar $K\bar{K}$ partial waves can be determined by combining $\pi^- p \to K\bar{K}n$ data with that for $\pi^+ n \to K\bar{K}$ $\pi^+ n \to K\bar{K}p$.[8] Furthermore, the requirement that the $I=1$ $K\bar{K}$ P wave be dominated by the high-mass tail of the ρ meson decaying into $K\bar{K}$ with a ρKK coupling given by SU(3) eliminates[8] all but one of

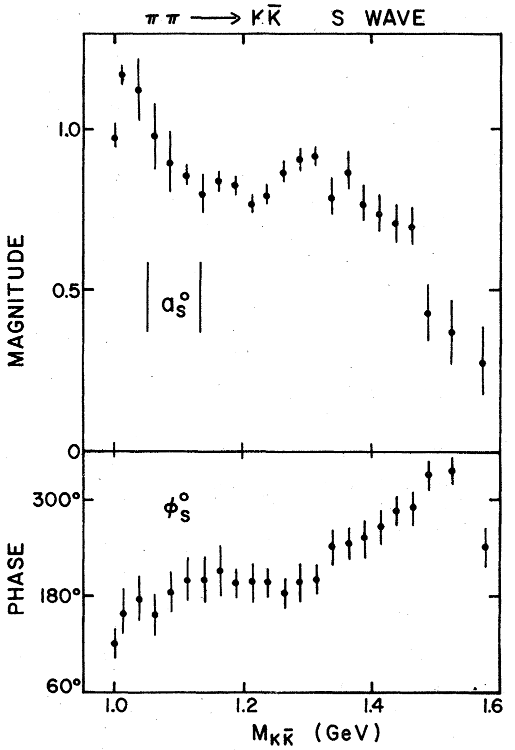

FIG. 3. The magnitude and phase of the $I=0$ $K\overline{K}$ S wave for solution I(b) of Ref. 8. One is the unitarity limit for the magnitude.

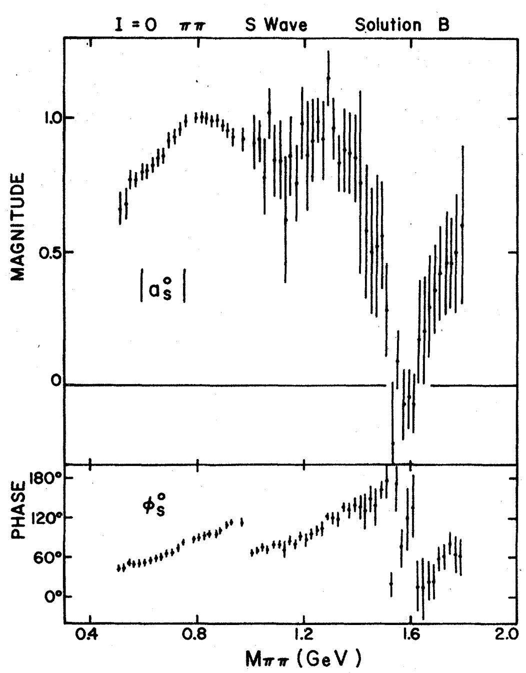

FIG. 4. The magnitude and phase of the $I=0$ $\pi\pi$ S wave for solution B of Ref. 15. Unitarity requires the magnitude to be less than one.

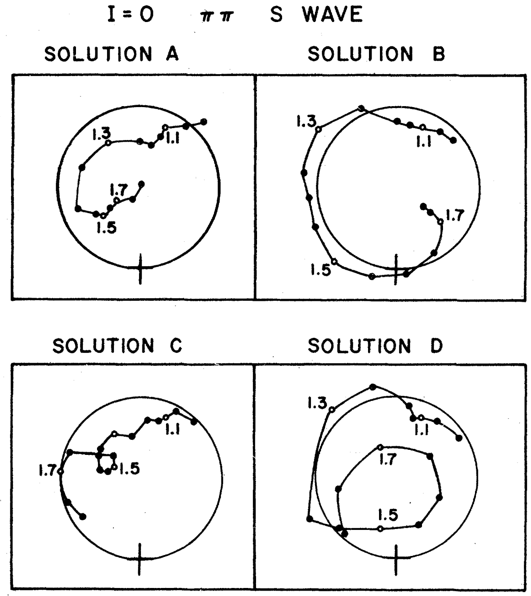

FIG. 5. Argand diagrams for the $I=0$ $\pi\pi$ S waves of Ref. 15 for all four partial-wave solutions.

the possible $K\overline{K}$ solutions. The magnitude and phase of the remaining isoscalar $\pi\pi \to K\overline{K}$ amplitude are shown in Fig. 3. The rapid rise and subsequent fall of the cross section just above $K\overline{K}$ threshold is generally attributed to the S^*, the bump at 1.3 GeV to the ϵ'.

