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Dependence of cluster size on inelasticity and strength of correlation on inelasticity and dispersion in rapidity in cosmic-ray interactions

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The dependence of cluster size on inelasticity in nucleon-nucleon interactions at cosmic-ray energies ($\sim 10^{12}$ eV) has been investigated. The cluster size has been found to be independent of inelasticity. The maximum number of pions in a cluster is found to be equal to four for all values of inelasticity. The variation in the strength of correlation (clustering) among the secondary particles produced in nucleon-nucleon interactions at cosmic-ray energies with respect to inelasticity and dispersion in rapidity is investigated. The dispersion parameter δ is found to be a weak parameter for measuring correlations and also shows no dependence on multiplicity. The strength of correlation is found to increase with inelasticity (K), and beyond $K = 0.5$ the strength of correlation remains constant.

I. INTRODUCTION

The cluster model[1] has attracted a considerable amount of attention due to its success in reproducing the recent experimental data at CERN ISR and Fermilab energies, particularly in the region of the nondiffractive component of the cross section. Although the cluster model has its origin in cosmic-ray physics (fireball model), little progress has been made towards understanding the basic characteristics of clusters. The first major problem is to understand whether cluster formation is a dynamical effect or else is a phenomenological artifice. Secondly, if the clusters are dynamical entities, their intrinsic characteristics such as their charge, mass, and multiplicity distribution should be determined. In a recent work[2] we have suggested a prescription for the determination of cluster sizes and have applied the method to cosmic-ray interactions. In order to investigate the nature of clusters a detailed study of cluster size and correlations *vis á vis* other parameters in multiparticle production is warranted. It would be quite interesting to understand the dependence of cluster size on inelasticity and the behavior of correlations with respect to dispersion in rapidity and inelasticity of the interactions. We present below the first such study in interactions at cosmic-ray energies ($\sim 10^{12}$ eV).

In the present work, we study the nucleon-nucleon interactions for a wide range of primary energies (0.1–2600 TeV) and charged-particle multiplicity (7–36). The complete details of the interactions have been given earlier.[3,4] The mean multiplicity and mean energy of the events for $E_p < 1$ TeV and $E_p > 1$ TeV are 14 and 0.5 TeV, and 17 and 118.7 TeV, respectively. The reactions studied are semi-inclusive processes since the rapidities of only the charged secondary particles were determined. The secondary particles in the central region have been considered here and the two leading particles on each end of the rapidity space have been neglected. Thus the secondary particles considered are those which contribute predominantly to the nondiffractive component of the total cross section. Since pions constitute a major fraction ($\sim 80\%$) of the secondary particles in high-energy interactions, in the following, a particle means a pion. In Sec. II A, the dependence of cluster size on inelasticity is discussed. The cluster size has been found to be independent of inelasticity. In Sec. II B 1, we discuss the dispersion parameter δ in the light of the nova model. The results show that δ is a weak parameter for measuring correlations. The variation of the strength of correlation with inelasticity (K) is discussed in Sec. II B 2. The strength of correlation increases with K, but beyond $K = 0.5$ it remains constant.

II. RESULTS AND DISCUSSION

A. Cluster-size characteristics

First, let us clarify the meaning of cluster size. The cluster size is directly dependent upon the number of particles constituting a cluster. Since a large majority of the produced particles in high-energy interactions constitute of pions, the cluster is comprised mostly of pions, although there is no prejudice against the formation of nucleon clusters as well. The clusters can be either charged or uncharged. Since the production of neutral pions in high-energy interactions is $\frac{1}{2}$ that of the charged pions, the charged clusters are also likely to contain neutral pions as well. A method for the determination of cluster size has been given earlier.[2] An obvious question to ask is that if clusters are produced and their sizes determined, then why don't we "see" them? The reason may be due to their very short lifetime, which may be shorter than that of the known resonances. Another difference between the resonances and clusters is that the number of particles decaying from a cluster can be more than those from a resonance. However, these differences do not preclude the suggestion that clusters may be considered to be "generalized resonances," because of their dynamical significance. In order to understand the characteristics of cluster size, its dependence upon inelasticity is discussed below.

