PHYSICAL REVIEW D VOLUME 19, NUMBER 8 15 APRIL 1979

Constrained local superfields

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(Received 1 December 1978)

By studying constrained local supermultiplets, we couple a nonlinear realization of supersymmetry to supergravity and investigate the super-Higgs mechanism.

Immediately after the formulation of supergravity,[1,2] attempts were made to couple it to various supersymmetric matter systems.[3] One system which attracted attention was the nonlinear realization of supersymmetry,[4] since such a coupling would permit the investigation of the super-Higgs mechanism.[5] Coupling was achieved in two and three dimensions,[6] but heretofore the problem has remained unsolved in four dimensions.[7,8]

Recently great progress has been made, both in coupling matter to supergravity,[9–11] and in our understanding of nonlinear realizations. Ivanov and Kapustnikov have investigated nonlinear realizations of global (rigid) supersymmetry and their relation to linear representations in great generality,[12] and in particular have shown that all nonlinear realizations are equivalent. Further, the explicit relation between the Volkov-Akulov model[4] and a constrained chiral superfield has been given.[13]

In this article, we use tensor calculus[9,10] to generalize the constrained chiral superfield[13] to the local case, and thus couple a nonlinear realization to supergravity. We investigate the super-Higgs mechanism, and discuss the uniqueness of our action. Although our entire analysis could be done using only the tensor calculus and working only with components, at times we have found it convenient to use the more compact superspace formalism.[14] Our notation is the two-component spinor notation of Ref. 15.

In Ref. 13, the imposition of the constraints

$$\Phi^2 = 0 \text{ and } \Phi = a\Phi\Pi_R^0\Phi^* , \tag{1}$$

where $\Phi = \mathcal{A} + 2\theta_A\chi^A + \theta^2\mathcal{F}$ is a (right) chiral superfield, was shown to yield a nonlinear realization of supersymmetry equivalent to the Volkov-Akulov model.[4] Here a is a constant of dimension (length)^2 and Π_R^0 is the global right chiral projector, which in chiral coordinates takes the simple form

$$\Pi_R^0 = \frac{1}{4}\frac{\partial}{\partial\theta^{A'}}\frac{\partial}{\partial\theta_{A'}} .$$

In global supersymmetry $\Pi_R^0\Phi^*$ is the kinetic multiplet.[16] Both the projector Π_R^0 and the kinetic multiplet have local analogs. The projector is[15]

$$\Pi_R = \frac{1}{4}\frac{\partial}{\partial\theta^{A'}}\frac{\partial}{\partial\theta_{A'}}\phi^2 ,$$

where

$$\phi^2 = 1 + \sqrt{2}\,\theta_A\psi_{A'}{}^{AA'} - \tfrac{1}{2}\theta^2(\psi'^2 + \tfrac{4}{3}\mathcal{S}^*) + \theta_A\theta_{A'}\sqrt{2}\,\rho^{AA'} - \theta^2\theta_{A'}\mu^{A'} + \theta'^2(\tfrac{1}{3}R)$$

and $\psi_{AMM'}$ is the spin-$\frac{3}{2}$ field, $\mathcal{S}$ is the complex spin-0 auxiliary field of the supergravity multiplet,

$$\rho^{AA'} \equiv \omega^A{}_{B'}{}^{B'A'} + \tfrac{1}{3}iA^{AA'} - \sqrt{2}\,\psi^{BAB'}\psi^{A'}{}_{BB'} ,$$

where

$$\tfrac{1}{2}(\omega_{MM'AB}\epsilon_{A'B'} + \omega_{MM'A'B'}\epsilon_{AB}) = \omega_{\mu ab}$$

is the spin connection with spin-$\frac{3}{2}$ torsion, $A_{MM'}$ is the axial-vector auxiliary field of the supergravity multiplet,

$$\begin{aligned}\mu^{A'} \equiv {}& -\frac{\sqrt{2}}{6}R_A{}^{AA'} - \tfrac{1}{3}i\psi^{B'BA'}A_{BB'} + \partial^{BB'}\psi^{A'}{}_{BB'} \\ & - \tfrac{1}{2}(\omega^{AB'}{}_A{}^B + \omega^{BC'B'}{}_{C'})\psi^{A'}{}_{BB'} \\ & - \tfrac{1}{2}\omega^{BB'}{}_{C'}{}^{A'}\psi^{C'}{}_{BB'} - \frac{\sqrt{2}}{2}\psi^{A'BB'}\psi_{B'}{}^{CC'}\psi_{BCC'} ,\end{aligned}$$

