Naked singularities, event horizons, and charged particles

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The motion of charged test particles in a Reissner-Nordström field is investigated. It is shown that naked singularities can be destroyed by shooting in suitably charged test particles to produce an event horizon around the source where none previously existed. It is also shown that existing event horizons cannot be destroyed by bombardment with charged test particles. In this manner existing naked singularities can be destroyed but not created while event horizons can be created but not destroyed. Furthermore, a simple explanation is given for radial test-particle oscillations—a phenomenon with no Newtonian analog.

I. INTRODUCTION

A question of interest in relativistic astrophysics is whether naked singularities and/or event horizons can be created or destroyed. This is closely connected with Penrose's cosmic censorship hypothesis whereby naked singularities tend to become clothed by event horizons as the universe evolves.

To investigate these questions we analyze the motion of radially moving charged test particles in a Reissner-Nordström field. A somewhat similar analysis has been carried out by Graves and Brill,[1] who were interested in a "pulsating wormhole throat." They considered particle orbits around a "wormhole" and showed that no test particle will ever reach the true singularity at $R=0$ if the magnitude of the charge-to-mass ratio q/m_0 is less than 1. Here we consider situations both with and without event horizons and we have found that radial test-particle oscillations are possible, a result with no classical analog. Also, conditions for the creation or destruction of event horizons and naked singularities via bombardment with charged test particles are investigated.

II. TEST-PARTICLE MOTION

The motion of charged test particles is given by the Lorentz force equation

$$\frac{d}{d\tau}\frac{dx^\mu}{d\tau}+\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}=-\epsilon_0 F^{\mu\nu}g_{\nu\beta}\frac{dx^\beta}{d\tau}, \tag{2.1}$$

where

$$\epsilon_0=q/m_0 \tag{2.1$'$}$$

is the ratio of the test particle's charge to rest mass and $F_{\mu\lambda}=A_{\lambda,\mu}-A_{\mu,\lambda}$, with A_μ being the electromagnetic vector potential. For one-dimensional motion (in the direction R) in a field described by a metric of the form

$$ds^2=g_{11}dR^2+g_{22}d\theta^2+g_{33}d\varphi^2+g_{44}dT^2 \tag{2.2}$$

with $A_\mu=(0,0,0,\psi)$, the Lorentz force equation (2.1) becomes

$$\frac{d^2R}{d\tau^2}+\left(\frac{g_{11,1}}{2g_{44}}+\frac{g_{44,1}}{2g_{44}}\right)\frac{dR}{d\tau}+\frac{g_{44,1}}{2g_{11}g_{44}}=-\epsilon_0\frac{\psi_{,1}}{g_{11}}\frac{dT}{d\tau}, \tag{2.3}$$

which has a first integral

$$\left(\frac{dR}{d\tau}\right)^2=\frac{g_{44}+(K-\epsilon_0\psi)^2}{-g_{44}g_{11}},\quad \frac{dT}{d\tau}=\frac{K-\epsilon_0\psi}{-g_{44}}, \tag{2.4}$$

where K is a constant of integration related to the energy per unit mass of the test particle. If (2.2) is a Reissner-Nordström field, so that

$$ds^2=\left(1-\frac{2M}{R}+\frac{Q^2}{R^2}\right)^{-1}dR^2+R^2(d\theta^2+\sin^2\theta d\phi^2)-\left(1-\frac{2M}{R}+\frac{Q^2}{R^2}\right)dT^2 \tag{2.5}$$

with $A_\mu=(0,0,0,Q/R)$, (2.3) and (2.4) reduce to

$$\frac{d^2R}{d\tau^2}=-\frac{(M-Q^2R^{-1})}{R^2}+\frac{\epsilon_0 Q}{R^2}(K-\epsilon_0 QR^{-1}), \tag{2.6a}$$

$$\left(\frac{dR}{d\tau}\right)^2=\frac{K^2-1}{R^2}\left\{R^2-\frac{2M[K\epsilon_0(Q/M)-1]}{K^2-1}R-\frac{Q^2(1-\epsilon_0^2)}{K^2-1}\right\}, \tag{2.6b}$$

which can further be rewritten as

$$\left(\frac{dR}{d\tau}\right)^2=\frac{K^2-1}{R^2}(R-R_+)(R-R_-), \tag{2.7}$$

where

$$R_\pm=\frac{M}{K^2-1}\{K\epsilon_0(Q/M)-1\pm[(K\epsilon_0(Q/M)-1)^2+(K^2-1)(1-\epsilon_0^2)Q^2/M^2]^{1/2}\}. \tag{2.8}$$

When $R_\pm$ are real positive numbers, they are equal to the turning radii of the test particle, i.e., the radii at which $(dR/d\tau)^2=0$.