Dependence of cluster size on inelasticity

The value of inelasticity (K) was determined for each event by the procedure followed by Edwards *et al.*[5] in cosmic-ray interactions. The average value of transverse momentum (P_T) of the secondary particles was assumed to be 0.35 GeV/c. Using the following relations,

$$\text{Momentum } (P) = P_T \csc\theta$$

and

$$\text{Energy } (E) = (P^2 + m^2)^{1/2},$$

where θ is the laboratory angle of the secondary particle and m is its mass, the magnitude of total energy ($\sum E_i$) used up in particle production was calculated. The inelasticity (K) can be defined as

$$K = \sum_{i=1}^{n} E_i / E_p, \qquad (1)$$

where n is the number of secondary particles in an event and E_p is the primary energy. The values of K and primary energy for each event are given in Table I.

The determination of cluster size is performed for three ranges of inelasticity. The events are grouped according to the values of K lying between 0.0–0.13, 0.13–0.34, and 0.34–0.82. The choice for grouping as above is dictated by the existence

TABLE I. The event type, primary energy, and inelasticity are given for each event (Ref. 3).

Sr. No.	Event type	Primary energy (TeV)	Inelasticity (K)	Sr. No.	Event type	Primary energy (TeV)	Inelasticity (K)
1	$0+36p$	644.52	0.02	27	$0+16p$	0.54	0.33
2	$0+16p$	0.24	0.30	28	$0+7n$	1.04	0.07
3	$0+9p$	70.34	0.01	29	$0+17p$	0.46	0.29
4	$0+9n$	0.57	0.23	30	$0+15p$	0.19	0.36
5	$0+22p$	16.19	0.08	31	$0+2n$	9402.09	0.00
6	$0+8p$	1.04	0.09	32	$0+7p$	2601.53	0.01
7	$0+13n$	1.77	0.14	33	$0+7n$	6.69	0.05
8	$0+30n,p$	3.09	0.33	34	$0+13p$	399.24	0.01
9	$0+16p$	3.69	0.10	35	$0+7p$	0.91	0.57
10	$0+11p$	0.17	0.24	36	$0+13p$	17.11	0.04
11	$0+14n$	0.51	0.48	37	$2+16p$	4.1	0.14
12	$0+9p$	0.33	0.18	38	$4+27p$	5.7	0.16
13	$0+9p$	2.62	0.05	39	$0+20n$	1.9	0.48
14	$0+13p$	7.68	0.08	40	$2+15p$	43.5	0.16
15	$0+10p$	21.74	0.02	41	$1+23p$	2.1	0.57
16	$0+13p$	0.87	0.12	42	$3+32p$	4.7	0.36
17	$0+15p$	1.13	0.34	43	$0+20p$	3.9	0.34
18	$0+18p$	0.47	0.22	44	$0+13p$	103.1	0.04
19	$0+12p$	0.12	0.24	45	$1+12p$	3.4	0.21
20	$0+8p$	0.58	0.11	46	$1+14p$	2.2	0.19
21	$0+14n$	0.60	0.18	47	$0+21n$	0.4	0.81
22	$0+15p$	25.19	0.03	48	$2+20p$	1.8	0.33
23	$0+16n$	15.53	0.05	49	$2+23p$	2.1	0.69
24	$0+15p$	0.12	0.82	50	$3+33p$	1.6	0.41
25	$0+19n$	0.48	0.35	51	$4+26p$	10.8	0.09
26	$0+11n$	2.72	0.07	52	$4+16p$	1.4	0.25

TABLE II. Values of χ^2 per degree of freedom (DOF) for single and double exponential distributions for different intervals of K.