The $\pi\pi$ isoscalar S wave[14,15] rises smoothly from threshold to just below 1 GeV, as can be seen in Fig. 4. The "up" solution between 700 and 900 MeV has been ruled out by comparison of $\pi^+\pi^-$ data[15] with that for $\pi^0\pi^0$.[16] For $\pi\pi$ masses above the S^*, all four solutions show resonance-like behavior; see the Argand diagrams of Fig. 5. At the present time, there exist no $\pi^0\pi^0$ data in the mass region above 1.2 GeV to distinguish between the various solutions. One might hope to select the physical solution by comparing with inelastic channels such as $K\overline{K}$, but the inelastic $\pi\pi$ cross-section predictions are so exceedingly sensitive to the choice of overall phase that all four $\pi\pi$ solutions are compatible with the $K\overline{K}$ data. Furthermore, the data are consistent with $K\overline{K}$ being the only important inelastic channel. One might hope that only one of the different partial-wave solutions would have acceptable analyticity properties. However, investigations of analyticity constraints[17] have not, in general, suceeded in selecting the physical solution.

As was the case for the strange scalar mesons discussed in Sect. II B, any realistic attempt to describe the $\pi\pi$ S wave must allow for the effect

TABLE II. Positions and couplings of the isoscalar $J^P = 0^+$ resonances, as determined from simultaneous fits to $\pi\pi$ (Ref. 15) and $K\bar{K}$ (Ref. 8) data.

Resonance	Solution	ReM (GeV)	$-$ImM (GeV)	$g_{K\bar{K}}/g_{\pi\pi}$
ϵ	A	0.80 ±0.10	0.50 ±0.20	0.2±0.2
	B	0.75 ±0.10	0.40 ±0.20	0.2±0.2
	C	0.72 ±0.10	0.50 ±0.15	0.1±0.2
	D	0.80 ±0.10	0.50 ±0.20	0.2±0.2
S^{*a}	A	1.005±0.003	0.004±0.002	1.8±0.5
	B	1.005±0.003	0.004±0.004	1.7±0.5
	C	1.006±0.002	0.004±0.004	1.8±0.5
	D	1.005±0.002	0.004±0.002	1.7±0.5
ϵ'	A	1.55 ±0.05	0.07 ±0.02	−1.7±0.3
	B	1.60 ±0.05	0.14 ±0.03	−0.5±0.2
	C	1.50 ±0.05	0.16 ±0.03	−2.4±0.4
	D	1.53 ±0.05	0.16 ±0.02	−2.0±1.0

[a]We quote here the position of the S^* pole on the second sheet; the third-sheet pole is farther removed from the physical region with $M = (0.99-0.01i)$ GeV.

of inelastic channels. Fortunately, $\pi\pi \rightarrow K\bar{K}$ data[6,8] are available and so the $\pi\pi$ fits are much more strongly constrained than were the $K\pi$. We fit, for each of the four $\pi\pi$ solutions, the $\pi\pi$ and $K\bar{K}$ mass using a K-matrix resonance plus background parametrization such as that of Eq. (8), but this time allowing for up to four pole terms. We found that three poles were, in fact, necessary to obtain a qualitatively acceptable description of the data. The resulting pole positions and residues (couplings) are listed in Table II. Once again, the errors quoted reflect systematic rather than statistical uncertainties. It is encouraging to note that the parameters of the $\epsilon(800)$ and $S^*(1005)$ are the same in the fits to all four solutions, which differ only for masses above 1.2 GeV. On the other hand, the ϵ' (~1550) mass, width, and couplings depend on the solution. We found no evidence for more than three resonances.

