PHYSICAL REVIEW D VOLUME 19, NUMBER 1 1 JANUARY 1979

Relation between scaling of semi-inclusive and inclusive reactions

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(Received 13 July 1978)

On the basis of the scaling law for the semi-inclusive differential cross section proposed by Dao *et al.* and the two-component mechanisms for production phenomena, we demonstrate that Feynman scaling for the inclusive differential cross section is good for the $x \neq 0$ region, but it is not valid at $x \simeq 0$. Our results are quite in accord with high-energy data for pp collisions.

After the proposal of a new scaling law for the semi-inclusive differential cross section by Dao *et al.*,[1] there have been several arguments[2-5] on the consequences of this semi-inclusive scaling (called scaling in the mean), especially on its relation to Feynman scaling. The authors of Refs. 1 and 3 found a discrepancy between scaling in the mean and Feynman scaling, on the basis of scaling in the mean for the semi-inclusive distributions, Koba-Nielsen-Olesen (KNO) scaling for the topological cross section,[6] and the energy-conservation sum rule [see Eq. (12a)]: Ernst and Schmitt (ES) pointed out[3] that the Lorentz-invariant inclusive cross section at $x=0$ (x is the usual Feynman scaling variable) should have an energy dependence $\langle n\rangle^2/\sqrt{s}$, where $\langle n\rangle$ is an average multiplicity and $\sqrt{s}$ is the total c.m. energy. ES claimed the above form is consistent with high-energy data on the violation of Feynman scaling at $x=0$ (Ref. 7) if $\langle n\rangle \propto s^{1/3}$. However, their result depends crucially on the choice of $\langle n\rangle \propto s^{1/3}$. In fact, it was shown by one of the present authors that such a choice is not allowed.[8] Subsequently, Yaes showed that the breaking of Feynman scaling in the $x \neq 0$ region is a result of scaling in the mean.[2] But it seems that Feynman scaling holds good up to $\sqrt{s}=53$ GeV except for the region around $x=0$.[7,9] In this context we proposed in a previous paper[5] a hopeful method by which the scaling for the semi-inclusive distributions is translated into Feynman scaling at asymptotic energies by means of the simple energy-conservation sum rule [cf. Eq. (12a)]. However, in the 40–300 GeV/c region this rule is not valid for the events with large multiplicities (we shall return this point later).

The purpose of this paper is to investigate the consequences of scaling in the mean based on two different production mechanisms. We shall show that scaling in the mean is consistent with the high-energy data[7,9] for inclusive cross sections; namely, that Feynman scaling holds in the $x \neq 0$ region and not in the $x \simeq 0$ region.

Let us consider π^- production in high-energy pp collisions as a typical reaction. Scaling in the mean states that the single-particle semi-inclusive differential cross section can be written as

$$\frac{\langle p_L\rangle_n \langle p_T\rangle_n}{n\sigma_n}\,\frac{d\sigma_n}{dp_L dp_T} = \phi\left(\frac{p_L}{\langle p_L\rangle_n},\ \frac{p_T}{\langle p_T\rangle_n}\right), \quad (1)$$

where σ_n is the n-particle cross section, and $\langle p_L\rangle_n (\langle p_T\rangle_n)$ is the average value of the magnitude of the c.m. longitudinal (transverse) momentum for the multiplicity n. It means that ϕ is independent of both $\sqrt{s}$ and n and depends only on the scaling variables ($p_L/\langle p_L\rangle_n$ and $p_T/\langle p_T\rangle_n$). The cross section satisfies the normalization condition

$$\int d^3p\, d\sigma_n/d^3p = n\sigma_n\,, \quad (2)$$

and $\langle p_L\rangle_n$ is defined by

$$\langle p_L\rangle_n = \frac{\int d^3p\, |p_L|\, d\sigma_n/d^3p}{\int d^3p\, d\sigma_n/d^3p}\,. \quad (3)$$