To display readily the properties of a radially moving charged test particle we have plotted in Fig. 1 the turning radii $R_\pm$ as a function of ϵ_0 for various values of K and the charge-to-mass ratio Q/M of the source of the field. We have taken Q and M as positive; the extension to negative values is straightforward. The shaded portions on the graphs are forbidden regions where $(dR/d\tau)^2<0$. Only positive values of the radius R of the test particle are considered.

The trends of $R_\pm$ are seen in Fig. 1 both as K increases at constant Q/M and as Q/M increases with constant K. For $Q/M\leqslant 1$ the horizons ($g_{44}=0$) given by

$$R_{H_\pm}=M[1\pm(1-Q^2/M^2)^{1/2}] \tag{2.9}$$

are shown as horizontal dashed lines. As is to be expected, there are no turning points within the horizons. All the curves intersect $R_T=0$ at $\epsilon_0=\pm1$. As we shall show in Sec. III this property together with the closure of the forbidden regions when $Q/M=1$ accounts for the inability to destroy a horizon by sending in charged test particles from infinity. Other interesting features of Fig. 1 are the straight lines that result at the transition value

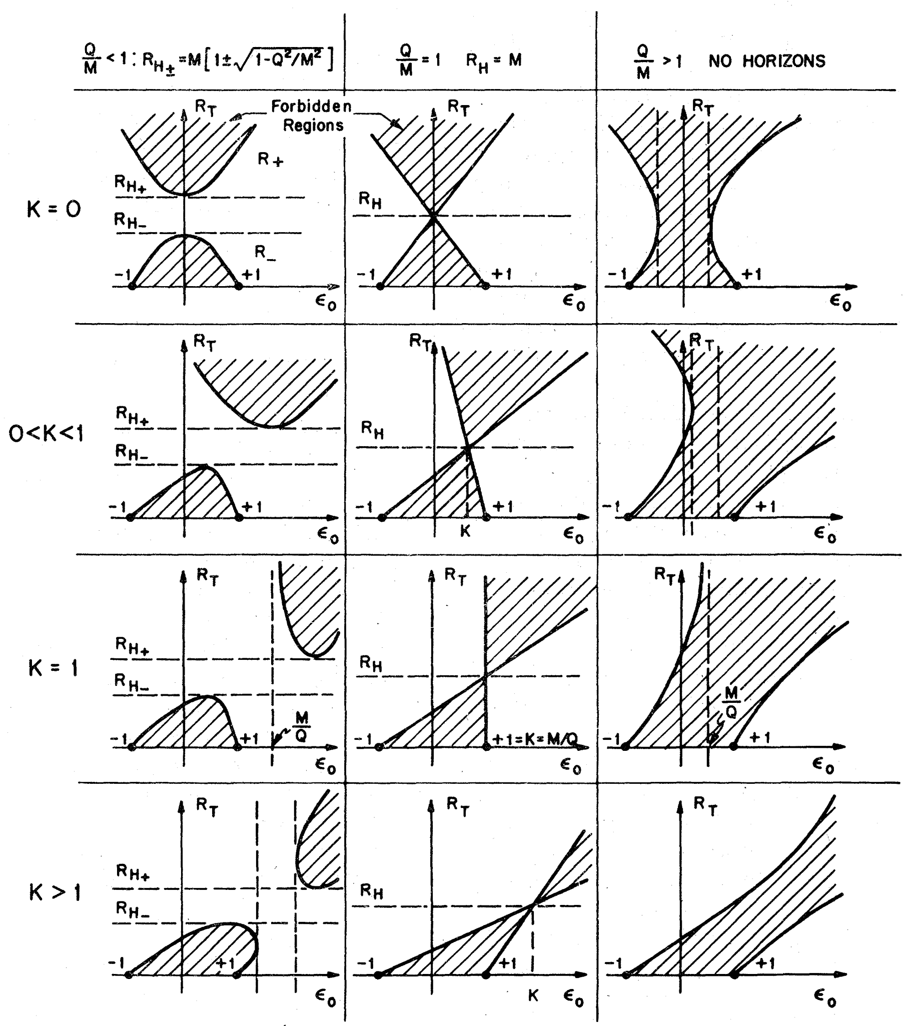

FIG. 1. Plots of the turning radii R_+ and R_- as a function of $\epsilon_0=q/m_0$ of a test particle for various values of K, a measure of the energy per unit mass of the test particles, and Q/M of the source. The shaded sections are forbidden regions where $(dR/d\tau)^2<0$ for the test particle. In all cases if $-1<\epsilon_0<+1$, it is impossible for a test particle to reach $R=0$. If $K<1$ a test particle does not have sufficient energy to reach $R=\infty$, while if $K\geqslant1$, a test particle can travel to $R=\infty$, with $K=1$ corresponding to the situation where the particle has zero velocity at $R=\infty$. For $K<1$, radial oscillations occur when $-1<\epsilon_0<+1$. Situations where a particle will remain motionless at a fixed radius correspond to locations where the curves have vertical tangents. Note that radial oscillations can occur for all values of Q/M so that the phenomenon has nothing to do with the existence of event horizons.

$Q/M = 1$ and the single vertical asymptote at $\epsilon_0 = M/Q$ when $K = 1$.