$R_{AMM'}$ is the spin-$\frac{3}{2}$ curvature in two-component form,[15] $R \equiv \mathcal{S} + 2\theta_A\eta^A + \theta^2\mathcal{R}$ is the Ricci scalar multiplet[10] in superfield form,

$$\theta^2 \equiv \theta_A\theta^A, \quad \theta'^2 \equiv \theta_{A'}\theta^{A'}, \text{ and } \psi'^2 \equiv \psi_{A'MM'}\psi^{A'MM'} . \tag{2}$$

We consider the local analog of the constraints (1)

$$\Phi^2=0 \quad \text{and} \quad \Phi=a\Phi\Pi_R\Phi^* , \tag{3a}$$

or, in components,

$$\begin{aligned}
&\mathcal{A}^2=0, \quad \mathcal{A}\chi^A=0, \quad \mathcal{A}\mathcal{F}=\chi^2, \quad \mathcal{A}=a\mathcal{A}(\mathcal{F}^*+\tfrac{1}{3}\mathcal{S}\mathcal{A}^*),\\
&\chi^A=a[-\sqrt{2}\,\mathcal{A}\tilde{D}^{AA'}\chi_{A'}+(\mathcal{F}^*+\tfrac{1}{3}\mathcal{S}\mathcal{A}^*)\chi^A],\\
&\mathcal{F}=a[\mathcal{F}(\mathcal{F}^*+\tfrac{1}{3}\mathcal{A}^*\mathcal{S})+\mathcal{A}\tilde{\Box}\mathcal{A}^*+2\sqrt{2}\,\chi_A\tilde{D}^{AA'}\chi_{A'}],
\end{aligned} \tag{3b}$$

where the kinetic multiplet

$$\Pi_R\Phi^*=\mathcal{F}^*+\tfrac{1}{3}\mathcal{A}^*\mathcal{S}-2\sqrt{2}\,\theta_A\tilde{D}^{AA'}\chi_{A'}+\theta^2\tilde{\Box}\mathcal{A}^*,$$

and

$$\begin{aligned}
\tilde{D}_{AA'}\chi^A &= \partial_{AA'}\chi^A+\frac{\sqrt{2}}{2}\psi_{B'BA'}(\partial^{BB'}\mathcal{A})-\tfrac{1}{2}\mathcal{F}\psi^A{}_{AA'}\\
&\quad +\tfrac{1}{2}\rho^*{}_{AA'}\chi^A+\frac{\sqrt{2}}{6}\mathcal{A}\eta_{A'},\\
\tilde{\Box}\mathcal{A} &= \Box\mathcal{A}-V^{*BB'}\partial_{BB'}\mathcal{A}+\tfrac{1}{3}\mathcal{A}\mathcal{R}^*+2\psi_A{}^{BB'}\partial_{BB'}\chi^A\\
&\quad +\chi_A\mu^A-\tfrac{1}{2}(\psi^2+\tfrac{4}{3}\mathcal{S})\mathcal{F},\\
V^{*BB'} &= -\Gamma^{BB'}+\frac{2i}{3}A^{BB'}-\frac{\sqrt{2}}{2}\psi^B{}_{MM'}\psi^{B'MM'},
\end{aligned} \tag{3c}$$

and

$-\Gamma^{BB'}=\Gamma^{\beta}=g^{\mu\rho}\Gamma^{\beta}_{\mu\rho}$

is the contracted affine connection. To solve these constraints we observe that $\mathcal{A}\chi^A=0$ can only be satisfied if $\mathcal{A}$ is proportional to χ^2 (this automatically ensures that $\mathcal{A}^2=0$). This leads us to the ansatz

