Is the $\underline{20}$-dominance model valid in charm decays, too?

Mituaki Katuya and Yoshio Koide
Laboratory of Physics, Shizuoka Women's University, Yada 409, Shizuoka 422, Japan
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It is pointed out that studies of $B(D\to l\nu X)/B(D\to K\pi)$ and $B(D\to K\pi\pi)/B(D\to K\pi)$ based on quantum chromodynamics lead to SU(4) $\underline{20}$-plet dominance for the weak interaction of current-current type. We predict $\Gamma(D^+\to\text{all})/\Gamma(D^0\to\text{all})\simeq 0.05$.

In the conventional scheme, the dominant term in the effective $\Delta S=\Delta C=1$ interaction is of the form

$$H_{\text{eff}}=\frac{G_F}{\sqrt{2}}\cos^2\theta_C \times\{\tfrac{1}{2}(c_++c_-)[\bar{d}\gamma_\mu(1-\gamma_5)u][\bar{c}\gamma^\mu(1-\gamma_5)s] +\tfrac{1}{2}(c_+-c_-)[\bar{d}\gamma_\mu(1-\gamma_5)s][\bar{c}\gamma^\mu(1-\gamma_5)u]\} +\text{H.c.}, \qquad (1)$$

where c_- is the enhancement factor for the $\underline{20}$ of SU(4) and c_+ the suppression factor for the $\underline{84}$ of SU(4). It is usually considered that the enhancement of $\underline{20}$ relative to $\underline{84}$ is considerably weaker in the $\Delta S=\Delta C=1$ term than in the $\Delta S=1$, $\Delta C=0$ term.[1,2] In fact, the experimental branching ratios[3] $B(D^0\to K^-\pi^+)=2.2\pm0.6\%$ and $B(D^+\to\bar{K}^0\pi^+)=1.5\pm0.6\%$ suggest $\Gamma(D^0\to K^-\pi^+)\simeq\Gamma(D^+\to\bar{K}^0\pi^+)$ together with a naive quark-parton-model prediction $\Gamma(D^0\to\text{all})\simeq\Gamma(D^+\to\text{all})$. It is usually taken that the experimental fact[3] $R(D^+)\simeq R(D^0)$ rules out the $\underline{20}$-dominance model[4] where

$$R(D^+)\equiv B(D^+\to K^-\pi^+\pi^+)/B(D^+\to\bar{K}^0\pi^+) =2.6\pm1.2$$

and

$$R(D^0)\equiv B(D^0\to\bar{K}^0\pi^+\pi^-)/B(D^0\to K^-\pi^+) =2.0\pm0.7,$$

because this model suppresses the mode $D^+\to\bar{K}^0\pi^+$ relative to the mode $D^0\to K^-\pi^+$ while it does not necessarily suppress the $D^+\to K^-\pi^+\pi^+$ relative to $D^0\to\bar{K}^0\pi^+\pi^-$.

Against this current conjecture, in this paper we point out that if we use the relation

$$c_-c_+^2=1, \qquad (2)$$

which is derived from the quantum-chromodynamics (QCD) calculation,[5] the experimental values[3,6] $B(D\to K\pi)$ and $B(D\to l\nu X)$ lead to $\Gamma(D^+\to\text{all})/\Gamma(D^0\to\text{all})\simeq0.05$, so that most of the D semileptonic events come from the charged D meson. Moreover, it is shown that the experimental value $R(D^+)=2.6$ together with the uniformity[7] of the population density of the Dalitz plot for $D^+\to K^-\pi^+\pi^+$ also confirms our conclusion.

Many authors[1,8] have investigated the weak decays of charmed particles by using the enhancement and suppression factors $c_\pm$ whose values are determined by assuming the number of quark flavors N_f, the quark-gluon fine-structure constant α_s, and the renormalization point μ. These parameters, however, cannot be determined unambiguously at present. Therefore, it seems very important to estimate the enhancement and suppression factors $c_\pm$ through the phenomenological analysis of the weak decays of charmed particles by employing the relation (2) which holds independently of the parameters N_f, α_s, and μ. We assume for the time being the validity of the relation (2) in the range $1\le c_-/c_+<\infty$, although an extraordinarily large value for c_-/c_+ is not likely.