As usual, we assume

$$\langle p_T\rangle_n = \text{const} \simeq \langle p_T\rangle\,. \quad (4)$$

According to the assumption (4), Eq. (1) is reduced to

$$\frac{\langle p_L\rangle_n}{n\sigma_n}\,\frac{d\sigma_n}{dp_L dp_T} = \frac{1}{\langle p_T\rangle}\,\phi\left(\frac{p_L}{\langle p_L\rangle_n},\, p_T\right). \quad (5)$$

The invariant inclusive cross section is given by a sum of the semi-inclusive cross sections as follows,

$$(E/\sigma)d\sigma/d^3p = (E/\sigma)\sum_n d\sigma_n/d^3p\,, \quad (6)$$

in which $\sigma = \sum_n \sigma_n$. Using the scaling form (5), we have

$$\frac{E}{\sigma}\frac{d\sigma}{d^3p}=E\sum_n \frac{n}{\langle p_L\rangle_n}\frac{\sigma_n}{\sigma}\Phi\left(\frac{p_L}{\langle p_L\rangle_n},p_T\right), \quad (7)$$

where

$$\Phi(p_L/\langle p_L\rangle_n,p_T)=\phi(p_L/\langle p_L\rangle_n,p_T)/(2\pi p_T\langle p_T\rangle). \quad (8)$$

Since we are interested in a longitudinal-momentum distribution in this article, we consider a differential cross section at the fixed value of p_T (say, $p_T=p_{T0}\sim\langle p_T\rangle$). We rewrite Eq. (7) as

$$(E/\sigma)(d\sigma/d^3p)_{p_T=p_{T0}} =(p_L^2+m_T^2)^{1/2}\sum_n (n\sigma_n/\langle p_L\rangle_n\sigma)\Phi(p_L/\langle p_L\rangle_n) \quad (9)$$

in terms of $m_T{}^2=p_{T0}{}^2+m^2$ (m is the mass of the produced particle).

It is known that the multiparticle production phenomena can be explained naturally by considering two-component production mechanisms[10]: (1) diffraction dissociation, which is expected to be energy independent and populate primarily low-multiplicity channels, (2) a nondiffractive mechanism which contributes mainly to high multiplicity channels at high energies and is responsible for experimentally observed logarithmic increase in average multiplicity. According to the two-component model analysis done by Białkowski and Miettinen (FM),[10] the negative-charged-prong cross sections can be described by means of the Poisson distribution $p(n)=\exp(-\langle n_\pi\rangle)\langle n_\pi\rangle^n/n!$ for a nondiffractive mechanism and the energy-independent diffraction cross section σ_n^D/σ which becomes negligibly small for $n\simeq N+1 (N\sim 4)$, as follows,

$$\sigma_n/\sigma=\alpha P(n)+\sigma_n^D/\sigma, \quad (10)$$

in which α means

$$\alpha=1-\sum_n \sigma_n^D/\sigma. \quad (11)$$

We assume, further, that the average longitudinal momentum $\langle p_L\rangle_n$ is given by

$$\langle p_L\rangle_n=\begin{cases}(1-a)\sqrt{s}/(3n) \text{ due to a diffractive mechanism.}^{11} & (12a)\\ b\simeq O(\langle p_T\rangle) \text{ due to a nondiffractive mechanism.} & (12b)\end{cases}$$

When one takes the diffractive production mechanism, $\langle p_L\rangle_n$ is determined by the energy-conservation sum rule (12a), while $\langle p_L\rangle_n$ becomes b (an n-independent value of the order of $\langle p_T\rangle$) if the nondiffractive mechanism dominates. The value b may have a weak-energy dependence. We suppose that particles are produced isotropically and independently of the direction of the incident beam when a production occurs through the nondiffractive mechanism which becomes dominant for large n at high energies. This picture is supported by the data of $\langle p_L\rangle_n$ in 300-GeV/c pp collisions[1] which show that $\langle p_L\rangle_n$ decreases with increasing n for $n_{ch}\lesssim 8$, whereas, it turns to be constant of the order of $\langle p_T\rangle$ for $n_{ch}\gtrsim 20$.

