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Inclusive multi-Pomeron corrections in the Regge-eikonal model: An application

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A recent formula for calculating multi-Pomeron exchange effects is applied to the Mueller-Regge expression for 0^- production from a 0^- beam, via charge exchange on protons. The target asymmetry and the differential cross section are calculated in an attempt to remedy the defects of a previous simpler model.

I. INTRODUCTION

There is both theoretical[1] and phenomenological[2-10] evidence to support the consideration of Regge cuts in one-particle-inclusive processes, particularly in the triple-Regge region.

In a recent series of papers[2-5] the reactions $\gamma + p \to \pi^{\pm,0} + X$ were studied. For $\pi^\pm$ production, a simple Regge-pole model predicts naturality dips as $p_\perp^2 \to 0$ and substantial absorption corrections were required to reproduce the experimentally observed peak. Preliminary data from DESY[6] for the target asymmetry in the reaction $\gamma + p\uparrow \to \pi^+ + X$ are nonzero and in reasonable agreement with the predictions of Ref. 4.

Further evidence for cuts is provided by consideration of the polarization of Λ's produced in the processes $p + \mathrm{Be} \to \Lambda + X$ and $p + \mathrm{Cu} \to \Lambda + X$. The measured polarization[7] is nonzero and relatively independent of nucleon number. A calculation of the polarization for $p + p \to \Lambda + X$[8] is in qualitative agreement with the beryllium data. A noninterfering Regge-pole model predicts zero polarization.

The absorption model of Refs. 9 and 10 considers initial-state rescattering in $a + b \to c + X$ (see Fig. 1) and, by analogy with two-body scattering, arrives at the formula

$$\Phi_X^{\mathrm{abs}}(\tau, s, M_X^2) = \int_0^\infty \Phi_X^R(\tau_1, s, M_X^2)\left[\frac{1}{\tau}\delta(\tau - \tau_1) - \frac{C}{2\lambda}\exp\left[\frac{-(\tau^2 + \tau_1^2)}{4\lambda}\right] I_0\left(\frac{\tau\tau_1}{2\lambda}\right)\right]\tau_1 d\tau_1 ,$$

where

$$\tau = 2k \sin\left(\frac{\theta_{\mathrm{c.m.}}}{2}\right),$$

k = c.m. three-momentum,

C = opacity,

λ = squared inverse of the radius of interaction for a, b elastic scattering.

We note the following points:

1. This model contains the assumption that $p_{c_\perp}$ is small or, equivalently, that the flip of helicity into the X state is small and is ignored. Thus, no target asymmetry is possible.
2. For $K^+ + p \to K^0 + X$ (Ref. 9), the further assumption of ρ-A_2 exchange degeneracy gives the pole-only expression and the negative-cut correction both real. This results in actual and presumably spurious zeros in the cross sections (when the two terms are equal in magnitude).
3. No account is taken in Fig. 1 of possible c-b elastic scattering, which is important in this context.[11]
4. The diagrams of Fig. 2(a) were considered while the diagrams of Fig. 2(b), which are of the same order in Pomeron exchange, were excluded.
5. Neither pole-only nor pole-with-cut correction reproduced the data[12] for $K^- + p \to \bar{K}^0 + X$.

The present paper reconsiders the reactions

$$\pi^\pm + p \to \pi^0 + X ,$$
$$\pi^\pm + p \to \eta + X ,$$
$$K^+ + p \to K^0 + X ,$$
$$K^- + p \to \bar{K}^0 + X ,$$

and uses Ref. 13 to remedy certain of the above defects and omissions.

II. FORMALISM

The derivation of Ref. 13 gives the following formula for spinless particles under the eikonal approximation[14] for the nonforward three-body scattering amplitude Y [Fig. 3(a)] with corrections given by the exchanges shown in Fig. 3(b). The derivation can be extended to the case of a spin-$\frac{1}{2}$ particle coupling to the spinless legs of the Pomerons, which is sufficient for our purposes (see

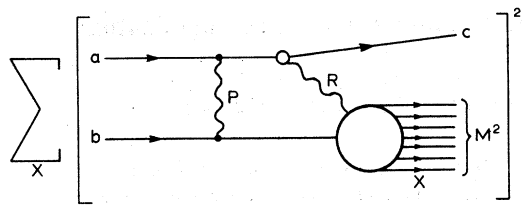

FIG. 1. Diagram used to illustrate the method of the initial-state absorption correction to the single-particle-inclusive reactions of Ref. 9.

