PHYSICAL REVIEW D VOLUME 19, NUMBER 8 15 APRIL 1979

List of grand unification gauge groups

Avadis Hacinliyan
Department of Physics, Bogazici University, Bebek, Istanbul, Turkey

Cihan Saclioglu*
Department of Physics, Yale University, New Haven, Connecticut 06520
(Received 17 October 1978)

We present a method for constructing the grand unification gauge group G (of the type G or G^P, where G is simple) corresponding to a given subgroup $G_{\text{flavor}} \times G_{\text{color}}$. The essence of the method is to count the leptoquarks and diquarks by generating the latter through Pauli-Gürsey transformations, to add that number to that of the generators of $G_{\text{flavor}} \times G_{\text{color}}$, and to compare the result with various possible G's via Diophantine equations. Specifically, we take $G_{\text{color}} = \text{SU}(3)$, or SU(4) with leptons representing the fourth color in the second case. For the flavor group, we consider generalizations of essentially all the examples of flavor groups proposed so far, subject to the constraints of lepton-quark flavor universality and an upper limit of 16 quark flavors so as not to destroy asymptotic freedom. The resulting system of roughly 200 Diophantine equations is easily numerically solved. After eliminating a large number of spurious solutions by matching the ranks of $G_{\text{flavor}} \times G_{\text{color}}$ and G, we exhibit the remaining grand unification candidates. The Georgi-Glashow $\text{SU}_L(2) \times \text{U}(1) \times \text{SU}_C(3) \subset \text{SU}(5)$ is found to be a special case of $\text{SU}_L(n) \times \text{U}(1) \times \text{SU}(3) \subset \text{SU}(n+3)$. The Fritzsch-Minkowski SO(10), the Segrè-Weldon SU(6), and Gürsey-Ramond-Sikivie E_6, E_7 models are recovered alongside a surprisingly small number of new possibilities. In particular, no semisimple unifying group of the form G^P is found for $2 \leq P \leq 4$. Finally, our results and methods are compared with those of a recent review on the same subject.

I. INTRODUCTION

Suppose we take the idea of grand unification[1,2] seriously. Then how do we find a grand unification gauge group $\mathcal{G}$ starting from a given "observable" subgroup $G_{\text{flavor}} \times G_{\text{color}}$ that may be more or less suggested by experiment? The answer essentially lies in the observation that the really new ingredient in grand unified models is the presence of leptoquark currents which cause transitions between leptons and quarks. Accordingly, adding the number of such new currents onto the number of the flavor and color currents and demanding that these match the number of generators of an appropriate $\mathcal{G}$ should be a workable strategy. This approach was the basis of two earlier articles[3,4] which dealt with the same problem in a much more limited way than is intended here.

Let us now enumerate and explain in more detail the assumptions (not all of which are independent) and the method that we will be working with: (i) For a single coupling constant to serve weak, electromagnetic, and strong interactions, $\mathcal{G}$ must be simple or of the form $G \times G \times \cdots \times G \equiv G^P$, with G simple. In the latter case, we must postulate a new discrete symmetry interchanging the various factors G. (ii) The strong-interaction gauge group is $\text{SU}_C(3)$, where the subscript C stands for color. (iii) The only observable particles are color singlets, and quarks are color triplets with the usual fractional charges. (iv) The full internal symmetry $\mathcal{G}$ of the quark Lagrangian becomes manifest at extraordinarily large energies (possibly of the order of the Planck mass), and in that limit the masses of at least the fermions can be neglected. (v) The previous assumption means[5] that the massless quark Lagrangian also has Pauli-Gürsey[6,7] symmetry in addition to the explicit symmetry described by $G_{\text{flavor}} \times G_{\text{color}}$. (vi) Combining (iv) and (v), we conclude that $\mathcal{G}$ is obtained by extending $G_{\text{flavor}} \times G_{\text{color}}$ via Pauli-Gürsey transformations. The latter are generated by diquark charges of the form $\int dv\, qq$ or $\int dv\, q^\dagger q^\dagger$, where q denotes a quark field operator. (vii) The color-triplet parts of the diquark charges supplement the color-triplet leptoquarks mentioned in the first paragraph to form diquark-leptoquark currents that make the proton unstable. Sufficient stability is attained by giving masses of the order of $G^{-1/2}$ to the corresponding vector bosons. (G is, of course, Newton's gravitational constant.) At this point one may either feel compelled to abandon this line of reasoning and attempt to include gravitation in the unified picture (possibly along the lines of extended supergravity[8]), or one may optimistically go on, as we shall here, if only to explore the consequences of the above framework.