Distribution type	$dn/dr=Ae^{-Br}+Ce^{-Dr}$ A	B	C	D	χ^2/DOF	$dn/dr=Ae^{-Br}$ A	B	χ^2/DOF
K=0.0–0.13, two particles	4.9	7.5	0.2	0.2	29.0/29	4.3	5.6	48.5/29
K=0.0–0.13, three particles	3.7	4.9	0.25	0.6	16.7/21	2.0	2.4	24.2/21
K=0.0–0.13, four particles	1.9	2.3	0.1	0.2	13.5/20	1.3	1.4	17.5/20
K=0.13–0.34, two particles	3.9	5.3	0.2	0.5	20.2/29	4.1	4.6	29.1/29
K=0.13–0.34, three particles	2.8	3.2	0.3	0.7	12.1/22	2.3	2.1	15.4/22
K=0.13–0.34, four particles	2.1	1.9	0.05	0.1	15.0/19	1.9	1.6	16.4/19
K=0.34–0.82, two particles	4.2	5.9	0.2	0.3	18.5/31	4.0	4.6	30.3/31
K=0.34–0.82, three particles	3.8	4.4	0.35	0.75	15.1/25	2.2	2.2	23.9/25
K=0.34–0.82, four particles	2.1	2.6	0.3	0.6	17.0/20	1.7	1.5	20.4/20

of nearly equal numbers of secondary particles in the three cases.

In order to determine the best theoretical curve which would fit the observed rapidity-difference distributions, single and double exponential distributions were tried using an IBM 370 computer. Various values of the parameters were tried in the equations. It was found that the equation which yielded the lowest χ^2 follows the form

$$dn/dr=Ae^{-Br}+Ce^{-Dr}\,. \tag{2}$$

The values of χ^2 and χ^2 per degrees of freedom for single and double exponential curves for different distributions of inelasticity are given in Table II.

Figure 1 shows the two-, three-, four-, five-, and six-particle rapidity-difference (r) distributions for inelasticity K=0.0–0.13. The numerical equations of the theoretical curves represented by solid lines in Fig. (1), for two-, three-, and four-particle rapidity-difference distributions are, respectively,

$$dn/dr=4.9e^{-7.5r}+0.2e^{-0.2r}\,, \tag{3a}$$

$$dn/dr=3.7e^{-4.9r}+0.25e^{-0.6r}\,, \tag{3b}$$

and

$$dn/dr=1.9e^{-2.3r}+0.1e^{-0.2r}\,. \tag{3c}$$

The dotted lines in Fig. (1) show the contributions of the two individual terms in Eqs. (3a), (3b), and (3c) and are also a measure of the slopes in each case. The five- and six-particle rapidity-difference distributions do not show a sharp peak and are not representable by an exponential curve. From this we conclude that the maximum number of charged particles constituting a cluster for K=0.0–0.13 is four although two- and three-particle clusters are also present in the interactions.

Following the same procedure, the rapidity-difference distributions were calculated for two, three, four, five, and six particles for interactions having K=0.13–0.34 and K=0.34–0.82 and are shown in Figs. 2 and 3, respectively. The equations of theoretical curves represented by solid lines in Fig. (2), for two-, three-, and four-particle distributions are, respectively,

$$dn/dr=3.9e^{-5.3r}+0.2e^{-0.5r}\,, \tag{4a}$$

$$dn/dr=2.8e^{-3.2r}+0.3e^{-0.7r}\,, \tag{4b}$$

and

$$dn/dr=2.1e^{-1.9r}+0.05e^{-0.1r}\,, \tag{4c}$$

and the equations of the theoretical curves represented by solid lines in Fig. (3), for two-, three-, and four-particle distributions are, respectively,