Appendix A)

$$H(t,s_{ab},M_X{}^2)=\int\frac{d^2\vec{Q}_a}{(2\pi)^2}\frac{d^2\vec{Q}_c}{(2\pi)^2}\frac{d^2\vec{Q}_{\bar a}}{(2\pi)^2}\frac{d^2\vec{Q}_{\bar c}}{(2\pi)^2}S(\vec{Q}_a,\vec{Q}_c)\times Y(t_{ac},\bar{t}_{ac},t_0,s_{ab},M_X{}^2)S^*(\vec{Q}_{\bar a},\vec{Q}_{\bar c})\,, \tag{2.1}$$

where

$$S=\int d^2\vec{B}_{ab}d^2\vec{B}_{cb}\exp\{i(\vec{Q}_a\cdot\vec{B}_{ab}+\vec{Q}_c\cdot\vec{B}_{cb})+i[\chi_{ab}(B_{ab})+\chi_{ac,b}(B_{ab},B_{cb})+\chi_{cb}(B_{cb})]\}\,, \tag{2.2}$$

and

$$S^*=\int d^2\vec{B}_{\bar a\bar b}d^2\vec{B}_{\bar c\bar b}\exp\{-i(\vec{Q}_{\bar a}\cdot\vec{B}_{\bar a\bar b}+\vec{Q}_{\bar c}\cdot\vec{B}_{\bar c\bar b})-i[\chi^*_{ab}(B_{\bar a\bar b})+\chi^*_{ac,b}(B_{\bar a\bar b},B_{\bar c\bar b})+\chi^*_{cb}(B_{\bar c\bar b})]\}.$$

The mixed eikonal phase is found to be small and

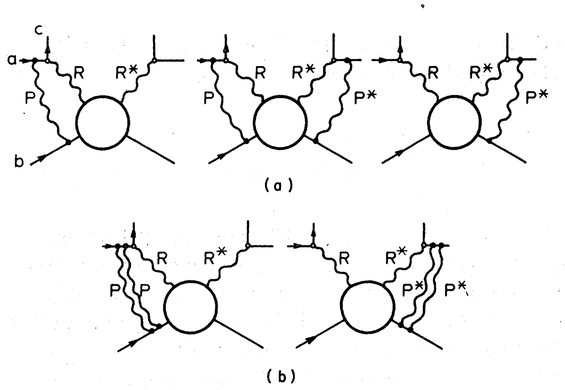

FIG. 2. (a) The three diagrams that are considered for initial-state rescattering in the model of Craigie and Kramer (Ref. 3). (b) Two rescattering corrections neglected by Ref. 3.

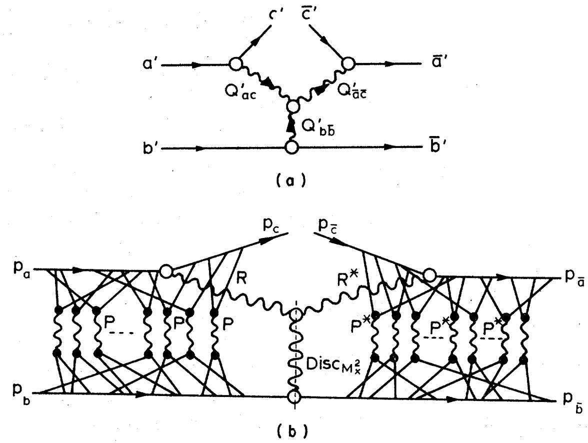

FIG. 3. (a) A representation of the input triple-Regge expression used in this paper. (b) A diagram of the class that can be accounted for by the formula of Ref. 13.

