Asymmetric creation of matter and antimatter in the expanding universe

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We consider a simple model in which the matter-antimatter asymmetry of the universe is brought about by an effective two-particle interaction that violates baryon-number conservation as well as *CP* invariance. The particle fields participating in the interaction are quantized, and their time development in an isotropically expanding universe is found to all orders in the coupling constant. Pair production by the asymmetric interaction, as well as symmetric production by the gravitational field of the expanding universe, appear simultaneously in the solution. Taking an initial state in which no particles participating in the asymmetric interaction are present, we find the created baryon-number density. We consider in more detail the case when the matter-antimatter asymmetry is produced during a stage when the radius of the universe is small with respect to its present value. We make numerical estimates of the created matter-antimatter asymmetry, and put limits on possible values of the parameters of this model.

I. INTRODUCTION

Observational evidence seems to indicate that there exists a fairly large-scale imbalance of matter and antimatter in the universe.[1] If the homogeneity of the universe includes that of the matter and antimatter distributions, then one would expect there to be more matter than antimatter in the universe as a whole. Such asymmetry could in principle be present in the initial conditions, but it is more satisfying to contemplate a model in which the initial conditions are symmetric and the matter-antimatter asymmetry arises through a dynamical mechanism. Models of that type also have the possibility of being tested against observations. The possibility that the mechanism for producing the asymmetry involves elementary-particle interactions which violate the conservation of baryon number and other symmetries has been raised or discussed in a number of papers.[2-11] Unified gauge theories of strong, weak, and electromagnetic interactions generally involve nonconservation of baryon number,[12,13] and such violations can also occur through instantons in the Weinberg-Salam model.[14]

In this paper, we consider a simple model of the baryon-number-nonconserving interaction involving a complex scalar field ϕ which carries a nonzero baryon number (which we take to be unity) and a complex scalar field ψ which has zero baryon number (which we treat as an antilepton[15] field). The asymmetric interaction is of the general form introduced in Ref. 9:

$$\lambda R(\phi^*\Lambda\psi+\psi^*\Lambda^*\phi)\,, \qquad (1.1)$$

where λ is a dimensionless coupling constant, R is the scalar curvature of spacetime, and Λ is a complex function of space and/or time. [In Ref. 9, $R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$ replaced R in Eq. (1.1) so that the interaction would not vanish in a black-hole metric.] This may be regarded as an effective Lagrangian summarizing a set of more fundamental interactions which allow decay of a baryon (or perhaps a quark) to an antilepton, or production of a baryon-lepton pair (since R becomes large when the temperature would be high in a cosmological metric, one could regard R as modeling the behavior of an interaction mediated by very massive particles); or the Lagrangian may be regarded as representing an interaction in which gravitons interact with the fields in such a way as to violate baryon-number conservation. Because of the small value of R today, even if conventional baryons participate in this type of interaction, their lifetime would be much larger than the known limits. For reasonable values of λ this interaction is significant only when R is large. The gravitational field is treated classically, while ϕ and ψ are quantized. The particles have arbitrary masses and additional baryon-number-conserving couplings to the scalar curvature. The metric is taken to be that of a spatially flat isotropic universe with an arbitrary expansion parameter (or "radius") $a(t)$. Our results would be only slightly altered in the spatially curved Robertson-Walker universes. During the radiation-dominated phase of the universe, R vanishes identically, so that this asymmetric interaction is absent except near the cosmological singularity or the "bounce," when it is significant, or near the present time, when it is negligible.

We find [for arbitrary $a(t)$ and $\Lambda(t)$] the general solution of this problem expressed as a series in powers of λ. Our solution exhibits both the purely gravitational symmetric creation of pairs,[16] and the baryon-number-nonconserving creation of pairs and decays caused by the asymmetric interaction (such pair creation is permitted because the gravitational metric acts as a time-dependent external field). For an arbitrary $a(t)$ in which the expansion is slow at early and late times, we give the baryon number density at late times to all orders in λ. [We treat $a(t)$ as though it were constant at early and late times, but the results are unchanged if $a(t)$ varies sufficiently slowly at those times that the particle creation rate at early and late times is negligible; an example of such an $a(t)$ would be one in which a cosmological "bounce" occurs.] We take the initial state to contain no particles of the type which participate in the asymmetric interaction (one could equally well take an initial state represented by a statistical density operator). Thus, all such particles present at late times are generated by the gravitational field and by the asymmetric interaction. We find that to all orders in λ the created baryon number density has no explicit dependence on the parameters which characterize the purely gravitational (symmetric) pair production. Nevertheless, the created baryon and lepton densities do have terms involving products of λ and those parameters.

The matter-antimatter asymmetry in this model first appears in order λ^2. Using our results to that order in λ, we estimate the magnitude of the asymmetry in a cosmological model in which the universe first contracts to a minimum and then expands to the present time. In obtaining our estimates, the mass of the lepton is regarded as negligibly small, and its purely gravitational interaction is taken as conformally invariant (so that no purely gravitational production of those leptons occurs).[16-18] The mass m of the baryon is taken to be sufficiently large that the purely gravitational production of baryons can be neglected (this also means that thermal equilibrium is not established for those particles). Alternatively, the baryon can be regarded as massless with conformally invariant coupling to gravity. The quantity $\Lambda(t)$ appearing in the interaction is taken to have absolute magnitude unity and to change phase by 2σ at $t=0$, where the minimum of the "bounce" occurs. Thus, the angle σ is a measure of the asymmetry of the interaction (when the phase of Λ is constant for all t, no asymmetric production occurs). Our estimates should remain valid, as shown in the Appendix, also if the phase of Λ changes by 2σ in a smooth manner during the period when the interaction is significant. The approximate magnitude of the created baryon number density is given as a function of m, λ, σ, and the quantity G, of dimension length squared, which characterizes the minimum of the bounce (G is not necessarily the Newtonian constant). The effect is found to depend only weakly on the baryon mass m. For various values of λ, G, and σ, the ratio of baryon number density to the total entropy density (divided by Boltzmann's constant) is estimated numerically, both for models in which the asymmetric interaction produces most of the energy and entropy density of the universe, and for models in which that energy and entropy density come mostly from other particles, and only the baryon number density results from the interaction term. In the former class of models the ratio of baryon number to entropy is independent of λ (to second order in λ), and λ is determined as a function of G and the Newtonian constant through self-consistency with the Einstein equation. The ratio of baryon number to entropy is conserved[19] in the standard isotropic cosmological model (this asymmetric production process is significant only at times near $t=0$, so that the usual predictions of the standard model are not affected). The observed value of that ratio, in the range 10^{-8} to 10^{-10}, is used to put limits on λ, σ, and G (the parameter characterizing the minimum dimension of the universe). For example, we find that $G^{1/2}$ cannot be larger than about 10^{-39} sec ($\approx 10^5\times$ Planck time) for such baryon-number-nonconserving interactions to be capable of generating the observed matter-antimatter asymmetry.

The results of our estimates are expected to be of the correct order of magnitude for any model in which the matter-antimatter asymmetry is produced mainly by interactions which are effectively described by the present Lagrangian at times when the expansion parameter $a(t)$ was small with respect to its present value. With the present interaction, the asymmetry does not vanish when the masses vanish. The considerations of Refs. 9, 10, and 20 concerning massless particles do not apply to this case, in which the external gravitational field induces asymmetric pair creation.

The general formalism and solution as a power series to all orders in λ is given in Secs. II and III. The second-order results and numerical estimates placing limits on the parameters of the model are given in Secs. IV and V. In the Appendix, Lorentzian forms for the scalar curvature and the phase change are considered.

II. GENERAL FORMALISM

We consider a system of two complex fields $\phi(x)$ and $\psi(x)$ described by the Lagrange density

$$\mathcal{L}=\sqrt{-g}[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi+g^{\mu\nu}\partial_\mu\psi^*\partial_\nu\psi-(m_1{}^2+\xi_1 R)\phi^*\phi - (m_2{}^2+\xi_2 R)\psi^*\psi-\lambda R(\phi^*\Lambda\psi+\psi^*\Lambda^*\phi)] \quad (2.1)$$

with R the scalar curvature. We interpret $\phi(x)$ as carrying baryon number +1 ("baryon field") and $\psi(x)$ as carrying zero baryon number ("antilepton field"). Then the last terms in (2.1) violate baryon-number conservation. We assume that the fields $\phi(x), \psi(x)$ participate in other interactions which conserve baryon number and establish their baryonic and leptonic nature respectively; since such interactions do not contribute to any possible baryon-antibaryon asymmetry, we do not consider them explicitly in this paper. Finally, the complex quantity Λ is a function of time and/or space; thus the last two terms in (2.1) also break CP invariance. Therefore, the Lagrangian (2.1) provides a simple framework for investigating the cosmological implications of baryon-number and CP nonconservation.

The invariance of the Lagrangian (2.1) under constant-phase transformations

$$\phi(x) \to e^{i\theta}\phi(x),$$
$$\psi(x) \to e^{i\theta}\psi(x)$$

implies the existence of a conserved current:

$$j^\mu(x) = i\sqrt{-g}\, g^{\mu\nu}(\phi^* \overleftrightarrow{\partial}_\mu \phi + \psi^* \overleftrightarrow{\partial}_\mu \psi), \tag{2.2}$$

where

$$\partial_\mu j^\mu(x) = 0. \tag{2.3}$$

Therefore, the quantity

$$\int_\sigma dS_\mu j^\mu(x) \equiv Q \tag{2.4}$$

is independent of the spacelike Cauchy hypersurface σ. It corresponds to the total charge if the particles are charged, and in general to the conserved sum of the baryon and antilepton numbers in this model.

The equations of motion are

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\phi) + (m_1{}^2 + \xi_1 R)\phi + \lambda R\Lambda\psi = 0, \tag{2.5a}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\psi) + (m_2{}^2 + \xi_2 R)\psi + \lambda R\Lambda^*\phi = 0. \tag{2.5b}$$

Instead of continuing with a general form of $g^{\mu\nu}$, we specialize to a spatially flat, statically bounded Robertson-Walker metric:

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j, \tag{2.6}$$

$$a(t) \to \begin{cases} a_+, & t > T \\ a_-, & t < -T \end{cases} \tag{2.7}$$

where $a_\pm$ are constants.

Since

$$R = 6(a^{-2}\dot{a}^2 + a^{-1}\ddot{a}),$$

Eqs. (2.7) imply that

$$R(t) = 0 \text{ for } t > |T|, \tag{2.8}$$

so that at early and late times spacetime becomes flat and one can define in and out Hilbert spaces in the conventional way. Notice also that the baryon- and CP-nonconserving interaction vanishes for $t > |T|$. To preserve the homogeneity and isotropy of the model, we assume that Λ is a function of time alone.

Since the metric (2.6) is spatially flat, it is convenient to decompose the fields in Fourier components. As a purely mathematical device, we imagine the system confined in a box of side L with periodic boundary conditions at the walls (the continuum limit $L \to \infty$ will be taken at the end); then

$$\phi(x) = \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}[La(t)]^{-3/2}\phi_{\vec{k}}(t), \tag{2.9}$$

$$\psi(x) = \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}[La(t)]^{-3/2}\psi_{\vec{k}}(t), \tag{2.10}$$

with

$$\vec{k} = \frac{2\pi}{L}(n_1, n_2, n_3), \tag{2.11}$$

and n_1, n_2, n_3 integers.

In terms of the Fourier components, the equations of motion reduce to

$$\frac{d^2}{dt^2}\phi_{\vec{k}}(t) + \left(\frac{k^2}{a^2} + m_1{}^2 + \xi_1 R - \frac{3}{2}\frac{\ddot{a}}{a} - \frac{3}{4}\frac{\dot{a}^2}{a^2}\right)\phi_{\vec{k}}(t) + \lambda R\Lambda\psi_{\vec{k}}(t) = 0, \tag{2.12}$$

$$\frac{d^2}{dt^2}\psi_{\vec{k}}(t) + \left(\frac{k^2}{a^2} + m_2{}^2 + \xi_2 R - \frac{3}{2}\frac{\ddot{a}}{a} - \frac{3}{4}\frac{\dot{a}^2}{a^2}\right)\psi_{\vec{k}}(t) + \lambda R\Lambda^*\phi_{\vec{k}}(t) = 0. \tag{2.13}$$

We also note that as a result of the canonical commutation relations, the operators $\phi_{\vec{k}}, \psi_{\vec{k}}$ obey at equal times

$$[\phi_{\vec{k}}(t), \dot{\phi}^*_{\vec{k}'}(t)] = i\delta_{\vec{k},\vec{k}'}, \tag{2.14a}$$

$$[\psi_{\vec{k}}(t), \dot{\psi}^*_{\vec{k}'}(t)] = i\delta_{\vec{k},\vec{k}'}, \tag{2.14b}$$

and all the other equal-time commutators among them are zero.