Our general method can now be summarized as follows: It centers on constructing diquarks from normal $q^\dagger q$ currents by subjecting the latter to Pauli-Gürsey transformations. This also gives us the number of diquarks. Since the number of flavor and color currents is known at the outset, what mainly remains is to check the total against the number of generators of various candidates for $\mathcal{G}$.

As to the specific applications, we wish to demonstrate that the approach we have outlined can be used with considerably differing types of flavor and color groups by actually deriving the corresponding grand unification groups whenever such unification is possible. We believe that it is quite worthwhile to keep our options concerning the flavor, color, or chiral structures of gauge theories as open as possible, given the recent very fluid state of the subject both on the theoretical and experimental fronts. Experimentally, the number of quark flavors has not[9] neatly stopped at four as was widely expected, and also, the existence of at least one new heavy lepton,[10] most likely with its own neutrino, is now well established. In addition, the recent highly energetic three- and four-muon events[11] are quite possibly related to yet newer quark and/or lepton flavors.[12] As is to be expected, there is even a wider variety of new possibilities in the theoretical field. Theories with up to twelve quark flavors[13] have been proposed, as well as theories such as $SU(3)\times U(1)$,[14] $SU_L(2)\times SU_R(2)$,[15] where the last one leads to the unifying group[16] SO(10) if the leptons are taken to represent the fourth color. Thus, to make our search for grand unification fairly complete, we consider generalizations of all the above cases and quite a few others not mentioned in the literature so far. However, we hope it will be clear to the reader that our method can also be easily adapted to find grand unification groups for flavor and color alternatives that we have not treated here.

A short preview of the main results has already been given in the abstract. What we find remarkable is that most of the solutions here have been discovered in earlier searches in which a small flavor group was the starting point. The fact that very few additional cases were found upon enlarging the flavor group seems to indicate that the grand unification scheme favors more economical flavor groups [except in the case of $SU_L(n)\times U(1)\times SU_C(3)\subset SU(n+3)$]. It is perhaps even more surprising that no semisimple group of the type $G^P(P\leqslant 4)$ provides a solution.

The plan of the paper is as follows. In Sec. II we introduce diquarks via Pauli-Gürsey transformations and indicate how they can be classified and counted by considering simple examples. In Sec. III we outline the alternatives we will consider. These involve a number of different internal-symmetry groups for the flavor group, different choices for the chiral structure of the flavor currents, different sets of diquark and leptoquarks, and two possibilities [namely $SU_C(3)$ and $SU(4)\supset SU_C(3)$] for the flavor-singlet subgroup of $\mathcal{G}$. We then set up our Diophantine equations. Section IV consists of a presentation of methods to eliminate most of the spurious solutions to the aforementioned equations, followed by tables in which the surviving solutions are exhibited. We discuss some mathematical and physical aspects of the theories we have found in Sec. V. We also briefly compare our methods and results with a recent paper of largely overlapping interest.

II. DIQUARKS AND LEPTOQUARKS

It is well known that when masses are neglected and internal-symmetry-breaking interactions turned off, a spin-$\frac{1}{2}$ particle q admits an extra "internal" symmetry. Particle and antiparticle states of the same helicity can be rotated into each other through Pauli-Gürsey transformations defined by

$$q\rightarrow aq+b\gamma_5 q^C\,,\quad |a|^2+|b|^2=1\,. \tag{1}$$

Here q^C denotes the charge-conjugate wave function. In the Dirac-Pauli representation, $q^C\equiv\gamma_2 q^*$. The same operation can naturally be extended to field operators. The fact that this three-parameter group is nothing but SU(2) justifies the expression rotated above. The one-parameter chirality transformation

$$q\rightarrow\exp(i\alpha\gamma_5)q \tag{2}$$

of course also preserves the helicity of a pure helicity state.