the remaining two phases are given by

$$\begin{aligned}\chi_{ab}&=\frac{1}{s_{ab}}\int\frac{d^2\vec{Q}_a}{(2\pi)^2}\exp(-i\vec{Q}_a\cdot\vec{B}_{ab})\beta_a(-\vec{Q}_a{}^2)\times\beta_b(-\vec{Q}_a{}^2)\xi_\alpha(-\vec{Q}_a{}^2)s_{ab}{}^{\alpha(-\vec{Q}_a{}^2)}\,,\\ \chi_{cb}&=\frac{1}{s_{cb}}\int\frac{d^2\vec{Q}_c}{(2\pi)^2}\exp(-i\vec{Q}_c\cdot\vec{B}_{cb})\beta_c(-\vec{Q}_c{}^2)\times\beta_b(-\vec{Q}_c{}^2)\xi_\alpha(-\vec{Q}_c{}^2)s_{cb}{}^{\alpha(-\vec{Q}_c{}^2)}\,.\end{aligned} \tag{2.3}$$

The $\vec{Q}$'s are two-dimensional momentum transfers and the B's are impact parameters. We note that

$$\begin{aligned}t_{ac}&=t_{\min}-\frac{1}{x}(\vec{p}_c+x\vec{Q}_a+\vec{Q}_c)^2\,,\\ \bar{t}_{ac}&=t_{\min}-\frac{1}{x}(\vec{p}_c+x\vec{Q}_{\bar a}+\vec{Q}_{\bar c})^2\,,\\ t_0&=-(\vec{Q}_a+\vec{Q}_c-\vec{Q}_{\bar a}-\vec{Q}_{\bar c})^2\,.\end{aligned} \tag{2.4}$$

For the parameters of elastic scattering symbolized by the χ's, we take

$$\begin{aligned}&\alpha(t)=1+\alpha'_P t\,,\quad\text{where}\quad\alpha'_P=0.25\,,\\ &\xi_\alpha=-\exp\left(-i\frac{\pi}{2}\alpha(t)\right).\end{aligned} \tag{2.5}$$

We set

$$\begin{aligned}&\beta_a(t)\beta_b(t)=D\exp(at)\,,\\ &\beta_c(t)\beta_b(t)=D'\exp(a't)\,,\end{aligned} \tag{2.6}$$

and we can relate these quantities to the opacity and the radius of interaction squared using the absorption model

$$D=4\pi Ca\,,$$

where C = opacity and

$$1=4a\lambda\,,$$

TABLE I. Absorption parameters.

Reaction	s (GeV^2)	C	λ [(GeV/c)$^{-2}$]
$\pi^- p$	100.0	0.669	0.0676
	25.0	0.691	0.0676
$\pi^+ p$	100.0	0.694	0.0729
	25.0	0.694	0.0729
$K^- p$	100.0	0.553	0.0676
	28.0	0.572	0.0676
	25.0	0.586	0.0676
$K^+ p$	100.0	0.551	0.0729
	28.0	0.516	0.0729
	14.0	0.516	0.0729

where λ = inverse of the radius of interaction squared. The values of C and λ used for the various rescatterings are given in Table I.

These identifications convert (2.3) to the Fourier transform of a Gaussian and yield

$$\chi_{ab} = i\frac{D}{4\pi A}\exp\left(-\frac{\vec{B}_{ab}^{\,2}}{4A}\right),$$
$$\chi_{cb} = i\frac{D'}{4\pi A'}\exp\left(-\frac{\vec{B}_{cb}^{\,2}}{4A'}\right), \tag{2.7}$$

where

$$A = a + \alpha_P'\ln(s_{ab}) - i\frac{\pi}{2}\alpha_P',$$
$$A' = a' + \alpha_P'\ln(s_{cb}) - i\frac{\pi}{2}\alpha_P'.$$

