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Gluon corrections to the Drell-Yan model

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We calculate the $O(\alpha_s)$ corrections to the Drell-Yan model for $pp \to \mu^+\mu^- X$ in quantum chromodynamics. We find in agreement with previous work that the infrared and mass singularities can be consistently absorbed into the quark structure functions. The remaining finite terms are negative and rather small for the initial-gluon graphs but positive and about equal to Drell-Yan term for the final- and virtual-gluon graphs.

I. INTRODUCTION

The Drell-Yan model[1] is generally successful in describing the mass and longitudinal-momentum distributions[2] of high-mass lepton pairs produced in hadronic collisions. However, the model does not explain the rather large average transverse momentum observed experimentally.[3] Altarelli, Parisi, and Petronzio[4] and Fritzsch and Minkowski[5] have suggested that this transverse momentum could arise from higher-order processes in quantum chromodynamics (QCD) such as quark-gluon scattering and gluon emission. In this paper we consider the $O(\alpha_s)$ corrections in QCD to the integrated cross section $d\sigma/dQ^2$ for a lepton pair of mass Q^2.

Our calculation is based on a generalization of the parton model suggested by Politzer[6] and extended by several others.[7,8] We treat the quarks and gluons as if they were free particles, calculate their interactions using QCD perturbation theory, and fold the resulting cross sections with quark and gluon distribution functions for the incident hadrons. Outgoing quarks and gluons are assumed to fragment into jets of hadrons with unit probability. In this picture final states in which the gluons carry a finite energy and are not collinear with other constituents are clearly distinct. Processes involving soft or collinear gluons are not distinguishable, however, and it is precisely these which produced infrared and mass singularities[9] as the gluon mass λ and the quark mass m respectively approach zero. The infrared singularities are removed by the Bloch-Nordsieck mechanism,[10] but the mass singularities still remain, leading to powers of $\log Q^2/m^2$ which invalidate the use of perturbation theory.

Politzer's[6] essential observation is that the mass singularities for at least the lowest-order corrections to deep-inelastic scattering and to lepton pair production are simply related, so that these singularities can be consistently absorbed into universal, Q^2-dependent quark distributions. He conjectures that this can be done to all orders, so that the corrections to the Drell-Yan model can be calculated perturbatively. Since with this prescription the integrations over phase space converge as the gluons become collinear, the problem of double counting is resolved, at least asymptotically. Amati, Petronzio, and Veneziano[8] have shown that Politzer's result can be obtained from general theorems on mass singularities plus Ward identities. They have also argued that the result can be extended to other processes and to the leading logarithms in all orders of perturbation theory. However, the cancellation of all mass singularities in this way has not been proved.

If the conjectured concellation does occur to all orders, then the lepton pair cross section is given by the Drell-Yan term expressed in terms of the (nonscaling) quark distributions measured in deep-inelastic scattering plus additional finite correction terms. In this paper we calculate the part of these correction terms coming from the $O(\alpha_s)$ graphs for lepton pair production and for deep-inelastic scattering. A complete calculation would also require summing nonleading logarithms from higher-order graphs. For example, if the cancellation of nonleading logarithms in higher order occurred only if one chose different scales, say Q^2 and cQ^2, for their arguments, then our result would be modified by $\log c$ terms. However, the cancellation of the logarithms would seem to have simple physical interpretation only if the effect of the higher-order graphs is essentially to build up the running coupling constant $\alpha_s(Q^2)$ and the quark distributions $q(x, Q^2)$ with the same scale in both processes.

Assuming that this is so, we find that initial gluons, except for the scaling violations which they produce in the quark distributions, so not make a large contribution even though they carry 50% of the momentum of the proton. The contributions of final and virtual gluons must be considered together to remove the infrared singularities. We find that they give a correction term which is positive and about equal to Drell-Yan term up to quite high masses. The cross section contains an unexpec-

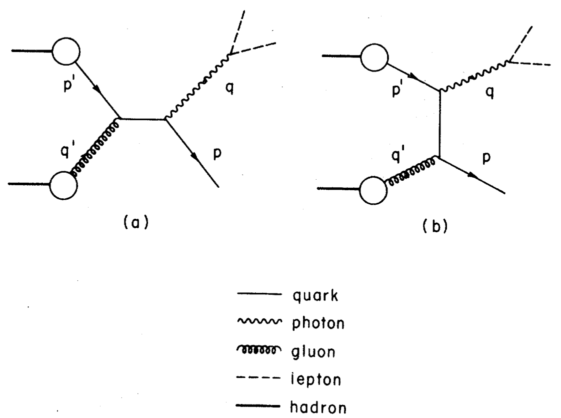

FIG. 1. Initial-gluon graphs for lepton pair production.

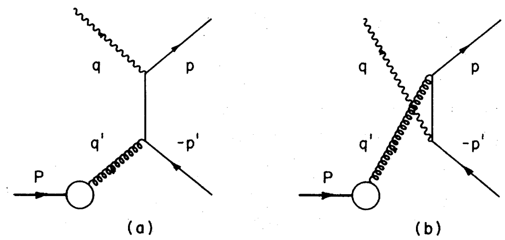

FIG. 2. Initial-gluon graphs for deep-inelastic scattering.

tedly large numerical factor which comes from integrating over the region $\hat{s} \simeq Q^2$, where $\hat{s}$ is the quark-antiquark invariant subenergy. Unfortunately, this is just the region in which there is a problem of double counting, so the interpretation of the result is unclear. The theroretically interesting conclusion that the Drell-Yan mechanism dominates at large enough Q^2 is of course unaffected. Another potentially important process is quark-quark scattering by gluon exchange with the emission of a lepton pair. We do not consider this process here because it is higher order in α_s, but it might be large because it can involve only valence quarks.