The simplest type of particle motion is no motion at all, i.e., the particle remains motionless in equilibrium at some fixed radius R. Classically this situation will occur when the inverse square gravitational and Coulomb forces are exactly equal, giving the condition

$$Qq = Mm_0 , \tag{2.10}$$

which is independent of the radius.

The general relativistic condition for equilibrium is obtained by setting both $dR/d\tau = 0$ and $d^2R/d\tau^2 = 0$ in (2.6), thereby giving

$$\begin{aligned} Qq &= (M - Q^2R^{-1})m_0(1 - 2M/R + Q^2/R^2)^{-1/2} \\ &= (M - Q^2R^{-1})m_0(-g_{44})^{-1/2} . \end{aligned} \tag{2.11}$$

If $Q/M = 1$, (2.11) reduces to $Qq = Mm_0$. Otherwise, in contrast with the classical result, the relativistic condition is radius dependent. The radius of a stationary particle is a special case of a turning radius. Various stationary radii are seen in Fig. 1 where the turning radii curves have vertical tangents. Examination of the curves near the stationary radii shows that the equilibrium is unstable, neutral, or stable, depending on whether respectively $Q/M < 1$ $(K > 1)$, $Q/M = 1$ $(K = 1)$, or $Q/M > 1$ $(K < 1)$.

It is seen from the curves of Fig. 1 that when $K < 1$ radial oscillations can occur in the allowed regions for all values of Q/M if $-1 < \epsilon_0 < +1$. Such radial oscillations have no classical analog. Classically the force on a charged test particle points either outward or inward and does not change direction as the radius of the particle varies. Furthermore, when $-1 < \epsilon_0 < +1$ it is impossible for the particle to reach $R = 0$ no matter how energetic it is. Again there is no classical analog, for classically a particle with zero or negative charge will experience an attractive force and will always reach $R = 0$. As we shall now show, this nonclassical motion of charged test particles arises because the electric field energy of the source has an effective mass which contributes to the Schwarzschild mass to interact with the mass of the test particle.

For a general static electrovac field described by the metric

$$ds^2 = g_{ij}dx^i dx^j - V^2 dT^2 , \tag{2.12}$$

Whittaker's theorem states that the total effective mass M_T inside a volume v_3 is given by[2]

$$M_T = \frac{\kappa}{8\pi} \int_{v_3} (T^i{}_i - T^4{}_4) V dv_3 \tag{2.13}$$

where the invariant volume element dv_3 is

$$dv_3 = \sqrt{\bar{g}}\, dx^1 dx^2 dx^3 \tag{2.14}$$

with $\bar{g}$ being the determinant of the spatial part of the metric, so that $-g = V^2\bar{g}$.