Finally, we obtain

$$\begin{aligned}H(t, s_{ab}, M_X^2) = \int \frac{d^2\vec{Q}_a}{(2\pi)^2}\frac{d^2\vec{Q}_c}{(2\pi)^2}\frac{d^2\vec{Q}_{\bar{a}}}{(2\pi)^2}\frac{d^2\vec{Q}_{\bar{c}}}{(2\pi)^2}&\left[(2\pi)^2\delta^2(\vec{Q}_a) + \sum_{n=1}^{\infty}\frac{(-1)^n}{nn!}\frac{D^n}{(4\pi A)^{n-1}}\exp\left(\frac{-\vec{Q}_a^{\,2}}{n}A\right)\right]\\
\times&\left[(2\pi)^2\delta^2(\vec{Q}_c) + \sum_{n=1}^{\infty}\frac{(-1)^n}{nn!}\frac{D'^n}{(4\pi A')^{n-1}}\exp\left(\frac{-\vec{Q}_c^{\,2}}{n}A'\right)\right] Y(t_{ac}, \bar{t}_{ac}, t_0, s_{ab}, M_X^2)\\
\times&\left[(2\pi)^2\delta^2(\vec{Q}_{\bar{a}}) + \sum_{n=1}^{\infty}\frac{(-1)^n}{nn!}\frac{D^n}{(4\pi A^*)^{n-1}}\exp\left(\frac{-\vec{Q}_{\bar{a}}^{\,2}}{n}A^*\right)\right]\\
\times&\left[(2\pi)^2\delta^2(\vec{Q}_{\bar{c}}) + \sum_{n=1}^{\infty}\frac{(-1)^n}{nn!}\frac{D'^n}{(4\pi A'^*)^{n-1}}\exp\left(\frac{-\vec{Q}_{\bar{c}}^{\,2}}{n}A'^*\right)\right].\end{aligned} \tag{2.8}$$

This formula was applied to evaluate all possible corrections mediated by the exchange of not more than two additional Pomerons (Fig. 4).

III. APPLICATION

Here we note that the target asymmetry[15] is given by

$$\Sigma = \frac{\mathrm{Disc}_{M_X^2}\langle -|T|+\rangle}{\mathrm{Disc}_{M_X^2}\langle +|T|+\rangle}, \tag{3.1}$$

where $\langle\lambda'|T|\lambda\rangle$ is the forward amplitude for $0^-0^-p \to 0^-0^-p$ and the protons have helicity λ and λ', respectively. The differential cross section is proportional to $\sum_\lambda\langle\lambda|T|\lambda\rangle$.

We begin by considering an M-function decomposition. The kinematics we use are shown in Fig. 5, which depicts the initial c.m. frame (a is the incoming 0^-, b is the incoming proton, and c is the outgoing 0^-). The decomposition is then given by[16]

$$H_{\lambda'\lambda}(t, s, M_X^2) = \frac{1}{2m}\bar{u}^{\lambda'}(p_b)(A + B\gamma\cdot p_a + C\gamma\cdot p_c + D\gamma\cdot p_a\gamma\cdot p_c)u^{\lambda}(p_b), \tag{3.2}$$

where m is proton mass, and if

$$p_b = (E_b, -\vec{p}), \quad p_a = (E_a, \vec{p}), \quad p_c = (E_c, \vec{q}),$$

then

$$H_{++} = A - [m^2 - E_b(E_b + E_a)]\frac{B}{m} + (E_bE_c + pq\cos\theta)\frac{C}{m} + (E_aE_c - pq\cos\theta)D$$

and

$$H_{+-} = (E_a + E_b)pq\sin\theta e^{-i\phi}\frac{D}{m}. \tag{3.3}$$

So, in the triple-Regge region, the function D effectively decouples kinematically from the nonflip amplitude and we have the motivation for our choice of "third leg" exchanges.

For the differential cross section we take the Pomeron and the f Reggeon. These exchanges are expected to dominate and do not couple strongly to a helicity-flip proton [Fig. 6(a)]. The 0^--0^--Reggeon vertex admits only ρ and A_2 exchanges for the different processes. The Pomeron and f Reggeon have the same signature and isospin so there will be no sign changes between the $\rho\rho f$, A_2A_2f and the $\rho\rho P$, A_2A_2P triple-Regge couplings. (See Table II.)