II. INITIAL GLUONS

In this section we consider the production of lepton pairs by the first-order graphs with initial gluons shown in Fig. 1 and the corresponding corrections to deep-inelastic scattering shown in Fig. 2.

We shall calculate these graphs treating the external quarks and gluons as being on the mass shell. Then we must give the quark a nonzero mass m; Politzer's result[6] implies that the limit $m \to 0$ exists for the final answer expressed in terms of the corrected structure functions. Since these graphs have no infrared divergence, the gluon mass can be set equal to zero.

The graphs in Fig. 1 are identical to those in an Abelian theory except for an overall factor of $\frac{1}{6}$ from the color average. The calculation is greatly simplified by using the conservation of the color and electromagnetic currents at the beginning. Then summation over the lepton spins and integration over their moments yields

$$\Pi_{\mu\nu}(q) = -\frac{e^2}{6\pi}\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right) \tag{2.1}$$

and the $q_\mu q$ term does not contribute, while the gluon spin average gives $-\frac{1}{2} g_{\mu\nu}$. The remaining spin sums and the integration over the quark-gluon center-of-mass scattering angle are straightforward. The result for the quark-gluon cross section is[11]

$$\frac{d\hat{\sigma}}{dQ^2} = \frac{\alpha^2 \alpha_s}{18} e_q{}^2 \frac{\hat{\tau}}{Q^4} \left\{ 2[(1-\hat{\tau})^2 + \hat{\tau}^2] \ln\left[\frac{Q^2}{m^2}\frac{(1-\hat{\tau})^2}{\hat{\tau}}\right] + (1+7\hat{\tau})(1-\hat{\tau}) \right\}, \tag{2.2}$$

where

$$\hat{s} = (p' + q')^2, \quad Q^2 = q^2, \quad \hat{\tau} = Q^2/\hat{s}, \tag{2.3}$$

e_q is the quark charge in units of e, $\alpha = e^2/4\pi$, and $\alpha_s = g^2/4\pi$. Terms which vanish as $m \to 0$ have been dropped.

We introduce bare quark, antiquark, and gluon distributions $q_0(x)$, $\bar{q}_0(x)$, and $g_0(x)$ for the incident hadrons.[12] These will be modified by interactions which produce both Q^2 dependence singularities as the quark mass approaches zero. Under the usual parton-model assumptions the quark-gluon cross section, Eq. (2.2), leads to a hadron-hadron cross section

$$\frac{d\sigma}{dQ^2} = \sum_q \int_0^1 dx_1 dx_2 \theta(x_1 x_2 s - Q^2)\{[q_0(x_1) + \bar{q}_0(x_1)]g_0(x_2) + (x_1 \leftrightarrow x_2)\}\frac{d\hat{\sigma}}{dQ^2}, \tag{2.4}$$

where $d\hat{\sigma}/dQ^2$ is evaluated at

$$\hat{s} = x_1 x_2 s \tag{2.5}$$

and the sum is over quark flavors. Note that $d\sigma/dQ^2$ contains the expected $\ln Q^2/m^2$ term.

The first-order corrections to deep-inelastic scattering involving initial gluons come from the graphs shown in Fig. 2. For these graphs we define a tensor

$$\hat{W}_{\mu\nu} = \frac{1}{2} \sum_{\text{states}} \langle g | J_\mu | q\bar{q} \rangle \langle q\bar{q} | J_\nu | g \rangle (2\pi)^4 \delta^4(p + p' - q - q') , \tag{2.6}$$

where the factor of $\frac{1}{2}$ comes from the two gluon helicities. A straightforward calculation yields

$$g^{\mu\nu} \hat{W}_{\mu\nu} = -2\alpha_s e_q^{\,2} \frac{\hat{s}^2 + Q^4}{(\hat{s} + Q^2)^2} \left(\ln \frac{\hat{s}}{m^2} - 1 \right) \tag{2.7}$$

and

$$q'^\mu q'^\nu \hat{W}_{\mu\nu} = \alpha_s e_q^{\,2} \hat{s} , \tag{2.8}$$

where now

$$\hat{s} = (q + q')^2 , \quad Q^2 = -q^2 > 0 . \tag{2.9}$$

Terms which vanish as $m \to 0$ have been dropped.

The corresponding tensor $W_{\mu\nu}$ for proton-photon scattering is a function of q and the proton momentum P. Its form is

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) W_1(x, Q^2) + \frac{1}{M^2} \left(P_\mu - q_\mu \frac{P \cdot q}{q^2} \right) \left(P_\nu - q_\nu \frac{P \cdot q}{q^2} \right) W_2(x, Q^2) \tag{2.10}$$

with

$$\nu = P \cdot q , \quad Q^2 = -q^2 , \quad M^2 = P^2 , \quad x = Q^2/2\nu . \tag{2.11}$$

From the usual parton-model prescription[15]

$$W_{\mu\nu} = \frac{1}{4\pi M} \int_x^1 \frac{dy}{y} g_0(y) \hat{W}_{\mu\nu} , \tag{2.12}$$

where

$$q'_\mu = y P_\mu , \quad \hat{s} = (y/x - 1) Q^2 . \tag{2.13}$$

Putting together Eqs. (2.7)–(2.13), we find that the initial-gluon graphs give

$$\frac{\nu}{M} W_2(x, Q^2) = \sum_q \frac{\alpha_s}{2\pi} e_q^{\,2} x \int_x^1 \frac{dy}{y} g_0(y) \left\{ \left[\left(1 - \frac{x}{y} \right)^2 + \left(\frac{x}{y} \right)^2 \right] \ln \left[\frac{Q^2}{m^2} \left(\frac{y}{x} - 1 \right) \right] - 1 + 8\frac{x}{y} - 8\frac{x^2}{y^2} \right\} . \tag{2.14}$$

