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CP-nonconserving effects in quantum chromodynamics

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The instanton-generated CP-violating perturbation is shown to have a form $3\bar{\theta}m_u m_d m_s/(m_u m_d + m_u m_s + m_d m_s)(\bar{\psi}i\gamma_5\psi)$ for $\bar{\theta} \ll 1$. The induced neutron electric dipole moment, evaluated in the framework of the MIT bag model, is $d_n = 8.2\times 10^{-16}\bar{\theta}\, e$ cm.

The structure of the quantum-chromodynamics Lagrangian $\mathcal{L}$ is completely fixed by general requirements of Lorentz invariance, local gauge invariance, renormalizability and spin content of fundamental fields[1]:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\theta - \mathcal{L}_M, \tag{1}$$

where

$$\mathcal{L}_0 = -\frac{1}{2g^2}\,\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + i\{\bar{\psi}_R(\partial\!\!\!/ + iA\!\!\!/)\psi_R + (R\to L)\}, \tag{2a}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu], \tag{2b}$$

$$\mathcal{L}_\theta = \theta S(x), \quad S = \frac{1}{32\pi^2}\,\mathrm{Tr}(F_{\mu\nu}F_{\rho\delta})\epsilon^{\mu\nu\rho\delta}, \tag{3}$$

$$\mathcal{L}_M = \bar{\psi}_R M\psi_L + \mathrm{H.c.} \tag{4}$$

The colored gluons and N flavored quarks have been represented by $A_\mu = A_\mu^i\Lambda^i$ $(2\,\mathrm{Tr}\Lambda^i\Lambda^j = 2\delta^{ij})$ and $\psi_{R,L} = \{\frac{1}{2}(1\pm\gamma_5)\psi^a,\ a=1,2,\ldots,N\}$, respectively. The local operator $S(x)$ in Eq. (3) determines the topological number $n=\int S(x)d^4x$, which is restricted to integer values and distinguishes topologically distinct gauge field configurations contributing to the Euclidean generating functional.[2] The existence of the nontrivial topological sectors $n\neq 0$ has profound implications.[3-5] In particular it implies an explicit breaking of $U_5(1)$ symmetry even in the zero-quark-mass limit $M=0$. This phenomena is described by the Adler-Bell-Jackiw anomaly[6]

$$\partial^\mu J_\mu^{50}(x) = NS(x) + 2i[\bar{\psi}_R(x)M\psi_L(x) - \mathrm{H.c.}], \tag{5}$$

where J_{50}^μ is the 9th axial component of Gell-Mann's currents $J_\mu^{5A} = \bar{\psi}\gamma_\mu\gamma_5\lambda^A\psi$, $J_\mu^A = \bar{\psi}\gamma_\mu\lambda^A\psi$ with $\mathrm{Tr}(\lambda^A\lambda^B) = \frac{1}{2}\delta^{AB}$, $A, B = 1,\ldots,N$ and $\lambda^0 = 1$.

The theory (1) without the quark-mass term $\mathcal{L}_M$ possesses the global symmetry $SU_R(N)\otimes SU_L(N)\otimes U(1)$ generated by the charges of above currents

$$Q_{R,L}^{A\neq 0} = \int (J_0^A \pm J_0^{5A})\,d^3x, \quad Q^{A=0} = \int J_0^{A=0}\, d^3x.$$

Evidently there exists an appropriate basis of quark fields:

$$\psi'_{R(L)} = U_{R(L)}\psi_{R(L)}, \quad U_{R(L)} \in SU_{R(L)}(N), \tag{6}$$

which reduces the mass matrix M to diagonal form with positive diagonal elements multiplied by an overall complex phase factor. Thus without loss of generality the M can be assumed to be

$$M_{ab} = m_a\delta_{ab}\exp(i\rho). \tag{7}$$

Note that the phase ρ cannot be rotated away. However, the theory (1) can be further simplified through the formal rotation $\psi\to\exp(-i\gamma_5\theta/N)\psi$ which takes into account the relation (5). Of course, the resulting theory is equivalent to the original one, and it is described by the Lagrangian [cf. Eqs. (1), (4), and (7)]

$$\tilde{\mathcal{L}} = \mathcal{L}_0 - \mathcal{L}_{\tilde{M}}, \tag{8}$$

with

$$\tilde{M}_{ab} = m_a\delta_{ab}\exp(i\tilde{\theta}), \quad \tilde{\theta} \equiv (\rho - 2\theta/N). \tag{9}$$

Clearly the theory (8) violates P and T if $\tilde{\theta}\neq 0$.