In summary then, we have found, by fitting simultaneously the $\pi\pi$ and $K\bar{K}$ data, evidence for three isoscalar resonances, the $\epsilon(800)$, $S^*(1005)$, and ϵ' (1540). We find that these three resonances, plus inelastic background, provide good qualitative descriptions of both the $\pi\pi$ and $K\bar{K}$ S waves.

III. SU(3) PROPERTIES OF THE SCALAR MESONS

The mass spectrum of the scalar mesons discussed in the preceding section is summarized in Fig. 6. Because of the lack of evidence for the $\delta'(1270)$ in $\pi\eta$ data and the existence of a $K\bar{K}$ partial-wave solution with no S-wave structure in the 1.3-GeV region,[7] we do not consider this state sufficiently well established to be included in Fig. 6. In contrast to the analysis of Ref. 4, we find three isoscalar states, one more than required to form an SU(3) nonet. The existence of this extra state poses a problem for the interpretation of all these states as either normal $q\bar{q}$ states in the same SU(6) $\times$O(3) supermultiplet as the A_2 and A_1 nonets or as the $q\bar{q}q\bar{q}$ bound states expected in the MIT bag model[1] or, presumably, any other model with confined quarks. We next explore whether SU(3) can be used to select a unique nonet of $q\bar{q}$ states from this surplus of 0^{++} candidates.

Mesons which are ordinary $q\bar{q}$ states are supposed to be members of SU(3) nonets; their masses and widths are therefore related by SU(3). We consider first the SU(3) constraints on the masses. We first assume that the $\delta(980)$ and $\kappa(1510)$ are the $I=1$ and $I=\frac{1}{2}$ members of an SU(3)

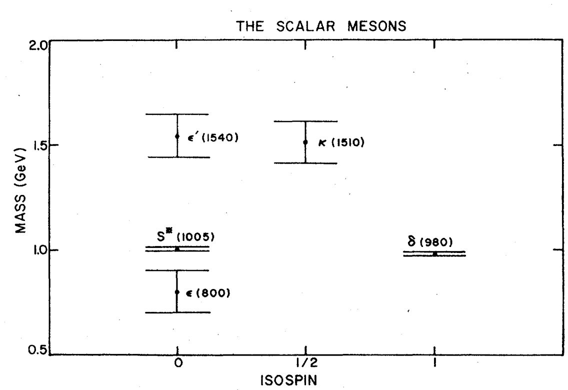

FIG. 6. The masses of the $J^P = 0^+$ resonances. The error bars include the spreads in values obtained from different partial-wave solutions.

octet and denote by σ_8 are isoscalar octet member and by σ_1 the SU(3) singlet. The physical isoscalar states, σ and σ', are then given in terms of the SU(3) eigenstates by

$$\begin{aligned}\sigma &= \sigma_8 \cos\theta + \sigma_1 \sin\theta ,\\ \sigma' &= -\sigma_8 \sin\theta + \sigma_1 \cos\theta .\end{aligned} \tag{9}$$

The particle masses are related to the octet isoscalar mass M_8, the singlet mass M_1, and the singlet-octet mass-mixing term M_{18}, by

$$\begin{aligned}M_\sigma &= M_8 \cos^2\theta + M_1 \sin^2\theta + 2M_{18} \sin\theta \cos\theta ,\\ M_{\sigma'} &= M_8 \sin^2\theta + M_1 \cos^2\theta - 2M_{18} \sin\theta \cos\theta ,\end{aligned} \tag{10}$$

where M_8 is given by the Gell-Mann–Okubo rule:

$$M_8 = \tfrac{1}{3}(4M_\kappa - M_\delta). \tag{11}$$

The requirement that the mass matrix be diagonal with respect to σ and σ' relates M_{18}, M_8, M_1, and θ by

$$M_{18} = \tfrac{1}{2}(M_8 - M_1)\tan 2\theta . \tag{12}$$

On the other hand, M_{18} can also be expressed in terms of quark wave functions, namely

$$M_{18} = -\frac{2\sqrt{2}}{3}(M_\kappa - M_\delta)\langle \psi_8/\psi_1 \rangle , \tag{13}$$

where $\langle \psi_8/\psi_1 \rangle$ is the overlap between singlet and octet spatial wave functions. The naive quark model has $\langle \psi_8/\psi_1 \rangle = 1$, $M_1 = \frac{1}{3}(2M_\kappa + M_\delta)$, and thence, via Eq. (12), $\theta \simeq -35^\circ$ or so called "magic" mixing. For the scalar mesons, the naive quark model thus has a σ of the same mass as the $\delta(980)$ and a σ' at a mass of $2M_\kappa - M_\delta$ which is over 2 GeV. In other words, no two of the isoscalar mesons of Fig. 6 can form a magically mixed SU(3) nonet including the $\delta(980)$ and $\kappa(1510)$.