We assume that the decay amplitudes can be expressed in terms of the factorized matrix elements. In order to obtain the factorized matrix elements, it is useful to write down the following effective Hamiltonian:

$$H_{\text{eff}}^{\text{hadron}}=\frac{G_F}{\sqrt{2}}\cos^2\theta_C(X_+J_{\mu 2}{}^1J_4^{\mu 3}+X_-J_{\mu 2}{}^3J_4^{\mu 1})+\text{H.c.}, \qquad (3)$$

where

$$J_{\mu i}^{J}=a_1(P_i^J)\partial_\mu P_i^J+v_2(P_i^J)(P\overleftrightarrow{\partial}_\mu P)_i^J +a_3(P_i^J)[(PP\partial_\mu P+\partial_\mu PPP)_i^J-\tfrac{1}{2}\delta_i^J\text{Tr}(PP\partial_\mu P)] +a_3'(P_i^J)[(P\partial_\mu PP)_i^J-\tfrac{1}{4}\delta_i^J\text{Tr}(PP\partial_\mu P)], \qquad (4)$$

and $X_\pm=(2c_+\pm c_-)/3$. Here we assume the conserved vector current (CVC) hypothesis, $v_2(P_i^J)=1$, and we set $a_1(\pi)=f_\pi$, $a_1(K)=f_K$, and $a_1(D)=f_D$.

$\Gamma(D\to l\nu X)/\Gamma(D\to K\pi)$

The $D\to K\pi$ amplitudes have already been estimated[1,8] by assuming the factorization of the matrix elements as follows:

$$A(D^0 \to K^- \pi^+) = G[X_+ f_\pi (m_D{}^2 - m_K{}^2) - X_- f_D (m_K{}^2 - m_\pi{}^2)], \quad (5)$$

$$A(D^+ \to \bar{K}^0 \pi^+) = G[X_+ f_\pi (m_D{}^2 - m_K{}^2) + X_- f_K (m_D{}^2 - m_\pi{}^2)], \quad (6)$$

where $G \equiv (G_F/\sqrt{2})\cos^2\theta_C$. From Eqs. (5) and (6) we can predict the total widths of D^0 and D^+ by using the data[3] $B(D^0 \to K^-\pi^+) = 2.2 \pm 0.6\%$ and $B(D^+ \to \bar{K}^0\pi^+) = 1.5 \pm 0.6\%$, respectively. Furthermore, if we adopt the popular value[9] $\Gamma_{SL} \equiv \Gamma(D^0 \to l\nu X) = \Gamma(D^+ \to l\nu X) = 2 \times 10^{11}\ \mathrm{sec}^{-1}$, we can predict the semileptonic branching ratios of D^0 and D^+.

Figure 1 illustrates $B(D^+ \to l\nu X)$ and $B(D^0 \to l\nu X)$ as functions of X_-/X_+.[10] Note that $B(D^0 \to l\nu X)$ has a maximum,[11] 2.5%, at $X_-/X_+ \simeq \frac{1}{3}$, so that most of the D semileptonic events must come from the charged D meson. As shown in Fig. 1, the world average[6] $B(D \to l\nu X) = 9.8 \pm 1.4\%$, which is averaged over the D^0 and D^+, leads to[9]

$$X_-/X_+ = -(0.59 \pm 0.05) \text{ or } c_-/c_+ = 7.8^{+1.5}_{-1.0} \quad (7a)$$

and

$$X_-/X_+ = -(0.81 \pm 0.02) \text{ or } c_-/c_+ = 19 \pm 2\,. \quad (7b)$$

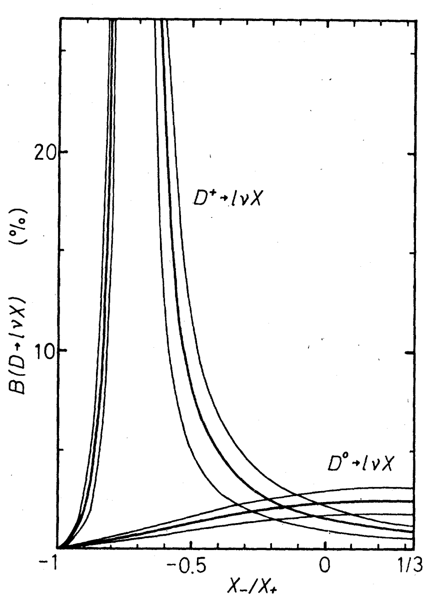

FIG. 1. Branching ratios $B(D^0 \to l\nu X)$ and $B(D^+ \to l\nu X)$ versus X_-/X_+ [$= (2c_+ - c_-)/(2c_+ + c_-)$]. $X_-/X_+ = \frac{1}{3}$ and $X_-/X_+ = -1$ are equivalent to $c_-/c_+ = 1$ and $c_-/c_+ = \infty$, respectively. Each of the three curves corresponds to three values $B(D^0 \to K^-\pi^+) = 2.8$, 2.2, 1.6% and $B(D^+ \to \bar{K}^0\pi^+) = 2.1$, 1.5, 0.9%, respectively.

