Heavy quarks in a spherical bag

William A. Ponce
Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003
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We use the vector meson $\Upsilon(9460)$ as input to the fixed-sphere MIT bag model. Among our results are a mass of 4700 MeV for the b quark, a zero-point energy that is a function of the heaviest quark mass inside the bag, and predictions for masses of the ground-state b-flavored hadrons. Series expansions of the bag equations for heavy quarks are employed. Improvement of the spectrum of ground-state hadrons in the charm sector is achieved.

I. INTRODUCTION

The MIT bag model[1] is one of the most powerful tools of hadron spectroscopy. The model has been around for several years and phenomenological analyses of the equations have been performed in many previous papers and for many different situations. The main feature of the model is its very simple way of achieving quark and gluon confinement, but owing to the flexibility of the model it has been used even for quasi-confinement[2] [broken quantum chromodynamics (QCD)].

The greatest success of the theory lies in its description of ground-state hadrons formed with light quarks[3] [SU(3) of flavor], where agreement with the experimental results is remarkable. However, further applications of the model to excited states, heavy-quark states, or nonspectroscopic calculations[4-6] have been very limited, and the results are unsatisfactory at present. Therefore, the model must be implemented with new ideas. A deeper understanding of the basic postulates and of the original parameters, such as the bag pressure B and the zero-point-energy parameter Z_0 must be achieved.

At present, it appears that the bag has been derived from basic QCD postulates,[7] so we might expect in the near future more insight and, hopefully, numerical results for B and Z_0. A previous attempt to calculate Z_0 from first principles[8] was unsuccessful, and we interpret the inability to find a result as a failure in the renormalization of the spherical-bag model as it now stands.

In this paper we argue in favor of a mass-dependent Z_0 while keeping the parameter B constant. We employ the experimental mass value of the charmed D^* meson to fix the parameters in a series expansion of Z_0 in powers of the heaviest quark mass inside the spherical bag.

The remainder of the paper is as follows. In Sec. II we present the bag formalism and apply it to the recently discovered $\Upsilon(9460)$, and as a result we present the spectrum of hadrons containing the new flavor. In Sec. III we carry out series expansions of the spherical-bag equations in powers of the quark masses and present alternative equations valid for heavy quarks inside the bag. In Sec. IV we address the problem of the zero-point energy and present numerical results of Z_0 for different quarks. Our conclusions are presented in Sec. V.

II. b-QUARK SPECTROSCOPY

In this section we present the equations of the MIT bag model and solve them for hadrons containing one or more b quarks. A complete account of this model can be found in Ref. 3.

As we shall discuss, part of the input to the numerical analysis described in this section is not derived until Sec. IV. We beg forbearance on the part of the reader for this seeming bit of illogic, but we wish to introduce the bag equations at this point, and it seems only natural to report on the attendant numerical work at the same time.

The equations are as follows: a linear boundary condition (LBC) gives the possible frequencies ω of a quark of mass m inside a bag of radius R

$$\tan[(\omega^2 - X^2)^{1/2}] = \frac{\kappa(\omega^2 - X^2)^{1/2}}{\omega - \kappa X + \kappa}\ , \tag{1}$$

where $X = mR$. The solutions to Eq. (1) for $\kappa = -1$ are the $1S, 2S, 3S, \ldots$ eigenstates and for $\kappa = +1$ are the $1P, 2P, 3P, \ldots$ eigenestates for a quark inside the bag. Higher values of angular momentum are not allowed in a fixed spherical bag. Equation (1) must be solved once for each quark or antiquark in the bag. The energy equation gives the mass of a hadron of radius R as a sum of several terms,

$$M(R) = E_q + E_V + E_0 + E_M + E_E\ , \tag{2}$$

where $E_q = \sum_i \omega_i/R$ is the kinetic energy of the quarks inside the bag, $E_V = 4\pi B R^3/3$ is the energy due to the bag pressure B (taken here as a universal constant); E_M and E_E are the magnetic and electric energies respectively of color gluons inside the bag; and $E_0 = -Z_0/R$ is the zero-point energy (Z_0 is the zero-point parameter). The nonlinear boundary condition (NLBC) that we get by minimizing $M(R)$ with respect to R,

$$\partial M(R)/\partial R\,|_{R=R_0}=0\,, \tag{3}$$

is the equation that ensures the equilibrium for the fixed sphere and fixes the radius R_0 of the bag. The mass of the physical hadron associated with the bag of radius R_0 is $M(R_0)$.