Now we consider the U(1) generator Q:

$$Q\equiv\int dv\,q^\dagger(\vec{x})q(\vec{x})\,, \tag{3}$$

where we now take q to be a fermionic field operator. The effect of (1) on (3), and (1) combined with (2), is to produce the new charges

$$D_1^\dagger\equiv\int dv\,q^\dagger\gamma_5\gamma_2 q^*\,,\quad D_1\equiv\int dv\,q^\dagger\gamma_2\gamma_5 q \tag{4}$$

and

$$D_2^\dagger\equiv\int dv\,q^\dagger\gamma_2 q^*\,,\quad D_2\equiv\int dv\,q^\dagger\gamma_2 q\,, \tag{5}$$

respectively. Clearly, in the above case the charges (5) identically vanish owing to fermion anticommutation relations and the fact that γ_2 is symmetric in the Dirac-Pauli representation. On the other hand, (4) survives since $\gamma_5\gamma_2$ is an antisymmetric matrix.

The above situation generalizes to quark fields with color and flavor in the following way. A diquark operator is nonzero if and only if it is antisymmetric under the combined interchange of flavor, color, and Dirac spinor indices. We now introduce a restriction on the color properties of diquarks. Since we expect they will be accompanied by leptoquarks which are necessarily color triplets, we only consider color-triplet or antitriplet diquarks. *Thus, with the color representation always constrained to be antisymmetric, symmetric (antisymmetric) flavor combinations go with* γ_2 $(\gamma_5\gamma_2)$. It would be very simple (except for increasing the number of alternatives) to also include color-sextet diquarks in this scheme, but we leave them out because they do not seem to be really necessary in a grand unified field theory. When the generators of $\mathcal{G}$ are applied on a representation containing the quarks and the leptons, the color charges shuffle the quark colors, the flavor charges do the same to quark and lepton flavors, while the leptoquarks change quarks and leptons into each other. The color-triplet diquarks in turn fit in nicely with the leptoquarks (both group theoretically and in possible physical processes such as proton decay into leptons and mesons). There is no similar need for color-sextet diquarks in this minimal picture.

Now let us illustrate the above remarks with a simple example. We choose a vectorlike[17] flavor group SU(n). Denoting flavor indices by Greek and color indices by Latin letters, the only diquarks that we admit are

$$D^i_{\alpha\beta}(S) \equiv \epsilon^{ijk}\int dv\, q^T_{\alpha j}\gamma_2 q_{k\beta} \sim \left(\frac{n(n+1)}{2}, 3^*\right), \qquad (6)$$

$$D^i_{\alpha\beta}(A) \equiv \epsilon^{ijk}\int dv\, q^T_{\alpha j}\gamma_5\gamma_2 q_{k\beta} \sim \left(\frac{n(n-1)}{2}, 3^*\right), \qquad (7)$$

and their Hermitian conjugates. The expressions in parentheses show the $\mathrm{SU}(n)_{\text{flavor}} \times \mathrm{SU}_C(3)$ transformation properties of these objects. Henceforth, we only consider quarks in the fundamental representation of vectorlike SU(n). If quarks in other flavor representations are introduced, there will be additional possibilities with different numbers of diquarks.

The situation is different if the flavor group is generated by charges of a definite chirality. As an example we consider $\mathrm{SU}_L(n)$. The left-handed quark fields then behave as the fundamental n representation while the right-handed fields are singlets. Let us now write our diquarks in the form

$$D^i(c) = \epsilon^{ijk}\int dv\, (q^L_j + q^R_j)^T\gamma_2(q^L_k + q^R_k) \qquad (8)$$

$$= 2\epsilon^{ijk}\int dv\, (q^{L^T}_j\gamma_2 q^R_k) \sim (n, 3^*)\,. \qquad (9)$$

Notice that now we have one set of charges and their number is given by a linear expression in n in contrast to (6) and (7) where the number of diquarks is quadratic in n. The above examples will suffice to guide us through the somewhat more complicated cases in the next section.