To evaluate M_T we make use of the result that for the metric form (2.12) the Einstein tensor G^μ_λ satisfies[3]

$$-\int_{v_3} (G^i{}_i - G^4{}_4) V dv_3 = \oint_{v_2} \frac{(V^2)_{,i}}{V} n^i dv_2 , \tag{2.15}$$

where v_3 is any portion of space bounded by the closed surface v_2, dv_3 and dv_2 are the respective invariant elements of volume and area, and n^i is the outward unit normal to v_2. By using the Einstein field equations $G^\mu_\lambda = -\kappa T^\mu_\lambda$, we find from (2.13) and (2.15) that

$$M_T = \frac{1}{8\pi} \oint_{v_2} \frac{(V^2)_{,i}}{V} n^i dv_2 . \tag{2.16}$$

We now consider spherical symmetry and let the closed surface v_2 be a sphere located outside the extent of the source so that the Reissner-Nordström metric (2.5) holds. Then we have

$$\begin{aligned} &V^2 = 1 - \frac{2M}{R} + \frac{Q^2}{R^2} , \\ &n^i = (V, 0, 0), \quad dv_2 = R^2 \sin\theta d\theta d\phi , \end{aligned} \tag{2.17}$$

which, when substituted into (2.16), gives M_T as

$$M_T = M - \frac{Q^2}{R} . \tag{2.18}$$

Comparison of (2.10) and (2.11) shows that the classical source mass is replaced in general relativity with the total effective mass $M_T = M - Q^2/R$. This is further evidenced from the first term on the right-hand side of (2.6a). The effective mass (2.18) found from Whittaker's theorem agrees with the result of de la Cruz and Israel[4] using junction conditions across thin shells. As R decreases, M_T also decreases because the electric field energy inside a sphere of radius R decreases. M_T goes to zero when $R = Q^2/M$ and becomes negative for smaller R.

We now examine the conditions under which this can occur. Letting E^μ_ν and M^μ_ν be the respective electrostatic and material parts of the stress-energy tensor $T^\mu_\nu = E^\mu_\nu + M^\mu_\nu$, we obtain from (2.13)

$$\begin{aligned} M_T = &\frac{\kappa}{8\pi} \int_{v_3} (E^i_i - E^4_4) V dv_3 \\ &+ \frac{\kappa}{8\pi} \int_{v_3} (M^i_i - M^4_4) V dv_3 . \end{aligned} \tag{2.19}$$

If we have only electric fields; so that $A_\mu = (0, 0, 0, \psi)$, the identity[5]

$$4\pi(E^i_i - E^4_4) = \frac{1}{V^2} g^{ij} \psi_{,i} \psi_{,j} \tag{2.20}$$

for the case of spherical symmetry becomes

$$4\pi(E_i^i - E_4^4) = (\psi_{,R})^2 \frac{g^{11}}{V^2}. \tag{2.21}$$

If the metric is well behaved inside the source, then (2.21) is positive-definite. Substituting (2.21) into (2.19) and breaking the integral into two parts, internal and external to the source defined by $R=R_s$, we obtain after setting $\psi = Q/R$ and $dv_3 = (R^2/V)\sin\theta dR d\theta d\varphi$ in the exterior region $R>R_s$,

$$M_T = -\frac{Q^2}{R} + \frac{Q^2}{R_s} + \frac{\kappa}{(4\pi)8\pi}\int_0^{R_s} (\psi_{,R})^2 \frac{g^{11}}{V} dv_3 + \frac{\kappa}{8\pi}\int_{v_3} (M_i^i - M_4^4) V\, dv_3. \tag{2.22}$$

External to the source ($R>R_s$) the sum of the first three terms in the expression (2.22) for M_T will be positive. Hence the only way M_T can be negative is if

$$\frac{\kappa}{8\pi}\int_{v_s} (M^i{}_i - M^4{}_4) V\, dv_3 < 0\,.$$

The expression

$$\frac{\kappa}{8\pi}\int_{v_s} (M^i{}_i - M^4{}_4) V\, dv_3$$

gives the mass of the source when it is uncharged. If we thereby designate

$$\rho_m = \frac{\kappa}{8\pi}(M^i{}_i - M^4{}_4)V \tag{2.23}$$

as the material density, we see that if ρ_m is everywhere positive, then the total effective mass $M_T = M - Q^2/R$ will also always be positive. (For other discussions of negative mass and mass density in connection with charged bodies, see de la Cruz and Israel[4] and Boulware[6].)