The flip term in the target asymmetry is given

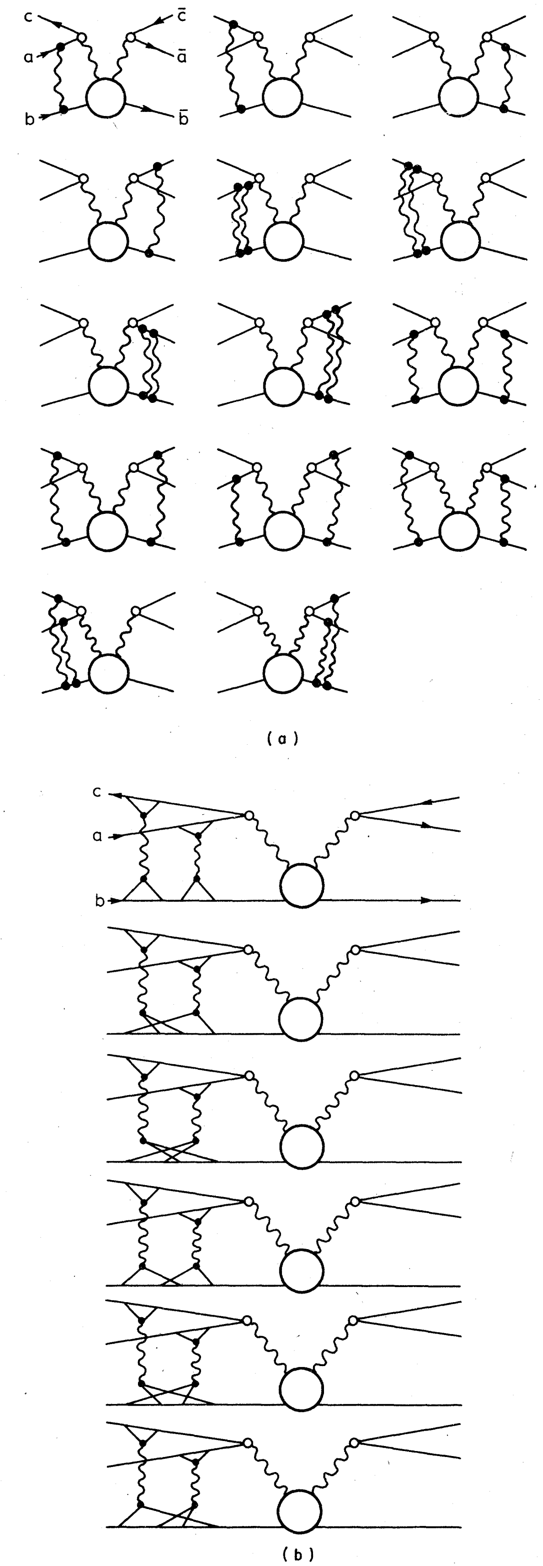

FIG. 4. (a) The rescattering-correction diagrams finally accounted for in the present paper. (b) A class of diagrams treated together in the eikonal approximation.

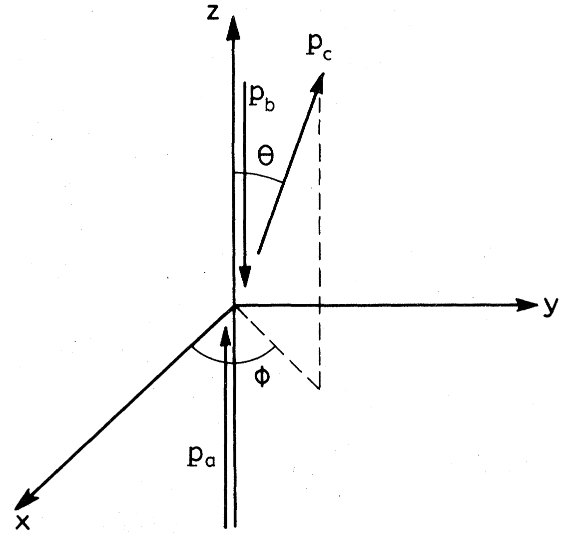

FIG. 5. The arrangement and center-of-mass kinematics used for the present calculations.

by the ρ Reggeon [Fig. 6(b)] since the helicity-flip coupling of the ρ to nucleons is large.