The diquarks (6)–(9) are obviously all flavor nonsinglets. When combined with their leptoquark parts they form currents corresponding to the gauge bosons which cause proton decay in second order. This is the reason why masses comparable to $G^{-1/2}$ have to be brought in. However, another type of leptoquark operator also occurs naturally in unified gauge theories. If we think of lepton number as a fourth color, we have three additional leptoquarks and their Hermitian conjugates. These are necessarily flavor singlets since the SU(4) of color and lepton number is assumed to commute with the flavor group. Being flavor singlets, they generally do not have accompanying diquarks. Hence, they cannot contribute to proton decay in second order but in fourth order. A trilinear coupling of three leptoquark bosons each of which tags on to one quark may result in the decay $p \to \nu_e\nu_e e^+$. This process, as well as the possible second-order decay $K^0 \to \mu^+ e^-$ can be made sufficiently rare by giving masses $\gtrsim G^{-1/6}$ (10^6 GeV) to these bosons.

Let us summarize the conclusions of this section:

(i) For a vectorlike SU(n) flavor group we have flavor-symmetric and -antisymmetric color-triplet diquarks. Including their Hermitian conjugates, there are respectively $3n(n+1)$ and $3n(n-1)$ of these charges. A third possibility is to consider them together, in which case we get $6n^2$.

(ii) For $\mathrm{SU}_L(n)$ or $\mathrm{SU}_R(n)$, the total number of diquarks and antidiquarks in $6n$.

(iii) With $\mathrm{SU}(4)_{\text{color}}$, there are three extra flavor-singlet leptoquarks and their conjugates. Commuting these leptoquarks and antileptoquarks we also get a new flavor-singlet leptonic charge. These make up the seven generators needed in going from $\mathrm{SU}(3)_{\text{color}}$ to $\mathrm{SU}(4)_{\text{color}}$. Notice that the introduction of flavor-singlet leptonic charges implies the existence of a new flavor-singlet interaction for quarks and leptons.

III. FLAVOR AND COLOR ALTERNATIVES

We now present the flavor and color groups that we will be dealing with in this paper. Our basic assumption here is one of lepton-quark flavor universality. Although this assumption is in accord with experimental observations involving the lighter quarks and leptons, it has not yet been tested for the heavier new quark[18,9] and lepton[10] species. Cases where leptons and quarks transform under separate flavor groups have also been considered[19]; in such models the observed lepton-quark universality has to be the result of a specially arranged pattern of symmetry breaking. We will not be treating such models in this paper, although our methods could be generalized to cover them as well.

For $\mathcal{G}$ we limit ourselves to $G^P(1 \leq P \leq 4)$ where G is (a) SU(k), (b) SO(k), (c) Sp(k) (k even). Thus for a given P, we have $P(k^2-1)$ generators in case (a), $Pk(k-1)/2$ in case (b), and $Pk(k+1)/2$ in case (c). We treat the exceptional groups separately since the flavor-color content of their generators has been already discussed elsewhere.[3,4,20,21]

We consider two choices for the subgroup of $\mathcal{G}$ that commutes with the flavor subgroup: (A) $SU_C(3)$, i.e., the color SU(3) and (B) SU(4). In the latter case $SU_C(3)$ is a subgroup of SU(4). We will sometimes loosely refer to this alternative as $SU(4)_{\text{color}}$ since leptons can be thought of as making up a fourth color. However, it should be kept in mind that this SU(4) does not share the important property of being exact that is enjoyed by the real color SU(3). Hence, the use of the term "color" for case (B) is somewhat improper—it is only meant to be a shorthand name for the flavor-singlet subgroup of $\mathcal{G}$.

Next come the possibilities concerning the chiral structure of the flavor group. We treat two main categories: (I) vectorlike flavor groups, (II) chiral flavor groups. We will more explicitly state the cases included under (II) later.

It remains now to list the flavor groups and the possible diquark combinations allowed by each of them. Starting with (A) (I), i.e., the case of $SU(3)_{\text{color}}$ and a vectorlike flavor group, we try the following for G_{flavor}: (1) SU(n), (2) SU(n) × U(1), (3) SU(n) × SU(n), (4) SU(n) × SU(n) × U(1). The motivation for picking the above groups is basically to generalize some popular and simple models proposed so far. They also represent the most symmetric and the most obvious alternatives. For example, (1) simply corresponds to n flavors and (2) might stand for n light and n heavy quarks, chosen in equal numbers for greater symmetry. One can easily invent more complicated cases and investigate their consequences in the same way if one so desires.