The parameters we use as input to these equations are $B^{1/4}=145$ MeV, $\alpha_c=0.55$ (the quark-gluon coupling constant), $m_u=0$, $m_d=0$, $m_s=279$ MeV, $m_c=1486$ MeV, and $Z_0=0.62$. The first five numbers are suggested by the light-hadron spectroscopy as given in Ref. 3. The values of m_c and Z_0 are taken according to Sec. IV in this paper. We fix the mass of the b quark to $m_b=4700$ MeV by solving the equations with $M(R_0)=9460$ MeV, the mass of the recently discovered Υ particle. At this point there are no parameters left, and we are able to calculate the spectrum of b-flavored mesons and baryons. This spectrum is presented in Table I. We find it convenient to employ the notation of SU(4) for the flavor content of the hadrons, where the extra quantum number in going from SU(3) to SU(4) is the "heaviness." Both the b and the c quarks carry one unit of "heaviness," and there is no further symmetry between them. Therefore, in the fundamental representation of SU(4) we can have on top of the four-dimensional pyramid either the c quark or the b quark. For example, one of the implications of such an assumption is that we can have four different states to place on top of the 20-dimensional pyramid of $\frac{3}{2}^+$ baryons. They are bbb, bbc, bcc, ccc. We calculate the mass of the first three but not of the fourth, which belongs to the charm sector. We postpone further discussions of the values in Table I to the Conclusions, where we also present arguments for the value of $\alpha_c=0.55$ used in our calculations.

III. SERIES EXPANSIONS

The limit $X=mR\to\infty$ in some of the equations for the spherical bag has been studied previously by Golowich.[9] Some of the conclusions from this study are that it is mathematically possible to take such

TABLE I. Mass spectrum of hadrons composed of one or more b quarks and ordinary quarks in lowest cavity mode (1S). We list for each particle its composition, its mass, the contributions of the five energy terms to its mass, the inverse radius for the bag, and the parameter $X_b=m_bR_0$.

State		Mass (MeV)	E_q (MeV)	E_v (MeV)	E_0 (MeV)	E_M (MeV)	E_E (MeV)	R_0^{-1} (MeV)	X_b
1^-	$b\bar{b}$	9460	9587	61	−195	6	0	311.5	15.1
	$b\bar{c}$	6388	6433	105	−162	9	2	260.6	18.0
	$b\bar{s}$	5431	5376	156	−141	14	26	228.1	20.6
	$b\bar{u}$	5299	5211	162	−140	16	50	225.2	20.9
0^-	$b\bar{b}$	9435	9620	48	−211	−22	0	337.2	13.9
	$b\bar{c}$	6347	6470	83	−175	−34	3	282.0	16.7
	$b\bar{s}$	5372	5415	127	−151	−48	28	244.1	19.3
	$b\bar{u}$	5232	5245	135	−148	−53	53	239.2	19.6
$\frac{1}{2}^+$	bbs	10142	10123	160	−140	−26	25	226.1	20.8
	bbu	10003	9954	168	−138	−30	50	222.7	21.1
	bcs	6988	6964	183	−134	−44	19	216.3	21.7
	bcu	6842	6793	191	−132	−51	41	213.4	22.0
	bss	6022	5864	246	−121	13	20	196.0	24.0
	buu	5735	5527	257	−120	27	44	193.2	24.3
	$b(ud)_A$	5555	5549	238	−123	−154	45	198.1	23.7
	$b(us)_S$	5880	5696	251	−121	19	35	194.6	24.2
	$b(us)_A$	5736	5728	224	−125	−127	36	202.1	23.3
$\frac{3}{2}^+$	bbb	14248	14320	91	−169	6	0	273.2	17.2
	bbc	11152	11162	129	−150	9	2	242.9	19.4
	bbs	10181	10100	179	−135	14	24	218.0	21.6
	bbu	10048	9933	185	−134	14	49	215.6	21.8
	bcc	8039	7997	165	−139	15	2	224.1	21.0
	bcs	7054	6928	213	−128	23	18	205.7	22.9
	bcu	6919	6760	218	−126	27	40	204.0	22.9
	bss	6051	5848	261	−119	42	20	192.2	24.5
	$b(su)_S$	5912	5680	266	−118	50	34	191.0	24.6
	buu	5769	5512	271	−119	61	44	189.8	24.8