For $t < -T$, one has

$$\phi_{\vec{k}}(t) = \frac{1}{\sqrt{2\omega_{1k}}}\left(a^{(\text{in})}_{\phi\vec{k}} e^{-i\omega_{1k}t} + b^{\dagger(\text{in})}_{\phi,-\vec{k}} e^{i\omega_{1k}t}\right) \quad (t < -T), \tag{2.15a}$$

$$\psi_{\vec{k}}(t)=\frac{1}{\sqrt{2\omega_{2k}}}\left(a^{(\text{in})}_{\psi\vec{k}}\,e^{-i\omega_{2k}t}+b^{\dagger(\text{in})}_{\psi-\vec{k}}\,e^{i\omega_{2k}t}\right)\quad(t<-T)\tag{2.15b}$$

with

$$\omega_{1,2k}=\left(\frac{k^2}{a_-^{\,2}}+m_{1,2}^{\;2}\right)^{1/2}.$$

Similarly, for $t>T$

$$\phi_{\vec{k}}(t)=\frac{1}{\sqrt{2\omega'_{1k}}}\left(a^{(\text{out})}_{\phi\vec{k}}\,e^{-i\omega'_{1k}t}+b^{\dagger(\text{out})}_{\phi-\vec{k}}\,e^{i\omega'_{1k}t}\right)\quad(t>T),\tag{2.16a}$$

$$\psi_{\vec{k}}(t)=\frac{1}{\sqrt{2\omega'_{2k}}}\left(a^{(\text{out})}_{\psi\vec{k}}\,e^{-i\omega'_{2k}t}+b^{\dagger(\text{out})}_{\psi-\vec{k}}\,e^{i\omega'_{2k}t}\right)\quad(t>T)\tag{2.16b}$$

with

$$\omega'_{1,2k}=\left(\frac{k^2}{a_+^{\,2}}+m_{1,2}^{\;2}\right)^{1/2}.$$

We will also need the solutions of Eqs. (2.12), (2.13) for $\lambda=0$; at all times,

$$\overset{(0)}{\phi}_{\vec{k}}(t)=a^{(\text{in})}_{\phi\vec{k}}\,\chi_{\phi\vec{k}}(t)+b^{\dagger(\text{in})}_{\phi-\vec{k}}\,\chi^*_{\phi k}(t),\tag{2.17}$$

$$\overset{(0)}{\psi}_{\vec{k}}(t)=a^{(\text{in})}_{\psi\vec{k}}\,\chi_{\psi k}(t)+b^{\dagger(\text{in})}_{\psi-\vec{k}}\,\chi^*_{\psi\vec{k}}(t),\tag{2.18}$$

where $\chi_{\phi k},\chi_{\psi k}$ are c-number solutions of

$$\frac{d^2}{dt^2}\chi_{\phi k}(t)+\left(\frac{k^2}{a^2}+m_1^{\,2}+\xi_1R-\frac{3}{2}\frac{\ddot a}{a}-\frac{3}{4}\frac{\dot a^2}{a^2}\right)\chi_{\phi k}(t)=0,\tag{2.19}$$

$$\frac{d^2}{dt^2}\chi_{\psi k}(t)+\left(\frac{k^2}{a^2}+m_2^{\,2}+\xi_2R-\frac{3}{2}\frac{\ddot a}{a}-\frac{3}{4}\frac{\dot a^2}{a^2}\right)\chi_{\psi k}(t)=0\tag{2.20}$$

satisfying the boundary conditions at $t<-T$:

$$\chi_{\phi k}(t)=\frac{1}{\sqrt{2\omega_{1k}}}e^{-i\omega_{1k}t}\quad(t<-T),\tag{2.21a}$$

$$\chi_{\psi k}(t)=\frac{1}{\sqrt{2\omega_{2k}}}e^{-i\omega_{2k}t}\quad(t<-T).\tag{2.21b}$$

The solution of Eqs. (2.19) and (2.20) are known for specific cases, and can be approximated under various conditions, as in Sec. IV below.

The quantities

$$\Delta_\phi(\vec{k};t,t')\equiv i\theta(t-t')[\overset{(0)}{\phi}_{\vec{k}}(t),\overset{(0)}{\phi}{}^*_{\vec{k}}(t')],\tag{2.22}$$

$$\Delta_\psi(\vec{k};t,t')\equiv i\theta(t-t')[\overset{(0)}{\psi}_{\vec{k}}(t),\overset{(0)}{\psi}{}^*_{\vec{k}}(t')]\tag{2.23}$$

are retarded Green's functions for Eqs. (2.19) and (2.20), respectively; for example,

$$\left[\frac{d^2}{dt^2}+\left(\frac{k^2}{a^2}+m_1^{\,2}+\xi_1R-\frac{3}{2}\frac{\ddot a}{a}-\frac{3}{4}\frac{\dot a^2}{a^2}\right)\right]\Delta_\phi(\vec{k};t,t')=\delta(t-t'),$$

as can be seen easily by means of the equal-time commutation relations obeyed by the $\overset{(0)}{\phi}_{\vec{k}},\overset{(0)}{\phi}{}^*_{\vec{k}}$ [Eq. (2.14)]. Using the decompositions (2.17), (2.18) we find

$$\Delta_\phi(\vec{k};t,t')=i\theta(t-t')[\chi_{\phi k}(t)\chi^*_{\phi k}(t')-\chi^*_{\phi k}(t)\chi_{\phi k}(t')],\tag{2.24}$$

$$\Delta_\psi(\vec{k};t,t')=i\theta(t-t')[\chi_{\psi k}(t)\chi^*_{\psi k}(t')-\chi^*_{\psi k}(t)\chi_{\psi k}(t')],\tag{2.25}$$

where $\theta(x)$ is unity for $x>0$ and zero for $x<0$.

These Green's functions allow us to convert Eqs. (2.12), (2.13) into integral equations:

$$\phi_{\vec{k}}(t)=\overset{(0)}{\phi}_{\vec{k}}(t)-\lambda\int_{-T}^{T}\Delta_\phi(\vec{k};t,t')R(t')\Lambda(t')\psi_{\vec{k}}(t')dt',$$

$$\psi_{\vec{k}}(t)=\overset{(0)}{\psi}_{\vec{k}}(t)-\lambda\int_{-T}^{T}\Delta_\psi(\vec{k};t,t')R(t')\Lambda^*(t')\phi_{\vec{k}}(t')dt'.$$

The limits of integration can be extended from $-\infty$ to ∞ because $R(t')$ vanishes for $|t'|>T$.

The boundary conditions (2.16a), (2.16b) have been incorporated in these equations. One can easily deduce the following decoupled set of equations satisfied by $\phi_{\vec{k}}(t),\psi_{\vec{k}}(t)$:

$$\phi_{\vec{k}}(t)=\overset{(0)}{\phi}_{\vec{k}}(t)-\lambda\int_{-\infty}^{\infty}\Delta_\phi(\vec{k};t,t')R(t')\Lambda(t')\overset{(0)}{\psi}_{\vec{k}}(t')dt'+\lambda^2\int_{-\infty}^{\infty}dt'\int_{-\infty}^{\infty}dt''\Delta_\phi(\vec{k};t,t')R(t')\Lambda(t')\Delta_\psi(\vec{k};t',t'')R(t'')\Lambda(t'')\phi_{\vec{k}}(t''),\tag{2.26}$$

$$\psi_{\vec{k}}(t)=\overset{(0)}{\psi}_{\vec{k}}(t)-\lambda\int_{-\infty}^{\infty}\Delta_\psi(\vec{k};t,t')R(t')\Lambda^*(t')\overset{(0)}{\phi}_{\vec{k}}(t')dt'+\lambda^2\int_{-\infty}^{\infty}dt'\int_{-\infty}^{\infty}dt''\Delta_\psi(\vec{k};t,t')R(t')\Lambda^*(t')\Delta_\phi(\vec{k};t',t'')R(t'')\Lambda(t'')\psi_{\vec{k}}(t'').\tag{2.27}$$

If we introduce

$$G_\phi(\vec{k};t,t')\equiv\langle t|G_\phi|t'\rangle=\Delta_\phi(\vec{k};t,t')R(t')\Lambda(t'),\tag{2.28}$$

$$G_\psi(\vec{k};t,t')\equiv\langle t|G_\psi|t'\rangle=\Delta_\psi(\vec{k};t,t')R(t')\Lambda^*(t'),\tag{2.29}$$

we can rewrite Eqs. (2.26), (2.27) in the compact form

$$\phi_{\vec{k}} = \overset{(0)}{\phi}_{\vec{k}} - \lambda G_\phi \overset{(0)}{\psi}_{\vec{k}} + \lambda^2 G_\phi G_\psi \phi_{\vec{k}} , \tag{2.30}$$

$$\psi_{\vec{k}} = \overset{(0)}{\psi}_{\vec{k}} - \lambda G_\psi \overset{(0)}{\phi}_{\vec{k}} + \lambda^2 G_\psi G_\phi \psi_{\vec{k}} . \tag{2.31}$$

Their solution is

$$\phi_{\vec{k}} = \sum_{n=0}^{\infty} \lambda^{2n} (G_\phi G_\psi)^n [\overset{(0)}{\phi}_{\vec{k}} - \lambda G_\phi \overset{(0)}{\psi}_{\vec{k}}] , \tag{2.32}$$

$$\psi_{\vec{k}} = \sum_{n=0}^{\infty} \lambda^{2n} (G_\psi G_\phi)^n [\overset{(0)}{\psi}_{\vec{k}} - \lambda G_\psi \overset{(0)}{\phi}_{\vec{k}}] . \tag{2.33}$$

For later convenience, we introduce

$$F \equiv \sum_{n=0}^{\infty} \lambda^{2n+1} (G_\psi G_\phi)^n , \tag{2.34}$$

$$\tilde{F} \equiv \sum_{n=0}^{\infty} \lambda^{2n+1} (G_\phi G_\psi)^n , \tag{2.35}$$

and rewrite the solutions (2.32), (2.33) as

$$\phi_{\vec{k}} = \overset{(0)}{\phi}_{\vec{k}} + G_\phi F(\lambda G_\psi \overset{(0)}{\phi}_{\vec{k}} - \overset{(0)}{\psi}_{\vec{k}}) , \tag{2.36}$$

$$\psi_{\vec{k}} = \overset{(0)}{\psi}_{\vec{k}} + G_\psi \tilde{F}(\lambda G_\phi \overset{(0)}{\psi}_{\vec{k}} - \overset{(0)}{\phi}_{\vec{k}}) . \tag{2.37}$$

Now we are ready to relate the in and out creation and annihilation operators by examining these equations at $t > T$. The functions $\chi_{\phi\vec{k}}, \chi_{\psi\vec{k}}$ for $t > T$ are related to those at $t < -T$ [Eqs. (2.21)] by Bogoliubov transformations:

$$\chi_{\phi\vec{k}}(t) = \frac{1}{\sqrt{2\omega'_{1k}}} (\alpha_{\phi k} e^{-i\omega'_{1k}t} + \beta_{\phi k} e^{i\omega'_{1k}t}) \quad (t > T) , \tag{2.38}$$

$$\chi_{\psi\vec{k}}(t) = \frac{1}{\sqrt{2\omega'_{2k}}} (\alpha_{\psi k} e^{-i\omega'_{2k}t} + \beta_{\psi k} e^{i\omega'_{2k}t}) \quad (t > T) , \tag{2.39}$$

where α and β are complex constants satisfying

$$|\alpha_{\phi k}|^2 - |\beta_{\phi k}|^2 = |\alpha_{\psi k}|^2 - |\beta_{\psi k}|^2 = 1 . \tag{2.40}$$

This identifies the positive- and negative-frequency components of the first terms on the right-hand side of Eqs. (2.36), (2.37). For the second terms, we have, using (2.28), (2.24),

$$\begin{aligned}
\langle t | G_\phi F(\lambda G_\psi \phi_k^{(0)} - \psi_k^{(0)}) | t' \rangle &= \int_{-\infty}^{\infty} dt'' \Delta_\phi(\vec{k}; t, t'') R(t'') \Lambda(t'') \langle t'' | F(\lambda G_\psi \phi_k^{(0)} - \psi_k^{(0)}) | t' \rangle \\
&= i\chi_{\phi k}(t) \int_{-\infty}^{\infty} dt'' \theta(t - t'') \chi^*_{\phi k}(t'') R(t'') \Lambda(t'') \langle t'' | F(\lambda G_\psi \phi_k^{(0)} - \psi_k^{(0)}) | t' \rangle \\
&\quad - i\chi^*_{\phi k}(t) \int_{-\infty}^{\infty} dt'' \theta(t - t'') \chi_{\phi k}(t'') R(t'') \Lambda(t'') \langle t'' | F(\lambda G_\psi \phi_k^{(0)} - \psi_k^{(0)}) | t' \rangle ,
\end{aligned}$$

and the positive- and negative-frequency components can be separated by means of Eqs. (2.38), (2.39).