It should be emphasized that if the instantons, representing nontrivial topological sectors $(n\neq 0)$ of gauge fields, were absent one would be able to render the quark-mass matrix diagonal *as well as* real and arrive at a theory which *automatically* conserves flavor *and* P and T. These symmetries, with the exception of isospin, would be respected by unified weak and electromagnetic interactions up to order e^2.[7]

Some suggestions have been made as to how to maintain this simple picture in the presence of instantons: The hypothesis of an extended $U_5(1)$ invariance of unified strong and weak Lagrangians entails the existence of a light Higgs boson—the axion. An alternative suggestion insists on a vanishing up-quark mass $m_u = 0$. Unfortunately both suggestions appear to be untenable on experimental grounds.[8]

In such an *impasse* one may attempt to *impose* strong CP conservation on the theory (1) by choosing $\tilde{\theta}=0$. However, this choice is unstable with respect to renormalization effects induced by the CP breaking of weak interactions. Depending on the soft or hard nature of the CP breaking the angles θ and ρ acquire finite or infinite shifts from their bare values.[9] Therefore, as the renormalization of $\tilde{\theta}$ is infinite one cannot avoid the presence in the theory of an additional adjustable parameter $\tilde{\theta}$. Whereas in the case of a finite renormalization one should be content if the calculated $\tilde{\theta}$ is consistent with an observed magnitude of CP violation.

Now I shall identify the actual mechanism of the strong CP breaking and construct the corresponding effective interaction $\delta\mathcal{L}(CP)$ amenable to perturbative treatment. I shall restrict myself to $N \leqslant 3$ flavors (u, d, s) ignoring effects due to heavy quarks $(c, b, \ldots)$.

The peculiar nature of the problem may be recognized by the following observation: CP-violating effects should disappear $\delta\mathcal{L}(CP) \to 0$ as the instanton angle $\tilde{\theta}$ or at least one of the quark masses, e.g., m_u, vanishes. This is easy to ascertain by an appropriate unitary transformation (6) reducing $\mathcal{L}_{\tilde{M}}$ to

$$\mathcal{L}_{\tilde{M}} \to \mathcal{L}'_{\tilde{M}} = (e^{3i\tilde{\theta}})m_u \bar{u}_R u_L + m_d \bar{d}_R d_L + m_s \bar{s}_R s_L + \text{H.c.} \tag{10}$$

This has the stated property that $\tilde{\theta}$ drops out as $m_u \to 0$. Hence $\delta\mathcal{L}(CP)$ is not just the CP-violating piece of $\mathcal{L}_{\tilde{M}}$ or $\mathcal{L}'_{\tilde{M}}$, e.g., $\delta\mathcal{L}(CP) \neq \mathcal{L}_{\tilde{M}}(CP) = -\sin\tilde{\theta}\, m_a(\bar{\psi}_a i\gamma_5\psi_a)$.

The approach described below is an adaptation of the Dashen's general theorem on the chiral perturbation theory.[10]

To proceed I will specify the nature of the chiral $SU_R(3)\otimes SU_L(3)$ symmetry. I will adopt the standard point of view that the axial charges $Q_R^A - Q_L^A$, $A = 1 \cdots 8$ are spontaneously broken. In this case the vacuum is infinitely degenerate and the naive perturbation theory is unreliable. To determine $\delta\mathcal{L}(CP)$, I have to select one vacuum out of the infinite set. Of course the physical content of the theory will be independent of this particular choice.

I shall consider only leading-order effects in the chiral-symmetry-breaking term $\mathcal{L}_{\tilde{M}}$. Therefore the vacuum may be considered to be the flavor-symmetric state $|\Omega\rangle$ fixed by

$$\langle\Omega|\bar{\psi}_R^a\psi_L^b|\Omega\rangle = -\Delta\delta_{ab}, \quad a, b = 1, 2, 3 . \tag{11}$$

Here $\Delta = |\Delta| e^{i\delta}$ is, in general, a complex quantity. However, in the case of CP-symmetric vacuum, the Δ is real and $\delta = 0$ or π.