We define the quark distribution functions in the proton by their relation to νW_2:

$$\frac{\nu}{M} W_2(x, Q^2) = \sum_q e_q^{\,2} x [q(x, Q^2) + \bar{q}(x, Q^2)] . \tag{2.15}$$

Clearly gluons contribute equally to the quark and antiquark distributions for all low-mass flavors. Hence

$$q(x, Q^2) = q_0(x) + \frac{\alpha_s}{4\pi} \int_x^1 \frac{dy}{y} g_0(y) \left\{ \left[\left(1 - \frac{x}{y} \right)^2 + \left(\frac{x}{y} \right)^2 \right] \ln \left[\frac{Q^2}{m^2} \left(\frac{y}{x} - 1 \right) \right] - 1 + 8\frac{x}{y} - 8\frac{x^2}{y^2} \right\} . \tag{2.16}$$

The Drell-Yan term for lepton pair production is

$$\frac{d\sigma}{dQ^2} = \sum_q \frac{4\pi\alpha^2}{9} e_q^{\,2} \frac{1}{Q^4} \int_0^1 dx_1 dx_2 \delta(x_1 x_2 - \tau) [q_0(x_1) \bar{q}_0(x_2) + \bar{q}_0(x_1) q_0(x_2)] , \tag{2.17}$$

where

$$\tau = Q^2/s . \tag{2.18}$$

We must reexpress this in terms of $q(x,Q^2)$ defined in Eq. (2.16) and add the contribution from quark-gluon scattering in Eq. (2.4). To $O(\alpha_s)$ the $\ln Q^2/m^2$ terms all cancel in agreement with Politzer's result.[6] The remaining parts can be written as

$$\frac{d\sigma}{dQ^2}=\frac{d\sigma^{(\mathrm{DY})}}{dQ^2}+\frac{d\sigma^{(i)}}{dQ^2}\,, \tag{2.19}$$

where $d\sigma^{(\mathrm{DY})}/dQ^2$ is given by Eq. (2.17) with $q_0(x)$ replaced by $q(x,Q^2)$, and

$$\frac{d\sigma^{(i)}}{dQ^2}=\sum_q \frac{\alpha^2\alpha_s}{9}\,e_q{}^2\frac{1}{Q^4}\int_0^1 dx_1dx_2\theta(x_1x_2-\tau)\{g(x_1,Q^2)[q(x_2,Q^2)+\bar{q}(x_2,Q^2)]+(x_1\leftrightarrow x_2)\}$$
$$\times\frac{\tau}{x_1x_2}\left\{\left[\left(1-\frac{\tau}{x_1x_2}\right)^2+\left(\frac{\tau}{x_1x_2}\right)^2\right]\ln\left(1-\frac{\tau}{x_1x_2}\right)+\frac{3}{2}-5\frac{\tau}{x_1x_2}+\frac{9}{2}\left(\frac{\tau}{x_1x_2}\right)^2\right\}. \tag{2.20}$$

This is the desired $O(\alpha_s)$ correction term from initial gluons.

It should be noted that we have chosen Q^2 as the momentum transfer at which to evaluate the structure functions and α_s. While this seems plausible, it is not necessary to make the $\ln Q^2/m^2$ terms cancel: any multiple of Q^2 would do. The correct scale is determined by the cancellation of non-leading logarithms in higher order.

It is instructive to consider the behavior of $d\sigma^{(i)}/dQ^2$ as $\tau\to 0$ for fixed Q^2. In this limit the integral is dominated by small x_1 and x_2. Suppose that for $x_1\to 0$ and $x_2\to 0$

$$\sum_q e_q{}^2\{g(x_1,Q^2)[q(x_2,Q^2)+\bar{q}(x_2,Q^2)]+(x_1\leftrightarrow x_2)\}\sim\frac{C}{x_1x_2}\,. \tag{2.21}$$

Then the integrations are elementary and yield

$$\frac{d\sigma^{(i)}}{dQ^2}\sim\frac{\alpha^2\alpha_s}{9}\frac{1}{Q^4}\frac{2C}{9}\ln\tau\,,\quad \tau\to 0\,. \tag{2.22}$$

It is not surprising that perturbation theory breaks down in this limit. Also, $d\sigma^{(i)}/dQ^2$ is evidently negative both for $\tau\to 0$ and for $\tau\to 1$, for which the logarithm in the integrand dominates. We shall see in Sec. IV that it is negative for all τ.

III. FINAL AND VIRTUAL GLUONS

The contributions from final and virtual gluons are separately infrared divergent and so must be considered together. The infrared singularities can be regulated in any convenient fashion. Since QCD is essentially Abelian to this order, we have chosen to give the gluons a nonzero mass λ and to renormalize on the quark mass shell. Then self-energy corrections to external lines need not be considered, and the infrared and mass singularities are clearly separated. We shall first let $\lambda\to 0$ and then $m\to 0$.

