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Hojman-Rosenbaum-Ryan-Shepley torsion theory and Eötvös-Dicke-Braginsky experiments

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Considering a modified form of local gauge invariance and minimal coupling, Hojman, Rosenbaum, Ryan, and Shepley obtained a dynamic torsion theory which allows propagation of torsion *in vacuo*. In this theory, the torsion is determined by the gradient of a scalar field ϕ. For the Sun, $\phi = 0.67\times 10^{-4}U$ where U is the Newtonian potential. In this field, test bodies with different electromagnetic energy contents behave differently. For aluminum and gold (or platinum), the gravitational accelerations would differ by $2\times 10^{-7}\vec{\nabla}U$. This implication disagrees with the null experiments of precisions $10^{-11}\vec{\nabla}U$ and $10^{-12}\vec{\nabla}U$ performed respectively by Roll, Krotkov, and Dicke and by Braginsky and Panov.

I. INTRODUCTION

In order to incorporate spin into gravitation theory, torsion arises naturally. Recently there has been considerable interest in finding possible interactions with torsion. For reviews, please see Hehl *et al.*,[1] and Ne'eman and Regge.[2]

Not satisfied with the nonpropagating character of the torsion in Einstein-Cartan-Sciama-Kibble (ECSK) theory of gravity, Hojman, Rosenbaum, Ryan, and Shepley (HRRS)[3] have proposed a propagating torsion theory. In this theory, they proposed a direct coupling between torsion and electromagnetic field. In the present paper, we look into some empirical implications of HRRS theory and demonstrate that it is in disagreement with experiments.

In Sec. II we review and discuss HRRS theory. Section III obtains the scalar field and torsion field of the Sun. Section IV calculates the test-body accelerations and shows that HRRS theory disagrees with Eötvös-Dicke-Braginsky experiments.

II. HOJMAN-ROSENBAUM-RYAN-SHEPLEY THEORY

To begin with, we review Hojman-Rosenbaum-Ryan-Shepley (HRRS) theory briefly. For easy reference, we adopt their notation and conventions. In general relativity one constructs a minimally coupled theory by letting the metric of special relativity $\eta_{\mu\nu}$ go to a general metric $g_{\mu\nu}$ and by replacing ordinary derivatives by covariant derivatives. For example, the usual definition of the electromagnetic field tensor $F_{\mu\nu}$ in general relativity is

$$F_{\mu\nu}=A_{\nu;\mu}-A_{\mu;\nu}\,, \tag{1}$$

where a semicolon signifies covariant differentiation involving the connection coefficients $\Gamma^{\alpha}{}_{\mu\nu}$

In the presence of nonsymmetric connection coefficients $\Gamma^{\alpha}{}_{\mu\nu}$, definition (1) is incompatible with the condition that the coupling of electromagnetism to torsion be invariant under the usual gauge transformation $A_\mu \to A'_\mu = A_\mu + \Lambda_{,\mu}$, where Λ is a scalar function. One solution is to define $F_{\mu\nu}$ as

$$F_{\mu\nu}=A_{\nu,\mu}-A_{\mu,\nu}=A_{\nu|\mu}-A_{\mu|\nu}\,, \tag{2}$$

where the bar symbol denotes a covariant derivative using the Christoffel symbols of the metric (ignoring torsion). This type of definition means that photons are decoupled from torsion.

By postulating the general principle that spinning particles both generate and react to torsion, HRRS argued that photons should also be coupled to torsion. Therefore, they proposed another solution by retaining the definition (1) but modifying the form of local gauge invariance in the presence of torsion. Starting with the minimal substitution and the gauge transformation of the form

$$\psi_{.\mu}=\psi_{,\mu}-iqb_\mu{}^\alpha A_\alpha\psi\,, \tag{3}$$

$$\psi\to\psi'=e^{iq\Lambda}\psi\,, \tag{4}$$

$$A_\mu\to A'_\mu=A_\mu+c_\mu{}^\alpha\Lambda_{,\alpha}\,, \tag{5}$$

with $b_\mu{}^\alpha$, Λ, and $c_\mu{}^\alpha$ functions of spacetime, they found that in order to have the minimal coupling be consistent with gauge invariance, the following relations must hold:

$$b_\mu{}^\sigma=e^{-\phi}\delta_\mu{}^\sigma\,, \tag{6}$$

$$c_\mu{}^\sigma=e^{\phi}\delta_\mu{}^\sigma\,, \tag{7}$$

$$T^\alpha{}_{\mu\nu}=\delta_\nu{}^\alpha\phi_{,\mu}-\delta_\mu{}^\alpha\phi_{,\nu}\,, \tag{8}$$

where ϕ is a scalar field and $T^\alpha{}_{\mu\nu}$ is the torsion tensor defined by

$$T^\alpha{}_{\mu\nu}=\Gamma^\alpha{}_{\nu\mu}-\Gamma^\alpha{}_{\mu\nu}\,. \tag{9}$$

With the above considerations and constraints, HRRS proposed the following Lagrangian density for their theory:

$$\mathcal{L} = \mathcal{L}'_\psi + \mathcal{L}_{g\phi} + \mathcal{L}_A , \tag{10}$$

with

$$\mathcal{L}'_\psi = -\frac{1}{4\pi}\,\psi^*_{.\mu}\psi^{.\,\mu}\sqrt{-g}$$

$$= -\frac{1}{4\pi}(\psi^*_{,\mu} + iqe^{-\phi}A_\mu\psi^*)(\psi^{,\,\mu} - iqe^{-\phi}A^\mu\psi)\sqrt{-g}\,, \tag{11}$$

$$\mathcal{L}_{g\phi} = \frac{1}{16\pi}\,R'\sqrt{-g}\,, \tag{12}$$

$$\mathcal{L}_A = -\frac{1}{16\pi}\,F_{\mu\nu}F^{\mu\nu}\sqrt{-g}\,. \tag{13}$$

$\mathcal{L}'_\psi$ is the gauge-invariant Lagrangian density for the complex scalar field. $\mathcal{L}_{g\phi}$ is the Lagrangian density for the gravitational field with torsion, where R' is the scalar curvature derived from the connection $\Gamma^\alpha{}_{\beta\gamma}$, which includes torsion. Expressed in terms of the scalar curvature R derived from the symmetric Christoffel connection $\{^\alpha{}_{\beta\gamma}\}$,

$$\mathcal{L}_{g\phi} = \mathcal{L}_g + \mathcal{L}_\phi + \text{total divergence}\,, \tag{14}$$

with

$$\mathcal{L}_g = \frac{1}{16\pi}\,R\sqrt{-g}\,, \tag{15}$$

$$\mathcal{L}_\phi = -\frac{3}{8\pi}\,\phi_{,\alpha}\phi^{,\,\alpha}\sqrt{-g}\,. \tag{16}$$

Combining (10)–(16), the total Lagrangian density can be expressed as

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi}\,(R - 6\phi_{,\alpha}\phi^{,\,\alpha} - F_{\mu\nu}F^{\mu\nu} - \psi^*_{.\mu}\psi^{.\,\mu})\,. \tag{17}$$

The definitions of $\psi_{.\mu}$, $F_{\mu\nu}$, $\Gamma^\alpha{}_{\beta\gamma}$ can be summarized as follows:

$$\psi_{.\mu} = \psi_{,\mu} - iqe^{-\phi}A_\mu\psi\,, \tag{18}$$

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} + A_\mu\phi_{,\nu} - A_\nu\phi_{,\mu}\,, \tag{19}$$

$$\Gamma^\alpha{}_{\beta\gamma} = \{^\alpha{}_{\beta\gamma}\} - \delta^\alpha{}_\gamma\phi_{,\beta} + g_{\beta\gamma}\phi^{,\,\alpha}\,, \tag{20}$$

where $\{^\alpha{}_{\beta\gamma}\}$ is the symmetric connection generated by the metric.

To generalize the Lagrangian density to include the spinor fields and other matter fields, we note that the following substitution rule is required and works for the HRRS scheme: Replace A_μ in the general-relativity counterparts by $e^{-\phi}A_\mu$. The torsion coupling to spinor fields can be neglected for our purposes.

The structure of HRRS theory is strong in the sense that this structure is dictated by the construction and no free parameter can be naturally introduced.