The result can be written in the form

$$\overset{(\text{out})}{a}_{\phi\vec{k}} = (\alpha_{\phi k} + h^{(1)}_{\phi k}) \overset{(\text{in})}{a}_{\phi\vec{k}} + (\beta^*_{\phi k} + h^{(2)}_{\phi k}) \overset{\dagger(\text{in})}{b}_{\phi -\vec{k}} + h^{(3)}_{\phi k} \overset{(\text{in})}{a}_{\psi\vec{k}} + h^{(4)}_{\phi k} \overset{\dagger(\text{in})}{b}_{\psi -\vec{k}} , \tag{2.41a}$$

$$\overset{\dagger(\text{out})}{b}_{\phi\vec{k}} = (\beta_{\phi k} + p^{(1)}_{\phi k}) \overset{(\text{in})}{a}_{\phi -\vec{k}} + (\alpha^*_{\phi k} + p^{(2)}_{\phi k}) \overset{\dagger(\text{in})}{b}_{\phi\vec{k}} + p^{(3)}_{\phi k} \overset{(\text{in})}{a}_{\psi -\vec{k}} + p^{(4)}_{\phi k} \overset{\dagger(\text{in})}{b}_{\psi\vec{k}} , \tag{2.41b}$$

$$\overset{(\text{out})}{a}_{\psi\vec{k}} = (\alpha_{\psi k} + h^{(1)}_{\psi k}) \overset{(\text{in})}{a}_{\psi\vec{k}} + (\beta^*_{\psi k} + h^{(2)}_{\psi k}) \overset{\dagger(\text{in})}{b}_{\psi -\vec{k}} + h^{(3)}_{\psi k} \overset{(\text{in})}{a}_{\phi\vec{k}} + h^{(4)}_{\psi k} \overset{\dagger(\text{in})}{b}_{\phi -\vec{k}} , \tag{2.41c}$$

$$\overset{\dagger(\text{out})}{b}_{\psi\vec{k}} = (\beta_{\psi k} + p^{(1)}_{\psi k}) \overset{(\text{in})}{a}_{\psi -\vec{k}} + (\alpha^*_{\psi k} + p^{(2)}_{\psi k}) \overset{\dagger(\text{in})}{b}_{\psi\vec{k}} + p^{(3)}_{\psi k} \overset{(\text{in})}{a}_{\phi -\vec{k}} + p^{(4)}_{\psi k} \overset{\dagger(\text{in})}{b}_{\phi\vec{k}} . \tag{2.41d}$$

With the definitions in Eqs. (2.46)–(2.48) below, the constants are given by

$$\begin{aligned}
h^{(1)}_{\phi k} &= \lambda \int (\textstyle\prod dt_i) z_{\phi k}(t_1) w(t_1) F(t_1, t_2) G_\psi(t_2, t_3) \chi_{\phi k}(t_3) , \\
h^{(2)}_{\phi k} &= \lambda \int (\textstyle\prod dt_i) z_{\phi k}(t_1) w(t_1) F(t_1, t_2) \times G_\psi(t_2, t_3) \chi^*_{\phi k}(t_3) , \\
h^{(3)}_{\phi k} &= - \int (\textstyle\prod dt_i) z_{\phi k}(t_1) w(t_1) F(t_1, t_2) \chi_{\psi k}(t_2) , \\
h^{(4)}_{\phi k} &= - \int (\textstyle\prod dt_i) z_{\phi k}(t_1) w(t_1) F(t_1, t_2) \chi^*_{\psi k}(t_2) ;
\end{aligned} \tag{2.42}$$

$$\begin{aligned}
p^{(1)}_{\phi k} &= \lambda \int (\textstyle\prod dt_i) z^*_{\phi k}(t_1) w(t_1) F(t_1, t_2) \times G_\psi(t_2, t_3) \chi_{\phi k}(t_3) , \\
p^{(2)}_{\phi k} &= \lambda \int (\textstyle\prod dt_i) z^*_{\phi k}(t_1) w(t_1) F(t_1, t_2) \times G_\psi(t_2, t_3) \chi^*_{\phi k}(t_3) , \\
p^{(3)}_{\phi k} &= - \int (\textstyle\prod dt_i) z^*_{\phi k}(t_1) w(t_1) F(t_1, t_2) \chi_{\psi k}(t_2) , \\
p^{(4)}_{\phi k} &= - \int (\textstyle\prod dt_i) z^*_{\phi k}(t_1) w(t_1) F(t_1, t_2) \chi^*_{\psi k}(t_2) ;
\end{aligned} \tag{2.43}$$

$$h^{(1)}_{\psi k}=\lambda\int(\textstyle\prod dt_i)z_{\psi k}(t_1)w^*(t_1)\tilde{F}(t_1,t_2)\times G_\phi(t_2,t_3)\chi_{\psi k}(t_3)\,,$$

$$h^{(2)}_{\psi k}=\lambda\int(\textstyle\prod dt_i)z_{\psi k}(t_1)w^*(t_1)\tilde{F}(t_1,t_2)\times G_\phi(t_2,t_3)\chi^*_{\psi k}(t_3)\,,\tag{2.44}$$

$$h^{(3)}_{\psi k}=-\int(\textstyle\prod dt_i)z_{\psi k}(t_1)w^*(t_1)\tilde{F}(t_1,t_2)\chi_{\phi k}(t_2)\,,$$

$$h^{(4)}_{\psi k}=-\int(\textstyle\prod dt_i)z_{\psi k}(t_1)w^*(t_1)\tilde{F}(t_1,t_2)\chi^*_{\phi k}(t_2)\,;$$

$$p^{(1)}_{\psi k}=\lambda\int(\textstyle\prod dt_i)z^*_{\psi k}(t_1)w^*(t_1)\tilde{F}(t_1,t_2)\times G_\phi(t_2,t_3)\chi_{\psi k}(t_3)\,,$$

$$p^{(2)}_{\psi k}=\lambda\int(\textstyle\prod dt_i)z^*_{\psi k}(t_1)w^*(t_1)\tilde{F}(t_1,t_2)\times G_\phi(t_2,t_3)\chi^*_{\psi k}(t_3)\,,\tag{2.45}$$

$$p^{(3)}_{\psi k}=-\int(\textstyle\prod dt_i)z^*_{\psi k}(t_1)w^*(t_1)\tilde{F}(t_1,t_2)\chi_{\phi k}(t_2)\,,$$

$$p^{(4)}_{\psi k}=-\int(\textstyle\prod dt_i)z^*_{\psi k}(t_1)w^*(t_1)\tilde{F}(t_1,t_2)\chi^*_{\phi k}(t_2)\,.$$

In the above formulas we used the abbreviations

$$z_{\phi k}(t)=i[\alpha_{\phi k}\chi^*_{\phi k}(t)-\beta^*_{\phi k}\chi_{\phi k}(t)]\,,\quad z_{\psi k}(t)=i[\alpha_{\psi k}\chi^*_{\psi k}(t)-\beta^*_{\psi k}\chi_{\psi k}(t)]\,,\tag{2.46}$$

$$w(t)=R(t)\Lambda(t)\,,\tag{2.47}$$

and

$$F(t_1,t_2)=\langle t_1|F|t_2\rangle\,.\tag{2.48}$$

III. BARYON-ANTIBARYON ASYMMETRY

In this section we are going to derive a formula for the baryon-antibaryon asymmetry using the results of Sec. II. It is essential for this calculation that the operators given in Eqs. (2.41) are indeed creation and annihilation operators, i.e., that they satisfy the correct commutation relations. This can be proven formally by means of the integral equations (2.26), (2.27); it is instructive however to verify it directly in terms of the explicit solutions (2.41).

Let us examine the relation

$$[a^{(\text{out})}_{\phi\vec{k}},a^{(\text{out})\dagger}_{\phi\vec{k}'}]=\delta_{\vec{k}\vec{k}'}\,.\tag{3.1}$$

Substituting the expressions (2.41) and using Eq. (2.40), this reduces to

$$2\,\mathrm{Re}(\alpha^*_{\phi k}h^{(1)}_{\phi k}-\beta_{\phi k}h^{(2)}_{\phi k})+|h^{(1)}_{\phi k}|^2-|h^{(2)}_{\phi k}|^2+|h^{(3)}_{\phi k}|^2-|h^{(4)}_{\phi\vec{k}}|^2=0\,.\tag{3.2}$$

Using Eqs. (2.42),

$$|h^{(1)}_{\phi k}|^2-|h^{(2)}_{\phi k}|^2=\lambda^2\int(\textstyle\prod dt_i)z_{\phi k}(t_1)z^*_{\phi k}(t_4)w(t_1)w^*(t_4)F(t_1,t_2)F^*(t_4,t_5)G_\psi(t_2,t_3)G^*_\psi(t_5,t_6)[\chi_{\phi k}(t_3)\chi^*_{\phi k}(t_6)-\chi_{\phi k}(t_6)\chi^*_{\phi k}(t_3)]\,.$$

From Eqs. (2.24), (2.28)

$$\chi_{\phi k}(t_3)\chi^*_{\phi k}(t_6)-\chi_{\phi k}(t_6)\chi^*_{\phi k}(t_3)=i[w^*(t_3)^{-1}G^*_\phi(t_6,t_3)-w(t_6)^{-1}G_\phi(t_3,t_6)]\,,\tag{3.3}$$

and from (2.34)

$$\int dt_2dt_3\lambda^2F(t_1,t_2)G_\psi(t_2,t_3)G_\phi(t_3,t_4)=F(t_1,t_4)-\lambda\delta(t_1-t_4)\,.\tag{3.4}$$

Therefore,

$$|h^{(1)}_{\phi k}|^2-|h^{(2)}_{\phi k}|^2=i\int(\textstyle\prod dt_i)z_{\phi k}(t_1)z^*_{\phi k}(t_3)w(t_1)w^*(t_3)F(t_1,t_2)F^*(t_3,t_4)[G_\psi(t_2,t_4)w^*(t_4)^{-1}-G^*_\psi(t_4,t_2)w(t_2)^{-1}]$$
$$-2\,\mathrm{Re}\left[i\lambda\int(\textstyle\prod dt_i)z_{\phi k}(t_1)w(t_1)F(t_1,t_2)G_\psi(t_2,t_3)z^*_{\phi k}(t_3)\right].\tag{3.5}$$

In a similar fashion, one can derive

$$|h^{(3)}_{\phi k}|^2-|h^{(4)}_{\phi k}|^2=-i\int(\textstyle\prod dt_i)z_{\phi k}(t_1)z^*_{\phi k}(t_3)F(t_1,t_2)F^*(t_3,t_4)[G_\psi(t_2,t_4)w^*(t_4)^{-1}-G^*_\psi(t_4,t_2)w(t_2)^{-1}]\,.\tag{3.6}$$

Finally, using the definition (2.46) of $z_{\phi k}$,

$$2\,\mathrm{Re}(\alpha^*_{\phi k}h^{(1)}_{\phi k}-\beta_{\phi k}h^{(2)}_{\phi k})=2\,\mathrm{Re}\left[i\lambda\int(\textstyle\prod dt_i)z_{\phi k}(t_1)w(t_1)F(t_1,t_2)G_\psi(t_2,t_3)z^*_{\phi k}(t_3)\right].\tag{3.7}$$

It is clear from these results that Eq. (3.2), and therefore Eq. (3.1), is correct. The proof of the remaining commutation relations proceeds along the same lines.