$$dn/dr=4.2e^{-5.9r}+0.2e^{-0.3r}\,, \tag{5a}$$

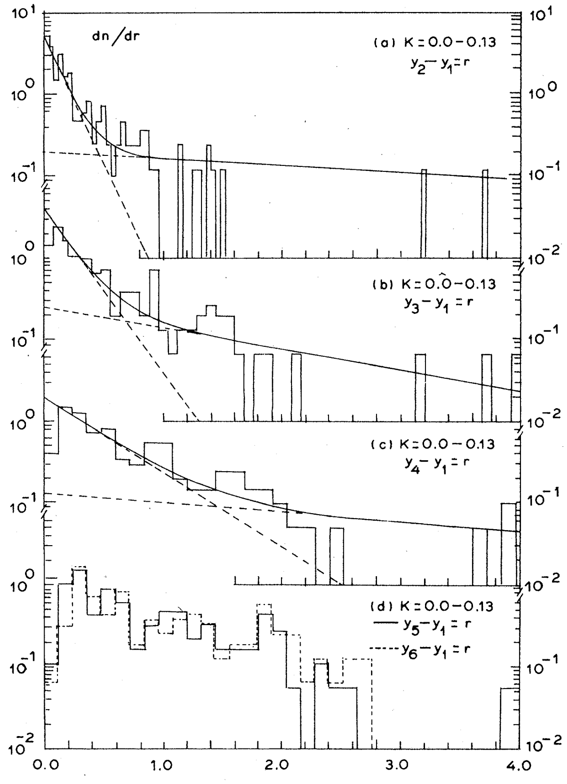

FIG. 1. Rapidity-difference (r) distributions of charged secondary particles having inelasticity K= 0.0–0.13 for (a) two adjacent particles, (b) first and third particle, (c) first and fourth particle, and (d) first and fifth particle (solid line) and first and sixth particle (dashed line). The solid curves in (a), (b), and (c) show the respective contributions of Eq. (3a), (3b) and (3c) in the text and dashed lines show the individual contributions of the two terms in the three equations.

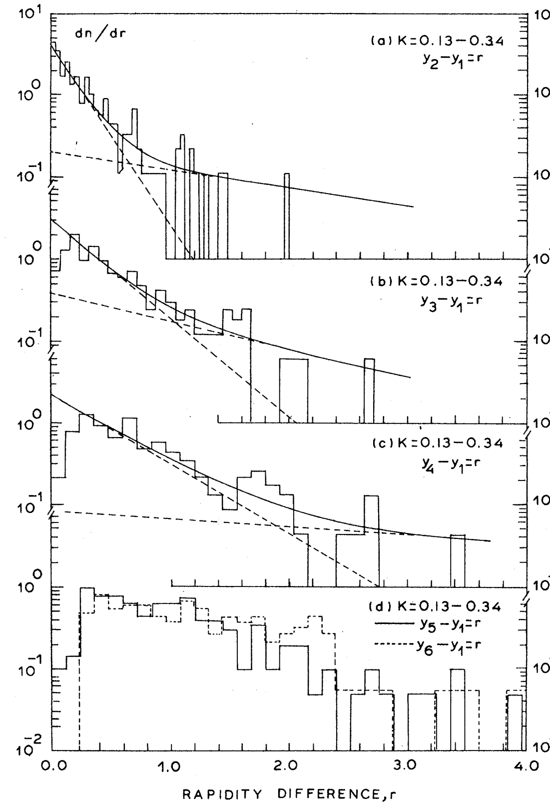

FIG. 2. Rapidity-difference (r) distribution of charged secondary particles having inelasticity K= 0.13–0.34 for (a) two adjacent particles, (b) first and third particle, (c) first and fourth particle, and (d) first and fifth particle (solid line) and first and sixth particle (dashed line). The solid curves in (a), (b), and (c) show the respective contributions of Eq. (4a), (4b), and (4c) in the text and dashed lines show the individual contributions of the two terms in the three equations.

$$dn/dr = 3.8e^{-4.4r} + 0.35e^{-0.75r}, \qquad (5b)$$

and

$$dn/dr = 2.1e^{-2.6r} + 0.3e^{-0.6r}. \qquad (5c)$$

The dotted lines represent the two exponential terms separately and are also a measure of the slopes in each case. It is seen from Figs. 2 and 3 that two-, three-, and four-particle correlations are present and five- and six-particle correlations are nonexistent in these interactions. Now it is evident that the maximum number of charged particles constituting a cluster for all the three ranges of K, viz., K=0.0–0.13, K=0.13–0.34, and K=0.34–0.82 is four, although two- and three-particle clusters are also present in the interactions. Thus we find that clusters of similar sizes are produced in different regions of K. This can be understood from observation of our data given in Table I. It is seen that K is independent of the primary energy. We have seen[2] that two- and three-particle clusters are produced in all the interactions having $E_p < 1$ TeV and $E_p > 1$ TeV, whereas four-particle clusters are produced only at $E_p > 1$ TeV. The observation of two-, three-, and four-particle clusters in all the regions of K is due to the contribution of interactions at $E_p < 1$ TeV and $E_p > 1$ TeV in all the K regions, since K is independent of the primary energy. Again, the observation of maximum cluster size of four particles in all the K regions is due to the contribution of events of $E_p > 1$ TeV in all the regions of K.