$$\mathcal{A}=\chi^2(a+a_{A'}\chi^{A'}+b\chi'^2),$$

and $\mathcal{F}$ an arbitrary function of χ and χ'. Systematically collecting terms of the same order in χ and χ', we obtain an explicit solution of the constraints (3). This solution, however, is neither manifestly supercovariant nor illuminating; we, therefore, rearrange our solution and eventually obtain the results

$$\begin{aligned}
\mathcal{A}(\chi,\chi') &= a\chi^2[1+2\sqrt{2}\,a^2\mathsf{A}+a^3(\mathsf{B}+8a\mathsf{A}^2)],\\
\mathcal{F}(\chi,\chi') &= \frac{1}{a}-\tfrac{1}{3}\mathcal{S}^*\mathcal{A}(\chi,\chi')-2\sqrt{2}\,a\mathsf{A}-8a^3\mathsf{A}\mathsf{A}^*-a^2\mathsf{B}\\
&\quad -4\sqrt{2}\,a^4(\mathsf{A}^*\mathsf{B}+4a\mathsf{A}^*\mathsf{A}^2)\\
&\quad -4\sqrt{2}\,a^4(\tfrac{1}{2}\mathsf{A}\mathsf{B}^*+4a\mathsf{A}\mathsf{A}^{*2})\\
&\quad -2a^5(\mathsf{B}\mathsf{B}^*+96a^2\mathsf{A}^2\mathsf{A}^{*2}+12a\mathsf{A}^{*2}\mathsf{B}+8a\mathsf{A}^2\mathsf{B}),
\end{aligned} \tag{4a}$$

where, for the constrained field, A and B are manifestly supercovariant objects:

$$\begin{aligned}
\mathsf{A}\equiv\chi^{A'}\tilde{D}_{AA'}\chi^A &= \chi^{A'}\mathsf{D}_{AA'}\chi^A+\chi^{A'}\chi^A(\tfrac{1}{2}\rho^*{}_{AA'}+\sqrt{2}\,a\psi_{B'BA'}\partial^{BB'}\chi_A)+a\chi'^2\left(-\frac{\sqrt{2}}{2}\psi_B{}^{BA'}\mathsf{D}_{AA'}\chi^A\right)\\
&\quad +a\chi^2\chi^{A'}\left[\frac{\sqrt{2}}{6}\left(\eta_{A'}+\frac{\sqrt{2}}{2}\mathcal{S}^*\psi^A{}_{AA'}\right)+2a^2\psi_{B'BA'}(\partial^{BB'}\chi^{C'})\mathsf{D}_{CC'}\chi^C\right]\\
&\quad +a\chi'^2\chi^A\left\{2a^2\psi^{B'BC'}(\partial_{BB'}\chi_A)(\partial_{CC'}\chi^C)+\psi_B{}^{BA'}\left[\frac{\sqrt{2}}{4}\rho^*{}_{AA'}-2a^2(\mathsf{D}_{CA'}\chi^C)(\mathsf{D}_{AC'}\chi^{C'})\right]\right\}\\
&\quad +a^3\chi^2\chi'^2\Big[\psi^{B'BC'}\{\partial_{BB'}(\mathsf{D}_{CC'}\chi^C)+\sqrt{2}\,(\partial_{BB'}\chi_{C'})[a^2(\mathsf{D}\chi)^2+\tfrac{1}{3}\mathcal{S}+2a^2(\partial^{CA'}\chi_C)(\mathsf{D}_{AA'}\chi^A)]\}\\
&\qquad -\tfrac{1}{3}(\partial_{BB'}\chi^B)\left(\eta^{B'}-\frac{\sqrt{2}}{3}\psi_A{}^{AB'}\mathcal{S}^*\right)\\
&\qquad +\sqrt{2}\,\psi_D{}^{DA'}\Big(\frac{\sqrt{2}}{4}\rho^*{}_{AA'}(\mathsf{D}^{AB'}\chi_{B'})+(\mathsf{D}_{AA'}\chi^A)[a^2(\mathsf{D}\chi')^2+\tfrac{1}{3}\mathcal{S}^*+2a^2(\partial^{CB'}\chi_{B'})(\mathsf{D}_{CC'}\chi^{C'})]\\
&\qquad\quad +a\psi_{B'BA'}[(\partial^{BB'}\chi^C)(\mathsf{D}_{CC'}\chi^{C'})-(\partial^{BB'}\chi^{C'})(\mathsf{D}_{CC'}\chi^C)]\Big)\Big],\\
\mathsf{B}\equiv\chi'^2\tilde{\Box}\mathcal{A} &= a^{-1}\chi'^2\big(-2[a^2(\mathsf{D}\chi)^2+\tfrac{1}{3}\mathcal{S}]+a\chi_A[2a\Box\chi^A+\mu^A-2aV^{*BB'}\partial_{BB'}\chi^A-8\sqrt{2}\,a^3(\partial_{BB'}\chi^A)(\partial^{BB'}\chi^{A'})(\mathsf{D}_{CA'}\chi^C)]\\
&\quad +a^2\chi^2\{\tfrac{1}{3}\mathcal{R}^*+\tfrac{1}{6}\mathcal{S}^*\psi^2+\tfrac{2}{9}\mathcal{S}\mathcal{S}^*-2\sqrt{2}\,a^2V^{*BB'}(\partial_{BB'}\chi^{A'})(\mathsf{D}_{AA'}\chi^A)+2\sqrt{2}\,a^2(\Box\chi^{A'})(\mathsf{D}_{AA'}\chi^A)\\
&\quad -4\sqrt{2}\,a^2(\partial_{BB'}\chi^{A'})[\partial^{BB'}(\mathsf{D}_{AA'}\chi^A)]+4a^2(\partial\chi')^2[a^2(\mathsf{D}\chi)^2+\tfrac{1}{3}\mathcal{S}-2a^2(\partial_{BA'}\chi^B)(\mathsf{D}^{AA'}\chi_A)]\}\big),
\end{aligned} \tag{4b}$$