Using B_μ and $H_{\mu\nu}$ defined by HRRS, i.e.,

$$B_\mu = e^{-\phi}A_\mu\,, \tag{21}$$

$$H_{\mu\nu} = B_{\nu,\mu} - B_{\mu,\nu} = e^{-\phi}F_{\mu\nu}\,, \tag{22}$$

we can put the generalized Lagrangian density, which includes all matter fields, into the form

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi}(R - 6\phi_{,\alpha}\phi^{,\,\alpha} - e^{2\phi}H_{\mu\nu}H^{\mu\nu}) + \mathcal{L}'_{\text{gr}}(A_\mu \to B_\mu)\,, \tag{23}$$

where $\mathcal{L}'_{\text{gr}}(A_\mu \to B_\mu)$ is the Lagrangian density in general relativity for matter fields other than electromagnetic (em) field with A_μ replaced by B_μ.

Between $(A_\mu, F_{\mu\nu})$ and $(B_\mu, H_{\mu\nu})$ in this theory as to which one should be interpreted as electromagnetic field, we mention two points: (i) In the absence of the scalar field ϕ, they agree, and (ii) according to the usual methods of measurement, we can see clearly from the substitution rule $A_\mu \to B_\mu$ in $\mathcal{L}'_{\text{gr}}$ of (23) that $(B_\mu, H_{\mu\nu})$ is the quantity measured. With this interpretation, the torsion in the original consideration only plays an intermediate role of arriving at the Lagrangian. Our conclusion in Sec. IV for test bodies does not depend on which interpretation we choose, e.g., which stress-energy tensor we choose (see Sec. IV).

III. SCALAR FIELD AND TORSION FIELD DUE TO THE SUN

From the variation of ϕ in $\mathcal{L}$ in (23), we obtain the field equation for ϕ:

$$\phi^{|\alpha}{}_\alpha - \tfrac{1}{6}e^{2\phi}H^{\mu\nu}H_{\mu\nu} = 0\,, \tag{24}$$

where the bar symbol denotes covariant derivatives using $\{^\alpha{}_{\beta\gamma}\}$. Using (22), Eq. (24) can be written in HRRS form derived from (17):

$$\phi^{|\alpha}{}_\alpha - \tfrac{1}{6}F^{\mu\nu}F_{\mu\nu} = 0\,. \tag{25}$$

HRRS theory has Newtonian limits for $g_{\mu\nu}$. Hence in the solar system, we have the following solution for $g_{\mu\nu}$:

$$g_{00} = -1 + 2U + O(U^2)\,, \tag{26}$$

$$g_{0i} = O(U^{3/2})\,, \tag{27}$$

$$g_{ij} = 1 + O(U)\,, \tag{28}$$

where U is the Newtonian gravitational potential and where $O(x)$ means of order x. Here we use Latin indices as spatial indices.

For the scalar field of the Sun, Eq. (24) or (25) reduces to

$$\nabla^2\phi = \tfrac{1}{3}(\vec{B}^2 - \vec{E}^2) \tag{29}$$

to first order in ϕ, where $\vec{B} = (H_{23}, H_{31}, H_{12})$ and $\vec{E} = (H_{10}, H_{20}, H_{30})$, and where we have absorbed the constant factor e^{ϕ_0} of the background field into the normalization of units [see discussions following (30)]. If, instead, $F_{\mu\nu}$ is used in defining $\vec{B}$ and $\vec{E}$, Eq. (29) remains the same except that no change of units is needed. To a good accuracy, the solar field is spherical and the solution of (29) outside the Sun is

$$\phi = \phi_0 - \frac{1}{12\pi}\frac{1}{r}\int_{\text{Sun}}(\vec{B}^2 - \vec{E}^2)d^3x$$
$$= \phi_0 - \frac{2}{3}\frac{1}{r}(\mathcal{E}_{\text{magnetic}} - \mathcal{E}_{\text{electric}}), \tag{30}$$

where $\mathcal{E}_{\text{magnetic}}$ and $\mathcal{E}_{\text{electric}}$ are respectively the total megnetic and electric energy of the Sun. In (30), ϕ_0 is the background field due to the Galaxy and the Universe. The variation of this background field is small compared with the variation of ϕ due to the Sun. Hence ϕ_0 can be treated as a constant in the solar system, and only appears as a constant factor $e^{-\phi_0}$ between the relation of B_μ and A_μ. Therefore, ϕ_0 can be set equal to zero by appropriate normalization of units.