a limit in the equations and that heavy quarks behave as nonrelativistic objects inside the bag. This suggests an approach based on a series expansion in inverse powers of the variable X which should simplify the mathematics and allow for greater ease in interpreting the ensuing equations. As a result, we shall see that the standard spherical-bag approach must ultimately break down for quarks of increasing mass.

Let us postulate the following expansion for the frequency ω of a quark inside a spherical bag occupying the $1S$ state:

$$\omega = X + a + b/X + c/X^2 + d/X^3 + O(X^{-4}) + \cdots .$$

We expect such an equation to converge to a finite value for $X > 1$. Using now the LBC and its properties in the limit $X \to \infty$, we get for the coefficients $a, b, c, d, \ldots$ the following values:

$$\omega = X + \pi^2/2X - \pi^2/2X^2 + \pi^2(1-\pi^2)/8X^3 + O(X^{-4}) + \cdots . \tag{4}$$

Comparing the values given by Eq. (4) with the exact solutions for the J/ψ meson for which $X = 5.74$, we see that the first three terms in the expansion give an ω which is less than 1% different from the exact solution, and for the Υ, for which $X = 15.09$, the difference is less than 0.05%. Therefore, such an expansion is acceptable for the charm, good for the b quark, and it must be very good for any heavier quark. Equation (4) corresponds to the LBC (1) in the exact model, and it is valid for any heavy quark inside a hadron (meson or baryon).

The other bag equations depend upon the number of different flavors contained within the bag. The simplest case is for flavor-singlet mesons (formed from a quark-antiquark pair of the same flavor) and for single-flavor baryons (e.g., ccc, bbb, etc.). For such hadrons, the electric energy is zero. The following equations are valid for these cases only.

The NLBC that we get by minimizing the energy is

$$4\pi BR^4 = -Z_0 + N\pi^2/X + (14.59A - 14.80N)/X^2 - (43.77N + 24.77A)/X^3 + \cdots , \tag{5}$$

where $N = 2$ for mesons, 3 for baryons; $A = 1$ for 1^- mesons, -3 for 0^- mesons, and 1.5 for baryons. The energy equation is thus

$$M = \frac{1}{R}\left[NX - 4Z_0/3 + 5N\pi^2/6X + (9.72A - N\pi^2)/X^2 + \cdots\right] . \tag{6}$$

In getting the numerical coefficients in Eqs. (5) and (6) we use not only Eq. (4) but also the following series expansion for the magnetic energy E_M in inverse powers of X:

$$E_M = \frac{A\alpha_c}{R}\left[8.84/X^2 - 11.26/X^3 + O(X^{-4}) + \cdots\right] . \tag{7}$$

The expressions for the magnetic and electric energy in this paper are derived directly from the corresponding expressions in Ref. 3 by performing series expansions like the one in Eq. (4). Also, we use numbers as coefficients in all our equations instead of analytic expressions because otherwise the expressions become intractably long. The value $\alpha_c = 0.55$ for the quark-gluon coupling constant is also used throughout this paper.

The first thing we see in these equations is the negative leading term, $-Z_0$, on the right-hand side of Eq. (5) ($Z_0 > 0$). However, the left-hand side of Eq. (5), which fixes the radius for the spherical bag, must be non-negative. Therefore, for fixed Z_0, there is an inconsistency as X increases beyond a certain value. If we want to use the spherical-bag model for heavy quarks, we must modify the original equations. Let us re-emphasize that the series expansions in this section deal with flavor-singlet mesons and single-flavor baryons only. Other hadrons such as flavored mesons are addressed in Sec. IV.