The operator which measures the baryon number in mode $\vec{k}$ at times $t>T$ is

$$N_{\phi\vec{k}}^{(\text{out})}-\bar{N}_{\phi\vec{k}}^{(\text{out})}=a_{\phi\vec{k}}^{\dagger(\text{out})}a_{\phi\vec{k}}^{(\text{out})}-b_{\phi\vec{k}}^{\dagger(\text{out})}b_{\phi\vec{k}}^{(\text{out})}. \tag{3.8}$$

In general, one considers a state expressed in terms of eigenvectors of the in-Hamiltonian (we work in the Heisenberg picture); such a state is represented by a density matrix

$$\rho=\sum_n |n,\text{in}\rangle\rho_n\langle n,\text{in}|\,.$$

The baryon-antibaryon asymmetry which develops in this state at $t>T$ is given by

$$\langle N_{\phi\vec{k}}^{(\text{out})}-\bar{N}_{\phi\vec{k}}^{(\text{out})}\rangle=\text{Tr}[\rho(N_{\phi\vec{k}}^{(\text{out})}-\bar{N}_{\phi\vec{k}}^{(\text{out})})]\,,$$

and can be calculated by using the known expansion of the out creation and annihilation operators in terms of the in-ones. Since in the present paper we want to illustrate the effects expected when the interaction (2.1) is present, we limit ourselves to the simplest case: The baryon-antibaryon excess produced in the in-vacuum.

Using (3.8) and (2.41a), (2.41b), we obtain

$$\Delta N_{\phi\vec{k}}\equiv\langle 0,\text{in}|(N_{\phi\vec{k}}^{(\text{out})}-\bar{N}_{\phi\vec{k}}^{(\text{out})})|0,\text{in}\rangle=2\,\text{Re}\beta_{\phi k}(h_{\phi k}^{(2)}-p_{\phi k}^{(1)*})+|h_{\phi\vec{k}}^{(2)}|^2+|h_{\phi\vec{k}}^{(4)}|^2-|p_{\phi\vec{k}}^{(1)}|^2-|p_{\phi\vec{k}}^{(3)}|^2\,. \tag{3.9}$$

We can simplify this expression considerably by using the methods presented earlier in the proof of the commutation relation (3.1):

$$|h_{\phi k}^{(2)}|^2-|p_{\phi k}^{(1)}|^2=2i\lambda^2\int(\textstyle\prod dt_i)w(t_1)w^*(t_4)F(t_1,t_2)F^*(t_4,t_5)G_\psi(t_2,t_3)G_\psi^*(t_5,t_6)\,\text{Im}[z_{\phi k}(t_1)z_{\phi k}^*(t_4)\chi_{\phi k}^*(t_3)\chi_{\phi k}(t_6)]\,. \tag{3.10}$$

One also has

$$\text{Im}[z_{\phi k}(t_1)z_{\phi k}^*(t_4)\chi_{\phi k}^*(t_3)\chi_{\phi k}(t_6)]=-\text{Re}[z_{\phi k}(t_1)z_{\phi k}^*(t_4)]\,\text{Im}[\chi_{\phi k}(t_3)\chi_{\phi k}^*(t_6)]-\text{Re}[\chi_{\phi k}(t_3)\chi_{\phi k}^*(t_6)]\,\text{Im}[z_{\phi k}^*(t_1)z_{\phi k}(t_4)]\,, \tag{3.11}$$

and, from (2.46), (3.3)

$$\begin{aligned}&\text{Im}[\chi_{\phi k}(t_3)\chi_{\phi k}^*(t_6)]=\tfrac{1}{2}[w^*(t_3)^{-1}G_\phi^*(t_6,t_3)-w(t_6)^{-1}G_\phi(t_3,t_6)]\,,\\&\text{Im}[z_{\phi k}^*(t_1)z_{\phi k}(t_4)]=-\text{Im}[\chi_{\phi k}^*(t_1)\chi_{\phi k}(t_4)]=-\tfrac{1}{2}[w^*(t_4)^{-1}G_\phi^*(t_1,t_4)-w(t_1)^{-1}G_\phi(t_4,t_1)]\,.\end{aligned} \tag{3.12}$$

Hence, if we define

$$A_{\phi k}(t_1,t_2)\equiv\text{Re}[z_{\phi k}(t_1)z_{\phi k}^*(t_2)]=A_{\phi k}(t_2,t_1)\,, \tag{3.13}$$

$$B_{\phi k}(t_1,t_2)\equiv\text{Re}[\chi_{\phi k}(t_1)\chi_{\phi k}^*(t_2)]=B_{\phi k}(t_2,t_1)\,, \tag{3.14}$$

we have

$$\begin{aligned}2\,\text{Im}[z_{\phi k}(t_1)z_{\phi k}^*(t_4)\chi_{\phi k}^*(t_3)\chi_{\phi k}(t_6)]=&-A_{\phi k}(t_1,t_4)[w^*(t_3)^{-1}G_\phi^*(t_6,t_3)-w(t_6)^{-1}G_\phi(t_3,t_6)]\\&+B_{\phi k}(t_3,t_6)[w^*(t_4)^{-1}G_\phi^*(t_1,t_4)-w(t_1)^{-1}G_\phi(t_4,t_1)]\,.\end{aligned} \tag{3.15}$$

Substituting Eq. (3.15) in Eq. (3.10) and using Eq. (3.4), and

$$\lambda^2\int dt_2dt_3G_\phi(t_1,t_2)F(t_2,t_3)G_\psi(t_3,t_4)=\tilde{F}(t_1,t_4)-\lambda\delta(t_1-t_4)\,, \tag{3.16}$$

we find

$$\begin{aligned}|h_{\phi k}^{(2)}|^2-|p_{\phi k}^{(1)}|^2=&-i\int(\textstyle\prod dt_i)w(t_1)w^*(t_3)A_{\phi k}(t_1,t_3)F(t_1,t_2)F^*(t_3,t_4)[G_\psi(t_2,t_4)w^*(t_4)^{-1}-G_\psi^*(t_4,t_2)w(t_2)^{-1}]\\&-2\int(\textstyle\prod dt_i)B_{\phi k}(t_2,t_4)\,\text{Im}[w(t_1)F(t_1,t_3)\tilde{F}^*(t_1,t_4)G_\psi(t_3,t_2)]\\&-2\lambda\int(\textstyle\prod dt_i)A_{\phi k}(t_1,t_3)\,\text{Im}[w(t_1)F(t_1,t_2)G_\psi(t_2,t_3)]\\&+2\lambda\int(\textstyle\prod dt_i)B_{\phi k}(t_1,t_3)\,\text{Im}[w(t_1)F(t_1,t_2)G_\psi(t_2,t_3)]\,.\end{aligned} \tag{3.17}$$

Another term in Eq. (3.9) is

$$|h_{\phi k}^{(4)}|^2 - |p_{\phi k}^{(3)}|^2 = 2i\int (\prod dt_i) w(t_1) w^*(t_3) F(t_1,t_2) F^*(t_3,t_4)\,\mathrm{Im}[z_{\phi k}(t_1) z_{\phi k}^*(t_3) \chi_{\psi k}^*(t_2)\chi_{\psi k}(t_4)]\,.$$

One has, in analogy with (3.15),

$$2\,\mathrm{Im}[z_{\phi k}(t_1) z_{\phi k}^*(t_3)\chi_{\psi k}^*(t_2)\chi_{\psi k}(t_4)] = -A_{\phi k}(t_1,t_3)[w(t_2)^{-1}G_\psi^*(t_4,t_2) - w^*(t_4)^{-1}G_\psi(t_2,t_4)] + B_{\psi k}(t_2,t_4)[w^*(t_3)^{-1}G_\phi^*(t_1,t_3) - w(t_1)^{-1}G_\phi(t_3,t_1)]\,, \quad (3.18)$$

with $B_{\psi k}$ defined as in (3.14) with $\chi_{\psi k}$ replacing $\chi_{\phi k}$. Then

$$|h_{\phi k}^{(4)}|^2 - |p_{\phi k}^{(3)}|^2 = -i\int (\prod dt_i) w(t_1) w^*(t_3) A_{\phi k}(t_1,t_3) F(t_1,t_2) F^*(t_3,t_4)[w(t_2)^{-1}G_\psi^*(t_4,t_2) - w^*(t_4)^{-1}G_\psi(t_2,t_4)] + 2\int (\prod dt_i) B(t_2,t_4)\,\mathrm{Im}[w^*(t_3) F(t_1,t_2) F^*(t_3,t_4) G_\phi(t_3,t_1)]\,. \quad (3.19)$$

Finally,

$$\beta_{\phi k}(h_{\phi k}^{(2)} - p_{\phi k}^{(1)*}) = 2i\lambda\beta_{\phi k}\int (\prod dt_i) z_{\phi k}(t_1)\chi_{\phi k}^*(t_3)\,\mathrm{Im}[w(t_1) F(t_1,t_2) G_\psi(t_2,t_3)]\,.$$

Using the identity

$$2\,\mathrm{Im}[\beta_{\phi k} z_{\phi k}(t_1)\chi_{\phi k}^*(t_2)] = B_{\phi k}(t_1,t_2) - A_{\phi k}(t_1,t_2)\,, \quad (3.20)$$

we obtain

$$2\,\mathrm{Re}[\beta_{\phi k}(h_{\phi k}^{(2)} - p_{\phi k}^{(1)*})] = -2\lambda\int (\prod dt_i)[B_{\phi k}(t_1,t_3) - A_{\phi k}(t_1,t_3)]\,\mathrm{Im}[w(t_1) F(t_1,t_2) G_\psi(t_2,t_3)]\,. \quad (3.21)$$

When we substitute Eqs. (3.17), (3.19), and (3.21) in (3.9), many cancellations occur and we obtain

$$\Delta N_{\phi \vec{k}} = -2\,\mathrm{Im}\left\{\int (\prod dt_i)[B_{\phi k}(t_2,t_4) w(t_1) F(t_1,t_3)\tilde{F}^*(t_1,t_4) G_\psi(t_3,t_2) - B_{\psi k}(t_2,t_4) w^*(t_3) F(t_1,t_2) F^*(t_3,t_4) G_\phi(t_3,t_1)]\right\}. \quad (3.22)$$

Using the identity

$$F G_\psi = G_\psi \tilde{F}\,, \quad (3.23)$$

we can rewrite (3.22) in a highly symmetric form

$$\Delta N_{\phi \vec{k}} = -2\,\mathrm{Im}\left\{\int (\prod dt_i)[B_{\phi k}(t_2,t_4) w(t_3)\tilde{F}(t_1,t_2)\tilde{F}^*(t_3,t_4) G_\psi(t_3,t_1) - B_{\psi k}(t_2,t_4) w^*(t_3) F(t_1,t_2) F^*(t_3,t_4) G_\phi(t_3,t_1)]\right\}, \quad (3.24)$$

where definitions of the various quantities appearing are given in Eq. (3.13), Eqs. (2.46)–(2.48), Eqs. (2.28)–(2.29), and Eqs. (2.34)–(2.35). This is our final form for the asymmetry.

In the remainder of this section we comment on its implications:

(i) The parameters $\alpha_{\phi k}$, $\beta_{\phi k}$, $\alpha_{\psi k}$, and $\beta_{\psi k}$ of Eqs. (2.38), (2.39), which represent the purely gravitational production of particles, do not enter in the expression for the asymmetry. They do appear in the formulas for $N_{\phi \vec{k}}$, $\bar{N}_{\phi \vec{k}}$, but they cancel out in the difference.

(ii) Since F and $\tilde{F}$ are of order λ, the asymmetry is of order λ^2; in fact, Eq. (3.24) involves only even powers of λ. Thus, changing the sign of λ in the Lagrangian (2.1) does not affect the result.