Thus, substituting Eqs. (10) and (12) into (9), and using

$$x=2p_L/\sqrt{s}, \quad (13)$$

we obtain

$$\frac{E}{\sigma}\frac{d\sigma}{d^3p}\bigg|_{p_T=p_{T0}}=\frac{3}{2(1-a)}(x^2+4m_T{}^2/s)^{1/2}\sum_{n=0}^N n^2\frac{\sigma_n^D}{\sigma}\Phi\left(\frac{3xn}{2(1-a)}\right)+\frac{\alpha}{2b}(sx^2+4m_T{}^2)^{1/2}\langle n_\pi\rangle\Phi\left(\frac{\sqrt{s}x}{2b}\right). \quad (14)$$

Since the experimental data[1] show that $\Phi(t)$ decreases rapidly as $|t|$ increases, we can see that the second term of Eq. (14) has a dominant contribution only at $x\simeq 0$ and vanishes for $|x|>0$. Therefore, at high energy Eq. (14) turns out to be

$$(E/\sigma)(d\sigma/d^3p)_{p_T=p_{T0}}\simeq\begin{cases}\alpha m_T\langle n_\pi\rangle\Phi(0)/b+0(1/\sqrt{s}) \text{ for } x=0, & (15a)\\ 3x/[2(1-a)\sigma]\sum_{n=0}^N n^2\sigma_n^D\Phi[3x/2(1-a)] \text{ for } |x|\gg m_T/\sqrt{s}. & (15b)\end{cases}$$

The above equations tell us that (1) the invariant inclusive cross section at $x=0$ increases as $\langle n_\pi\rangle$ when the energy increases, which is consistent to the data of the scaling violation,[7] and (2) the Feynman scaling holds for $|x|\gg m_T/\sqrt{s}$ [notice σ_n^D/σ and $\Phi\{3xn/2(1-a)\}$ are energy independent]. Furthermore, using the numerical values of the parameters of two-component model ($\alpha,\sigma_n^D/\sigma$, etc.) obtained by FM[10] and taking $a\simeq 0.75$, $b\simeq 0.30$–0.36 GeV/c,[12] and (see Ref. 13)

$$\phi(t)\propto e^{-t}, \quad (16)$$

we get the inclusive cross section for all x as is shown in Figs. 1 and 2. Though there is some

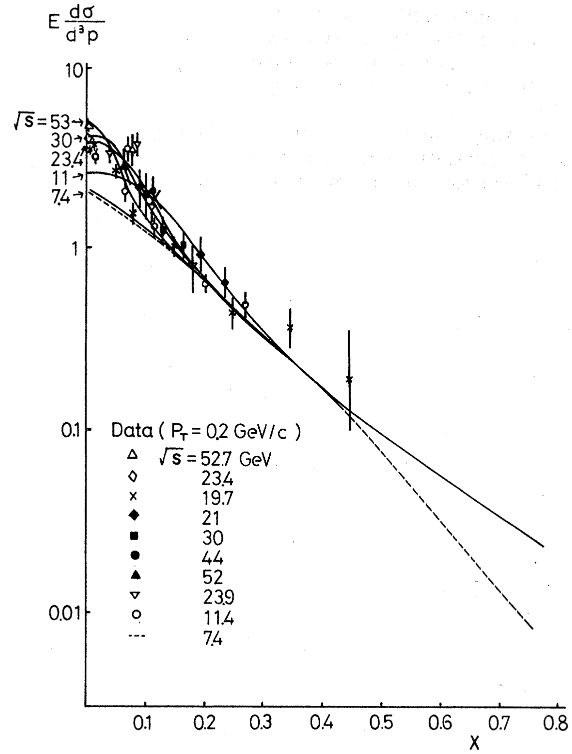

FIG. 1. A comparison of our calculation with the high-energy data of x distributions in $pp \to \pi^- + X$. The solid line is our estimate from $\sqrt{s} = 7.4$ to $\sqrt{s} = 53$ GeV. The data are taken from the figures in Ref. 9.