For the Sun, the macroscopic and atomic electromagnetic energy content is negligible as compared to its nuclear electromagnetic energy content. The nuclear magnetic energy is small compared to the nuclear electric energy. So

$$\phi = \frac{2}{3}\frac{M}{r}\frac{\mathcal{E}_{\text{ne}}}{M} = \frac{2}{3}\frac{\mathcal{E}_{\text{ne}}}{M}U, \tag{31}$$

where $\mathcal{E}_{\text{ne}}$ is the total nuclear electric energy. The composition of outer layers of the Sun is,[4] by fractional mass,

$$X(\text{H}) = 0.71,$$
$$Y(\text{He}) = 0.265, \tag{32}$$
$$Z \text{ (other elements)} = 0.025.$$

The Sun has converted all its deuterium to ^{3}He. The isotopic ratio ^{3}He/^{4}He, as derived from the solar wind value by Apollo and Surveyer data,[5] is 4×10^{-4}. From the Weizsäcker semiempirical mass formula, the Coulomb energy of a nucleus is $a_c Z(Z-1)/A^{1/3}$ where $a_c (= 3e^3/5r_0)$ can be determined from high-energy electron-scattering experiments to have the value 0.807 MeV.[6] Using this formula, the Coulomb energy of ^{4}He is 1.02 MeV, 2.7×10^{-4} of its rest mass. Assuming that the total solar composition is similar to that of outer layers and that the average fraction of Coulomb energy for heavier elements in the composition is roughly that of ^{16}O (i.e., 1.2×10^{-3}), we arrive at the following value for the ratio $\mathcal{E}_{\text{ne}}/M$:

$$\frac{\mathcal{E}_{\text{ne}}}{M} = 1.0\times10^{-4}. \tag{33}$$

If the solar interior has more ^{4}He than average, then the $\mathcal{E}_{\text{ne}}/M$ ratio should be higher. Combining (31) and (33), we find

$$\phi = 0.67\times10^{-4}U. \tag{34}$$

Using (20) and (34), the torsion field in the solar system is

$$T^\alpha{}_{\mu\nu} = 0.67\times10^{-4}(\delta_\nu{}^\alpha U_{,\mu} - \delta_\mu{}^\alpha U_{,\nu}). \tag{35}$$

IV. TEST-BODY ACCELERATIONS AND THE EÖTVÖS-DICKE-BRAGINSKY EXPERIMENTS

In this section we will follow the method introduced in a previous paper[7] to calculate the test-body accelerations. First we define stress-energy tensor density, four-momentum, and center of mass. According to the canonical Lagrangian formulation, the electromagnetic stress-energy tensor density can be defined as

$$\mathcal{T}_\mu{}^{\nu\,(\text{em})} = -B_{\lambda,\mu}(\partial\mathcal{L}_A/\partial B_{\lambda,\nu}) + \delta_\mu{}^\nu\mathcal{L}_A$$
$$= \frac{1}{4\pi}e^{2\phi}\left(\frac{\partial B_\lambda}{\partial x^\mu}H^{\nu\mu} - \tfrac{1}{4}\delta_\mu{}^\nu H_{\lambda\alpha}H^{\lambda\alpha}\right)\sqrt{-g}, \tag{36}$$

modulo a total divergence term. The total stress-energy tensor density is

$$\mathcal{T}_\mu{}^\nu = \mathcal{T}_\mu{}^{\nu\,(\text{em})} + \mathcal{T}_\mu{}^{\nu\,(\text{pf})}, \tag{37}$$

where $\mathcal{T}_\mu{}^{\nu\,(\text{pf})}$ is the usual stress-energy tensor density of other fields and particles. The four-momentum vector of a test body is

$$P_\mu = \int \mathcal{T}_\mu{}^0 d^3x. \tag{38}$$

Defining the center of mass as

$$X^\mu = \int x^\mu \mathcal{T}_0{}^0 d^3x/P_0, \tag{39}$$

one can readily show that

$$\dot{X}^\mu = P^\mu/P^0 \tag{40}$$

for a test body for which the second-order derivatives of ϕ can be neglected.