B. Strength of correlation

The strength of correlation measures the "strength" with which the secondary particles are correlated. The value of the slope in the

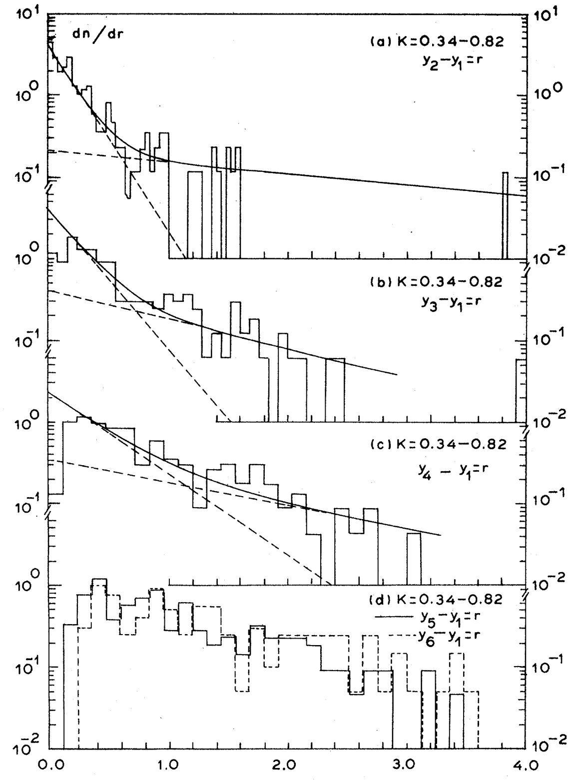

FIG. 3. Rapidity-difference (r) distribution of charged secondary particles having inelasticity $K=0.34-0.82$ for (a) two adjacent particles, (b) first and third particle, (c) first and fourth particle, and (d) first and fifth particle (solid line) and first and sixth particle (dashed line). The solid curves in (a), (b), and (c) show the respective contributions of Eqs. (5a), (5b), and (5c) in the text and dashed lines show the individual contributions of the two terms in the three equations.

first term of Eq. (2) is a measure of the strength of correlation[2] in the first region of the rapidity-difference distribution. The larger the number of small values of r, the larger the value of the slope and hence the correlation. As the particles are more and more closely spaced in rapidity, the rapidity difference will be smaller and smaller, leading to an increase in the value of the slope of the rapidity-difference distribution. Whereas the existence of a strong correlation strongly supports cluster formation, a weak correlation indicates that cluster production is not the dominant mode of particle production. Since strength of correlation is an intrinsic characteristic of clusters, it would be interesting to investigate the relationship of this parameter with other parameters in high-energy interactions, as described below.

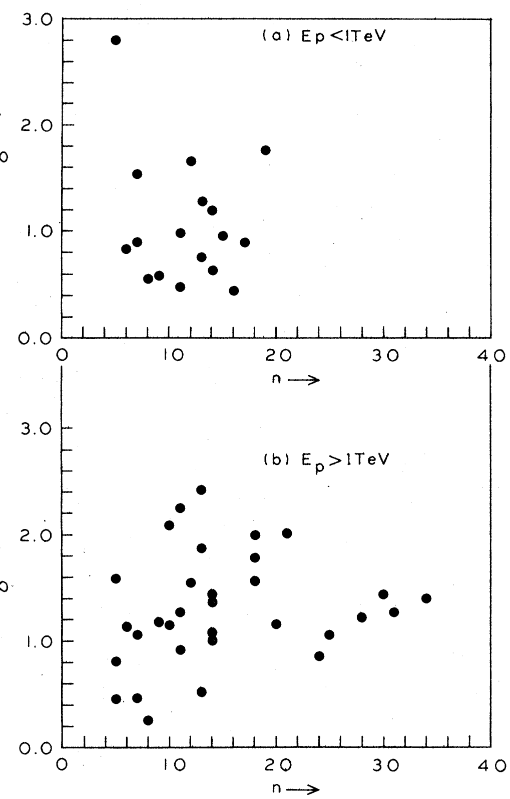

FIG. 4. Variation of dispersion in rapidity $\delta(Y)$, with the number of charged secondary particles after eliminating the contribution of the two leading particles, for primary energy E_p (a) less than 1 TeV and (b) greater than 1 TeV, respectively.

