Forward-backward asymmetry in $e^- + e^+ \to q + \bar{q} \to \pi, K, D, \ldots, + X$

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The forward-backward asymmetry A_M due to weak-electromagnetic interference effects in the inclusive process $e^- + e^+ \to M + X$, where M is a pseudoscalar meson, is calculated in terms of the axial-vector weak neutral-current couplings b_q of the quarks composing M, their electric charges Q_q, and the ratio $D(x)$ of their $(q \to M)$ fragmentation probabilities. Thus, if b_q and Q_q are considered known, (i) a measurement of A_M effectively determines $D(x)$, and (ii) for mesons composed of a light quark and a heavy quark a theoretical argument indicates that A_M is independent of $D(x)$ so that the result of a measurement of A_M is immediately predictable.

Recently, with the puspose of finding the correct model that describes the weak and electromagnetic interactions, much attention has been given to weak-interaction effects in electromagnetic processes. In general, the interference between electromagnetic and weak-neutral-current amplitudes will produce helicity-dependent cross sections and forward-backward asymmetries. The first of these effects is intrinsically parity violating and cannot arise in any order if the electromagnetic amplitude alone is considered. Its existence, therefore, is a clear signal for the presence of a weak neutral current. With this in mind, several processes have been previously investigated. Thus, in polarized-electron–nucleon or polarized-electron–nucleus scattering a dependence of the cross section on the incident electron's helicity has been sought (and found[1]). In $e^- + e^+ \to l^- + l^+$ one could search for the longitudinal polarization of the lepton and, if l is the heavy lepton τ, this longitudinal polarization would manifest itself in the angular distribution of the τ decays.[2] On the other hand, the forward-backward asymmetry is not intrinsically parity violating. It can also arise from higher-order electromagnetic corrections, and these contributions must be subtracted before the weak-neutral-current contribution can be extracted from the observed asymmetry. Both the longitudinal polarization and the forward-backward asymmetry calculations have been carried out in detail for the process $e^- + e^+ \to \mu^- + \mu^+$,[3] and the situation in this case is well understood.

In the process $e^- + e^+ \to q + \bar{q}$ $(q = u, d, s, c, b, \ldots)$ these interference effects can provide a way of determining the weak-neutral-current couplings and electric charges of the quarks. The weak-neutral-current couplings have already been determined[4] for the u and d quarks from neutrino reaction data. On the other hand, as long as the "sea quarks" within the nucleons are ignored the determination of these couplings for the heavier $s, c, b, \ldots$ quarks must proceed in some other direction. This is the motivation of the present work: to obtain information about the weak-neutral-current couplings and electric charges of the heavier quarks from the interference effects in $e^-e^+ \to s\bar{s}, c\bar{c}, b\bar{b}, \ldots$. In all cases the interference effects in $e^-e^+ \to q\bar{q}$ are related to the corresponding interference effects in $e^-e^+ \to$ hadrons through the specification of an appropriate (quark $\to$ hadron) fragmentation probability which, if the quark weak-neutral-current couplings and electric charges are considered known, can be extracted from suitable interference-effect observations.

In this note we have restricted ourselves to the forward-backward asymmetry. In principle, the calculation of this asymmetry in the $q\bar{q}$ case is the same as the calculation in the $\mu^-\mu^+$ case. In practice, however, what is observed are the hadrons associated with q and $\bar{q}$, and not q and $\bar{q}$ themselves. For this reason we have specifically calculated the forward-backward asymmetry of M in $e^- + e^+ \to M +$anything (X), where M $(M = \pi, K, D, \ldots)$ is the pseudoscalar meson associated with the produced quark (or antiquark). Interference effects in $e^- + e^+ \to M + \bar{M}$ and $B + \bar{B}$, where B is a spin-$\frac{1}{2}$ baryon, have been previously investigated.[5] We feel, however, that there are some advantages in considering the inclusive process. This process is more convenient from an experimental point of view because the cross section will not be inhibited by form factors, as will the cross sections for $M\bar{M}$ and $B\bar{B}$. On the theoretical side we have a phenomenologically well tested model that allows us to make a definite connection between the inclusive process and the free-quark process, whereas for the processes $M\bar{M}$ and $B\bar{B}$ this connection would be more difficult to establish; stated in another way there is no known model which will establish a relation between the form factors and the quark couplings that is generally valid. Our calculations will be based on the following Lagrangian