$\Gamma(D \to K\pi\pi)/\Gamma(D \to K\pi)$

The $D^+ \to K^-\pi^+\pi^+$ amplitude is

$$A(D^+(q) \to K^-(p)\pi^+(k_1)\pi^+(k_2)) = G\{2[3X_- + X_+ f_\pi (a_3 - a_3')] m_D E - [2X_- + X_+ f_\pi (a_3 - a_3')](m_D{}^2 + m_K{}^2) + 2(X_- - X_+ f_\pi a_3') m_\pi{}^2\}, \quad (8)$$

where $E = p_0$ and $a_3 = a_3(F)$, $a_3' = a_3'(F)$. The uniformity[7] of the population density of the Dalitz plot requires the condition

$$X_-/X_+ = -f_\pi (a_3 - a_3')/3\,. \quad (9)$$

Then the amplitude (8) becomes

$$A(D^+ \to K^-\pi^+\pi^+) = G[X_-(m_D{}^2 + m_K{}^2 - m_\pi{}^2) - X_+ f_\pi (a_3 + a_3') m_\pi{}^2]\,. \quad (10)$$

If we neglect the second term of Eq. (10), that is, if

$$|f_\pi (a_3 + a_3')| \ll |X_-/X_+| (m_D{}^2 + m_K{}^2 - m_\pi{}^2)/m_\pi{}^2\,, \quad (11)$$

then we can calculate the value $R(D^+)$ as follows:

$$R(D^+) = \frac{(\Omega/64\pi^3 m_D)|A(D^+ \to K^-\pi^+\pi^+)|^2}{(p/8\pi m_D{}^2)|A(D^+ \to \bar{K}^0\pi^+)|^2} = 0.133 \times (X_-/X_+)^2/(X_-/X_+ + 0.73)^2\,, \quad (12)$$

where Ω is the $D^+ \to K^-\pi^+\pi^+$ phase volume $\frac{1}{2} \times 0.236$ GeV^2 and p is the center-of-mass momentum in the decay $D^+ \to \bar{K}^0\pi^+$. As illustrated in Fig. 2, the experimental value $R(D^+) = 2.6 \pm 1.2$ leads to the result

$$X_-/X_+ = -(0.60^{+0.02}_{-0.04}) \text{ or } c_-/c_+ = 7.9^{+0.5}_{-0.9} \quad (13a)$$

and

$$X_-/X_+ = -(0.94^{+0.06}_{-0.05}) \text{ or } c_-/c_+ = 68^{+\infty}_{-31}\,. \quad (13b)$$

It is worthwhile noting that the solution (13a) is in very good agreement with the solution (7a). Hereafter, we adopt the solution $X_-/X_+ = -0.6$, since the other solutions (7b) and (13b) are inconsistent and $c_-/c_+ \simeq 20 \sim 70$ is too large.

Now let us show that we can realize the enhancement of the amplitude $A(D^0 \to \bar{K}^0\pi^+\pi^-)$ relative to the amplitude $A(D^+ \to K^-\pi^+\pi^+)$ by adjusting the parameter $(a_3 + a_3')$ under the condition (9). For simplicity, we assume $f_\pi a_3 \equiv f_\pi a_3(F) \simeq f_K a_3(D) \simeq f_D a_3(K)$ and $f_\pi a_3' \equiv f_\pi a_3'(F) \simeq f_K a_3'(D) \simeq f_D a_3'(K)$. Then we get

$$A(D^0(q) \to \bar{K}^0(p)\pi^+(k_1)\pi^-(k_2)) = G\tfrac{1}{2}X_-\{3[(m_D{}^2 + m_K{}^2 - m_\pi{}^2) - 2(1 - X_-/X_+) m_D E] - f_\pi (a_3 + a_3')[m_D{}^2 + m_K{}^2 + m_\pi{}^2/(X_-/X_+)]\}\,. \quad (14)$$