IV. ZERO-POINT ENERGY

When we compare the solutions to the spherical bag in the charm sector found in the literature[10,11] with the experimental results, we can see that there is poor agreement between them. One immediately thinks part of the problem is that for heavy quarks the shape of the bag is nonspherical. Unfortunately, a nonspherical bag has not yet been solved[12] because of the mathematical difficulty of the problem. Therefore, it would be very useful to modify the spherical model in such a way that it could be used for heavy quarks.

At this point we want to postulate an X-dependent Z_0 such that $\lim_{X\to\infty} Z_0 = 0$. At first sight this seems a heretical suggestion. It means that the value of Z_0 for a given bag is determined by the masses of the quarks contained within, particularly of the heaviest quark. However, we feel that it is a reasonable compromise between maintaining the simple spherical-bag approach, yet allowing for a consistent set of equations for very massive quarks. Our feeling is strengthened by the enhanced agreement of the model with experiment, as we shall show.

Guided by these ideas, we propose the following series expansion for Z_0:

$$Z_0 = H/X + C/X^2 + E/X^3 + \cdots , \tag{8}$$

TABLE II. Masses of the c quark and of the 1^- meson D^* for different values of Z_0. The values for m_c are determined by placing the J/ψ meson at 3095 MeV.

Z_0	m_c (MeV)	M_{D^*} (MeV)
1.9	1562	1962
1.7	1530	1980
1.5	1500	1999
1.3	1471	2018

where the coefficients $H, C, E, \ldots$ must be fixed by means of theoretical considerations. There are several possible ways to proceed, each one giving a different set of coefficients and different Z_0 values. For example, we can imagine that the first term always dominates (even for charm) and take $C = E = 0$, while fixing the value for H phenomenologically, or we can postulate $H = 0$, fix C, and neglect E, etc. The approach we have chosen is to assume $H, C \neq 0$ and neglect higher-order terms, thereby requiring two conditions to fix our two parameters. One condition comes from performing a best fit to the J/ψ and D^* masses employing Z_0 as a free parameter in the exact bag equations (and *not* the heavy-quark expansion). This analysis is summarized in Table II and yields a best-fit value $Z_0 = 1.4$ along with $X = 5.74$. This value for X corresponds to the J/ψ particle and characterizes the entire charm sector in our approach. Thus we obtain the constraint

$$1.4 = \frac{H}{5.74} + \frac{C}{(5.74)^2} \,. \tag{8a}$$

Additional aspects of our charm analysis are presented in Table III. As we can see, there is an improvement in the results for the charm mesons and for the $\frac{1}{2}^+$ $c(ud)_A$ baryon with respect to previous published values which used the same model but with a different Z_0 value.[10] However, notice that nothing is achieved for the 0^- $(c\bar{c})$ meson. It is these results, together with the mentioned inconsistency of Eq. (5), that motivate us to take seriously the series expansion in Eq. (8).

In order to obtain a second condition for the parameters H, C, it would be ideal to employ data from the b-quark sector analogous to that from the c-quark sector. Unfortunately, this is not possible because experimental data regarding b-flavored mesons are as yet lacking. Therefore, we have chosen the following somewhat arbitrary line of reasoning which involves unflavored 0^- mesons. Beginning with the pion, these states have been traditionally difficult to treat and the 0^- meson $\eta_c(2830)$ is no exception. As mentioned above, we cannot obtain a reasonable mass estimate for this state in our model without upsetting the remainder of our predictions. Apparently we are leaving out a significant piece of physics as regards 0^- mesons. It turns out to be possible to arrange that our heavy-quark expansion is parametrized to break down for only the 0^- mesons as follows. As before, we consider only flavor-singlet mesons or single-flavor baryons.