$$L=-e(J_\mu-\bar{e}\gamma_\mu e)A^\mu-g_Z(N_\mu+\bar{e}\Gamma^e_\mu e)Z^\mu ,$$

where

$$J_\mu=\sum_{\text{quarks}} Q_q\bar{q}\gamma_\mu q ,$$

$$N_\mu=\sum_{\text{quarks}}\bar{q}\Gamma^q_\mu q ,$$

and

$$\Gamma^f_\mu=\gamma_\mu(a_f+b_f\gamma_5)\quad (f=e,q) .$$

If we specialize to the Weinberg-Salam[6] (WS) model we have the relations

$$g_Z=\frac{g}{2\cos\theta_W}=\frac{e}{2\cos\theta_W\sin\theta_W} ,$$

$$M_Z\cos\theta_W=M_W ,$$

$$\frac{G}{\sqrt{2}}=\frac{g_Z{}^2}{2M_Z{}^2}=\frac{g^2}{8M_W{}^2}=\frac{e^2}{8M_W{}^2\sin^2\theta_W} ,$$

$$a_f=I_f^{(3)}-2\sin^2\theta_W Q_f ,$$

$$b_f=-I_f^{(3)} ,$$

where $I_f^{(3)}=\pm\frac{1}{2}$ is the weak isospin of the (left-handed) fermion ($I_e^{(3)}=I_d^{(3)}=I_s^{(3)}=\cdots=-\frac{1}{2}$).

The cross section for $e^-+e^+\to q+\bar{q}$ is given by

$$d\sigma_q(p_q)=\left(\frac{d\sigma^{(0)}}{d\Omega}\right)d\Omega dE_q\delta(E_q-\tfrac{1}{2}\sqrt{s})(1+A_q) , \qquad (1)$$

where the purely electromagnetic cross section $d\sigma^{(0)}/d\Omega$ and the forward-backward asymmetry A_q are given by

$$\frac{d\sigma^{(0)}}{d\Omega}=\frac{\alpha^2Q_q{}^2}{4s}(1+\cos^2\theta-\xi^2\sin^2\theta\cos2\phi) ,$$

$$A_q=\frac{4b_eb_q}{Q_q}B(s)\frac{2\cos\theta}{1+\cos^2\theta-\xi^2\sin^2\theta\cos2\phi} . \qquad (2)$$

ξ is the transverse polarization of the electron, whose direction is taken as the x axis and

$$B(s)\equiv\left(\frac{g_Z{}^2}{2M_Z{}^2}s\right)\left(\frac{-1}{4\pi\alpha}\right)\left(\frac{-1}{1-s/M_Z{}^2}\right) .$$

The asymmetry A_μ of the μ^- in $e^-+e^+\to\mu^-+\mu^+$ is obtained from Eq. (2) with the substitution: $Q_q\to Q_\mu=-1$ and $b_q\to b_\mu$. This gives

$$\frac{A_q}{A_\mu}=-\frac{1}{Q_q}\frac{b_q}{b_\mu} \qquad (3)$$

and reduces, in the WS model, to

$$\left(\frac{A_q}{A_\mu}\right)_{\text{WS}}=\frac{2I_q^{(3)}}{Q_q} . \qquad (4)$$

In the framework of the quark-parton model the cross section for $e^-+e^+\to M+X$, where $M=[Q\bar{q}]$, is given by

$$d\sigma_M(p_M)=\int dx\left[d\sigma_Q\left(\frac{1}{x}p_M\right)D_{M/Q}(x)+d\sigma_{\bar{q}}\left(\frac{1}{x}p_M\right)D_{M/\bar{q}}(x)\right] , \qquad (5)$$

$D_{M/Q}(x)$ ($D_{M/\bar{q}}(x)$) being the fragmentation probability that the quark Q ($\bar{q}$) will form the meson M with momentum $p_M=xp_Q$ ($xp_{\bar{q}}$). Substituting Eq. (2) in Eq. (5) we obtain

$$\frac{d\sigma_M}{dxd\Omega}=\frac{\alpha^2}{4s}(1+\cos^2\theta-\xi^2\sin^2\theta\cos2\phi)\times[Q_Q{}^2D_{M/Q}(x)+Q_q{}^2D_{M/\bar{q}}(x)](1+A_M) , \qquad (6)$$

where

$$A_M=\frac{Q_Q{}^2A_Q+Q_q{}^2A_{\bar{q}}D(x)}{Q_Q{}^2+Q_q{}^2D(x)}=\frac{Q_Q{}^2A_Q-Q_q{}^2A_qD(x)}{Q_Q{}^2+Q_q{}^2D(x)} , \qquad (7)$$

$$x=\frac{2E_M}{\sqrt{s}} ,$$

and we have defined

$$D(x)\equiv\frac{D_{M/\bar{q}}(x)}{D_{M/Q}(x)} . \qquad (8)$$

Equation (7) has two important limiting forms which we discuss separately.