We first consider the final-gluon graphs for lepton pair production in Figs. 3(a) and 3(b). We have

$$\begin{aligned}
&\hat{s}=(p+p')^2\,,\quad Q^2=q^2\,,\\
&\hat{t}=m^2-2p_0q_0+Q^2+2|\vec{p}|\,|\vec{q}\,|\cos\theta\,,\\
&\hat{u}=m^2-2p_0'q_0+Q^2-2|\vec{p}\,'|\,|\vec{q}\,|\cos\theta\,,
\end{aligned} \tag{3.1}$$

where in the center of mass is the scattering angle and

$$\begin{aligned}
&p_0=p_0'=\tfrac{1}{2}(\hat{s})^{1/2}\,,\\
&|\vec{p}\,|=|\vec{p}\,|=\tfrac{1}{2}(\hat{s}-4m^2)^{1/2}\,,\\
&q_0=\frac{\hat{s}+Q^2-\lambda^2}{2(\hat{s})^{1/2}}\,,\quad q_0'=\frac{\hat{s}+\lambda^2-Q^2}{2(\hat{s})^{1/2}}\,,\\
&\vec{q}=\vec{q}\,'=\frac{[(\hat{s}-Q^2+\lambda^2)^2-4\hat{s}\lambda^2]^{1/2}}{2(\hat{s})^{1/2}}\,.
\end{aligned} \tag{3.2}$$

The color average produces a factor of $\frac{4}{9}$. Carry-

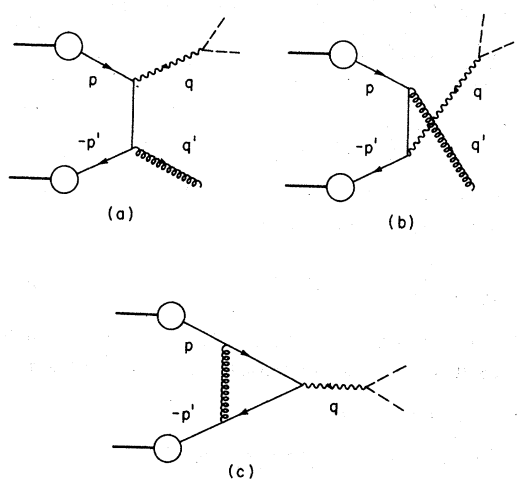

FIG. 3. Final- and virtual-gluon graphs for lepton pair production.

ing out the spin sums and the integrations over phase space, we find

$$\frac{d\hat{\sigma}}{dQ^2}=\frac{16\alpha^2\alpha_s}{27}e_q^{\,2}\frac{1}{\hat{s}^2Q^2}\left[\frac{\hat{s}^2+Q^4}{4p_0q_p-2Q^2}\ln\left(\frac{2p_0q_0-Q^2+2|\vec{p}\,||\vec{q}\,|}{2p_0q_0-Q^2-2|\vec{p}\,||\vec{q}\,|}\right)-(\hat{s}-Q^2)-\frac{2m^2Q^2}{2p_0q_0-Q^2-2|\vec{p}\,||\vec{q}\,|}\right] \tag{3.3}$$

up to terms which do not contribute as $\lambda\to 0$ and $m\to 0$. The infrared singularities arise from the region $\hat{s}\approx Q^2$. For $\hat{s}>(1+\epsilon)Q^2$, where ϵ is an arbitrarily small number, we can simply set $\lambda=0$. For $(Q+\lambda)^2<\hat{s}<(1+\epsilon)Q^2$ we can treat $\hat{s}$ as a constant when integrating over the quark distributions. In the limit $\lambda\to 0$ the $\hat{s}$ integral becomes simple:

$$\int_{(Q+\lambda)^2}^{(1+\epsilon)Q^2}d\hat{s}\frac{d\hat{\sigma}}{dQ^2}=\frac{32\alpha^2\alpha_s}{27}e_q^{\,2}\frac{1}{Q^2}\left[\ln\left(\frac{\epsilon Q}{2\lambda}\right)\ln\left(\frac{Q^2}{m^2}\right)-\ln\left(\frac{\epsilon m}{\lambda}\right)+f_1\left(\frac{Q^2}{m^2}\right)\right], \tag{3.4}$$

where

$$f_1(z)=\int_1^\infty\frac{dx}{x}\ln\left[\frac{[x+(x^2-1)^{1/2}]^2}{4(x^2-1)+z}\right]. \tag{3.5}$$

From the Appendix,

$$f_1(z)\sim-\tfrac{1}{4}\ln^2 z+\ln 2\ln z+c_1\,,\quad z\to\infty \tag{3.6}$$

where c_1 is a constant which we have not bothered to evaluate because it cancels out in the final answer.

We must add the interference term between the virtual-gluon graph shown in Fig. 3(c) and the lowest-order graph. The color average gives a factor of $\frac{4}{9}$. Subtracting the vertex function at $q^2=0$, we find for $\lambda\to 0$

$$\frac{d\hat{\sigma}}{dQ^2}=-\frac{16\alpha^2\alpha_s}{27}e_q^{\,2}\frac{1}{Q^2}\delta(\hat{s}-Q^2)\left[\ln\left(\frac{Q^2}{\lambda^2}\right)\ln\left(\frac{Q^2}{m^2}\right)-\ln\left(\frac{m^2}{\lambda^2}\right)-\tfrac{3}{2}\ln\left(\frac{Q^2}{m^2}\right)-\pi^2+2+2\,\mathrm{Re}f_2\left(-\frac{Q^2}{m^2}-i\epsilon\right)\right], \tag{3.7}$$

where

$$f_2(z)=\int_0^\infty\frac{dx}{(x^2+1)^{1/2}}\ln\left\{\frac{[x+(x^2+1)^{1/2}]^2}{4x^2+z}\right\}. \tag{3.8}$$

From the Appendix

$$f_2(z)\sim-\tfrac{1}{4}\ln^2 z-\frac{\pi^2}{12}\,,\quad z\to\infty. \tag{3.9}$$