(iii) If the phase of $\Lambda(t)$ is constant,

$$\Lambda(t) = |\Lambda(t)| e^{i\sigma}\,, \quad (3.25)$$

Eqs. (2.28), (2.29) show that

$$G_\phi = e^{i\sigma}|G_\phi|\,,$$

$$G_\psi = e^{-i\sigma}|G_\psi|\,,$$

and one sees from (2.34), (2.35) that F and $\tilde{F}$ are real. Therefore, each term in the integrand of Eq. (3.24) is real, and the asymmetry vanishes. If (3.25) holds, one can adjust the phases of $\psi(x)$, $\phi(x)$, so that *CP* is conserved; we thus have verified that matter-antimatter asymmetry exists if and only if *CP* is violated.

(iv) The model simplifies considerably when

$$m_1 = m_2\,, \quad \xi_1 = \xi_2\,. \quad (3.26)$$

Then

$$\chi_{\phi k}=\chi_{\psi k}\equiv\chi\,,$$
$$G_\phi=G_\psi^*\equiv G\,,$$
$$\tilde{F}=F^*\,, \tag{3.27}$$
$$B_{\phi k}=B_{\psi k}\equiv B_k\,,$$

and Eq. (3.24) reduces to

$$\Delta N_{\phi\vec{k}}=4\,\mathrm{Im}\left[\int\Big(\prod dt_i\Big)B_k(t_2,t_4)w^*(t_3)F(t_1,t_2)\times F^*(t_3,t_4)G(t_3,t_1)\right]\,, \tag{3.28}$$

which is nonvanishing in general. However, if (3.26) is satisfied, and if other interactions do not determine which linear combinations of ϕ and ψ are the actual baryon and lepton fields, then the identification of the asymptotic states is ambiguous; any constant unitary transformation

$$\begin{pmatrix}\tilde{\phi}\\ \tilde{\psi}\end{pmatrix}=U\begin{pmatrix}\phi\\ \psi\end{pmatrix} \tag{3.29}$$

leaves the equations of motion invariant in the asymptotic region ($t>|T|$).

Let us write Eqs. (2.41) in the form

$$\begin{bmatrix}a_{\phi\vec{k}}\\ a_{\psi\vec{k}}\\ b^\dagger_{\phi-\vec{k}}\\ b^\dagger_{\psi-\vec{k}}\end{bmatrix}^{(\mathrm{out})}=T\begin{bmatrix}a_{\phi\vec{k}}\\ a_{\psi\vec{k}}\\ b^\dagger_{\phi-\vec{k}}\\ b^\dagger_{\psi-\vec{k}}\end{bmatrix}^{(\mathrm{in})}, \tag{3.30}$$

and define

$$\hat{U}=\begin{pmatrix}U&0\\0&U\end{pmatrix}. \tag{3.31}$$

If we redefine $\tilde{\phi}$, $\tilde{\psi}$ as the baryon and antilepton fields, respectively, and denote by a tilde the in-states defined by these fields, we have

$$\begin{aligned}\Delta\tilde{N}_{\phi k}&\equiv\langle\tilde{0},\mathrm{in}|\tilde{a}^{\dagger\,(\mathrm{out})}_{\phi\vec{k}}\tilde{a}^{(\mathrm{out})}_{\phi\vec{k}}-\tilde{b}^{\dagger\,(\mathrm{out})}_{\phi\vec{k}}\tilde{b}^{(\mathrm{out})}_{\phi\vec{k}}|\tilde{0},\mathrm{in}\rangle\\&=|(\hat{U}T\hat{U}^\dagger)_{13}|^2+|(\hat{U}T\hat{U}^\dagger)_{14}|^2\\&\quad-|(\hat{U}T\hat{U}^\dagger)_{31}|^2-|(\hat{U}T\hat{U}^\dagger)_{32}|^2\,.\end{aligned}$$

Now choose

$$U=\frac{1}{\sqrt{2}}\begin{pmatrix}1&e^{i\sigma}\\-e^{-i\sigma}&1\end{pmatrix}, \tag{3.32}$$

with σ an arbitrary real constant phase. Then

$$\begin{aligned}\Delta\tilde{N}_{\phi k}=\tfrac{1}{2}[\,&|T_{13}|^2+|T_{14}|^2+|T_{23}|^2+|T_{24}|^2\\&-|T_{31}|^2-|T_{32}|^2-|T_{41}|^2-|T_{42}|^2]\\&+\mathrm{Re}[e^{-i\sigma}(T_{13}T_{23}^*+T_{14}T_{24}^*-T_{31}T_{41}^*-T_{32}T_{42}^*)]\,.\end{aligned} \tag{3.33}$$

On the other hand, using Eqs. (3.27) in (2.42)–(2.45), we find that

$$h^{(1)}_{\phi,k}=p^{(2)*}_{\psi,k}\,,\quad h^{(3)}_{\phi,k}=p^{(4)*}_{\psi,k}\,,$$
$$h^{(2)}_{\phi,k}=p^{(1)*}_{\psi,k}\,,\quad h^{(4)}_{\phi,k}=p^{(3)*}_{\psi,k}\,,$$

and similar relations with ϕ and ψ interchanged. Then, using the explicit form of the T_{ij}'s,

$$T_{13}=\beta_k^*+h^{(2)}_{\phi k}\,,\quad T_{31}=\beta_k+p^{(1)}_{\phi k}\,,$$
$$T_{14}=h^{(4)}_{\phi k}\,,\quad T_{32}=p^{(3)}_{\phi k}\,,$$
$$T_{23}=h^{(4)}_{\psi k}\,,\quad T_{41}=p^{(3)}_{\psi k}\,,$$
$$T_{24}=\beta_k^*+h^{(2)}_{\psi k}\,,\quad T_{42}=\beta_k+p^{(1)}_{\psi k}\,,$$

we obtain the relations

$$T_{13}=T_{42}^*\,,\quad T_{14}=T_{41}^*\,,\quad T_{23}=T_{32}^*\,,\quad T_{24}=T_{31}^*\,.$$

When these are substituted in (3.34), we find

$$\Delta\tilde{N}_{\phi k}=0\,.$$

We therefore conclude that, if the masses and ξ's for both fields are equal and if other interactions do not require the original ϕ and ψ to be baryon or antilepton fields, then the baryon-antibaryon asymmetry is spurious; it can be eliminated by any change of basis of the form (3.29), (3.32).

Finally, we remark that in this model, the matter-antimatter asymmetry is twice the baryon-antibaryon asymmetry calculated above. This follows from the existence of the conserved charge, Eq. (2.4). If we evaluate it in the in- and out-region and set the two expressions equal, we obtain

$$\begin{aligned}&a^{\dagger\,(\mathrm{in})}_{\phi\vec{k}}a^{(\mathrm{in})}_{\phi\vec{k}}-b^{\dagger\,(\mathrm{in})}_{\phi\vec{k}}b^{(\mathrm{in})}_{\phi\vec{k}}+a^{\dagger\,(\mathrm{in})}_{\psi\vec{k}}a^{(\mathrm{in})}_{\psi\vec{k}}-b^{\dagger\,(\mathrm{in})}_{\psi\vec{k}}b^{(\mathrm{in})}_{\psi\vec{k}}\\&\quad=a^{\dagger\,(\mathrm{out})}_{\phi\vec{k}}a^{(\mathrm{out})}_{\phi\vec{k}}-b^{\dagger\,(\mathrm{out})}_{\phi\vec{k}}b^{(\mathrm{out})}_{\phi\vec{k}}\\&\qquad+a^{\dagger\,(\mathrm{out})}_{\psi\vec{k}}a^{(\mathrm{out})}_{\psi\vec{k}}-b^{\dagger\,(\mathrm{out})}_{\psi\vec{k}}b^{(\mathrm{out})}_{\psi\vec{k}}\,.\end{aligned} \tag{3.34}$$

Since $\phi(x)$ is the antilepton field, the lepton number in the in-vacuum state is

$$\Delta N_{\psi k}=\langle 0,\mathrm{in}|b^{\dagger\,(\mathrm{out})}_{\psi\vec{k}}b^{(\mathrm{out})}_{\psi\vec{k}}-a^{\dagger\,(\mathrm{out})}_{\psi\vec{k}}a^{(\mathrm{out})}_{\psi\vec{k}}|0,\mathrm{in}\rangle\,,$$

and Eq. (3.35) gives

$$\Delta N_{\phi k}-\Delta N_{\psi k}=0\,.$$

Therefore, the difference between the number of particles and antiparticles is

$$\Delta N_k\equiv\Delta N_{\phi k}+\Delta N_{\psi k}=2\Delta N_{\phi k}\,. \tag{3.35}$$

IV. RESULTS TO SECOND ORDER

It is clear from Eq. (3.24) that the matter-antimatter asymmetry first appears in second order in λ. It will be useful for making specific estimates to give the various results explicitly to second order. To that order, one finds from Eqs. (2.41), or directly from the integral Eqs. (2.26)–(2.27), that

$$a^{(\text{out})}_{\phi\vec{k}} = [\alpha_{\phi k}(1+i\lambda^2 H_1) - i\lambda^2\beta^*_{\phi k}H_3]a^{(\text{in})}_{\phi\vec{k}} + [\beta^*_{\phi k}(1-i\lambda^2 H_4) + i\lambda^2\alpha_{\phi k}H_2]b^{\dagger\,(\text{in})}_{\phi,-\vec{k}}$$
$$- i\lambda[\alpha_{\phi k}I_1 - \beta^*_{\phi k}I_3]a^{(\text{in})}_{\psi\vec{k}} - i\lambda[\alpha_{\phi k}I_2 - \beta^*_{\phi k}I_4]b^{\dagger\,(\text{in})}_{\psi,-\vec{k}}\,, \tag{4.1a}$$

$$b^{\dagger\,(\text{out})}_{\phi\vec{k}} = [\alpha^*_{\phi k}(1-i\lambda^2 H_4) + i\lambda^2\beta_{\phi k}H_2]b^{\dagger\,(\text{in})}_{\phi,\vec{k}} + [\beta_{\phi k}(1+i\lambda^2 H_1) - i\lambda^2\alpha^*_{\phi k}H_3]a^{(\text{in})}_{\phi,-\vec{k}}$$
$$+ i\lambda[\alpha^*_{\phi k}I_4 - \beta_{\phi k}I_2]b^{\dagger\,(\text{in})}_{\psi\vec{k}} + i\lambda[\alpha^*_{\phi k}I_3 - \beta_{\phi k}I_1]a^{(\text{in})}_{\psi-\vec{k}}\,, \tag{4.1b}$$

$$a^{(\text{out})}_{\psi\vec{k}} = [\alpha_{\psi k}(1+i\lambda^2 H_{\psi 1}) - i\lambda^2\beta^*_{\psi k}H_{\psi 3}]a^{(\text{in})}_{\psi\vec{k}} + [\beta^*_{\psi k}(1-i\lambda^2 H_{\psi 4}) + i\lambda^2\alpha_{\psi k}H_{\psi 2}]b^{\dagger\,(\text{in})}_{\psi,-\vec{k}}$$
$$- i\lambda[\alpha_{\psi k}I_1^* - \beta^*_{\psi k}I_2^*]a^{(\text{in})}_{\phi\vec{k}} - i\lambda[\alpha_{\psi k}I_3^* - \beta^*_{\psi k}I_1^*]b^{\dagger\,(\text{in})}_{\phi,-\vec{k}}\,, \tag{4.1c}$$

$$b^{\dagger\,(\text{out})}_{\psi\vec{k}} = [\alpha^*_{\psi k}(1-i\lambda^2 H_{\psi 4}) + i\lambda^2\beta_{\psi k}H_{\psi 2}]b^{\dagger\,(\text{out})}_{\psi\vec{k}} + [\beta_{\psi k}(1+i\lambda^2 H_{\psi 1}) - i\lambda^2\alpha^*_{\psi k}H_{\psi 3}]a^{(\text{in})}_{\psi,-\vec{k}}$$
$$+ i\lambda[\alpha^*_{\psi k}I_4^* - \beta_{\psi k}I_3^*]b^{\dagger\,(\text{in})}_{\phi\vec{k}} + i\lambda[\alpha^*_{\psi k}I_2^* - \beta_{\psi k}I_1^*]a^{(\text{in})}_{\phi,-\vec{k}}\,, \tag{4.1d}$$

where

$$I_1 = \int_{-\infty}^{\infty} dt\, R(t)\Lambda(t)\chi^*_{\phi k}(t)\chi_{\psi k}(t)\,, \tag{4.2a}$$

$$I_2 = \int_{-\infty}^{\infty} dt\, R(t)\Lambda(t)\chi^*_{\phi k}(t)\chi^*_{\psi k}(t)\,, \tag{4.2b}$$

$$I_3 = \int_{-\infty}^{\infty} dt\, R(t)\Lambda(t)\chi_{\phi k}(t)\chi_{\psi k}(t)\,, \tag{4.2c}$$

$$I_4 = \int_{-\infty}^{\infty} dt\, R(t)\Lambda(t)\chi_{\phi k}(t)\chi^*_{\psi k}(t)\,, \tag{4.2d}$$

and

$$H_1 = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt'\, R(t)\Lambda(t)\chi^*_{\phi k}(t)R(t') \times \Lambda^*(t')\Delta_\psi(k,t,t')\chi_{\phi k}(t')\,, \tag{4.3a}$$

$$H_2 = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt'\, R(t)\Lambda(t)\chi^*_{\phi k}(t)R(t') \times \Lambda^*(t')\Delta_\psi(k,t,t')\chi^*_{\phi k}(t')\,, \tag{4.3b}$$

$$H_3 = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt'\, R(t)\Lambda(t)\chi_{\phi k}(t)R(t') \times \Lambda^*(t')\Delta_\psi(k,t,t')\chi_{\phi k}(t')\,, \tag{4.3c}$$

$$H_4 = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt'\, R(t)\Lambda(t)\chi_{\phi k}(t)R(t') \times \Lambda^*(t')\Delta_\psi(k,t,t')\chi^*_{\phi k}(t')\,, \tag{4.3d}$$

and the $H_{j\psi}$ $(j=1,2,3,4)$ are defined by interchanging Λ with Λ^* and the subscripts ϕ and ψ in the H_j defined above.