Regarding the number of various diquark combinations for (A) (I), we recall from Sec. II that (A) (I) (1) and (A) (I) (2) have already been handled. For these we can have (i) flavor-symmetric diquarks, $3n(n+1)$ in number, (ii) flavor-antisymmetric diquarks, $3n(n-1)$ in number, and (iii) a mixture of (i) and (ii) with a total of $6n^2$ diquarks. It is not difficult to obtain all possible color-triplet diquarks corresponding to (A) (I) (3) and (A) (I) (4) by considering all combinations of the five basic cases: flavor symmetric or antisymmetric in the first or the second SU(n) ($2\times 2=4$ cases) plus $6n^2$ diquarks with one quark from the first SU(n) and the other from the second SU(n). We obtain 31 possible combinations by choosing one, two, three, four, or all five at a time of the above five cases. Some of the numbers of diquarks for the 31 possibilities turn out to be identical and we end up with 13 distinct expressions which we list below: (i) $6n(n-1)$, (ii) $6n(n+1)$, (iii) $12n^2$, (iv) $6n^2$, (v) $9n^2-3n$, (vi) $9n^2+3n$, (vii) $3n(n-1)$, (viii) $3n(n+1)$, (ix) $12n^2+6n$, (x) $12n^2-6n$, (xi) $15n^2+3n$, (xii) $15n^2-3n$, (xiii) $18n^2$.

The chiral groups of (A) (II) are easier to handle. We again limit ourselves to the most familiar cases: (1) $SU_L(n)$, (2) $SU_L(n)\times U(1)$, (3) $SU_L(n)\times SU_R(n)$, (4) $SU_L(n)\times SU_R(n)\times U(1)$. We have seen in Sec. II that the number of diquarks for (1) and (2) are $6n$. For (3) and (4) the $\epsilon_{ijk}q_L^j\gamma_2 q_R^k$ structure gives $3\times n\times n$ which must be doubled to include the conjugate generators, $6n^2$ all in all.

We are finally ready to set up our Diophantine equations. The right-hand sides of the equations will successively consist of the expressions (a), (b), (c). The left-hand sides will be given by the number of generators of G_{flavor} plus that of the generators of G_{color} [8 for SU(3), 15 for SU(4)] and of the diquarks. We list the left-hand sides of (A). To find the equations for (B) one just adds on seven to each case.

(A) (I) (1) (i) $n^2+7+3n(n+1)$, (ii) $n^2+7+3n(n-1)$, (iii) n^2+7+6n^2,

(2) (i) $n^2+8+3n(n+1)$, (ii) $n^2+8+3n(n-1)$, (iii) n^2+8+6n^2,

(3) (i) $2n^2+6+6n(n-1)$, (ii) $2n^2+6+6n(n+1)$, (iii) $2n^2+6+12n^2$,

(iv) $2n^2+6+6n^2$, (v) $2n^2+6+9n^2-3n$, (vi) $2n^2+6+9n^2+3n$,

(vii) $2n^2+6+3n(n-1)$, (viii) $2n^2+6+3n(n+1)$, (ix) $2n^2+6+12n^2+6n$,

(x) $2n^2+6+12n^2-6n$, (xi) $2n^2+6+15n^2+3n$, (xii) $2n^2+6+15n^2-3n$,

(xiii) $2n^2+6+18n^2$,

(4) add 1 to (i)–(xiii) above;

(II) (1) n^2+7+6n,

(2) n^2+8+6n,

(3) $2n^2+6+6n^2$,

(4) $2n^2+7+6n^2$.

It is easy to see that equating each expression above to (a), (b), and (c) and repeating this procedure for case (B) results in $36\times 2\times 3=216$ equations in the unknowns n, P, and k. After choosing some practical limits on n (16 flavors maximum to preserve asymptotic freedom[22]) and P (≥ 4), it is a straightforward matter to solve these equations numerically on even a hand-held programmable calculator. We present the solutions in the next section.