Thus for $Q^2/m^2\to\infty$ the sum of the final- and virtual-gluon cross sections is

$$\begin{aligned}\frac{d\hat{\sigma}}{dQ^2}=&\frac{16\alpha^2\alpha_s}{27}e_q^{\,2}\frac{1}{\hat{s}^2Q^2}\theta(\hat{s}-(1+\epsilon)Q^2)\left[\left(\frac{\hat{s}^2+Q^4}{\hat{s}-Q^2}\right)\ln\left(\frac{\hat{s}}{m^2}\right)-\left(\frac{\hat{s}^2+Q^4}{\hat{s}-Q^2}\right)\right]\\&+\frac{16\alpha^2\alpha_s}{27}e_q^{\,2}\frac{1}{Q^2}\delta(\hat{s}-Q^2)\left[(2\ln\epsilon+\tfrac{3}{2})\ln\left(\frac{Q^2}{m^2}\right)-2\ln\epsilon+\frac{2\pi^2}{3}-2+2c_1\right],\end{aligned} \tag{3.10}$$

which as expected has no singularity as $\lambda\to 0$. The corresponding hadronic cross section is with the usual parton-model assumptions

$$\frac{d\sigma}{dQ^2}=\sum_q\int_0^1 dx_1dx_2\theta(x_1x_2s-Q^2)[q_0(x_1)\bar{q}_0(x_2)+(x_1\to x_2)]\frac{d\hat{\sigma}}{dQ^2}\,. \tag{3.11}$$

We must consider the corresponding corrections to deep-inelastic scattering, which we use to define the quark distribution functions. The variables for the final-gluon graphs shown in Figs. 4(a) and 4(b) are

$$\begin{aligned}&\hat{s}=(p+q)^2\,,\quad Q^2=-q^2\,,\\&\hat{t}=2m^2-2p_0p_0'+2|\vec{p}\,||\vec{p}\,'|\cos\theta\,,\\&\hat{u}=m^2+\lambda^2-2p_0q_0'-2|\vec{p}\,||\vec{q}\,'|\cos\theta\,,\end{aligned} \tag{3.12}$$

where in the center of mass θ is the scattering angle and

$$p_0=\frac{\hat{s}+m^2+Q^2}{2(\hat{s})^{1/2}}\,,\quad q_0=\frac{\hat{s}-Q^2-m^2}{2(\hat{s})^{1/2}}\,,\quad |\vec{p}\,|=|\vec{q}\,|=\frac{[(\hat{s}+Q^2-m^2)^2+4m^2Q^2]^{1/2}}{2(\hat{s})^{1/2}}\,,$$

$$p_0'=\frac{\hat{s}+m^2-\lambda^2}{2(\hat{s})^{1/2}}\,,\quad q_0'=\frac{\hat{s}+\lambda^2-m^2}{2(\hat{s})^{1/2}}\,,\quad |\vec{p}\,'|=|\vec{q}\,'|=\frac{[\hat{s}-m^2-\lambda^2-4m^2\lambda^2]^{1/2}}{2(\hat{s})^{1/2}}\,. \tag{3.13}$$

Defining a tensor $\hat{W}_{\mu\nu}$ for these graphs in analogy with Eq. (2.6), we find

$$g^{\mu\nu}\hat{W}_{\mu\nu}=\frac{8\alpha_s}{3}\,e_q^{\,2}\left\{-\left(\frac{\hat{s}}{\hat{s}+Q^2}+\frac{2Q^2}{\hat{s}-m^2}\right)\ln\left(\frac{2p_0q_0'-\lambda^2+2|\vec{p}\,||\vec{q}\,'|}{2p_0q_0'-\lambda^2-2|\vec{p}\,||\vec{q}\,'|}\right)+\frac{|\vec{p}\,'|}{(\hat{s})^{1/2}}\left[\frac{4Q^2-4p_0q_0'}{\hat{s}-m^2}+\frac{4m^2Q^2}{(\hat{s}-m^2)^2}+\frac{4m^2Q^2}{(2p_0q_0'-\lambda^2)^2-4|\vec{p}\,|^2|\vec{p}\,'|^2}\right]\right\} \tag{3.14}$$

and

$$p^\mu p^\nu\hat{W}_{\mu\nu}=\frac{2\alpha_s}{3}\,e_q^{\,2}(\hat{s}+Q^2) \tag{3.15}$$

up to terms which do not contribute as $\lambda\to 0$, $m\to 0$. In this case the infrared singularities arise from $\hat{s}\approx m^2$. For $\hat{s}>\epsilon Q^2$ we can set $\lambda=0$, while for $(m+\lambda)^2<\hat{s}<\epsilon Q^2$ we can ignore the variation of the quark distributions. Then as $\lambda\to 0$ we find

$$\int_{(m+\lambda)^2}^{\epsilon Q^2}d\hat{s}\,g^{\mu\nu}\hat{W}_{\mu\nu}=\frac{8\alpha_s}{3}\,e_q^{\,2}\left[2\ln\left(\frac{Q^2}{\lambda^2}\right)\ln\left(\frac{Q^2}{m^2}\right)-2\ln\left(\frac{m^2}{\lambda^2}\right)+\ln^2\left(\frac{Q^2}{m^2}\right)+(2\ln\epsilon-4\ln 2-\tfrac{3}{2})\ln\left(\frac{Q^2}{m^2}\right)+2f_1\left(\frac{Q^4}{m^4}\right)-\ln^2\epsilon-\tfrac{3}{2}\ln\epsilon-2\zeta(2)+2\right], \tag{3.16}$$

where $f_1(z)$ is the function defined in Eq. (3.5) and $\zeta(2)=\pi^2/6$ is a Riemann ζ function.