The correct perturbation $\delta\mathcal{L}$ should be selected from unitary-equivalent quark-mass terms [cf. Eq. (6)],

$$\mathcal{L}_{\tilde{M}}(U_{R,L}) \equiv \bar{\psi}_R U_R^\dagger \tilde{M} U_L \psi_L + \text{H.c.}, \quad U_{R,L} \in SU_{R,L}(3) . \tag{12}$$

The $\delta\mathcal{L}$ should satisfy an obvious condition, namely it should not cause instability of the vacuum.[11] Hence $\delta\mathcal{L}$ can induce only a minimum possible shift of the vacuum energy, i.e.,

$$\langle\Omega|\delta\mathcal{L}|\Omega\rangle = \underset{U_{R,L} \in SU_{R,L}(3)}{\text{Min}} \langle\Omega|\mathcal{L}_{\tilde{M}}(U_{R,L})|\Omega\rangle . \tag{13}$$

It is important to emphasize that matrices $U_{R,L}$ are elements of a *special* unitary group $\det[U_{R,L}] = 1$.

The statement (13) is the content of Dashen's elegant theorem alluded to above [See also Ref. 12)]. Since the vacuum is flavor symmetric it is sufficient to consider pure chiral transformations $U_R^\dagger = U_L$ and assume that $\delta\mathcal{L}$ in Eq. (13) is flavor diagonal. Then it is easy to show that[13]

$$\delta\mathcal{L} = \bar{\psi}_R^a(\mu_a + i\omega)\psi_L^a e^{-i\delta} + \text{H.c.}, \tag{14}$$

where μ_a and ω are assumed to be *real* parameters fixed by

$$m_a{}^2 = \mu_a{}^2 + \omega^2 , \tag{15a}$$

$$\bar{\theta} \equiv \tilde{\theta} + \delta = \tfrac{1}{3}\arg\prod_a(\mu_a + i\omega) , \tag{15b}$$

$$\mu_a > 0 \text{ or } \sum_a \mu_a{}^{-1} < 0 \text{ for } \mu_1 < 0,\ \mu_{2,3} > 0, \text{ and cycl. perm.} \tag{15c}$$

According to Eq. (14) the correct form of the CP-violating perturbation is $\delta\mathcal{L}(CP) = \mp\, \omega\bar{\psi} i\gamma_5\psi$ which accounts for the entire CP violation as $\delta = 0, \pi$. A physically interesting limit is given by $|\bar{\theta}| \ll 1$ in which case one obtains[14]

$$\delta\mathcal{L}(CP) = \mp \frac{3m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s}\, \bar{\theta}(\bar{\psi} i\gamma_5\psi), \tag{16}$$

$$|\bar{\theta}| \ll 1 \quad \delta = 0, \pi.$$

A priori the case $|\pi \pm \bar{\theta}| \ll 1$ may also be of physical interest although it has some peculiar features; in contradistinction to the case $|\bar{\theta}| \ll 1$ the condition $|\pi \pm \bar{\theta}| \ll 1$ requires the spectrum of Goldstone bosons to be nonadditive in quark masses $m_a \approx |\mu_a|$.[15]

Now I will apply Eq. (16) to evaluate the neutron electric dipole moment

$$\vec{d}_n = \langle n|\vec{d}|n\rangle = \int d^3x\, \langle n|\vec{x}Q(\vec{x})|n\rangle , \tag{17}$$

where $Q(x) = \bar{\psi}\gamma_0 Q\psi$, and Q is the quark electric charge matrix. Evidently the P- and CP-violating interaction $\delta\mathcal{L}(CP)$ induces an asymmetry in the charge distribution inside the neutron. This means that the physical neutron n along with pure $\frac{1}{2}^+$ state $|N\rangle$ contains a small admixture of a state of opposite parity $|N\rangle$, i.e.,

$$|n\rangle = |N\rangle + \sum_{*} (M^* - M)^{-1} |N^*\rangle \langle N^* | \delta L(CP) | N\rangle ,$$
$$\delta L(CP) = \int \delta\mathcal{L}_{CP}(\vec{x})\, d^3x \ , \tag{18}$$

where M and M^* are masses of $\frac{1}{2}^+$ and $\frac{1}{2}^-$ states, respectively. Thus Eq. (17) becomes

$$\vec{d}_n = 2 \sum_{*} (M^* - M)^{-1} \mathrm{Re}\langle N | \vec{d} | N^*\rangle \langle N^* | \delta L(CP) | N\rangle , \tag{19}$$

which is reminiscent of the electroproduction amplitude of a ninth pseudoscalar meson $\eta' \sim \bar{\psi}\gamma_5\psi$ at threshold. The expression (19) is expected to be saturated by low-lying resonances $N^*(\frac{1}{2}^-) = S'_{11}, S''_{11}$. Hence the problem is reduced to the evaluation of one-particle matrix elements $\langle N|\vec{d}|S_{11}\rangle$ and $\langle N|\delta L|S_{11}\rangle$.