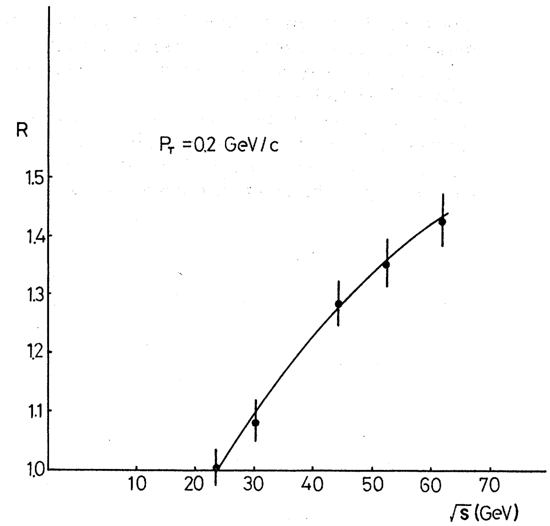

FIG. 2. Our fit to the data of the rise of π^- distribution at $x = 0$: $R = (Ed\sigma/d^3p)_{x=0}/(Ed\sigma/d^3p)_{x=0}$(at $\sqrt{s} = 23.4$ GeV). The data are taken from Ref. 7. The solid curve shows our model calculations.

discrepancy between the data and the result obtained in this work at the large x region ($x \gtrsim 0.5$), it is because of our too simple model. In fact, this discrepancy may be improved by a more realistic model. For instance, if we take $\Phi(t) \propto \exp(-At - Bt^2)$ instead of (16), we may be able to get a good agreement with experiment at large x. However, it is not our purpose to obtain a detailed fit by increasing the number of parameters, but to study the consequences from the scaling in the mean based on naive considerations. Therefore, it is worth noticing that, in spite of our simple model, we can reproduce well the gross features of single-pion distributions in pp collisions as is shown in Figs. 1 and 2. In this way, we get the following results as our conclusions: (1) The semi-inclusive scaling is compatible with the data of the inclusive reactions. (2) Feynman scaling for $x \neq 0$ region is due to the diffractive production mechanism. (3) The violation of Feynman scaling (the increase of the inclusive differential cross section with the energy increasing) at $x \simeq 0$ is due to the nondiffractive mechanism. (4) The rise of the cross section at $x = 0$ is proportional to average multiplicity. (5) When KNO scaling is assumed for the topological cross sections in place of the two-component picture [Eq. (10)], we can not obtain a result which is consistent with the present high-energy data of inclusive reactions. Therefore, we can say that KNO scaling is not valid, but only a temporary accident as far as scaling in the mean holds.

The authors would like to thank Professor S. Hori, Professor E. Yamada, and Professor A. Wakasa for their kind comments and criticism.

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[11] This form is essentially the same as that presented in

Ref. 2 except for the factor a which is a correction due to the elastic events, transverse momentum, mass effects etc. Notice, also, n represents the number of negative charged particles (π^-).

[12]In order to get an agreement between our result and the data concerning the violation of the Feynman scaling, we need to choose b which depends weakly on the incident energy. The value of b changes from 0.30 to 0.36 GeV/c as $\sqrt{s}$ increases from 7 to 60 GeV. A weak energy dependence has also been observed in the high-energy cosmic-ray data of $\langle p_T \rangle$. See D. Cline, F. Halzen, and J. Luthe, Phys. Rev. Lett. 31, 491 (1973).

[13]This simple form is suggested by the data of the single-particle semi-inclusive differential cross section[1] and the normalization condition[2].