It is the apparent negative values of M_T when $R<Q^2/M$ that give rise to the repulsive forces [cf. (2.6a)] observed for $-1<\epsilon_0<+1$ in Fig. 1. A straightforward calculation from (2.1) shows, however, that repulsive forces will arise only when $R<Q^2/M$, so these repulsive forces will not be present if ρ_m is positive. Moreover, if $Q^2/M^2<1$, then since $R_{H-}<Q^2/M<R_{H+}$, the inner horizon R_{H-} will lie inside the source if ρ_m is positive.

It should be pointed out that, even for arbitrary nonspherical fields, if the source is charged the Schwarzschild mass seen at infinity is not obtained by integrating ρ_m over the volume of the source. Instead the Schwarzschild mass seen at infinity, where a Reissner-Nordström field obtains, is given by[5]

$$M = \int_{v_s}\left[\frac{\kappa}{8\pi}(M^i{}_i - M^4{}_4)V + \mu\psi\right] dv_3\,,$$

where μ is the proper charge density. Thus the total mass density is

$$\rho = \frac{\kappa}{8\pi}(M^i{}_i - M^4{}_4)V + \mu\psi\,.$$

It is seen that even if $M_\nu^\mu = 0$ so that no actual "matter" is present, the charge alone in the source will produce a Schwarzschild mass.[5,7]

III. CREATION AND DESTRUCTION OF EVENT HORIZONS

In this section we discuss the effects on a Reissner-Nordström field when charged test particles are sent radially from infinity into the source. Although we shall only treat test particles, all our results will still hold if, instead of particles, thin shells are used, thereby maintaining spherical symmetry at all times. For discussions of thin-shell motions see Refs. 4, 6, 7, and 8. What we shall show is that starting with a source with $Q/M>1$, so that there are no event horizons present, there is no difficulty in creating horizons by injecting suitably charged test particles into the source. On the other hand, if initially there exist two horizons because $Q/M<1$ we find that it is then impossible to destroy the horizons by sending in charged test particles.

Consider first a source with $Q/M>1$. From Fig. 1 the turning radii for the case $K=1$ are plotted in Fig. 2. The value $K=1$ corresponds to particles with zero velocity at infinity, i.e., the particles are being "dropped" into the source from infinity. From Fig. 1 it is seen that motion down to $R=0$ is allowed for test particles with charge-to-rest-mass ratio $q/m_0 = \epsilon_0 < -1$. Dropping in such particles will lower the charge-to-mass ratio Q/M of the source for two reasons: (1) The negative charge q of the test particle reduces the charge Q of the source, and (2) the positive mass m_0 of the test particle increases the mass M of the source.

The changes in the turning radii as more and more particles with $\epsilon_0<-1$ are dropped into the source are shown in Figs. 2(a) through 2(f). The single horizon corresponding to $Q/M=1$ is seen being formed in Fig. 2(c). This horizon then splits into the two event horizons when $Q/M<1$ until finally the charge of the source is reduced to zero, giving a Schwarzschild field.

Consider now what happens if we start out with an initially uncharged source and try to raise its charge-to-mass ratio and thereby destroy the horizons by sending in suitably charged test particles. In order to try to make $Q/M>1$ so that there are no horizons, we choose test particles with $\epsilon_0 = q/m_0 > 1$ so that each particle will supply more charge than mass when it reaches the source.