In contrast to the naive quark model in which M_1, the mixing angle, and $\langle \psi_8/\psi_1 \rangle$ are determined, SU(3) alone contains no information on M_1, the mixing angle, or $\langle \psi_8/\psi_1 \rangle$. However, it is clear that $\langle \psi_8/\psi_1 \rangle$ must vanish in the limit of no singlet-octet mixing, that is, for θ of 0° or 90°. If we, furthermore, require $\langle \psi_8/\psi_1 \rangle = 1$ for $\theta \simeq -35^\circ$, we can satisfy all these requirements by setting

$$\langle \psi_8/\psi_1 \rangle = -\frac{3}{2\sqrt{2}} \sin 2\theta . \tag{14}$$

We can now calculate M_σ and $M_{\sigma'}$ as functions of the mixing angle if we know M_κ and M_δ and use the prescription of Eq. (14) for $\langle \psi_8/\psi_1 \rangle$. The results of such a calculation are shown in Fig. 7 where the widths of the σ and σ' mass bands correspond to allowing the κ mass to vary by ± 100 MeV from its central value of 1.51 GeV. It is evident from Fig. 7 that the only pair of isoscalar mesons of Fig. 6 which could belong in the same SU(3) nonet with the $\kappa(1510)$ and $\delta(980)$ are the $\epsilon(800)$ and $\epsilon'(1540)$ and that the mixing angle θ must be approximately $\pm 20^\circ$.[18]

Before deciding on which, if any, of the isoscalar mesons of Fig. 6 belong in an SU(3) nonet with the $\kappa(1510)$ and $\delta(980)$, we must also investigate the SU(3) constraints on the resonance widths. The partial width for the decay of a scalar meson a into two pseudoscalar mesons b and c can be expressed in terms of the octet coupling constant g_8, the singlet coupling g_1, the mixing angle θ, and the SU(3) Clebsch-Gordan coefficients C^a_{bc}, namely

$$\Gamma(a \to bc) = \frac{q}{M_a^2}(C^a_{bc} g_8)^2 \tag{15}$$

for a pure octet state and

$$\begin{aligned}\Gamma(\sigma \to bc) &= \frac{q}{M_\sigma^2}(C^{\sigma_8}_{bc} g_8 \cos\theta + C^{\sigma_1}_{bc} g_1 \sin\theta)^2 ,\\ \Gamma(\sigma \to bc) &= \frac{q}{M_{\sigma'}^2}(-C^{\sigma_8}_{bc} g_8 \sin\theta + C^{\sigma_1}_{bc} g_1 \cos\theta)^2\end{aligned} \tag{16}$$

for the isoscalar resonances. Here q is the magnitude of the pseudoscalar-meson momentum in the resonance rest frame. The measured scalar-meson widths are

$$\begin{aligned}\Gamma(\delta \to \eta\pi) &= \tfrac{1}{5}\frac{q}{M_\delta^2} g_8^2 ,\\ \Gamma(\kappa \to K\pi) &= \tfrac{9}{20}\frac{q}{M_\kappa^2} g_8^2 ,\\ \Gamma(\sigma \to \pi\pi) &= \frac{q}{M_\sigma^2}\left[-(\tfrac{3}{5})^{1/2} g_8 \cos\theta + \frac{\sqrt{3}}{2\sqrt{2}} g_1 \sin\theta\right]^2 ,\\ \Gamma(\sigma \to K\bar{K}) &= \frac{q}{M_\sigma^2}\left(\frac{1}{\sqrt{10}} g_8 \cos\theta + \tfrac{1}{2} g_1 \sin\theta\right)^2 ,\\ \Gamma(\sigma' \to \pi\pi) &= \frac{q}{M_{\sigma'}^2}\left[+(\tfrac{3}{5})^{1/2} g_8 \sin\theta + \frac{\sqrt{3}}{2\sqrt{2}} g_1 \cos\theta\right]^2 ,\\ \Gamma(\sigma' \to K\bar{K}) &= \frac{q}{M_{\sigma'}^2}\left(-\frac{1}{\sqrt{10}} g_8 \sin + \tfrac{1}{2} g_1 \cos\theta\right)^2 .\end{aligned} \tag{17}$$