Case I. One of the fragmentation probabilities is much larger than the other:

$$D_{M/Q}(x)\gg D_{M/\bar{q}}(x);\quad D(x)\ll 1 .$$

Equation (7) yields in this case

$$A_M=A_Q . \qquad (9)$$

Case II. The fragmentation probabilities are equal:

$$D_{M/Q}(x)=D_{M/\bar{q}}(x);\quad D(x)=1 .$$

In this case Eq. (7) yields

$$A_M=\frac{Q_Q{}^2A_Q-Q_q{}^2A_q}{Q_Q{}^2+Q_q{}^2} . \qquad (10)$$

We have considered these two particular cases because we believe that case I applies to the "heavy" mesons containing the $c,b,\ldots$ quarks, while case II applies to the "light" mesons composed of the u,d,s quarks.

Let us consider the light mesons first and, for definiteness, restrict the discussion to the WS model. Assuming SU(3) and charge-conjugation invariance of the strong-interaction Hamiltonian, we have the relations of case II:

$$D_{\pi^+/u}=D_{\pi^-/d}=D_{\pi^+/\bar{d}} ,$$

$$D_{K^-/s}=D_{K^+/u}=D_{K^-/\bar{u}} ,$$

$$D_{\bar{K}^0/s}=D_{K^0/d}=D_{\bar{K}^0/\bar{d}} .$$

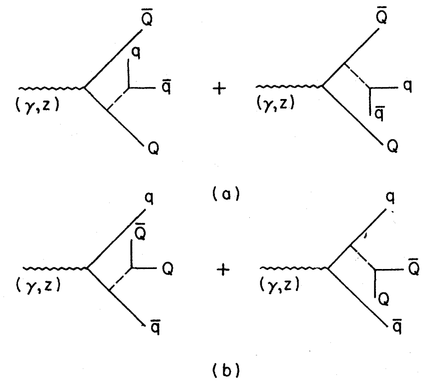

FIG. 1. Model for the fragmentation of the quarks Q and $\bar{q}$ to form the meson $M = [Q\bar{q}]$. The broken line represents a gluon exchanged between Q and q.

Then, using Eqs. (10) and (4), and remembering that $Q_s = Q_d$, we obtain

$$\left(\frac{A_{K^+}}{A_\mu}\right)_{\rm WS} = \left(\frac{A_{\pi^+}}{A_\mu}\right)_{\rm WS} = -\left(\frac{A_{\pi^-}}{A_\mu}\right)_{\rm WS} = -\left(\frac{A_{K^-}}{A_\mu}\right)_{\rm WS} = \frac{Q_u + Q_d}{Q_u^2 + Q_d^2} = \frac{3}{5}, \tag{11}$$

$$(A_{K^0})_{\rm WS} = 0 .$$

If we assume $D_{K^-/s} \gg D_{K^-/\bar{u}}$ and $D_{\bar{K}^0/s} \gg D_{\bar{K}^0/\bar{d}}$, we get $(A_{\bar{K}^0})_{\rm WS} = (A_{K^-})_{\rm WS} = (-1/Q_s)A_\mu$ in contrast to Eq. (11), but such an assumption cannot be justified in this case.

To justify the assumption that case I applies to the heavy mesons we consider a simple model in which the fragmentation process is dominated by the diagrams in Fig. 1. Diagrams (a) contribute to $D_{M/Q}$ and diagrams (b) to $D_{M/\bar{q}}$. Neglecting the transverse momentum of Q and $\bar{q}$ inside M, we have

$$E_Q = M_Q \gamma_M, \quad E_{\bar{q}} = M_q \gamma_M,$$

$$\vec{p}_Q = M_Q \vec{v}_M \gamma_M, \quad \vec{p}_{\bar{q}} = M_q \vec{v}_M \gamma_M,$$

which give

$$p_Q = \frac{M_Q}{M_M} p_M, \quad p_{\bar{q}} = \frac{M_q}{M_M} p_M, \quad M_M = M_Q + M_q . \tag{12}$$