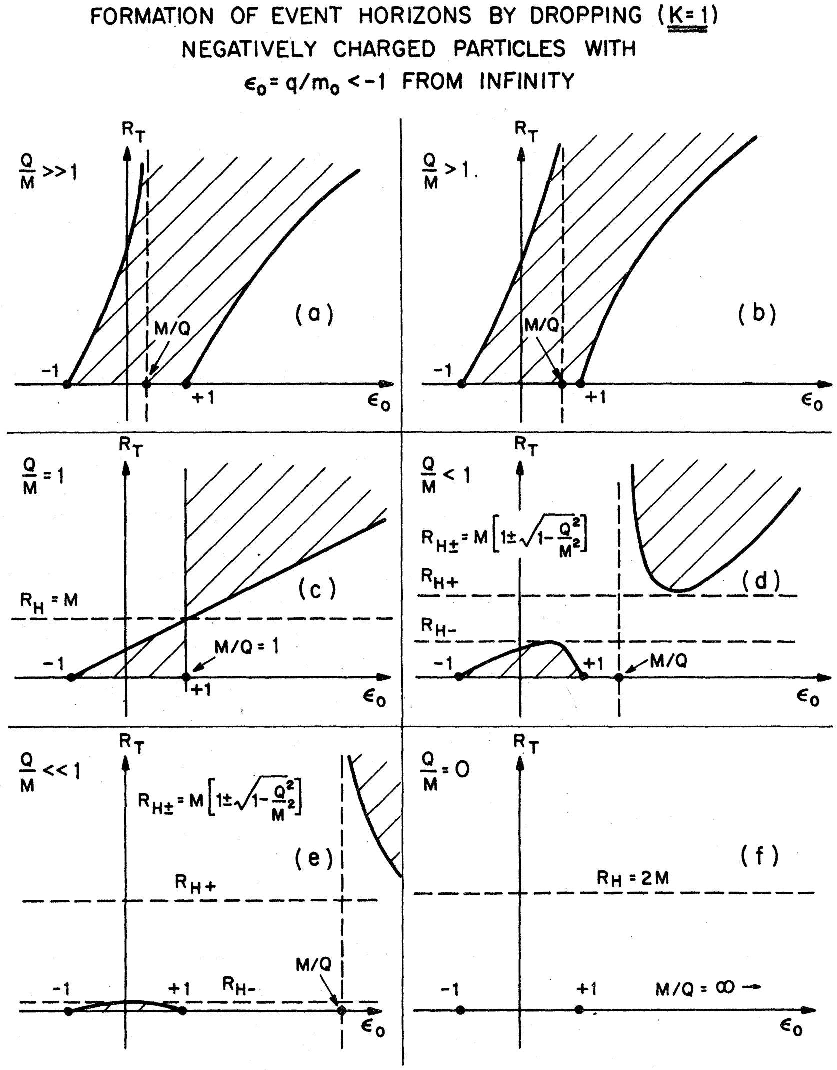

FIG. 2. Creation of event horizons by dropping from infinity negatively charged particles with $\epsilon_0=q/m_0<-1$. As successive particles with $q/m_0<-1$ reach the source, the value of Q/M of the source continuously decreases, as shown in the progression from (a) to (f). (c) shows the creation of an event horizon, which bifurcates as shown in (d) and (e). When the lower horizon merges with the origin, as shown in (f), an uncharged Schwarzschild source is formed. It should be noted for $Q/M\geq 1$ that if $q/m_0\geq -1$ a particle dropped from infinity will not reach the source, but will be repelled back to infinity, since the turning radius is greater than zero.

The sequence of events is shown in Fig. 3. Starting with particles with $\epsilon_0>1$ dropped ($K=1$) into the source from infinity, it is seen that as the charge-to-mass ratio Q/M of the source builds up the maximum possible value of ϵ_0 decreases as the asymptotic M/Q decreases. Thus, it is seen that as the critical value $Q/M=1$ is approached the allowed window closes, as shown in Fig. 3(d), barring further particles with $\epsilon_0>1$ from getting inside the horizon. Any further particles dropped from infinity would have to be such that $\epsilon_0<1$, which would decrease rather than increase Q/M.