If the state is the in-vacuum $|0,\text{in}\rangle$, then the expected number of baryons in mode $\vec{k}$ at late times is

$$\langle N^{(\text{out})}_{\phi\vec{k}}\rangle = \langle 0,\text{in}|a^{\dagger\,(\text{out})}_{\phi\vec{k}}a^{(\text{out})}_{\phi\vec{k}}|0,\text{in}\rangle$$
$$= |\beta_{\phi k}|^2 + \lambda^2\{|I_2|^2 + |\beta_{\phi k}|^2(|I_2|^2+|I_3|^2) + 2\,\text{Re}[\alpha_{\phi k}\beta_{\phi k}(iH_2 - I_2I_4^*)]\}\,, \tag{4.4}$$

and the number of antibaryons is

$$\langle \bar{N}^{(\text{out})}_{\phi k}\rangle = \langle 0,\text{in}|b^{\dagger\,(\text{out})}_{\phi\vec{k}}b^{(\text{out})}_{\phi\vec{k}}|0,\text{in}\rangle$$
$$= |\beta_{\phi k}|^2 + \lambda^2\{|I_3|^2 + |\beta_{\phi k}|^2(|I_2|^2+|I_3|^2) - 2\,\text{Re}[\alpha^*_{\phi k}\beta^*_{\phi k}(iH_3 + I_1^*I_3)]\}\,, \tag{4.5}$$

where we have made use of Eq. (2.40), and of the identities

$$2\,\text{Im}H_1 = |I_1|^2 - |I_2|^2\,, \tag{4.6}$$

$$2\,\text{Im}H_4 = |I_3|^2 - |I_4|^2\,. \tag{4.7}$$

where Im denotes the imaginary part. One has similar expressions for the average numbers of leptons and antileptons in mode $\vec{k}$. The expectation value of the baryon number in mode $\vec{k}$ is thus

$$\Delta N^{(\text{out})}_{\phi\vec{k}} = \langle N^{(\text{out})}_{\phi\vec{k}}\rangle - \langle \bar{N}^{(\text{out})}_{\phi k}\rangle = \lambda^2(|I_2|^2 - |I_3|^2)\,, \tag{4.8}$$

where use has been made of the identity

$$H_2 - H_3^* = iI_2I_3^* - iI_2I_4^*\,. \tag{4.9}$$

Equation (4.8) may also be obtained directly from Eq. (3.24) to second order in λ. The number of baryons in Eq. (4.4) involves both the purely gravitational particle creation through the appearance of α and β, as well as a term which remains when β vanishes. The latter term is due to the production of baryon-lepton pairs by the interaction involving λ. This pair production is energetically forbidden in flat spacetime (even if R did not vanish), but is possible here because of the time-dependent external gravitational field. It is only this term that contributes to the baryon number of Eq. (4.8), which is therefore essentially independent of the purely gravitational component of the particle production. As noted earlier, α and β do not appear in $\Delta N^{(\rm out)}_{\phi k}$ to all orders in λ.

The above expectation values all refer to the volume $(La_+)^3$. Therefore, the average number of baryons per unit volume at late times, obtained by summing over all modes and going to the continuum limit, is

$$\langle N^{(\rm out)}_\phi\rangle = \lim_{L\to\infty}(La_+)^{-3}\sum_{\vec k}\langle N^{(\rm out)}_{\phi\vec k}\rangle$$
$$= (2\pi^2 a_+{}^3)^{-1}\int_0^\infty dk k^2\langle N^{(\rm out)}_{\phi\vec k}\rangle \ . \quad (4.10)$$

Similarly, the average density of antibaryons at late times is

$$\langle \bar N^{(\rm out)}_\phi\rangle = (2\pi^2 a_+{}^3)^{-1}\int_0^\infty dk k^2\langle \bar N^{(\rm out)}_{\phi\vec k}\rangle \ , \quad (4.11)$$

and the average baryon number density at late times is

$$\Delta N^{(\rm out)}_\phi = (2\pi^2 a_+{}^3)^{-1}\int_0^\infty dk k^2 \Delta N^{(\rm out)}_{\phi\vec k}$$

or

$$\Delta N^{(\rm out)}_\phi = \lambda^2(2\pi^2 a_+{}^3)^{-1}\int_0^\infty dk k^2(|I_2|^2-|I_3|^2) \ . \quad (4.12)$$

V. ESTIMATE OF THE MAGNITUDE OF THE EFFECT

In this section, we estimate the baryon-antibaryon asymmetry in a specific example, assuming that the lepton is sufficiently light that its mass can be neglected. In particular, let

$$m_2 = 0, \quad \xi_2 = \tfrac{1}{6} \ . \quad (5.1)$$

Then there is exactly zero production of these leptons purely by the gravitational field[15-17] although there will be production caused by the interaction term. Thus,

$$|\alpha_{\psi k}| = 1, \quad |\beta_{\psi k}| = 0 \ , \quad (5.2)$$

and the exact solution of the zeroth-order lepton field equation [Eq. (2.20)] is

$$\chi_{\psi k}(t) = [2\omega_k(2,t)]^{-1/2}\exp\left[-i\int_{t_-}^t \omega_k(2,t')dt'\right] \ , \quad (5.3)$$

where

$$\omega_k(2,t) = k/a(t) \ , \quad (5.4)$$

and t_- is an arbitrary constant time earlier than $-T$ [Eq. (5.3) differs from the $\chi(t)$ obeying the boundary condition of Eq. (2.21) by an unimportant constant phase]. In this case, the expressions to second order in λ for the number density of leptons and antileptons created by the interaction simplify to

$$\langle N^{(\rm out)}_\psi\rangle = (2\pi^2 a_+{}^3)^{-1}\int_0^\infty dk k^2\langle b^\dagger_{\psi\vec k} b_{\psi\vec k}\rangle$$
$$= \lambda^2(2\pi^2 a_+{}^3)^{-1}\int_0^\infty dk k^2 |I_2|^2 \quad (5.5)$$

and

$$\langle \bar N^{(\rm out)}_\psi\rangle = (2\pi^2 a_+{}^3)^{-1}\int_0^\infty dk k^2\langle a^\dagger_{\psi\vec k} a_{\psi\vec k}\rangle$$
$$= \lambda^2(2\pi^2 a_+{}^3)^{-1}\int_0^\infty dk k^2 |I_3|^2 \ , \quad (5.6)$$

where N_ψ refers to leptons and $\bar N_\psi$ to antileptons in accordance with our definitions.

The expression for the created baryon number density is given by Eq. (4.12), namely

$$\Delta N^{(\rm out)}_\phi = \lambda^2(2\pi^2 a_+{}^3)^{-1}\int_0^\infty dk k^2(|I_2|^2-|I_3|^2) \ . \quad (5.7)$$

(This also equals the created lepton number density, as the difference of the baryon and lepton numbers is conserved.) Production of baryon-lepton pairs by such an interaction in flat spacetime is forbidden by energy conservation, but in the expanding universe the coupling to the time-dependent gravitational field makes the process possible.

The baryon under consideration is assumed to satisfy conditions such that one can neglect the creation of baryon-antibaryon pairs purely by the gravitational field. That pair creation will be negligible if the baryon mass m_1 is never much less than $|\dot a/a|$, or alternatively, if $m_1\approx 0$ and $\xi_1 = \frac{1}{6}$. In either case, one has (exactly, if $m_1 = 0$, $\xi_1 = \frac{1}{6}$)

$$|\alpha_{\phi k}| \approx 1, \quad |\beta_{\phi k}| \approx 0 \quad (5.8)$$

and the WKB solution

$$\chi_{\phi k}(t) \approx [2\omega_k(1,t)]^{-1/2}\exp\left[-i\int_{t_-}^t \omega_k(1,t')dt'\right], \quad (5.9)$$

where

$$\omega_k(1,t) = \left(\frac{k^2}{a^2(t)} + m_1{}^2\right)^{1/2} \ . \quad (5.10)$$

In this approximation one has $\langle N^{(\rm out)}_\phi\rangle = \langle N^{(\rm out)}_\psi\rangle$ and $\langle \bar N^{(\rm out)}_\phi\rangle = \langle \bar N^{(\rm out)}_\psi\rangle$, as given in Eqs. (5.5) and (5.6), respectively. The quantities I_2 and I_3 take the form

$$I_2=\frac{1}{2}\int_{-\infty}^{\infty}dt\,\frac{R(t)\Lambda(t)}{[\omega_k(1,t)\omega_k(2,t)]^{1/2}}\times\exp\left\{i\int_{t_-}^{t}dt'[\omega_k(1,t')+\omega_k(2,t')]\right\}\qquad(5.11)$$

and

$$I_3=\frac{1}{2}\int_{-\infty}^{\infty}dt\,\frac{R(t)\Lambda(t)}{[\omega_k(1,t)\omega_k(2,t)]^{1/2}}\times\exp\left\{-i\int_{t_-}^{t}dt'[\omega_k(1,t')+\omega_k(2,t')]\right\}.\qquad(5.12)$$

If the only particles created significantly were the baryon and lepton under consideration (or a number of types of similar particles), and the other types of particles are formed through subsequent decays, then the ratio of the baryon number to the total number of particles and antiparticles created initially would be

$$\frac{\Delta N_\phi^{(\mathrm{out})}}{\sum\langle N\rangle}=\frac{\int_0^\infty dk k^2(|I_2|^2-|I_3|^2)}{2\int_0^\infty dk k^2(|I_2|^2+|I_3|^2)}.\qquad(5.13)$$

This ratio is independent of the interaction strength λ, which would have to be chosen so that the total created energy density is consistent with the Einstein equations. The entropy density of the created particles divided by Boltzmann's constant would be of the order of $\sum\langle N\rangle$, so that Eq. (5.13) gives the conserved ratio of the baryon number density to the entropy density. The choice of $\Lambda(t)$, or in particular its phase, would determine that ratio, which is observed to be in the range 10^{-8} to 10^{-10}.

More generally, other particles, not involved in the asymmetry interaction will be created by the expansion of the universe, and one may expect the density of those particles and antiparticles to be the dominant contribution to $\sum\langle N\rangle$, while $\Delta N_\phi^{(\mathrm{out})}$ is still determined by the asymmetric interaction through Eq. (5.7). Both possibilities are considered below.