The generalization of the derivation of Ref. 13 to include the proton spin (Appendix A) allows us to use (2.8) for the flip amplitude with the replacement

$$\begin{aligned} Y_\rho(t_{ac}, \bar{t}_{ac}, t_0, s, M_X^2) &= \bar{u}^-(p_{\bar b})(\not{p}'_{\bar b} + m)\Gamma^{+-}_{b\bar b}(\not{p}'_b + m) \\ &\times u^+(p_b)\tilde{Y}_\rho(t_{ac}, \bar{t}_{ac}, t_0, s, M_X^2) , \quad (3.4) \end{aligned}$$

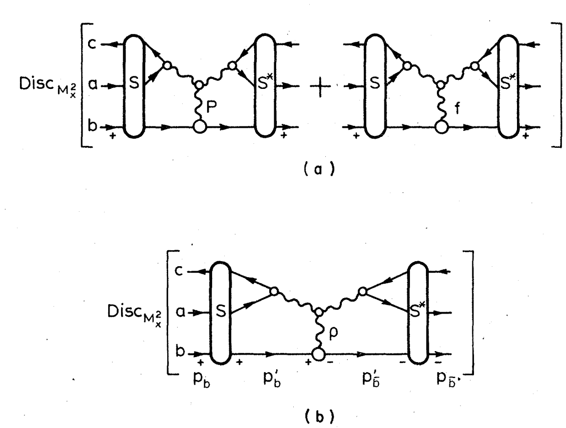

FIG. 6. (a) The two discontinuities taken to give rise to the differential cross section in the present model. The Pomeron and f Reggeon contribute to the Reggeon-particle b nonflip scattering. (b) The discontinuity which forms the target asymmetry. The Reggeon-particle b scattering is mediated by the ρ Reggeon which is taken to predominantly flip the spin of particle b.

where $\Gamma^{+-}_{b\bar{b}}$ is some gamma matrix formed from the available vectors. The spinor expression reproduces the threshold behavior given in Ref. 16 and gives a factor of $\sqrt{-t_0}$ as demanded by a factorizing Regge-pole vertex.[17] We replace the spinor expression by a multiplicative factor of sufficient generality to produce this behavior. Such a factor is[4]

$$p_{c_\perp}\cos\phi + Q_{a_1} + Q_{c_1} - i(p_{c_\perp}\sin\phi + Q_{a_2} + Q_{c_2}) - [p_{c_\perp}\cos\phi + Q_{\bar{a}_1} + Q_{\bar{c}_1} - i(p_{c_\perp}\sin\phi + Q_{\bar{a}_2} + Q_{\bar{c}_2})], \tag{3.5}$$

since it gives both the correct ϕ and $p_{c_\perp}$ dependences and goes to zero like $\sqrt{-t_0}$. Our calculation is performed with the azimuthal angle of the produced particle, ϕ, to be zero. The ordering of the factor corresponds to the + – ordering in the definition of the target asymmetry.

It remains to evaluate the off-forward [Fig. 3(a)] triple-Regge expressions

$$Y_\rho(t_{ac}, \bar{t}_{ac}, t_0, s, M_X^2),$$

$$Y_f(t_{ac}, \bar{t}_{ac}, t_0, s, M_X^2),$$

and

$$\tilde{Y}_\rho(t_{ac}, \bar{t}_{ac}, t_0, s, M_X^2).$$

A detailed consideration of the $\pi\pi p \to \pi\pi p$ off-forward triple-Regge terms is given in Appendix B; the generalization to all other reactions is straightforward. We use isospin to determine the relative signs of the flip (ρ) to the nonflip ($P+f$) amplitude (Table III). There is also an isospin factor of $(\frac{2}{3})^{1/2}$ which enters for all the reactions. Strong-exchange degeneracy is assumed between the ρ and the A_2 and the SU(3) factors are given in Table III.

Further details of the integral calculations are given in Appendix C.