Calculations can be carried out in the framework of the MIT bag model which is adequate for our purposes. Indeed $\vec{d}$ as well as δL are soft operators which do not couple to Goldstone bosons. Hence the matrix elements in question are expected to be insensitive to the chiral-symmetry aspects inadequately incorporated in the bag model. On the other hand, the results of a phenomenological analysis of a photoexcitation of nucleon resonances have been, in general, encouraging.[16] Predictions of transition rates have been consistent with experiment within a factor of two or three. The discrepancy may be largely attributed to the neglect of nucleon-recoil effects. Fortunately, these effects are absent in the above matrix elements.

In the bag model low-lying negative-parity states are a mixture of three-quark configurations $(1S_{1/2})^2(1P_{1/2})$ and $(1S_{1/2})^2(1P_{3/2})$ described by the representation $(70, 1^-)$ of $SU(6)\otimes O(3)$. The remaining $(56, 1^-)$ states should be ignored since they are an artifact of the center-of-mass motion of the entire bag.[17]

The bag model will be employed in its most naive version [cf. Ref. (16)]; the bag will be assumed to be static with spherically symmetric boundaries. The quadratic boundary condition will be ignored. The interaction of quarks will be also ignored.

The Hamiltonian of the bag should be diagonalized in the subspace $(70, 1^-)$. Hence j-j quark wave functions provide a convenient basis for state vectors.[18,16] Straightforward calculations yield

$$d_n = 32.7\times 10^{-3} e \frac{3 m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s} R^2 \bar{\theta} , \tag{20}$$

where the bag radius $R \approx (140\ \mathrm{MeV})^{-1}$. Standard estimates for quark mass ratios[19] $m_d/m_u = 1.8$, $m_s/m_d = 20$, and the bag value $m_s \approx 300$ MeV imply

$$d_n = 8.2\times 10^{-16}\bar{\theta}\ e\,\mathrm{cm}. \tag{21}$$

Thus the present experimental upper bound $d_n < 10^{-24}$ e cm requires

$$\bar{\theta} < 10^{-9}. \tag{22}$$

What are the effects of weak and electromagnetic interactions on our results (16) and (20)? These are determined by a short-distance expansion of a scalar- and vector-current correlation function which renormalizes operators in Eq. (2) and induces terms of higher dimension $(\bar{\psi}\psi\bar{\psi}\psi)$ into the effective Lagrangian $\delta\mathcal{L}$. The latter, however, are suppressed by a factor (m_N/m_W).[3] Hence, the effect of weak and electromagnetic interactions is essentially reduced to the renormalization of m_a and $\bar{\theta}$ in Eqs. (16) and (20).

It is not difficult to see that the conclusion holds even if $\bar{\theta}$ *and* m_a arise entirely from weak and electromagnetic interactions. In particular *CP*-violating milliweak interactions generated by an exchange of Higgs bosons will induce $\bar{\theta} \sim (10^{-3} G_F) m_H^2 < 10^{-5}$ which is in a serious conflict with the bound in Eq. (22). One possible direction for a resolution of this problem would be to find other mechanisms for *CP* violation.[20]

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[11]Recall that the magnetic field no matter how small it is, when applied to a ferromagnet, cannot be treated perturbatively unless it is paralled to the magnetization. The ground state of a ferromagnet is unstable with respect to external magnetic fields aligned in other directions. For a lucid discussion, see Ref. 10.

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[13]The extremal problem (13) was solved earlier by Dashen (Ref. 10) for the special case $m_1 = m_2$, $\delta = 0(\pi)$, and $\tilde{\theta} = 0$. Later Dashen's solution was generalized by J. Nuyts, Phys. Rev. Lett. 26, 1604 (1971). Nuyts showed that the most general form of the quark-mass term is given by Eq. (14), however, he did not derive subsidiary relations in Eq. (15): Note that Eqs. (15c) represent necessary and sufficient conditions for the extremal solution (14) to be a local minimum. The elegant parametrization of the results given in Eqs. (14) and (15) was kindly suggested to me by S. Coleman.

[14]A result similar to Eq. (16) has been also obtained by M. Peskin within the dilute-instanton-gas approximation (private communication).

[15]For $|\pi \pm \bar{\theta}| \ll 1$ the extremal problem (13) also has a solution, with $\omega^2 = \mu_1\mu_2 + \mu_1\mu_3 + \mu_2\mu_3$ which leads to too large *spontaneous CP* violation. In a special case this was first noted by Dashen (Ref. 10). The above expression for ω also was derived by M. A. B. Beg, Phys. Rev. D 4, 3810 (1971).

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