To make estimates in a simple case, consider a universe which contracts to a minimum value $a(t)=a_0$ at $t=0$, and then expands into the present universe. Then $t<-T$ refers to a time before the universe started contracting, or when the contraction was slow, and $t>T$ refers to a time when the expansion has ceased or become slow, such as the present time. Possible causes of this "bounce" might involve violation of the energy conditions of the singularity theorems through effects such as cosmological particle creation or black hole evaporation.[21-25] For simplicity, we take the state vector to be the vacuum for $t<-T$, but more generally one could use a density matrix to describe the state of the system, or one could include some classical background matter. The construction of a self-consistent model during the contraction phase would probably require such complications, which we avoid here (alternatively one might start with an initially anisotropically expanding universe, and have the anisotropy rapidly damped by the particle creation[26,27]).

The scalar curvature

$$R=6(\dot a^2a^{-2}+\ddot a a^{-1})\qquad(5.14)$$

will go as t^{-2} during the period when $a(t)$ follows a power law (such as $t^{2/3}$ for a dust-filled universe). During the radiation-dominated phase R vanishes identically ($a\propto t^{1/2}$), so that the interaction vanishes during that period. Near $t=0$, during the period of the bounce, $a(t)$ departs from a power law, and $R(t)$ becomes large and positive. We suppose that the expansion near $t=0$ is characterized by a quantity, G, of dimension length squared in the present units ($\hbar=c=1$). If the minimum is characterized by the Planck time (5.4×10^{-44} sec), then G is the Newtonian gravitational constant, but it need not have that value. Thus, we suppose that the scalar curvature $R(t)$ reaches its maximum value of order G^{-1} during a time interval of order $G^{1/2}$ near $t=0$, and that $R(t)$ is relatively negligible at other times. During that time interval, $a(t)$ is roughly of the order of a_0, its minimum value. In summary, for the purpose of making estimates, we take

$$R(t)\sim G^{-1}\text{ and }a(t)\sim a_0\text{ for }-G^{1/2}\lesssim t\lesssim G^{1/2},\qquad(5.15)$$

and $R(t)$ negligible outside that interval. We take $\Lambda(t)$ to have a simple form in which $|\Lambda(t)|=1$ and $\Lambda(t)=\Lambda^*(-t)$. In particular, let

$$\Lambda(t)=\begin{cases}e^{-i\sigma}\text{ for }t>0\\ e^{i\sigma}\text{ for }t<0\,,\end{cases}\qquad(5.16)$$

where σ is a real constant. In the Appendix, we consider a Lorentzian form, $R(t)=(t^2+G)^{-1}$, for the scalar curvature, and a similar smoothly varying function for $\Lambda(t)$ which undergoes a phase change of -2σ in the interval from $t=-G^{1/2}$ to $t=G^{1/2}$. We find that our results are not changed appreciably.

Then Eqs. (5.11) and (5.12) yield

$$I_2\approx\frac{G^{-1}\exp(i\delta)}{[\omega_k(1,0)\omega_k(2,0)]^{1/2}\omega_{12}(0)}\times\{\sin\sigma+\sin[G^{1/2}\omega_{12}(0)-\sigma]\}\qquad(5.17)$$

and

$$I_3 \approx \frac{G^{-1}\exp(-i\delta)}{[\omega_k(1,0)\omega_k(2,0)]^{1/2}\omega_{12}(0)} \times\{-\sin\sigma + \sin[G^{1/2}\omega_{12}(0)+\sigma]\}\,, \tag{5.18}$$

where

$$\omega_{12}(t) \equiv \omega_k(1,t)+\omega_k(2,t) \tag{5.19}$$

and

$$\delta \equiv \int_{t_-}^{0} dt'\omega_{12}(t')\,. \tag{5.20}$$

The baryon number density for $t>T$ is then given by Eq. (5.7) as

$$\Delta N_\phi^{(\text{out})} = \frac{\lambda^2 G^{-2}\sin(2\sigma)}{\pi^2 a_+^3}\int_0^\infty dk\, \frac{k^2 \sin[G^{1/2}\omega_{12}(0)]\{1-\cos[G^{1/2}\omega_{12}(0)]\}}{\omega_k(1,0)\omega_k(2,0)[\omega_{12}(0)]^2}\,. \tag{5.21}$$

As the baryon nonconservation is negligible for $t>G^{1/2}$, the baryon number density $\Delta N_\phi(t)$ for $t>G^{1/2}$ is given by the same expression with a_+ replaced by $a(t)$. Thus, putting $m_1=m$, $m_2=0$, and changing the variable of integration to $y=(k^2a_0^{-2}m+1)^{1/2}$, we have

$$\Delta N_\phi(t) = [a_0/a(t)]^3\lambda^2\pi^{-2}G^{-3/2}\sin(2\sigma)F(m)\,, \tag{5.22}$$

where

$$F(m) \equiv (Gm^2)^{-1/2}\int_0^\infty dy\, \frac{\sin\{(Gm^2)^{1/2}[y+(y^2-1)^{1/2}]\} - \frac{1}{2}\sin\{2(Gm^2)^{1/2}[y+(y^2-1)^{1/2}]\}}{[y+(y^2-1)^{1/2}]^2}\,. \tag{5.23}$$

The factor of $[a_0/a(t)]^3$ takes into account the effect of the expansion on the density. Defining

$$g=(Gm^2)^{1/2}\,, \tag{5.24}$$

one finds that

$$F(m) = \tfrac{1}{2}\,[(1+\tfrac{2}{3}g^2)\,\mathrm{Ci}(2g) - (1+\tfrac{1}{6}g^2)\,\mathrm{Ci}(g)] - \tfrac{1}{12}\,[2(g+g^{-1})\sin(2g) - \cos(2g) - (g+4g^{-1})\sin g + \cos g]\,, \tag{5.25}$$

where $\mathrm{Ci}(x)$ is the cosine integral,

$$\mathrm{Ci}(g) = -\int_g^\infty dt\,t^{-1}\cos t = \gamma + \ln g + \sum_{n=1}^{\infty}\frac{(-1)^n g^{2n}}{2n(2n)!}\,, \tag{5.26}$$

with $\gamma = 0.577\ldots$, the Euler constant. Notice that when $m=0$,

$$F(0) = 2^{-1}\ln 2 = 0.347\,. \tag{5.27}$$

We have calculated $F(m)$ for a representative set of values of m ranging from $G^{-1/2}$ to 0, and find the following values:

$$F(G^{-1/2}) = 0.124,\quad F(0.8G^{-1/2}) = 0.173\,,\quad F(0.5G^{-1/2}) = 0.249,\quad F(0.1G^{-1/2}) = 0.341\,. \tag{5.28}$$

The quantity $F(m)$ changes by less than a factor of 3 in the range of interest, so that ΔN_ϕ has only a weak mass dependence.

In our model, ΔN_ϕ does not vanish when $m=0$. The considerations of Refs. 9, 10, and 20, according to which ΔN_ϕ would be expected to vanish when the mass is zero, do not apply to the present interaction in which gravity acts as an external field which induces asymmetric pair production.

Taking $m=0.5G^{-1/2}$ as a representative value, the baryon number density of Eq. (5.22) is

$$\Delta N_\phi(t) = [a_0/a(t)]^3\lambda^2 G^{-3/2}\sin(2\sigma)(2.5\times 10^{-2})\,, \tag{5.29}$$

where $t \gtrsim G^{1/2}$.

Using the expressions for I_2 and I_3 in Eqs. (5.17) and (5.18), one finds that the sum of the number densities of baryons and antibaryons created by the interaction is

$$\langle N_\phi(t)\rangle + \langle \bar{N}_\phi(t)\rangle = [a_0/a(t)]^3 2\lambda^2 G^{-3/2}\pi^{-2}H(m)\,, \tag{5.30}$$

where

$$H(m) \equiv (2-\cos 2\sigma)(3g)^{-1} - \cos 2\sigma\{\tfrac{1}{6}\,[\sin 2g + 2(g+g^{-1})\cos 2g] + (1+\tfrac{2}{3}g^2)\,\mathrm{si}(2g)\} - (1-\cos 2\sigma)\{\tfrac{1}{6}\,[\sin g + (g+4g^{-1})\cos g] + (1+\tfrac{1}{6}g^2)\,\mathrm{si}(g)\}\,, \tag{5.31}$$

where g was defined in Eq. (5.24), and

$$\mathrm{si}(g) = -\int_g^\infty dt t^{-1} \sin t$$
$$= -\frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} g^{2n-1}}{(2n-1)(2n-1)!} . \qquad (5.32)$$

One finds the following values of $H(m)$:

$$H(G^{-1/2}) = 0.80 - (0.40)\cos 2\sigma,$$
$$H(0.8G^{-1/2}) = 0.92 - (0.39)\cos 2\sigma,$$
$$H(0.5G^{-1/2}) = 1.13 - (0.24)\cos 2\sigma, \qquad (5.33)$$
$$H(0.1G^{-1/2}) = 1.47 - (0.09)\cos 2\sigma,$$
$$H(0) = \pi/2 = 1.57 .$$

The sum of the number densities for leptons and antileptons is also given by Eq. (5.30). For $m = 0.5G^{-1/2}$, one has

$$\langle N_\phi(t)\rangle + \langle \bar{N}_\phi(t)\rangle = [a_0/a(t)]^3 \lambda^2 G^{-3/2}[0.23 - 0.05\cos(2\sigma)] . \qquad (5.34)$$

We remark that this result includes only the particles produced by the interaction, and would not be valid if purely gravitational production of these particles were appreciable. On the other hand, our estimate of ΔN_ϕ, Eq. (5.29), would remain approximately valid because the only effect of gravitational production on ΔN_ϕ is through modification of Eq. (5.9) which enters in the calculations of I_2 and I_3.

Taking G to be the Newtonian constant, the Einstein equation with zero cosmological constant requires a total energy density $\rho(t)$ of

$$\rho(t) = \frac{3}{8\pi G}\left(\frac{\dot{a}(t)}{a(t)}\right)^2 . \qquad (5.35)$$

For $t \approx G^{1/2}$ when $(\dot{a}/a)^2 \approx G^{-1}$, this gives

$$\rho \approx (0.12)G^{-2} \quad (t \approx G^{1/2}) . \qquad (5.36)$$

In the simple model in which the baryons and leptons participating in the asymmetric interaction are the only particles created near $t = 0$, if one requires consistency with the Einstein equation for $t \gtrsim G^{1/2}$ then Eq. (5.36) effectively determines λ. The average energy of a created lepton at $t \approx G^{1/2}$ would be roughly the same $(0.5G^{-1/2})$ as the baryon mass, so that the total energy density of the created particles and antiparticles is approximately given by the expression in Eq. (5.34) multiplied by $G^{-1/2}$. Since $(a_0/a)^3 \approx 1$ when $t \approx G^{1/2}$, consistency with (5.36) requires

$$\lambda \approx 0.8. \qquad (5.37)$$

Then the perturbation result would not be very accurate, but on dimensional grounds one would expect that the exact results do not differ by orders of magnitude from these perturbation theory results. The conserved ratio of the baryon number density to the entropy density is given by Eq. (5.13) in this simple model, or using Eqs. (5.34) and (5.29), by

$$\frac{\Delta N_\phi}{\sum\langle N\rangle} = \frac{0.025\sin(2\sigma)}{[0.46 - 0.10\cos(2\sigma)]} . \qquad (5.38)$$

For the observed range of 10^{-8} to 10^{-10} this gives

$$10^{-7} \gtrsim \sigma \gtrsim 10^{-9} . \qquad (5.39)$$

On the other hand, if λ is small with respect to 0.8, then the pairs created by the asymmetric interaction would provide only a small fraction of the energy density required by the Einstein equation. Let us suppose that the required energy density comes from a hot gas of particles and antiparticles having masses small with respect to $G^{-1/2}$ and contributing zero to the total baryon number. These particles could result, for example, from pair production by the time-changing gravitational field near $t \approx 0$. Then the energy density at $t \approx G^{1/2}$ is that of blackbody radiation,

$$\rho = n\pi^2 k_B^4 T^4/30, \qquad (5.40)$$

where n is the number of different kinds of particles present (counting each spin and charge separately), T is the temperature at that time and k_B is Boltzmann's constant. Thus, T is determined by Eq. (5.35) or (5.36). The entropy density at that time is (ignoring factors of $\frac{7}{8}$ for fermionic constituents),

$$s = 2n\pi^2 k_B^4 T^3/45 , \qquad (5.41)$$

so that

$$s/k_B = (2/45)n^{1/4}\pi^{1/2}(30\rho)^{3/4} . \qquad (5.42)$$

Taking $n^{1/4} \approx 1$ and substituting Eq. (5.36) (the Einstein equation) for ρ, one finds

$$s/k_B \approx 0.21\,G^{-3/2} \qquad (5.43)$$

at $t \approx G^{1/2}$. Then the conserved ratio of baryon number density to entropy density is obtained from Eqs. (5.29) and (5.43) as

$$\frac{\Delta N_\phi}{(s/k_B)} \approx \lambda^2(0.12)\sin(2\sigma). \qquad (5.44)$$

with $\lambda = \frac{1}{137}$, the fine-structure constant, one obtains the observed range, 10^{-8} to 10^{-10}, for

$$10^{-3} \gtrsim \sigma \gtrsim 10^{-5}, \quad \lambda = \tfrac{1}{137} . \qquad (5.45)$$

With $\lambda = (1/137)^2$ one finds that the ratio in Eq. (5.44) cannot be larger than 0.34×10^{-9}, and that when that ratio is in the range from 0.34×10^{-9} to 10^{-10} the angle σ is in the range

$$\tfrac{1}{4}\pi \gtrsim \sigma \gtrsim 0.15, \quad \lambda = (1/137)^2 . \tag{5.46}$$

For smaller values of λ, one cannot produce a ratio of baryon number density to entropy density which is sufficiently large to be in the observed range in the present model.