Even by "firing" ($K>1$) test particles into the source, it is still not possible to raise Q/M above unity. Figure 3(e) shows the turning radii for $Q/M=1$ and $K>1$, where it is seen that it is indeed possible to get particles with $\epsilon_0>1$, if not down to $R=0$, at least inside the horizon where it might be argued that the horizon would then be destroyed. If, however, $K>1$, the total mass carried in from infinity is not m_0 but instead $m_0(1-w^2)^{-1/2}$, where w is the velocity of the particle at infinity so that we have

$$m_\infty=m_0(1-w^2)^{-1/2}=Km_0\,. \tag{3.1}$$

If we therefore plot the turning radii as a function

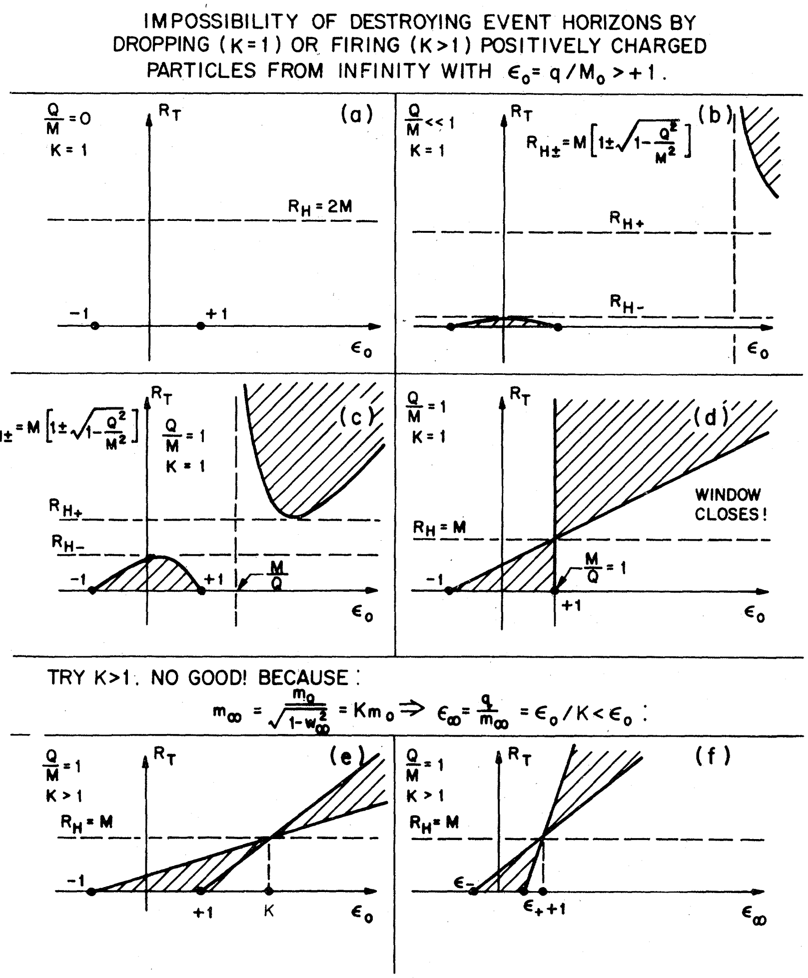

FIG. 3. Impossibility of destroying event horizons by dropping ($K=1$) or firing ($K>1$) from infinity positively charged particles with $\epsilon_0 = q/m_0 > +1$. (a) shows an uncharged Schwarzschild source with one event horizon. As charged particles with $q/m_0 > 1$ are dropped from infinity and reach the source, the Q/M ratio of the source builds up, resulting in two event horizons, as shown in Figs. 2(b) and 2(c). This process can be continued until $Q/M = 1$ is approached. At this point the allowed window for particles with $q/m_0 > 1$ closes, as shown in (d), making it impossible for such particles to fall within the event horizon. Thus the closing of the window prevents the destruction of the horizon and the exposure of a naked singularity. The event horizon cannot be destroyed even by firing test particles ($K>1$) from infinity. (e) shows that for $K>1$ it is possible to get particles with $q/m_0 > 1$ inside the event horizon. However, because a particle has nonzero velocity at infinity ($K>1$), its effective mass is not q/m_0 but q/m_∞, where $m_\infty = m_0/(1-w^2)^{1/2} > m_0$. This relativistic mass increase lowers the effective charge to mass ratio of a particle at infinity, so that, no matter what the value of $K>1$, it is still impossible, as shown in (f), to make $Q/M > 1$ and to destroy the event horizon.

of $\epsilon_\infty = q/m_\infty = \epsilon_0/K$ rather than $\epsilon_0 = q/m_0$ we obtain Fig. 3(f), which shows that no matter what the value of K, it is impossible to get particles with $q/m_\infty > 1$ even inside the horizon to make $Q/M > 1$ and thereby destroy the horizon.