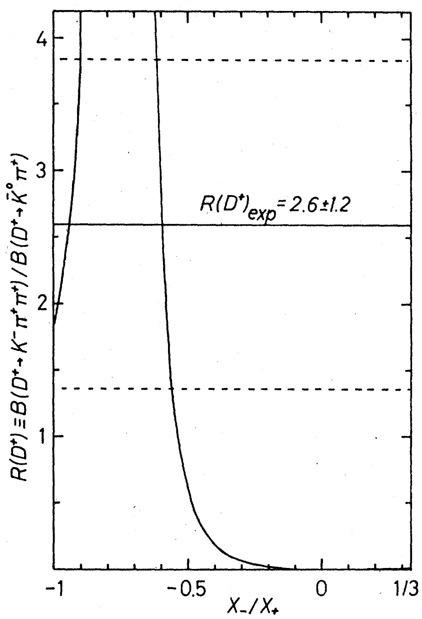

FIG. 2. The ratio $R(D^+)\equiv B(D^+\to K^-\pi^+\pi^+)/B(D^+\to\bar{K}^0\pi^+)$ versus X_-/X_+.

Since the $D^0\to\bar{K}^0\pi^+\pi^-$ decay mode does not exhibit either K^* or ρ production at a substantial level,[12] the experimental value $R(D^0)=2.0$ and Eq. (14) lead to $f_\pi(a_3+a_3')\simeq -1.8$ and 5.1 for $X_-/X_+=-0.6$. These values are well satisfactory with the condition (11).

Under the condition (9), the amplitude of the $D^0\to K^-\pi^+\pi^0$ becomes

$$\sqrt{2}A(D^0(q)\to K^-(p)\pi^+(k_1)\pi^0(k_2)) = G[X_+(4-3X_-/X_+)(1+X_-/X_+)m_D(\omega_2-\omega_1) + \tfrac{1}{2}X_-(m_D{}^2+m_K{}^2-m_\pi{}^2)-\tfrac{1}{2}X_+(a_3+a_3')f_\pi m_\pi{}^2]\,. \tag{15}$$

If the observed mode $D^0\to K^-\pi^+\pi^0$ has no ρ or K^* component, the predicted ratio $\Gamma(D^0\to K^-\pi^+\pi^0)/\Gamma(D^0\to K^-\pi^+)$ becomes 0.3, which is considerably small in comparison with the experiments.[13] Therefore, the observed mode $D^0\to K^-\pi^+\pi^0$ must be dominated by intermediate states ρ^+ and/or K^*.[14]

CONCLUSION

Studies of $B(D\to l\nu X)/B(D\to K\pi)$ and $B(D\to K\pi\pi)/B(D\to K\pi)$ based on the QCD relation (2) lead to the enhancement of 20 relative to 84 by a factor of $\simeq 8$:

$$X_-/X_+\simeq -0.6 \text{ or } C_-/C_+\simeq 8\ (c_-\simeq 4, c_+\simeq 0.5)\,. \tag{16}$$

Our predictions are as follows[15,16]:

$$\begin{aligned} &\Gamma(D^+\to \text{all})\simeq 1\times10^{12}\ \text{sec}^{-1}\,,\\ &\Gamma(D^0\to \text{all})\simeq 2\times10^{13}\ \text{sec}^{-1}\,,\\ &\Gamma(D^+\to \text{all})/\Gamma(D^0\to\text{all})\simeq 0.05\,,\\ &B(D^+\to l\nu X)\simeq 20\%,\quad B(D^0\to l\nu X)\simeq 1\%\,,\\ &B(D^0\to \bar{K}^0\pi^0)\simeq 0.7\%\,. \end{aligned} \tag{17}$$

ACKNOWLEDGMENT

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[10]Here we set $f_D=f_K(=1.28f_\pi)$ tentatively since the amplitude (5) is insensitive to f_D/f_π.

[11]The usefulness of the relation (2) in the study of nonleptonic decays has been pointed out by one of the

authors (Y.K). He has illustrated the ratios $B(D^{+,0} \to Kl\nu)/B(D^{+,0} \to K\pi)$ versus X_-/X_+, and has predicted $B(D^0 \to K^- l^+ \nu)/B(D^0 \to K^- \pi^+) \leq 0.64$. Y. Koide, Phys. Rev. D 18, 1644 (1978).

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