Using Eqs. (5) and (8), we obtain the modified NLBC

$$4\pi BR^4 = (N\pi^2 - 2H)/X + (14.59A - 14.80N - 3C)/X^2 + O(X^{-3}) + \cdots \,, \tag{9}$$

and from Eqs. (6) and (8) the modified energy equation

$$M = \frac{1}{R}\,[NX + 5(N\pi^2 - 2H)/6X + (9.72A - N\pi^2 - 2C)/X^2 + \cdots] \,. \tag{10}$$

Note that we have recovered the property $\lim_{X\to\infty} R = 0$ in Eq. (9), which we interpret as the series expansion for R in powers of X. In order for Eq. (9) not to have a leading negative term, we must require that $0 \leq H \leq \pi^2$, the upper limit being strongly suggestive because it ensures a larger bag radius for baryons than for mesons in those hadrons which consist entirely of heavy nonrelativistic quarks. Moreover, by choosing $H = \pi^2$, we see in Eq. (9) that for a single-flavor baryon the leading term is of order X^{-1} and, as required, positive. However, for a flavor-singlet meson the leading order goes

TABLE III. Masses of hadrons in the charm sector of SU(4). The bag parameters used were $B^{1/4} = 145$ MeV, $\alpha_c = 0.55$, $m_u = m_d = 0$, $m_s = 279$ MeV, $m_c = 1486$ MeV, $Z_0 = 1.4$.

State	Mass (MeV)	State	Mass (MeV)	State	Mass (MeV)
1^- $c\bar{c}$	3095	$\frac{1}{2}^+$ cuu	2380	$\frac{3}{2}^+$ cuu	2481
$c\bar{s}$	2141	$c(ud)_A$	2243	$c(su)_S$	2624
$c\bar{u}$	2009	$c(su)_S$	2530	css	2764
0^- $c\bar{c}$	2971	$c(su)_A$	2425	ccu	3630
$c\bar{s}$	1957	css	2678	ccs	3764
$c\bar{u}$	1800	ccu	3511	ccc	4747
		ccs	3664		

as X^{-2} and furthermore has a coefficient which is positive for 1^- mesons but negative for 0^- mesons. This choice of parametrization then makes manifest in the heavy-quark expansion our prejudice that current methods for describing 0^- flavor-singlet mesons are really inadequate and require additional, explicit contributions to the energy. At any rate, we then infer from Eq. (8a) a value for C,

$$C=-10.52\,.$$

Although rather arbitrary, our approach does have the advantage of allowing for a variable Z_0 while keeping the number of new parameters down. From here on, the value of Z_0 is *fully determined* by Eq. (8) for still heavier quarks. In particular, we find for the b sector a value $Z_0\approx 0.62$ as used in Sec. II. [In getting the value $Z_0\approx 0.62$ we use $X_b=15.09$, the value corresponding to the $\Upsilon(9460)$.]

Another case we can study in the context of the series expansions is that of flavored mesons containing a heavy quark (antiquark) and a massless antiquark (quark). The NLBC and the energy equations for such cases are

$$4\pi BR^4=2.30+(3.02A+8.39-2H)/X - (7.51+3C+1.86A)/X^2+\cdots\,, \tag{11}$$

$$M=\frac{1}{R}\left[X+3.07+(2.52A+6.99-5H/3)/X - (1.24A+2C+5.00)/X^2+\cdots\right]. \tag{12}$$

where $A=1$ for 1^- mesons and -3 for 0^- mesons as before. H, C are the same as in Eq. (8). The LBC for the heavy quark is Eq. (4) and for the light quark becomes $\omega=2.04$. In getting the numerical coefficients in Eqs. (11) and (12), we use for the magnetic and electric energies of a flavored meson formed of a heavy quark (antiquark) and a massless antiquark (quark) the expansions

$$E_M=\frac{A\alpha_c}{R}(2.74/X-1.13/X^2+\cdots)\,, \tag{13}$$

$$E_E=\frac{\alpha_c}{R}(0.47-1.34/X+4.42/X^2+\cdots)\,. \tag{14}$$

In a similar manner we can derive the equations for hadrons composed of mixtures of different heavy quarks (c, b, t) and massless quarks[13] (u, d).

When we solve Eqs. (7) to (14) including terms up to order X^{-2}, we get solutions that differ less than 1% with respect to the exact solutions in the b sector, and less than 5% in the charm sector.