IV. SOLUTIONS

The above approach initially yields a rather large number of solutions. Fortunately, the majority of these are simply numerical coincidences and do not correspond to actual embeddings. A few simple additional tests help to eliminate the spurious solutions. These are (i) to ensure that G has more generators than either the flavor group or the color group in the cases where $\mathcal{G}=G^P$ for some $P>1$, (ii) to check that rank $(G_{\text{flavor}})+\text{rank}(G_{\text{color}})=\text{rank}\,\mathcal{G}$. The latter test is possible as $G_{\text{flavor}}\times G_{\text{color}}$ is extended to $\mathcal{G}$ by introducing color-triplet generators which cannot contribute to the rank of $\mathcal{G}$. A good example for the former is the solution (though with $P>4$) SU(2) (Ref. 26) to (A) (II) (3) for $n=3$: $8\times 3\times 3+6=26\times 3=78$. However, this is clearly inadmissible since $\text{SU}_C(3)\not\subset\text{SU}(2)$. An example for the second test is the solution $\mathcal{G}=\text{SU}(15)$ to (A) (I) (1) (i) for $n=7$: $49+7+3\times 7\times(7+1)=15^2-1=224$. This is ruled out since the rank of the left-hand side is 8 while that of the right-hand side is 14.

The remaining solutions, though much reduced in number, still include some false ones. For example, if there is a solution that passes (i) and (ii) and if it is of the type $\mathcal{G}=[\text{SO}(2l+1)]^p$ for some l and p, then $\mathcal{G}=[\text{Sp}(2l)]^p$ is also a possible solution. Thus the roughly ten surviving cases must be examined individually to see whether the adjoint representations of the suspect $\mathcal{G}$'s have the correct decomposition with respect to $G_{\text{flavor}}\times G_{\text{color}}$. The representation tables of Patera and Sankoff[23] help considerably in this last step.

We now exhibit the actual solutions. There is one series of solutions that can be obtained in analytic form. When we attempt the embedding (A) (II) (2) (n^2+6n+8 generators) in $\mathcal{G}=\text{SU}(k)$ we must have $n^2+6n+8=(n+2)(n+4)=(k-1)(k+1)$ which clearly gives $n+3=k$. Thus the Georgi-Glashow $\text{SU}_L(2)\times\text{U}(1)\times\text{SU}_C(3)\subset\text{SU}(5)$ is seen to be a special case of $\text{SU}_L(n)\times\text{U}(1)\times\text{SU}_C(3)\subset\text{SU}(n+3)$. It is very easy to verify that the above series is a real embedding consistent with our assumptions. The rest of our results are shown below.

(A) (I)
[Vectorlike flavor, color SU(3)]

G_{flavor}	$\mathcal{G}$
(1) (i) one flavor	G_2
SU(3)	F_4
(ii) SU(6)	E_7
(2) (ii) SU(3) × U(1)	SU(6)
(iii) SU(2) × U(1)	SO(9)
(3) (iv) SU(3) × SU(3)	E_6

(A) (II)
[Chiral flavor, color SU(3)]

(2)	$\text{SU}_L(n)\times\text{U}(1)$	SU($n+3$)
(3)	$\text{SU}_L(3)\times\text{SU}_R(3)$	E_6

(B) (I)
[Vectorlike flavor, color SU(4)]

(1) (i) SU(2)	SO(9)
(ii) SU(4)	SO(12)
(3) (iv) SU(2) × SU(2)	SO(10)

(B) (II)
[Chiral flavor, color SU(4)]

(3)	$\text{SU}_L(2)\times\text{SU}_R(2)$	SO(10)

The exceptional groups G_2, F_4, E_7, shown in the above table, have been previously[3,4] constructed with the methods described in this article. The decompositions of the adjoint representations of G_2, F_4, E_6, and E_7 given in Refs. 20 and 21 agree with our findings. E_8 could not have been obtained in our scheme since its flavor group is SU(3) × SU(3) × SU(3).

V. DISCUSSION

We have searched for grand unification groups corresponding to a large number of flavor groups and an even larger number of choices of diquark combinations: $216\times 4=864$ (counting each P separately) Diophantine equations were considered. Since we allowed up to 16 quark flavors, the number of possibilities was actually roughly an order of magnitude higher, around 10 000. It is then a relief to discover that the real solutions are as few in number as shown in the previous section. It is also surprising that no semisimple cases of the form G^P have survived for $1<P\leq 4$. We have

systematically not gone beyond $P > 4$, not only out of considerations of practicality but also because the lack of a clear physical motivation for the discrete symmetry connecting the various factors G of G^P makes such models appear somewhat artificial. In addition, there is a greater tendency for spurious solutions as P increases. Among the cases examined, no solution with $P > 2$ even passed the rank test.