$$\mathsf{D}_{AA'}\chi_B=\partial_{AA'}\chi_B-\frac{1}{2a}\psi_{BAA'},$$

and

$$\Box=-\partial^{MM'}\partial_{MM'}.$$

Substituting (4b) into (4a) yields $\mathcal{A}$ and $\mathcal{F}$ explicitly in terms of χ and χ'. This gives the following nonlinear realization of the local supersymmetry algebra[17] (the supergravity transformations are given in Ref. 9):

$$\delta\chi^A=\mathcal{F}(\chi,\chi')\epsilon^A-\sqrt{2}\,\epsilon_{A'}\partial^{AA'}\mathcal{A}(\chi,\chi')+\sqrt{2}\,\epsilon_{A'}\psi_B{}^{AA'}\chi^B=\epsilon^A+\cdots \tag{5a}$$

(cf. Ref. 7). Clearly, there exists a gauge (U gauge) where $\chi=0 \Rightarrow \mathfrak{A}=0$ and $\mathfrak{F}=1/a$.

The action invariant under the transformation (5a) is found by applying the density formula[9]:

$$\begin{aligned} I&=\int d^4x\{\mathcal{L}_{SG}-\frac{e}{4a}[\mathfrak{F}(\chi,\chi')-\sqrt{2}\,\psi^{A'}{}_{AA'}\chi^A+\mathfrak{A}(\chi,\chi')(\mathcal{S}^*+\tfrac{1}{2}\psi^{A'}{}_{AA'}\psi_{B'}{}^{AB'}+\tfrac{1}{2}\psi_{B'AA'}\psi^{A'AB'})+\text{c.c.}]\} \\ &=\int d^4x\left[\mathcal{L}_{SG}+\frac{\sqrt{2}\,e}{2}(\chi^A D_{AA'}\chi^{A'}+\chi^{A'}D_{AA'}\chi^A)+\cdots\right], \end{aligned} \tag{5b}$$

where $\mathcal{L}_{SG}$ is the supergravity Lagrangian.[9]