At this point we lack only the value of Z_0 for hadrons containing one or more s quarks as the heaviest quark. For this purpose we cannot use the expansion of Eq. (8) because $X_s\cong 1.5$, and we do not have enough terms in the series to get an accurate value for Z_0 for such small X_s (we are not even sure if the series converges for such a value). Another way of getting Z_0 for the s sector is using phenomenology, i.e., trying to fit a value of $Z_0 < 1.84$ that will improve the previous results. We have done so and have found there is no unique Z_0 that will definitely improve the results for all of the strange particles. For example, a value of $Z_0=1.80$ will bring the $\frac{3}{2}^+$ baryons very close to their experimental results, but is inadequate for the other hadrons. A smaller value suffices for the $\frac{1}{2}^+$ baryons, but then it would be too small for the $\frac{3}{2}^+$ baryons, etc. One alternative here is to think of a different Z_0 for particles with different strangeness, but it would mean introducing more parameters into the theory.

TABLE IV. Quark masses and associated values for Z_0 in the MIT spherical-bag model with Z_0 variable.

Quark	Mass (MeV)	Z_0
u, d	0	1.84
s	265–279	1.70–1.84
c	1486	1.40
b	4700	0.62

To conclude this section we summarize in Table IV the quark masses given by the model and the associated Z_0 values.

V. CONCLUSIONS

In the first part of this paper we solved the bag equations for the b-quark sector in very much the same way that they were solved in Ref. 10 for the charm sector, the main difference being the different value of Z_0 chosen. How good is the spectrum we generated? Only future experimental results can answer this question and prove or disprove the idea of a mass-dependent Z_0.

Let us discuss some of the results in Table I. Generally speaking, they are in qualitative accord with results predicted by other models like Regge trajectories[14] and phenomenological potentials.[15] From Table I we see that the lightest particle carrying the b quark will be the 0^- meson at 5.23 GeV. This means that the threshold for producing pairs of b-flavored mesons lies above the Υ', Υ'' masses.[16] Therefore, we predict that these states do *not* decay into b-flavored meson pairs. Also we point out the very small spacing between the masses of the 0^- and 1^- flavor-singlet mesons that we predict (only 25 MeV). This is due to the fact that the magnetic energy becomes very small for heavy quarks [see Eqs. (7) and (13)] giving a reduced hyperfine splitting. Although we believe that the last statement will still be valid in a more complete model, we hesitate taking too seriously our 1^{-}-0^{-} mass-difference prediction because, as emphasized previously, we think that our understanding of the 0^{-} flavor-singlet mesons is less than complete. Finally, notice from Table I that in the b sector, the electric energy has become as important as the magnetic energy for hadrons composed of a mixture of light quarks and heavy quarks. For hadrons composed only of heavy quarks (c, b, t), the electric energy is still small, as we can see from Table I and from the following series expansion for the electric energy of a hadron formed of two different heavy flavors:

$$E_E = \frac{\alpha_c}{R}\left[(1.00/X_1 - 1.00/X_2)^2 + O(X^{-3}) + \cdots\right]. \tag{15}$$

In Sec. III we presented series expansions of the bag equations. The nice thing about these expansions is that they expose very clearly the possible mathematical problems of the model, and in addition give us a much simpler set of equations for very heavy quarks. Our conclusions from such expansions are that the model as originally defined will break down first for the 0^{-} mesons, next for the 1^{-} mesons, and finally for the baryons. Going back to the question formulated above, we suspect that the spectrum calculated in Sec. II is not good for the 0^{-} mesons, but is we hope acceptable for the 1^{-} mesons and that it must be as good as the phenomenological calculations for the light baryons.