We have ignored the small η-η' mixing in calculating $\Gamma(\delta \to \eta\pi)$. Note also that we cannot include the partial widths into states involving η' in this SU(3) comparison. We performed a least-squares fit to the masses and widths of the $\epsilon(800)$, $\epsilon'(1540)$, $\delta(980)$, and $\kappa(1510)$ to see whether the masses and widths were compatible with SU(3) and to determine the singlet-octet mixing angle. In these fits, as in Fig. 6, the errors on the resonance masses, as well as the widths, were increased from those of Tables I and II to account for the differences between different $\pi\pi$ and $K\pi$ partial-wave solutions. We found that the $\epsilon(800)$ and $\epsilon'(1540)$ masses and widths were in fact such that these two isoscalars could be included in an SU(3) nonet with the $\kappa(1510)$ and $\delta(980)$. The SU(3) mixing angle was found to

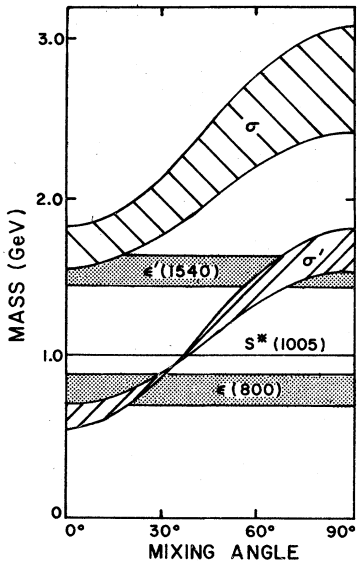

FIG. 7. SU(3) limits on the isoscalar σ and σ' meson masses from the $\kappa(1510)$ and $\delta(980)$ masses.

be $-21°$, in excellent agreement with that calculated from the masses alone. Attempts to include the $S^*(1005)$ in the same nonet as the κ and δ resulted in unacceptably high values for χ^2.

We have seen that the $\kappa(1510)$, $\delta(980)$, $\epsilon(800)$, and $\epsilon'(1540)$ can be classified as members of an SU(3) nonet. However, the mass difference between the $\kappa(1510)$ and $\delta(980)$, which is generally attributed to the mass difference between strange and non-strange quarks, is four times as large as expected from the $K^*(890)$ - $\rho(770)$ or the $K^*(1435)$ - $A_2(1300)$ mass splittings. Another problem arising from this SU(3) classification of the scalar mesons is the SU(3) interpretation of the $S^*(1005)$. If it is an isolated SU(3) singlet then we would have $g(S^*\to K\bar{K}) = (\frac{2}{3})^{1/2} g(S^*\to\pi\pi)$, but we know that this ratio of coupling constants is in fact 2.

The masses and couplings of $q\bar{q}q\bar{q}$ bound states, unlike those of simple $q\bar{q}$ states, are not constrained by SU(3). One might therefore hope that such an interpretation of the states of Fig. 6 would not suffer from the problems associated with the $q\bar{q}$ interpretation discussed above. In fact, the approximate mass degeneracy of the $\delta(980)$ and $S^*(1005)$ is an immediate consequence of the $q\bar{q}q\bar{q}$ interpretation of Ref. 1. The canonical mass difference of about 120 MeV between strange and nonstrange quarks then leads in this model to the prediction of an isoscalar meson with a mass of about 760 MeV. The $\epsilon(800)$ is an excellent candidate for this state. Furthermore, the large width of the $\epsilon(800)$ is expected in this four-quark model, while the narrow widths of the δ and S^* are only due to their proximity to $K\bar{K}$ threshold. However, this model requires a broad strange scalar meson at a mass of about 900 MeV. There is no experimental evidence for such a resonance; the $\kappa(1510)$ is much too massive to be included in this four-quark "nonet."