In the U gauge the action (5b) reduces to

$$I_U=\int d^4x\left(\mathcal{L}_{SG}-\frac{e}{2a^2}\right). \tag{5c}$$

The supergravity Lagrangian $\mathcal{L}_{SG}$ can be taken to include a separately invariant term,

$$\mathcal{L}_m=me(\mathcal{S}^*+\tfrac{1}{2}\psi^{A'}{}_{AA'}\psi_{B'}{}^{AB'}+\tfrac{1}{2}\psi_{B'AA'}\psi^{A'AB'}+\text{c.c.}), \tag{5d}$$

and necessarily contains the auxiliary spin-0 field in the combination $-(e/3)\mathcal{S}\mathcal{S}^*$; integrating out the auxiliary field $\mathcal{S}$ (still in the U gauge) leads to a cosmological term, $e(3m^2-1/2a^2)$, which vanishes for $m^2=1/6a^2$, in agreement with Ref. 7, and leaves the spin-$\frac{3}{2}$ field with a mass $m=1/a\sqrt{6}$. In a general gauge, the relevant terms quadratic or lower in χ' are

$$\mathcal{L}_2=e\left[-\tfrac{1}{3}\mathcal{S}\mathcal{S}^*+m(\mathcal{S}^*+\mathcal{S})-\frac{1}{2a^2}-\frac{\sqrt{2}}{2a}(\chi^{A'}\psi^A{}_{AA'}+\chi^A\psi^{A'}{}_{AA'})-\tfrac{1}{3}(\mathcal{S}^*\chi^2+\mathcal{S}\chi'^2)\right].$$

Again, integrating out $\mathcal{S}$, we find

$$\mathcal{L}_2'=e\left[3m^2-\frac{1}{2a^2}-m(\chi^2+\chi'^2)-\frac{\sqrt{2}}{2a}(\chi^{A'}\psi^A{}_{AA'}+\chi^A\psi^{A'}{}_{AA'})\right],$$

in agreement with Refs. 7 and 8. To see the super-Higgs mechanism (the absorption of the Goldstone spinor by the spin-$\frac{3}{2}$ field) one can either go to the U gauge (as we did above) or redefine the spin-$\frac{3}{2}$ field to absorb explicitly the Goldstone spinor by[1] $\psi_{AMM'}\to\psi_{AMM'}+(1/\sqrt{3})\epsilon_{AM}\chi_{M'}-(\frac{2}{3})^{1/2}m^{-1}\partial_{MM'}\chi_A+\cdots$ (where we have taken $m^2=1/6a^2$). This discussion parallels precisely that given in Ref. 1 for the general coupling of the linear chiral multiplet to supergravity.

The most general Hermitian locally supersymmetric action for a self-interacting chiral multiplet without higher derivative terms quadratic in the fields can be written as[5,14]

$$I=\int d^4x\left[\mathcal{L}_{SG}+\left(\int d^2\theta\,\mathcal{E}[V(\Phi)+W(\Phi,\Pi_R\Phi^*)]+\text{c.c.}\right)\right], \tag{6a}$$

where

$$\begin{aligned} V(\Phi)&=\sum_{i=0}^{\infty}v_i\times(\Phi)^i, \\ W(\Phi,\Pi_R\Phi^*)&=\sum_{i,j=1}^{\infty}w_{ij}\times(\Phi)^i\times(\Pi_R\Phi^*)^j, \end{aligned} \tag{6b}$$

and $\mathcal{E}$ is the chiral density of Ref. 14.

When we impose the constraints (3), Eqs. (6b) reduce to

$$V(\Phi)=v_0+v_1\times\Phi,$$
$$W(\Phi,\Pi_R\Phi^*)=w_1\times\Phi,$$

where

$$w_1=\sum_{j=1}^{\infty}w_{1j}\times(a)^j. \tag{6c}$$

For $2a\times(v_1+w_1)=-1$, this is precisely the action (5b) with the separately invariant term $\mathcal{L}_m$ of Eq. (5d) for $m=v_0$. In this sense our action is unique. One might also consider including terms of the form $U(\Pi_R\Phi^*)$ in the action. However, this would lead to higher derivative terms quadratic in the fields. Such terms are expected to violate causality in the classical theory and unitarity in the quantized theory.