Most of the final solutions we have obtained are in fact models that have been proposed and examined previously. Hence we will not dwell upon their particle contents and phenomenological predictions in detail, but will refer the reader to the original papers. Thus the SU(6) of (A) (I) (2) (ii) is nothing but the Segrè-Weldon[24] theory, the $SU(n+3)$ of (A) (II) (2) a generalization of the Georgi-Glashow[2] SU(5). The E_6 model of the (A) (II) (3) has been put forward by Gürsey, Ramond, and Sikivie[20] and the vectorlike E_7 of (A) (I) (1) (ii) has been constructed by Gürsey and Ramond.[21] The SO(10) theory of (B) (II) (3) is that of Fritzsch and Minkowski[16] while the vectorlike version of it in (B) (I) (3) (iv) and vectorlike E_6 of (A) (I) (3) (iv), though new, are presumably phenomenologically unacceptable.[25,26] Out of the remainder, G_2 has no room for flavor and F_4 only allows a vectorlike theory as does SO(9). In fact, if we demand a flavor chiral group, the only solutions are $SU_L(3) \times SU_R(3) \times SU_C(3) \subset E_6$, $SU_L(2) \times SU_R(2) \times SU_C(3) \subset SO(10)$, and $SU_L(n) \times U(1) \times SU_C(3) \subset SU(n+3)$.

Comparing what we have done so far with a recent article by Gell-Mann, Ramond, and Slansky,[19] we find that although our methods differ considerably, we arrive at the same conclusions when we start from the same assumptions, principally that of quark-lepton flavor universality. With this restriction, the above authors find three different classes of solutions: (i) the exceptional groups F_4, E_6, and E_7, (ii) the series[27] $SU(n) \times U(1) \times SU_C(3) \subset SU(n+3)$, (iii) the series $SO(n-6) \times U(1) \times SU_C(3) \subset SO(n)$. We have already seen that the first two classes are included among our solutions. Although the third class seems to have been excluded from our approach by the fact that we have not considered $SO(l)$-type flavor groups, our solutions $\mathcal{G} = SO(9)$, SO(10), SO(12) are in fact exactly of this type. This is possible because of the well-known isomorphies $SO(3) \approx SU(2)$, $SO(4) \approx SU(2) \times SU(2)$, and $SO(6) \approx SU(4)$.

The above-described agreement in results is somewhat surprising when we contrast the methods employed in the two searches. The authors of Ref. 19 start from $\mathcal{G}$ and a representation of $\mathcal{G}$ that contains the quarks and the leptons. They then derive G_{flavor} by picking $SU_C(3)$ out of $\mathcal{G}$. We, on the other hand, start from the adjoint representation of $G_{\text{flavor}} \times G_{\text{color}}$ and construct the adjoint representation of $\mathcal{G}$. In Ref. 19, the chiral structure of G_{flavor} is not important in the beginning, whereas in our scheme chiral and nonchiral cases are handled separately from the start. Pauli-Gürsey transformations play a central role in our paper while they are not even mentioned in Ref. 19. It might appear that the $SU(4)_{\text{color}}$ possibility is left out in Ref. 19, but this is not entirely correct. The $SU_C(3) \times U(1)$ in (iii) in the above paragraph emerges as a subgroup of $SU_C(4) \approx SO(6)$. However, it is true that $SU_C(4)$ is not considered in connection with nonorthogonal gauge groups in Ref. 19 and neither is the possibility that $\mathcal{G}$ may be of the type G^P. Strangely, these alternatives do not actually lead to new solutions consistent with our assumptions either. On the other hand, a class of models where leptons and quarks transform under separate flavor groups is treated in Ref. 19 and omitted altogether here. This summarizes the overlapping and nonoverlapping aspects of the two treatments.

ACKNOWLEDGMENTS

We would like to thank Dr. Yoichiro Nambu for making us aware of the importance of diquark charges. We are also grateful to Dr. Feza Gürsey for his enlightening answers to our various inquiries and for a reading of the manuscript. We are glad to acknowledge the partial support provided for this work by the Scientific and Technical Research Council of Turkey. Finally, one of us (C. S.) wishes to express his gratitude for the pleasant hospitality of the High Energy Theory Group of Yale University where the present article was completed. This work was supported in part by the U. S. Department of Energy under Contract No. EY-76-C-02-3075.