The invariantly defined horizon areas, $A_\pm = 4\pi R_\pm^2$, can change when charged test particles reach the source. The change in the horizon radii is found from (2.9) to be

$$dR_\pm = \frac{dM}{(1-Q^2/M^2)^{1/2}} \times \left[(1-Q^2/M^2)^{1/2} \pm \left(1 - \frac{Q}{M}\frac{dQ}{dM}\right)\right]. \tag{3.2}$$

Examination of a test particle's four-momentum at large radius shows that $dQ/dM = \epsilon_0/K$, so that (3.2) can be written as

$$dR_{\pm}=\frac{dM}{(1-Q^2/M^2)^{1/2}}\left[(1-Q^2/M^2)^{1/2}\pm\left(1-\frac{Q}{M}\frac{\epsilon_0}{K}\right)\right]. \tag{3.3}$$

From Fig. 1 it is seen that the maximum possible value of ϵ_0 for a particle from infinity to reach the outer event horizon R_+ is given by the larger of

$$\epsilon_{0v}=\frac{M}{Q}[K\pm(K^2-1)^{1/2}(1-Q^2/M^2)^{1/2}], \tag{3.4}$$

which correspond to the vertical tangents of the turning points in Fig. 1. Substitution of (3.4) into (3.3) gives

$$dR_{+v}=dM\left[1\mp\frac{(K^2-1)^{1/2}}{K}\right], \tag{3.5a}$$

$$dR_{-v}=dM\left[1\pm\frac{(K^2-1)^{1/2}}{K}\right]. \tag{3.5b}$$

For particles with $\epsilon_0\geqslant 0$ coming from $R=\infty$ we have $K\geqslant 1$, so it is seen from (3.5) that both R_+ and R_-, and the corresponding horizon areas, will increase as more and more particles reach the horizon. This is in agreement with a theorem of Hawking,[9] who showed that, if certain weak energy conditions hold, when particles collide the area of the final event horizon will be larger than the sum of the areas of the event horizons of the individual particles.

Comparison of Eqs. (3.3), (3.4), and (3.5) shows that if $K=1$, i.e., if particles are "dropped" from $R=\infty$, $dR_+\geqslant dR_-$, so that although the two increasing horizon radii R_+ and R_- will get closer and closer as they approach the value $R_\pm=M$, they never actually coalesce. In order to get the two horizons to coalesce, it is necessary to "fire" particles from $R=\infty$, i.e., have $K>1$, in which case it is possible to make $dR_->dR_+$ so that the inner horizon can "overtake" the outer horizon.

From Eq. (3.3) it is seen that for positive mass test particles, $dM>0$, R_+ will always increase as test particles reach the source. The value of R_-, however, can either increase or decrease depending on the sign and size of ϵ_0; this is in accord with the discussion relating to Fig. 2 showing that it is possible to make R_- decrease to zero.

In the above, we have considered only test particles with positive mass. From Fig. 3 or Eq. (3.3) it is seen that nonphysical negative mass test particles $dM<0$, can be used to destroy the outer horizon, R_+, and thereby produce a naked singularity. Production of naked singularities in spaces without horizons has been demonstrated by Boulware[6] using collapsing charged thin shells with negative mass density, but having a positive gravitational mass as seen by an observer at infinity.

IV. CONCLUSIONS

The motion of charged test particles in a Reissner-Nordström field has been investigated. It was found that the attractive or repulsive nature of the force exerted by the field on a test particle varies with the radius of the test particle. As a result, under certain conditions, radial test-particle oscillations are possible, a result with no Newtonian analog. This nonclassical behavior was shown to result because the motion of a charged test particle is influenced by the effective mass of the source's electric field energy, which varies with radius. The effects on the horizons of the Reissner-Nordström field from sending charged test particles into the source are also investigated. For radially moving test particles it was found that the injection of suitably charged test particles can produce horizons around a source where none previously existed. However, once horizons exist, it was shown that they cannot be destroyed by the injection of charged test particles.

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