Finally, suppose that the dimension G which characterizes the minimum of the expansion is different from the Newtonian constant, which we now denote by G_N. Then Eq. (5.36) is replaced by

$$\rho = \frac{3}{8\pi}(GG_N)^{-1} \text{ at } t \approx G^{1/2} , \tag{5.47}$$

immediately after the "bounce." Then two possibilities must be considered. First, if the energy density of the universe is created mainly by the asymmetric interaction, it follows that the created energy density is again of the order of magnitude of Eq. (5.34) multiplied by 2 (to include the leptons) and by $0.5\,G^{-1/2}$ (the approximate average energy per created baryon or lepton). Thus, the energy density of the created particles is

$$\rho \approx 0.2\lambda^2 G^{-2} \text{ at } t \approx G^{1/2} . \tag{5.48}$$

In order for Eqs. (5.47) and (5.48) to agree one must have

$$\lambda \approx 0.8(G/G_N)^{1/2} . \tag{5.49}$$

Thus, if the bounce were characterized by a length much larger than the Planck length, $G_N^{1/2}$, then the value of λ would be unreasonably large [for example, with $G \approx (10^{-13}\text{ cm})^2 \approx 10^{40}\,G_N$ the value of λ would be of order 10^{20}]. The second possibility to be considered is that the energy density required by Eq. (5.47) comes from other sources. Then Eq. (5.43) becomes

$$s/k_B \approx 0.21\,(GG_N)^{-3/4} , \tag{5.50}$$

while Eq. (5.29) is not altered. Using those results at $t \approx G^{1/2}$ when $a_0/a \approx 1$, one finds for the conserved ratio of baryon number density to entropy density:

$$\frac{\Delta N_\phi}{(s/k_B)} \approx \lambda^2(0.12)\sin(2\sigma)(G_N/G)^{3/4} . \tag{5.51}$$

Then if we make the reasonable assumption that $\lambda^2 \sin 2\sigma < 1$, it follows that the ratio $\Delta N_\phi/(s/k_B)$ will lie in the observed range of 10^{-8} to 10^{-10} only if

$$G < 10^{10} G_N . \tag{5.52}$$

Thus, for a model of the present type to be capable of generating the matter-antimatter asymmetry of the universe, the time characterizing the stage of the expansion when the violation of baryon number conservation occured must be no greater than about 10^5 Planck times (about 10^{-39} sec). Thus, models in which the universe never was characterized by a time close to the Planck time are not capable of generating the observed asymmetry by the mechanism proposed here. On the other hand, it is clear from the above considerations that one can construct reasonable asymmetric interactions which will generate the observed asymmetry, provided the minimum characteristic time is not much larger than the Planck time.

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APPENDIX

In this Appendix, we repeat the estimates of Sec. V for a different form of the scalar curvature and of $\Lambda(t)$. Specifically, we choose

$$R(t) = \frac{1}{G + t^2} , \tag{A1}$$

$$\Lambda(t) = e^{-i\sigma(t)} , \tag{A2}$$

$$\sigma(t) = \epsilon\,\frac{G^{1/2} t}{G + t^2} . \tag{A3}$$

In accordance with our previous estimates, we assume ϵ to be very small.

The reason for this calculation is twofold: We want to see how sensitive our results are to the specific choice of $R(t)$ and $\Lambda(t)$, and to examine whether the discontinuities in $R(t)$ and the phase of $\Lambda(t)$ used in Sec. V affected the estimates significantly.

Using the same approximations as in Sec. V, we find that, to first order in ϵ,

$$|I_2|^2 \approx \frac{1}{4\omega_1(0)\omega_2(0)} \int_{-\infty}^{+\infty} dt_1 \int_{-\infty}^{+\infty} dt_2 R(t_1)R(t_2)[1 - i\sigma(t_1) + i\sigma(t_2)] \exp[-i(t_2 - t_1)\omega_{12}(0)] ,$$

$$|I_3|^2 \approx \frac{1}{4\omega_1(0)\omega_2(0)} \int_{-\infty}^{+\infty} dt_1 \int_{-\infty}^{+\infty} dt_2 R(t_1)R(t_2)[1 - i\sigma(t_1) + i\sigma(t_2)] \exp[i(t_2 - t_1)\omega_{12}(0)] .$$

[I_2 and I_3 are defined in Eqs. (4.2b), (4.2c).] Substitution of Eqs. (A1)–(A3) yields

$$|I_2|^2 = \frac{\pi^2 G^{-1}}{4\omega_1(0)\omega_2(0)}\,[1+\epsilon G^{1/2}\omega_{12}(0)] \times \exp[-2G^{1/2}\omega_{12}(0)]\,, \tag{A4}$$

$$|I_3|^2 = \frac{\pi^2 G^{-1}}{4\omega_1(0)\omega_2(0)}\,[1-\epsilon G^{1/2}\omega_{12}(0)] \times \exp[-2G^{1/2}\omega_{12}(0)]\,. \tag{A5}$$

When these expressions are substituted in Eqs. (4.10), (4.11), and (4.12), one obtains

$$\Delta N_\phi = \lambda^2\left(\frac{a_0}{a_+}\right)^3 \frac{\epsilon G^{-3/2}}{4}\,F'(m)\,, \tag{A6}$$

$$\langle N_\phi + \overline{N}_\phi\rangle = \lambda^2\left(\frac{a_0}{a_+}\right)^3 \frac{G^{-3/2}}{4}\,H'(m)\,, \tag{A7}$$

with

$$F'(m) = \tfrac{1}{8}(2g+1)e^{-2g} - \tfrac{1}{2}g^2 E_1(2g)\,, \tag{A8}$$

$$H'(m) = \tfrac{1}{4}e^{-2g} - \tfrac{1}{2}gE_2(2g)\,, \tag{A9}$$

and

$$g = (Gm^2)^{1/2}\,,$$

$$E_n(2g) = \int_1^\infty dz\,\frac{e^{-2gz}}{z^n}\,.$$

Representative values of $F'(m)$, $H'(m)$ are

$$F'(G^{-1/2}) = 0.027,\quad F'(0.8G^{-1/2}) = 0.038,$$
$$F'(0.5G^{-1/2}) = 0.065,\quad F'(0.1G^{-1/2}) = 0.117,$$
$$F'(0) = 0.125,$$

and

$$H'(G^{-1/2}) = 0.007,\quad H'(0.8G^{-1/2}) = 0.012,$$
$$H'(0.5G^{-1/2}) = 0.055,\quad H'(0.1G^{-1/2}) = 0.176\,,$$
$$H'(0) = 0.250.$$

To compare with the results of Sec. V, we identify the small parameter ϵ in Eq. (A3) with the parameter 2σ in Eq. (5.16), since the phase changes by that amount in the interval from $t \approx -G^{1/2}$ to $t = G^{1/2}$. Then we find that the ratio of the baryon number density as calculated in this Appendix over the one calculated in Sec. V ranges from 0.528 to 0.889 as the mass changes from $m = G^{-1/2}$ to $m = 0$. The corresponding ratio for the density of baryons plus antibaryons ranges from 0.022 to 0.196 when the mass is varied within the above limits.

In the context of the approximations made in this paper, the difference between the two sets of estimates is unessential. This is particularly true in the case of our estimates of the baryon number density ΔN_ϕ.

We therefore conclude that our estimates are relatively insensitive to the special choice of $R(t)$ and $\Lambda(t)$. In particular, the nonvanishing result for $m = 0$ is not an artifact of the discontinuities in $R(t)$, $\Lambda(t)$ used in Sec. V.

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[1]A review is given in G. Steigman, Ann. Rev. Astron. Astrophys. 14, 339 (1976).

[2]S. Weinberg, in *Lectures on Particles and Field Theory*, edited by S. Deser and K. Ford (Prentice-Hall, Englewood Cliffs, New Jersey, 1964), p. 482.

[3]A. D. Sakharov, Zh. Eksp. Teor. Fiz. Pis'ma Red. 5, 32 (1967) [JETP Lett. 5, 24 (1967)].

[4]V. A. Kuzmin, Zh. Eksp. Teor. Fiz. Pis'ma Red. 12, 335 (1970) [JETP Lett. 12, 228 (1970)].

[5]L. Parker, in *Asymptotic Structure of Space-Time*, edited by F. P. Esposito and L. Witten (Plenum, New York, 1977), pp. 118, 160; Nature 261, 20 (1976).

[6]M. Yoshimura, Phys. Rev. Lett. 41, 381 (1978).

[7]A. Ignatiev, N. Krasnikov, V. Kuzmin, and A. Tavkhelidze, Phys. Lett. 76B, 436 (1978).

[8]S. Dimopoulos and L. Susskind, Phys. Rev. D 18, 4500 (1978).

[9]B. Toussaint, S. B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D 19, 1036 (1979).

[10]S. Weinberg, Harvard Univ. Report No. HUTP-78/A040 (unpublished).

[11]J. D. Bekenstein and U. Ben Yaacov (unpublished).

[12]J. Pati and A. Salam, Phys. Rev. D 8, 1240 (1973); 10, 275 (1974).

[13]H. Georgi and S. Glashow, Phys. Rev. Lett. 32, 438 (1974).

[14]G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976); Phys. Rev. D 14, 3432 (1976).

[15]Charge conservation and the form of the coupling (1.1) imply that ϕ and ψ have the same charge. Since traditionally the oppositely charged proton and electron are regarded as particles, we call ψ the antilepton field.

[16]L. Parker, Ph.D. thesis, Harvard University, 1966 (unpublished); Phys. Rev. Lett. 21, 562 (1968); Phys. Rev. 183, 1057 (1969); Phys. Rev. D 3, 346 (1971).

[17]For a discussion of conformal invariance in that context see, for example, L. Parker, Phys. Rev. D 7, 976 (1973).

[18]L. Parker, in *Proceedings of NATO Advanced Study Institute on Gravitation*, edited by S. Deser and M. Levy (Plenum, New York, in press), Sec. 6.

[19]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), Chap. 15; Ya. B. Zeldovich and I. D. Novikov, *Structure and Evolution of the Universe* (in Russian) (Nauka, Moscow, 1975); C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), Chap. 22.

[20]C. N. Yang and C. P. Yang, ITP Stony Brook Report No. SB78-51 (unpublished).

[21]H. Nariai and K. Tomita, Prog. Theor. Phys. 46, 776

(1971).
[22]L. Parker and S. A. Fulling, Phys. Rev. D 7, 2357 (1973).
[23]F. Lund, Phys. Rev. D 8, 3253 (1973).
[24]J. D. Bekenstein, Ann. Phys. (N.Y.) 82, 535 (1974); Phys. Rev. D 11, 2072 (1975).
[25]In Ref. 9, violation of the energy conditions by black-hole evaporation is suggested as a possible mechanism for a cosmological bounce.
[26]Ya. B. Zeldovich, Zh. Eksp. Teor. Fiz. Pis'ma Red. 12, 443 (1970) [JETP Lett. 12, 307 (1970)].
[27]B. L. Hu and L. Parker, Phys. Rev. D 17, 933 (1978), and references given there.