Recently, it was proposed by Johnson[17] that for bags with heavy quarks, the bag pressure B which determines the position of the boundary is determined by the color fields rather than by the quark kinetic energy. Analyzing our Eqs. (9) and (11) (where we solve for B as a function of the other parameters in the model), we see that such a result is manifested in Eq. (9) when applied to flavor-singlet mesons ($N=2$). For this case the equation reads

$$4\pi BR^4 = (14.59A - 14.80N - 3C)/X^2 + O(X^{-3})\cdots, \qquad C<0$$

where the terms containing the parameters A and C are due to the color magnetic and zero-point energies, respectively (the effects of the color fields in the model), and the term containing N is due to the kinetic energy of the heavy quarks. Observe that the effects of color dominate the effects of the kinetic energy for the case of 1^{-} mesons. For 0^{-} mesons, again the color dominates, but the equation breaks down because of a leading negative term. For the case of baryons, Eq. (9) up to the leading term reads

$$4\pi BR^4 = (N\pi^2 - 2H)/X + \cdots,$$

where both effects, the one due to the kinetic energy ($N=3$) and the one due to the color fields ($H=\pi^2$) are present in the leading order, but the kinetic energy dominates slightly. For the case of flavor mesons composed of one heavy quark and one light quark, Eq. (11) is valid. There we find up to leading terms

$$\begin{aligned}4\pi BR^4 &= 2.30 + O(X^{-1}) + \cdots\\ &= 2.04 + 0.26 + \cdots,\end{aligned}$$

where the number 2.04 is due to the kinetic energy of the light quark (mass $=0$), and the value 0.26 is due to the color electric energy. The effects of the kinetic energy of the heavy quark will be present only at the next order where there will also be terms due to the color electric, magnetic, and zero-point energies. As we can see, here our conclusion is that the kinetic energy of the light quark dominates, giving a picture of a very light quark moving very fast around the center of mass, and a heavy quark moving very slowly.

Next we discuss the question of what value to use for the quark-gluon coupling constant α_c. According to light-hadron spectroscopy the value $\alpha_c = 0.55$ is strongly recommended, but owing to the fact that QCD is an asymptotically free theory, such a coupling constant is expected to be scale dependent. When the theory is renormalizable and perturbation analysis is permitted, there exist well known formulas[18] which give the value of α_c as a function which decreases for increasing momentum transfer carried by the gluons. As we mentioned in the Introduction, the spherical bag as it now stands is not renormalizable, so strictly speaking we are not allowed to use any formula coming from or related to renormalization theory. If we want to introduce a running coupling constant, we must do it *ad hoc* by introducing extra parameters that must be fixed phenomenologically. Let us suppose we have done that and we have a formula for α_c as a function of the momentum transfer: The question is then what to use for a momentum transfer when heavy quarks are present inside the bag, i.e., for the specific problem of doing hadron spectroscopy? To scale momentum transfer with the value of the largest quark mass inside the bag is not proper because in calculating the gluon magnetic and electric energies, only exchange diagrams are considered (see Ref. 3). If we assume that the momentum transfer is proportional to the kinetic energy carried by the quarks, then it must be smaller for heavy nonrelativistic quarks than for light quarks. If we assume that the momentum transfer is related to the bag radius,[19] since in our approach the bag radius is relatively constant in going from light to heavy hadrons, then the average momentum transfer is also relatively constant. In conclusion, we feel that the introduction of a running coupling constant requires at least the introduction of two more parameters, which must be fixed phenomenologically at the present moment. Such a study will be more properly done when more experimental data regarding heavy quarks are available. This can only improve our results. For the time being, we continue to employ $\alpha_c = 0.55$ as suggested from light-hadron spectroscopy with the expectation that it is not much different for the case of heavy quarks and for the particular problem of hadron spectroscopy.

Finally, in Sec. IV, we argue in favor of a mass-dependent Z_0 parameter in the context of the spherical bag as a necessary condition to apply the spherical-bag equations to heavy quarks. Such an assumption is a mathematical one and not necessarily a law of nature. It could be that a mass-dependent Z_0 is a way of arranging things to take care of a running quark-gluon coupling constant. Or it could be that Z_0 is only shape dependent and that it becomes mass dependent via the fact that heavy quarks deform the sphericity of the bag. Then our approach is seen as an intermediate stage between the present spherical bag and the yet-to-be-formulated nonspherical one.[20] In any case we believe in a variable Z_0 as a clue regarding one of the many more ingredients we should add to the theory.

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