Note that the definition (36) of the electromagnetic stress-energy tensor is not symmetric. To symmetrize it together with $\mathcal{T}_\mu{}^{\nu\,(\text{pf})}$, we can add a total divergence to obtain

$$\mathcal{T}_\mu{}^{\nu\,(\text{em})} = \frac{1}{4\pi}e^{2\phi}(H_{\mu\lambda}H^{\nu\lambda} - \tfrac{1}{4}\delta_\mu{}^\nu H_{\lambda\alpha}H^{\lambda\alpha})\sqrt{-g}. \tag{41}$$

This will not affect the definition of P_μ in (38). Note also that if we use A_μ instead of B_μ to define $\mathcal{T}_\mu{}^{\nu\,(\text{em})}$ in (36), the difference is of order $\vec{\nabla}\phi$ (aside from a renormalization of units). Subsequently, the difference in test-body accelerations is of order $\phi_{,\mu\nu}$ which can be neglected. Therefore, alternations of the definitions of the stress-energy tensor will not affect our results.

From the Euler-Lagrange equations, we derive the matter-response equation

$$\mathcal{T}^{\nu}_{\mu,\nu} = \partial \mathcal{L}'/\partial x^{\mu}\,, \tag{42}$$

where $\mathcal{L}' = \mathcal{L}_A + \mathcal{L}'_{gr}(A_\mu \to B_\mu)$. From Eq. (42) we show that

$$\dot{P}_\mu = -\frac{1}{16\pi}(\sqrt{-g}\,e^{2\phi})_{,\mu}\int(2\vec{B}^2 - 2\vec{E}^2)d^3x + \tfrac{1}{2}g_{\lambda\delta,\mu}\int \mathcal{T}^{\lambda\delta\,(\mathrm{pf})}d^3x\,, \tag{43}$$

for a test body. Now choose a "local inertial frame" of $g_{\mu\nu}$ such that the test body is at rest at the moment considered and the Christoffel symbols vanish at the location of the body. Then (43) reduces to

$$\dot{P}_\mu = 2(\mathcal{E}_e - \mathcal{E}_m)\phi_{,\mu}\,, \tag{44}$$

where $\mathcal{E}_m$ and $\mathcal{E}_e$ are respectively the total magnetic and electric energy of the test body. From Eq. (40) we see in this local frame

$$\dot{P}_\mu = d(P^0\dot{X}_\mu)/dt = P^0\ddot{X} = m\ddot{X}_\mu\,, \tag{45}$$

where m is the mass of the test body. Combining (44) and (45), we obtain

$$\ddot{X}_\mu = 2\frac{\mathcal{E}_e - \mathcal{E}_m}{m}\,\phi_{,\mu}\,. \tag{46}$$

Therefore, in HRRS theory, test bodies of different electromagnetic content would accelerate differently in the solar field. For aluminum and gold (or platinum), the magnetic energy is small compared to electric energy. The $\mathcal{E}_e/M$ ratios for aluminum and gold (or platinum) are

$$(\mathcal{E}_e/M)_{\mathrm{Al}} = 1.7\times 10^{-3}\,, \tag{47}$$

$$(\mathcal{E}_e/M)_{\mathrm{Au\,(or)\,Pt}} = 4.5\times 10^{-3}\,. \tag{48}$$

Thus, on earth,

$$(\ddot{X}_i)_{\mathrm{Au\,(or)\,Pt}} - (\ddot{X}_i)_{\mathrm{Al}} = 2\left[\left(\frac{\mathcal{E}_e}{M}\right)_{\mathrm{Au\,(or)\,Pt}} - \left(\frac{\mathcal{E}_e}{M}\right)_{\mathrm{Al}}\right]\phi_{,i} = 2\times 10^{-7}U_{,i}\,, \tag{49}$$

where the Latin index i ranges from 1 to 3. But according to the precision experiments of Roll, Krotkov, and Dicke,[8] and Braginsky and Panov,[9] the accelerations of aluminum and gold or platinum do not differ respectively by 1 part in 10^{11} or 10^{12} of $U_{,i}$ in the solar gravitational field.

In the earth's gravitational field, $\phi \sim 10^{-3}U_\oplus$ ($U_\oplus$ = earth gravitational potential), and matter of different composition would fall differently by $10^{-6}g$ to $10^{-5}g$. This is in violation with the original Eötvös experiment and with the present-day geophysical measurements with accuracy $10^{-7}g$.

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