1. Dispersion in rapidity

The behavior of dispersion in rapidity with respect to multiplicity is considered here for interactions having a primary energy less than 1 TeV and greater than 1 TeV. In order to calculate the values of average rapidity Y, we consider a subset of N particles with rapidities Y_j. An entire event may be considered as a subset. The average rapidity of the subset is defined as

$$\bar{Y}=\frac{1}{N}\sum_{j=1}^{N}Y_j\,. \tag{6}$$

The dispersion of the subset was calculated from the relation

$$\delta(Y)=\left[\frac{1}{N-1}\sum_{j=1}^{N}(\bar{Y}-Y_j)^2\right]^{1/2}. \tag{7}$$

The rapidity (Y_j) of the secondary particles was determined as followed in an earlier work.[3] Using the above relation, the value of the dispersion parameter δ was calculated for each event. The two leading particles at the two ends of rapidity space for every event were eliminated in the determination of the dispersion parameter. This was done to eliminate the contribution of diffractively produced particles. Figures 4(a) and 4(b) show the variation of $\delta(Y)$ with the number of charged secondary particles ($n=n_s-2$) in an event for primary energy less than 1 TeV and greater than 1 TeV, respectively. It is clear that no dependence of $\delta(Y)$ on the multiplicity is observed. We know that the strength of correlation shows an increase[2] with a decrease in the number of particles in a cluster which in turn is dependent upon the event multiplicity. The absence of any dependence of $\delta(Y)$ on n as seen from Figs. 4(a) and 4(b) shows that $\delta(Y)$ is a weak parameter for measuring the strength of correlation. The other factor that contributes to make $\delta(Y)$ a weak parameter is that the existence of several clusters within an event cannot be identified, partly because the products of different clusters may overlap in rapidity and partly because neutrals go undetected in most experiments. It is not possible to identify separately the contribution from different clusters. Chao and Quigg[6] have argued that the problem of identification of particles belonging to a cluster is greatly aggravated if there is production of more than one cluster in an event. Thus $\delta(Y)$ cannot be taken as a quantitative measure to determine the cluster formation at ultrahigh energies where the possibility of the production of more than one cluster per event is non-negligible.

Berger *et al.*[7] have suggested that the magnitude of $\delta(Y)$ can be used to discrimate between fragmentation-type and multiperipheral-type models at $E_p \geq 100$ GeV/c. On the basis of the nova fragmentation model,[7,8] high-energy interactions with $\delta(Y) \lesssim 0.9$ indicate the formation of single-nova (cluster) events. At relatively low energies ($E_p \lesssim 30$ GeV/c), most events yield $\delta(Y) \lesssim 0.9$, which is attributed to the constraint of energy and momentum conservation. If the nova decays isotropically, then $\langle n \rangle_M \alpha M$, where M is the nova (cluster) mass and n is the number of particles decaying from the nova. The multiperipheral[7] or independent-emission[7] type models predict a vanishingly small cross section with $\delta(Y) \lesssim 0.9$ as the available energy in the center-of-mass system grows. The multiperipheral-type models lead to the behavior $\langle n \rangle \propto \ln M$ and hence $d\sigma/dM^2 \propto 1/S$, where $\sqrt{s}$ is the available energy in the center-of-mass system. Such models which are dominated by longitudinal phase space predict that $\langle \delta(Y) \rangle_M \propto \ln M$. It is evident from Figs. 4(a) and 4(b) that the fraction of events with $\delta(Y) \lesssim 0.9$ is $\sim 40\%$ and $\sim 17\%$ for $E_p < 1$ TeV and $E_p > 1$ TeV, respectively. Our observations are not unexpected if it is assumed that low-mass clusters give rise to low values of $\delta(Y)$. Hence the higher fraction of events with $\delta(Y) \lesssim 0.9$ for events with $E_p < 1$ TeV as compared to those with $E_p > 1$ TeV may be due to the larger contribution of low-mass clusters in the former events. This is borne out by our earlier work[2] where it is shown analytically that low-mass clusters contribute more predominantly at $E_p < 1$ TeV as compared to that at $E_p > 1$ TeV. Our results are in agreement with the proposition of Berger *et al.*[7] that the value of $\delta(Y)$ can reflect the validity of different models, as discussed above.