We also assume that $\vec{p}_{\bar{Q}} = -\vec{p}_Q$ and $\vec{p}_q = -\vec{p}_{\bar{q}}$. The essential difference between diagrams (a) and (b) is that in (a) the off-mass-shell quark is Q and the gluon momentum is $p_q + p_{\bar{q}}$, while in (b) the off-mass-shell quark is q and the gluon momentum is $p_Q + p_{\bar{Q}}$. Apart from factors which are essentially the same for all diagrams, we have

$$\text{Diags. (a)} \approx \frac{1}{(p_q + p_{\bar{q}})^2}\left[\frac{1}{(p_Q - p_{e^-} - p_{e^+})^2 - M_Q^2} + \frac{1}{(p_{e^-} + p_{e^+} - p_{\bar{Q}})^2 - M_Q^2}\right],$$

$$\text{Diags. (b)} \approx \frac{1}{(p_Q + p_{\bar{Q}})^2}\left[\frac{1}{(p_q - p_{e^-} - p_{e^+})^2 - M_q^2} + \frac{1}{(p_{e^-} + p_{e^+} - p_{\bar{q}})^2 - M_q^2}\right],$$

which, using Eq. (12) and putting $E_M \approx \sqrt{s}/2$ yields

$$\frac{\text{Diags. (b)}}{\text{Diags. (a)}} = \left(\frac{M_q}{M_Q}\right)^2 \left[\frac{1 - (M_Q/M_M)}{1 - (M_q/M_M)}\right].$$

Assuming then that case I applies to the heavy mesons, we obtain from Eqs. (9) and (4)

$$\left(\frac{A_{D^+}}{A_\mu}\right)_{\rm WS} = \left(\frac{A_{D^0}}{A_\mu}\right)_{\rm WS} = -\left(\frac{A_{D^-}}{A_\mu}\right)_{\rm WS} = -\left(\frac{A_{\bar{D}^0}}{A_\mu}\right) = \frac{2I_c^{(3)}}{Q_c} = \frac{3}{2}, \tag{13}$$

etc., so that, in particular, the asymmetries of D^+ and D^0 are predicted to be equal. On the other hand, if we assume $D_{D/\bar{q}} = D_{D/c}$, which corresponds to the badly broken SU(4) symmetry, and remember that $Q_c = Q_u$, we get

$$(A_{D^0})_{\rm WS} = 0$$

in contrast to Eq. (13).

From the point of view of an experimental test of the above relations, we note that the asymmetry of the heavy mesons cannot be directly measured and in fact manifests itself only in an asymmetry of their decay products. This last asymmetry must be calculated by folding the production cross section for the heavy meson with the angular distribution for its decay. The resultant expression will have an explicit dependence on the energy of the decay product, and will not satisfy the simple relations given in Eq. (7). On the other hand, the asymmetry of the most energetic decay product will coincide with the asymmetry of the heavy meson since the fastest product is emitted parallel to the heavy meson. Thus, in $D \to K\pi$ or $D \to K\pi\pi$

$$A_K(E_{\rm max}) = A_D ,$$

so that A_D is a directly measurable quantity.

Equation (7) can also be considered from another point of view. Instead of making definite *a priori* assumptions about the fragmentation probabilities, we can adopt specific charge assignments for the quarks, i.e., the conventional $Q_u = Q_c = \frac{2}{3}$ and $Q_d = Q_s = Q_b = -\frac{1}{3}$. A measurement of the asymmetry A_M, together with Eqs. (7) and (4), can then be regarded as an effective determination of the fragmentation probability ratio $D(x)$ defined in Eq. (8).

A further process in which weak-electromagnetic interference effects will manifest themselves is $e^- + e^+ \to B + X$. Here, in addition to a forward-backward asymmetry, a longitudinal polarization of the baryon is also expected. A systematic approach to the calculation of this longitudinal polarization and forward-backward asymmetry (as well as the forward-backward asymmetry in $e^- + e^+ \to M + X$) consists of setting down a general formula for the appropriate transition probability in terms of structure functions whose form is delimited by symmetry considerations. The quark-parton model can then be used to express the structure functions in terms of quark couplings and fragmentation probabilities with results identical, in the case $e^- + e^+ \to M + X$, to those obtained above. A study of these and other related questions (e.g., $e^- + e^+ \to$ vector meson $+ X$) are currently being prepared for publication.

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