*Permanent address: Department of Physics, Bogazici University, Bebek, Istanbul, Turkey.

[1]J. C. Pati and A. Salam, Phys. Rev. Lett. 31, 661 (1973).

[2]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

[3]C. Saclioglu, Phys. Rev. D 15, 2267 (1977).

[4]C. Saclioglu, Phys. Rev. D 17, 1598 (1978).

[5]Y. Nambu, lectures given at the International Summer School in High Energy Physics, Erice, Italy, 1972 (unpublished).
[6]W. Pauli, Nuovo Cimento 6, 204 (1957).
[7]F. Gürsey, Nuovo Cimento 7, 411 (1958).
[8]See P. Fayet and S. Ferrara, Phys. Rep. 32C, 249 (1977) and the references therein.
[9]S. W. Herb *et al.*, Phys. Rev. Lett. 39, 252 (1977).
[10]M. L. Perl *et al.*, Phys. Rev. Lett. 35, 1489 (1975).
[11]B. C. Barish *et al.*, Phys. Rev. Lett. 38, 577 (1977); A. Benvenuti *et al.*, *ibid.* 38, 1110 (1977).
[12]V. Barger *et al.*, Phys. Rev. Lett. 38, 1190 (1977); C. H. Albright, J. Smith, and J. Vermaseren, *ibid.* 38, 1187 (1977); A. Zee, F. Wilczek, and S. B. Treiman, Phys. Lett. 68B, 369 (1977); M. Barnett and L. N. Chang, *ibid.* 72B, 233 (1977).
[13]A. De Rújula, H. Georgi, and S. L. Glashow, Ann. Phys. (N.Y.) 109, 258 (1977).
[14]G. Segrè and J. Weyers, Phys. Lett. 65B, 243 (1976); B. W. Lee and S. Weinberg, Phys. Rev. Lett. 38, 1237 (1977).
[15]J. Pati and A. Salam, Phys. Rev. D 10, 275 (1974); R. N. Mohapatra and D. P. Sidhu, Phys. Rev. Lett. 38, 667 (1977); H. Fritzsch and P. Minkowski, Nucl. Phys. B103, 61 (1976).
[16]H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) 93, 193 (1975).
[17]H. Georgi and S. L. Glashow, Phys. Rev. D 6, 429 (1972).
[18]J.-E. Augustin *et al.*, Phys. Rev. Lett. 33, 1650 (1974); J. Aubert *et al.*, *ibid.* 33, 1404 (1974).
[19]M. Gell-Mann, P. Ramond, and R. Slansky, Rev. Mod. Phys. 50, 721 (1978).
[20]F. Gürsey, P. Ramond, and P. Sikivie, Phys. Lett. 60B, 177 (1976).
[21]F. Gürsey and P. Sikivie, Phys. Rev. Lett. 36, 775 (1976); P. Ramond, Nucl. Phys. B110, 214 (1976).
[22]H. Politzer, Phys. Rev. Lett. 26, 1346 (1973); D. Gross and F. Wilczek, *ibid.* 26, 1343 (1973).
[23]J. Patera and D. Sankoff, *Tables of Branching Rules for Representations of Simple Lie Algebras* (Les Presses de l'Universite de Montreal, Montreal, 1973).
[24]G. Segrè and H. Weldon, Hadronic J. 1, 424 (1978).
[25]A. Benvenuti *et al.*, Phys. Rev. Lett. 37, 1039 (1976); J. Blietschau *et al.*, Nucl. Phys. B118, 218 (1977).
[26]W. Atwood *et al.*, preliminary result presented by C. Prescott at the Trieste Conference, 1978 (unpublished).
[27]The reader may wonder why we obtain only the flavor-chiral version of this series. The reason is that we have assumed all quarks belong to one or more $(n, 3_c)$ representations in the case of SU(n). If we also allow flavor-singlet quarks in $Q^i \sim (1, \overline{3}_c)$, we can again get the needed $6n$ diquarks in the form $q^i_\alpha \gamma_2 Q^j \epsilon_{kij}$, where $\alpha = 1, 2, \ldots, n$. Of course, in the flavor chiral case, the right-handed quarks are natural-flavor singlets and *must* be introduced from the beginning. In the vector-like case, the Q hadrons will not respond to weak and electromagnetic forces and thus be stable against such decays.