The virtual-gluon graph shown in Fig. 4(c) contributes only for $\hat{s}=m^2$. It gives for $\lambda\to 0$

$$g^{\mu\nu}\hat{W}_{\mu\nu}=-\frac{8\alpha_s}{3}\,e_q^{\,2}\delta(\hat{s}-m^2)Q^2\left[2\ln\left(\frac{Q^2}{\lambda^2}\right)\ln\left(\frac{Q^2}{m^2}\right)-2\ln\left(\frac{m^2}{\lambda^2}\right)-3\ln\left(\frac{Q^2}{m^2}\right)+4f_2\left(\frac{Q^2}{m^2}\right)+4\right], \tag{3.17}$$

where $f_2(z)$ is the function defined in Eq. (3.8), and

$$p^\mu p^\nu\hat{W}_{\mu\nu}=0\,. \tag{3.18}$$

The total coefficient of $\delta(\hat{s}-m^2)$ is obtained by adding Eq. (3.16) and Eq. (3.18), and in this sum the $\ln\lambda$ singularities from the infrared divergences all cancel. The $\ln^2(Q^2/m^2)$ terms also cancel because we have chosen the upper limit on $\hat{s}$ in Eq. (3.16) to be ϵQ^2. Defining the quark distribution by νW_2 as we did for initial gluons, we find for $Q^2/m^2\to\infty$

$$q(x,Q^2)=q_0(x)+\frac{2\alpha_s}{3\pi}\int_{(1+\epsilon)x}^{1}\frac{dy}{y}\,q_0(y)\left\{\left[\frac{x^2+y^2}{y(y-x)}\right]\ln\left[\frac{Q^2}{m^2}\frac{y^2}{x(y-x)}\right]+\frac{1}{2}\left(\frac{y-8x}{y-x}\right)+\frac{3x}{y}\right\}+\frac{2\alpha_s}{3\pi}\,q_0(x)\left[(2\ln\epsilon+\tfrac{3}{2})\ln\left(\frac{Q^2}{m^2}\right)-\ln^2\epsilon-\tfrac{7}{2}\ln\epsilon-\tfrac{5}{2}+2c_1\right] \tag{3.19}$$

where c_1 is the constant in Eq. (3.6).

According to Politzer's prescription, the complete correction term from final and virtual gluons is obtained by reexpressing the Drell-Yan term, Eq. (2.17), in terms of $q(x,Q^2)$ and adding the contribution from Eq. (3.10). The result can be written as

$$\frac{d\sigma}{dQ^2}=\frac{d\sigma^{(\mathrm{DY})}}{dQ^2}+\frac{d\sigma^{(f)}}{dQ^2}\,, \tag{3.20}$$

where $d\sigma^{(\mathrm{DY})}/dQ^2$ is again the Drell-Yan term with $q_0(x)$ replaced by $q(x,Q^2)$, and

$$\frac{d\sigma^{(f)}}{dQ^2}=\sum_q \frac{16\alpha^2\alpha_s}{27}\, e_q^{\,2}\frac{1}{Q^4}\int_0^1 dx_1dx_2\theta(x_1x_2-(1+\epsilon)\tau)[q(x_1,Q^2)\bar{q}(x_2,Q^2)+(x_1\leftrightarrow x_2)]$$
$$\times\frac{\tau}{x_1x_2}\left\{\frac{1}{x_1x_2}\left(\frac{x_1^{\,2}x_2^{\,2}+\tau^2}{x_1x_2-\tau}\right)\left[\ln\left(1-\frac{\tau}{x_1x_2}\right)-1\right]-\frac{1}{2}\left(\frac{x_1x_2-8\tau}{x_1x_2-\tau}\right)-\frac{3\tau}{x_1x_2}\right\}$$
$$+\sum_q \frac{16\alpha^2\alpha_s}{27}\, e_q^{\,2}\frac{1}{Q^4}\int_0^1 dx_1dx_2\delta(x_1x_2-\tau)[q(x_1,Q^2)\bar{q}(x_2,Q^2)+(x_1\leftrightarrow x_2)]\ \tau\left(\ln^2\epsilon+\tfrac{3}{2}\ln\epsilon+\frac{2\pi^2}{3}+\frac{1}{2}\right). \quad (3.21)$$

As for the initial-gluon case, we have assumed here that the appropriate momentum transfer for the structure functions and α_s is Q^2, and this assumption can be verified only by considering nonleading logarithms in higher order.

To remove the artificial parameter ϵ, we carry out an integration by parts[13] on the first integral in Eq. (3.21). Then the $\ln\epsilon$ terms all cancel, leaving

$$\frac{d\sigma^{(f)}}{dQ^2}=-\sum_q \frac{8\alpha^2\alpha_s}{27}\, e_q^{\,2}\frac{1}{Q^4}\int_0^1 dx_1dx_2\theta(x_1x_2-\tau)$$
$$\times\left\{\frac{\partial}{\partial x_1}[x_1q(x_1,Q^2)]\bar{q}(x_2,Q^2)+q(x_1,Q^2)\frac{\partial}{\partial x_2}[x_2\bar{q}(x_2,Q^2)]+(x_1\leftrightarrow x_2)\right\}$$
$$\times\left\{\ln^2\left(1-\frac{\tau}{x_1x_2}\right)+\left[\frac{3}{2}-\left(1-\frac{\tau}{x_1x_2}\right)-\frac{1}{2}\left(1-\frac{\tau^2}{x_1^{\,2}x_2^{\,2}}\right)\right]\ln\left(1-\frac{\tau}{x_1x_2}\right)\right.$$
$$\left.+\frac{3}{2}\frac{\tau}{x_1x_2}+\frac{3}{4}\frac{\tau^2}{x_1^{\,2}x_2^{\,2}}\right\}+\frac{4}{3}\frac{\alpha_s}{\pi}\left(\frac{2\pi^2}{3}-\frac{7}{4}\right)\frac{d\sigma^{(\mathrm{DY})}}{dQ^2}\,, \quad (3.22)$$

where the last term comes from integrals containing a $\delta(x_1x_2-\tau)$. This term involves a surprisingly large numerical factor,

$$\frac{2\pi^2}{3}-\frac{7}{4}\approx 4.83\,, \quad (3.23)$$

coming from the constant terms which remain after the logarithms have canceled. Because of this factor, there is a large contribution from $x_1x_2=\tau$, which unfortunately is just the region corresponding to soft gluons for which perturbation theory is not reliable.