2. *Variation of the strength of correlation with respect to inelasticity*

The variation of the strength of correlation with the rapidity-difference distribution for three ranges of inelasticity is investigated. Figures 1(a), 2(a), and 3(a) show the rapidity difference distribution for $K=0.0-0.13$, $K=0.13-0.34$, and $K=0.34-0.82$, respectively. The numerical equations of the theoretical curves represented by solid lines for the rapidity-gap distribution in figures 1(a), 2(a), and 3(a) are given by Eq. (3a), (4a), and (5a), respectively.

In Figs. 1(a), 2(a), and 3(a), a double exponential curve is drawn through the three distributions for varying values of inelasticity. The value of the slope in the first exponential term decreases from $B=7.5$ for the case $K=0.0-0.13$ to $B=5.3$ for $K=0.13-0.34$. The value of the slope B for $K=0.13-0.34$ is nearly the same as that for $K=0.34-0.82$. It is seen from a comparison of Fig. 1(a) with Fig. 2(a) that with the increase in the value of K the strength of correlation tends to decrease. This shows that for low values of K, B is dependent upon the manner

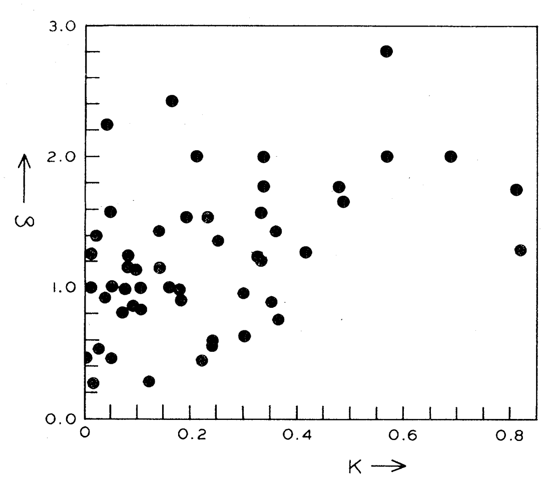

FIG. 5. Variation of dispersion in rapidity $\delta(Y)$, with inelasticity (K) of the interactions.

in which energy is taken by the leading particle, since the elasticity (η, fraction of primary energy retained by the incident particle) $=1-K$. This may be called the leading-particle effect. The weak inelasticity implies a high elasticity of the events, where the leading particle carries off a substantial portion of the primary energy.

We have found[2] that the strength of correlation is maximum for lowest-size clusters (two particles) and then shows a decrease as the cluster size increases. Due to the weak inelasticity, the energy shared among the secondary particles is small; they tend to have a small rapidity and hence give rise to a peak at low values of the rapidity difference. The peak at low rapidity difference is due to low cluster size and hence the strength of correlation in such cases is large. As the inelasticity increases, the peak in the rapidity-difference distribution shifts towards higher values indicating larger cluster size and thus showing a decrease in the strength of correlation. From Eqs. (4a) and (5a) we see that the value of B is nearly the same in both cases. Beyond a certain value of K, the production of low and high cluster sizes may be equally frequent resulting in the strength of correlation becoming independent of K.

The variation of $\delta(Y)$ with K is shown in Fig. 5. Although the value of slope B varies with K up to the value ~0.34, yet we find from Fig. 5 that $\delta(Y)$ does not show any dependence on K. This again shows that $\delta(Y)$ is not a strong parameter for measuring correlations.

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