The other difficulty arising from the classification of the $S^*(1005)$, $\delta(980)$, and $\epsilon(800)$ as four-quark states is the interpretation of the $\epsilon'(1540)$ and $\kappa(1510)$. If we assume that these two resonances are ordinary $q\bar{q}$ states and assume the usual value of 120 MeV for the difference between strange- and nonstrange-quark masses, the masses and partial widths of the $\epsilon'(1540)$ then determine the masses and widths of the remaining nonet members. We also assume that the nonet is magically mixed, since the mixing angle is only weakly constrained by the ϵ' and κ. These assumptions then imply the existence of (a) an isovector scalar meson with a mass of 1360 MeV and width of about 280 MeV and (b) an isoscalar resonance with a mass of 1350 MeV, width of 600 MeV, and $g_{K\bar{K}}/g_{\pi\pi}$ of about 20%. These predictions do not depend strongly on the value of the mixing angle. Moreover, if the strange- nonstrange quark mass difference is larger than 120 MeV, these δ and ϵ-like states will be even lighter. If such states exist, they should be seen in $\pi\pi$ and/or $K\bar{K}$ scattering. However, the analysis of existing $\pi\pi$ and $K\bar{K}$ data in Sec. II C shows no evidence for an extra $I=0$ state around 1300 MeV in either $\pi\pi$ or $K\bar{K}$ scattering. We consider the lack of evidence for such a state, as well as the absence of the $\kappa(900)$, to be serious shortcomings of an interpretation of the $\epsilon(800)$, $\delta(980)$, and $S^*(1005)$ as $q\bar{q}q\bar{q}$ states and the $\epsilon'(1540)$ and $\kappa(1510)$ as the ordinary $q\bar{q}$ states in the same SU(6) supermultiplet as the A_2 and A_1 nonets.

We have just seen that the intrepretation of the $\epsilon'(1540)$ and $\kappa(1510)$ as ordinary $q\bar{q}$ states with the usual value of 120 MeV for the mass difference between strange and nonstrange quarks requires the existence of an isovector scalar meson with a mass of 1360 MeV. By analogy with the strange and isoscalar mesons discussed in Sec. II, one might also expect this isovector resonace to have a mass higher than the position of the bump in the partial-wave amplitude. If this is so, then the so-called $\delta'(1270)$ (Refs. 6 and 7) might well have a mass of 1340 MeV and be the "missing" $q\bar{q}$ partner of the $\epsilon'(1540)$ and $\kappa(1510)$.

IV. SUMMARY AND CONCLUSIONS

Coupled-channel fits to the mass dependence of the $J^{PC}=0^{++}$ $K\pi$ partial-wave result in a reasonably narrow resonance, $\kappa(1510)$. There is no evidence for any other $K\pi$ S-wave resonance with a mass of less than 1.9 GeV. Simultaneous fits to $\pi\pi$ and $I=0$ $K\bar{K}$ S waves yield a broad $\epsilon(800)$, a narrow $S^*(1005)$, and a reasonably narrow $\epsilon'(1540)$.

The masses *and* widths of the $\kappa(1510)$, $\epsilon'(1540)$, and $\delta(980)$ are such that these states could constitute an SU(3) nonet of ordinary $q\bar{q}$ states with a singlet-octet mixing angle of $-21°$. However, the S^* couplings preclude the possibility of its being an SU(3) singlet and therefore a gluon-gluon bound state. On the other hand, the interpretation of the $S^*(1005)$, $\delta(980)$, and $\epsilon(800)$ as $q\bar{q}q\bar{q}$ states and the $\kappa(1540)$, $\epsilon'(1540)$, and possible $\delta(1300)$ as $q\bar{q}$ states is in serious difficulty because of the lack of evidence for a $\kappa(900)$ or an $\epsilon'(1300)$.

Data on $K\eta$, $K\eta'$, $\pi\eta$, and $\eta\eta$ channels will enable a better determination of the branching ratios of the scalar mesons, but it seems likely that our inability to interpret them simply as either $q\bar{q}$ or $q\bar{q}q\bar{q}$ states will remain. We therefore conclude that the scalar mesons may well be complex mixtures of two-quark states, four-quark states, and possibly even gluon-gluon bound states.

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[18]Using quadratic, rather than linear, mass formulas does not significantly change these results.