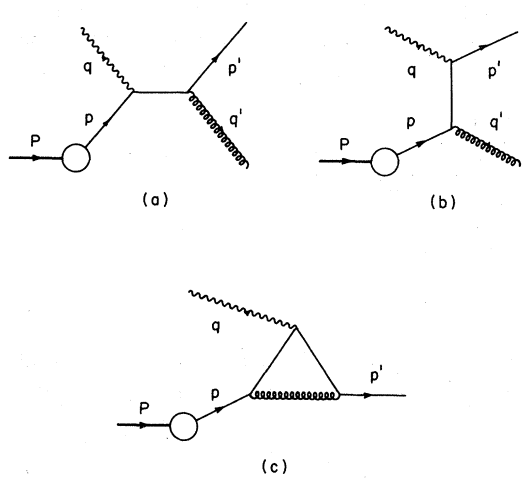

FIG. 4. Final- and virtual-gluon graphs for deep-inelastic scattering.

IV. NUMERICAL RESULTS

In the previous sections we have found that to $O(\alpha_s)$ the cross section for lepton pairs is given by the Drell-Yan term plus two correction terms, all of which depend on the quark and gluon distributions at the given Q^2. Scaling violations in these distributions have been studied by several authors.[14] The equations are somewhat complicated, and the scaling violations are rather small at moderate values of x. Since we are interested mainly in the size of the correction terms relative to the Drell-Yan term, we shall therefore greatly simplify the numerical calculation by ignoring the Q^2 dependence of the quark and gluon distributions.[15] This will of course distort the shape of the cross section as a function of Q^2, but the shape is also sensitive to the antiquark distribution, which is poorly known.

For the quark distributions we use the parameterization of Peierls, Trueman, and Wang,[16]

$$u(x)=1.79(1-x)^3(1+2.3x)/\sqrt{x}+s(x)\,,$$
$$d(x)=1.107(1-x)^{3.1}/\sqrt{x}+s(x)\,, \quad (4.1)$$
$$s(x)=\bar{u}(x)=\bar{d}(x)=\bar{s}(x)=0.15(1-x)^7/x\,.$$

Little is known about the gluon distribution except that it should satisfy the energy sum rule,

$$\sum_q\int_0^1 dx\,x[q(x)+\bar{q}(x)]+\int_0^1 dx\,xg(x)=1\,. \quad (4.2)$$

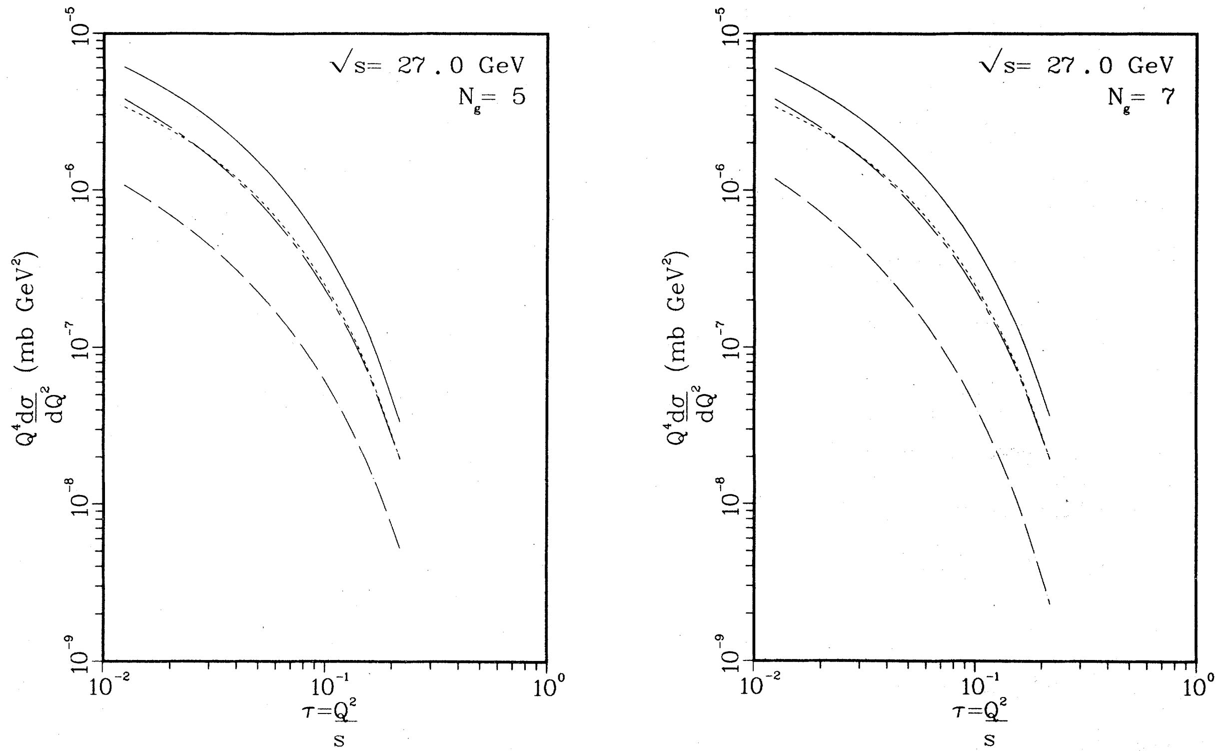

FIG. 5. Lepton pair production cross sections for $(s)^{1/2}=27$ GeV and $n_g=5,7$. Dashed curve (-----): Drell-Yan term. Chain-dot curve (– · – ·): Magnitude of negative initial-gluon term, Eq. (2.20). Chain-dash curve (– – – –): Final- and virtual-gluon term, Eq. (3.22). Solid curve (—): Sum of Drell-Yan term and $O(\alpha_s)$ corrections.