From this nonlinear realization, we can easily construct others. For example, consider a general real scalar ("vector") superfield V which satisfies the constraints

$$V^2=0 \text{ and } V=-\frac{a}{8}V\times(\mathfrak{D}_A\Pi_R\mathfrak{D}^A+\mathfrak{D}_{A'}\Pi_L\mathfrak{D}^{A'})V. \tag{7}$$

Using the relations

$$\Phi^2=0\Rightarrow\Phi\mathfrak{D}\Phi=0$$

and

$$\mathfrak{D}_{A'}\Phi=0\Rightarrow\Phi\mathfrak{D}_{A'}\mathfrak{D}_A\Phi=\Phi\{\mathfrak{D}_{A'},\mathfrak{D}_A\}\Phi\propto\Phi\mathfrak{D}_{AA'}\Phi=0,$$

where $\mathfrak{D}_A$, $\mathfrak{D}_{A'}$, $\mathfrak{D}_{AA'}$ are the curved superspace covariant derivatives,[14,15] one can show that the constraints (7) are satisfied if $V=a\Phi\Phi^*$. In the notation of Ref. 17, this can be written as $V=a\Phi\times\Phi$. V is a natural nonlinear realization for analyzing the super-Higgs mechanism in supersymmetric gauge theories.[11]

We have presented a nonlinear realization of supersymmetry coupled to supergravity and used it to demonstrate the super-Higgs mechanism. We have shown that our action is unique modulo higher-order derivative terms (and, of course, field redefinitions). Finally, we have suggested a generalization which should be useful in studying the super-Higgs mechanism in supersymmetric gauge theories.

ACKNOWLEDGMENTS

We would like to thank Professor S. Deser and Professor B. Zumino for bringing these problems to our attention. U. L. would also like to thank Professor S. Deser for his hospitality. M. R. would like to thank Professor T. T. Wu for his support. The work of U. L. was supported in part by NSF Grant No. PHY-78-09644 to Brandeis University. The work of M. R. was supported in part by the U.S. Department of Energy under Contract No. EY-76-S-02-3227 with Harvard University.

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[1]D. Z. Freedman, P. van Nieuwenhuizen, and S. Ferrara, Phys. Rev. D 13, 3214 (1976).

[2]S. Deser and B. Zumino, Phys. Lett. 62B, 335 (1976).

[3]See, for example, S. Ferrara, D. Z. Freedman, P. van Nieuwenhuizen, P. Breitenlohner, F. Gliozzi, and J. Scherk, Phys. Rev. D 15, 1013 (1977).

[4]D. V. Volkov and V. P. Akulov, Phys. Lett. 46B, 109 (1973).

[5]A. Das, M. Fischler, and M. Roček, Phys. Rev. D 16, 3427 (1977); E. Cremmer, B. Julia, J. Scherk, P. van Nieuwenhuizen, S. Ferrara, and L. Girardello, Phys. Lett. 79B, 231 (1978).

[6]T. Dereli and S. Deser, J. Phys. A 10, L149 (1977); 11, L27 (1978).

[7]S. Deser and B. Zumino, Phys. Rev. Lett. 38, 1433 (1977).

[8]In an interesting attempt to shed light on the problem, B. Zumino [Nucl. Phys. B127, 189 (1977)] generalized the Volkov-Akulov nonlinear realization of Ref. 4 to anti-de Sitter space.

[9]S. Ferrara and P. van Nieuwenhuizen, Phys. Lett. 76B, 404 (1978).

[10]S. Ferrara and P. van Nieuwenhuizen, Phys. Lett. 78B, 573 (1978).

[11]K. S. Stelle and P. C. West, Phys. Lett. 77B, 376 (1978).

[12]E. A. Ivanov and A. A. Kapustnikov, JINR Dubna Report No. E2-10765, 1977 (unpublished); J. Phys. A (to be published); also, B. Zumino and M. Roček, unpublished results.

[13]M. Roček, Phys. Rev. Lett. 41, 451 (1978).

[14]See, for example, W. Siegel and S. J. Gates, Jr., Nucl. Phys. B147, 77 (1979); J. Wess and B. Zumino, Phys. Lett. 79B, 394 (1978).

[15]M. Roček and U. Lindström, Phys. Lett. 79B, 217 (1978).

[16]P. Fayet and S. Ferrara, Phys. Rep. 32C, 249 (1977).

[17]K. S. Stelle and P. C. West, Phys. Lett. 74B, 330 (1978); S. Ferrara and P. van Nieuwenhuizen, *ibid.* 74B, 333 (1978).