Intuitively, however, we expect it to be peaked at low x like the antiquark distribution. We therefore take

$$g(x)=0.483(n_g+1)(1-x)^{n}g/x\,, \tag{4.3}$$

where the coefficient is fixed by the energy sum rule. The Brodsky-Farrar[17] counting rules suggest $n_g=5$, but we have also tried other values. Finally, we use for α_s the running coupling constant for four quark flavors,[18]

$$\alpha_s(Q^2)=\frac{12\pi}{25}\,\frac{1}{\ln(Q^2/\mu^2)}\;; \tag{4.4}$$

analysis of scaling violations gives[19]

$$\mu=0.5\ \text{GeV}. \tag{4.5}$$

Using these distributions we have calculated numerically for $\sqrt{s}=27$ GeV the Drell-Yan term, the initial-gluon correction term from Eq. (2.20), and the final- and virtual-gluon correction term from Eq. (3.22). In Fig. 5 we show the results both for $n_g=5$, the canonical value, and for $n_g=7$, matching the sea distribution. For both values of n_g the correction term from initial gluons is negative and rather small, while that from final gluons is large. Since this large value results mainly from the large numerical factor in the last term of Eq. (3.22), the final-gluon term should have a similar relative magnitude in $\pi^{\pm}p\to l^+l^-X$ and $\bar{p}p\to l^+l^-X$. We have already remarked that this term may not be reliable because the main contribution comes from $x_1x_2=\tau$, which is the soft-gluon region. Nevertheless, it will be interesting to look for deviations from the Drell-Yan model as better data on the proton sea-quark distribution or on $\bar{p}p\to l^+l^-X$ become available.

Note added. After this work was completed, we received papers by R. K. Ellis *et al.*,[20] and by Libby and Sterman[21] arguing for the factorization of mass singularities to all orders. We also received papers by Abad and Humpert[22] and by Altarelli, Ellis, and Martinelli[23] calculating the finite parts of the $O(\alpha_s)$ graphs.

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APPENDIX

In this appendix we analyze the functions $f_1(z)$ and $f_2(z)$ introduced in Sec. III. Equation (3.5) defines $f_1(z)$ as an analytic function of z with a cut along the negative real axis. Since the constant term cancels in the final result, we need only calculate the logarithmic terms as $z\to\infty$. For any $z>4$ we have

$$f_1(z)=\int_1^\infty dx\frac{1}{x}\ln\left(\frac{4x^2}{4x^2+z-4}\right)+\int_1^\infty dx\frac{1}{x}\ln\left\{\frac{[x+(x^2-1)^{1/2}]^2}{4x^2}\right\}. \tag{A1}$$

Letting $x=x'(z-4)^{1/2}$ in the first integral gives

$$\begin{aligned}f_1(z)&=\int_{1/(z-4)^{1/2}}^1 dx'\frac{1}{x'}\ln\left(\frac{4x'^2}{4x'^2+1}\right)+\text{const}\\&=-\ln^2\left[\frac{1}{(z-4)^{1/2}}\right]-2\ln 2\ln\left[\frac{1}{(z-4)^{1/2}}\right]-\int_{1/(z-4)^{1/2}}^1 dx'\frac{1}{x'}\ln(1+4x'^2)+\text{const}.\end{aligned} \tag{A2}$$

As $z\to+\infty$ the integral in the last line has a finite limit, so we obtain Eq. (3.6).

Equation (3.8) likewise defines $f_2(z)$ as an analytic function of z with a cut along the negative real axis. We need its behavior including the constant term as $z\to\infty$. Letting

$$x'=x+(x^2+1)^{1/2},\quad \frac{dx'}{x'}\frac{dx}{(x^2+1)^{1/2}}, \tag{A3}$$

we obtain

$$f_2(z)=\int_1^\infty dx'\frac{1}{x'}\ln\left(\frac{x'^2}{x'^2+z-2}\right)-\int_1^\infty dx'\frac{1}{x'}\ln\left[1+\frac{1}{x'^2(x'^2+z-2)}\right]. \tag{A4}$$

The second integral vanishes as $z\to\infty$. Substituting $x'=x''(z-2)^{1/2}$ in the first integral gives

$$\begin{aligned}f_2(z)&\sim\int_{1/(z-2)^{1/2}}^1 dx''\frac{1}{x''}\ln\left(\frac{x''^2}{x''^2+1}\right)+\int_1^\infty dx''\frac{1}{x''}\ln\left(\frac{x''^2}{x''^2+1}\right)\\&=-\ln^2\left[\frac{1}{(z-2)^{1/2}}\right]-2\int_0^1 dx''\frac{1}{x''}\ln(1+x''^2)\\&\sim-\tfrac{1}{4}\ln^2 z-\tfrac{1}{2}\zeta(2),\end{aligned} \tag{A5}$$

where $\zeta(2)=\pi^2/6$ is a Riemann ζ function. This asymptotic formula is valid in the cut z plane.

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