IV. DISCUSSION

Recently, several methods for calculating absorption or Regge-cut corrections to single-particle-inclusive distributions have been discussed. Craigie and Kramer[3] have indicated that exceptionally large absorption-type corrections in the a-b channel are required to reproduce the $\gamma p \to \pi^{\pm 0}X$ data, which do not exhibit the $p_\perp^2$ dip predicted by simple pole naturality considerations. One possible explanation of this fact is that their calculation contains only one term with interactions on both incoming and outgoing sides and only this type of term could fill in the forward dip.

Pumplin[11] gives arguments to show that c-b rescattering corrections will be important and shows that, in his scheme, it is not possible to fill in the forward dip in $\gamma p \to \pi^{\pm}X$ with reasonable absorption parameters. Pumplin also calculates the absorbed $\pi^- p \to \pi^0 X$ amplitude, which will be compared with our own later. Capella *et al.*[18] use an eikonal approach in the a-b channel, valid at $x \simeq 1$, which indicates that the resulting corrections will be much stronger than in two-body reactions. However, Azcarate[19] has performed a calculation on $pp \to nX$ in which he claims the data are incompatible with absorption of the "strong" Capella type, but compatible with that of the "weak" Craigie type. Goldstein and Owens[20] have made an alternative derivation of impact-parameter–type absorption in the a-b channel which is a small improvement on the Craigie type theoretically, although they admit it should present few differences phenomenologically.

All these approaches lead to similar results and treat one channel (either a-b or c-b) preferentially. Alternatively, Paige, Sidhu, and Trueman, in a series of papers,[21,22] develop a Reggeon-calculus method and pick out the Regge-cut terms which they feel will be dominant. These terms have one further Reggeon exchanged on a triple-Regge graph. Their estimates give these cut graphs as possibly 30% the size of the original pole graphs. The interesting point of this formulation is that it ensures a similar treatment of both the a-b and c-b channels, but because of the complexity of the formulation it is difficult to include higher-order terms.

Bartels and Kramer[23] have also used Reggeon calculus, but in the eikonal approximation, to study

TABLE II. Regge parameters.

Trajectory	Intercept	Slope [(GeV/c)$^{-2}$]	Signature	Isospin
P	1.0	0.25	+1	0
ρ	0.47	0.905	−1	1
A_2	0.47	0.905	+1	1
f	0.4	1.0	+1	0

TABLE III. SU(3) factors and relative signs of triple-Regge couplings.

Reaction	SU(3) factors ρ	A_2	Relative sign
$\pi^- p \to \pi^0 X$	2	0	−
$\pi^- p \to \eta X$	0	$2/\sqrt{3}$	−
$\pi^+ p \to \pi^0 X$	2	0	+
$\pi^+ p \to \eta X$	0	$2/\sqrt{3}$	+
$K^+ p \to K^0 X$	$\sqrt{2}$	$\sqrt{2}$	+
$K^- p \to \bar{K}^0 X$	$-\sqrt{2}$	$\sqrt{2}$	−

the importance of both multi-Pomeron exchange and "enhanced" (more than one triple-Pomeron vertex) graphs. They conclude that not only must several terms of the eikonal expansion be included at lower energies, but also that at higher energies enhanced graphs could become important. Bartels and Kramer also make the point that, from their approach, our formulation appears to be double counting by adding both the a-b and c-b channel eikonal phases. They are probably correct for $x = 1$, but at $x < 1$ there must be substantial longitudinal momentum flowing down at least one Reggeon and it is not clear that their argument holds. It must be stressed that our approach is a hybrid s channel $(3 \to 3)$ while theirs is a t channel $(4 \to 2)$ and so direct comparisons are not freely available. So we conclude from the literature that the absorption approach takes account of either the a-b or the c-b rescattering channels while the Reggeon-calculus approach indicates that at least some higher Pomeron terms must be included. The present formulation is radically different from Reggeon calculus and can reproduce some of the absorption-type formulas by setting equal to zero one or another of the eikonal phases; there is also the option of including various higher-order corrections which seem likely to be important.

Initially, we expected the size of the correction, induced by our formula, to be considerably larger than that of a Craigie-Kramer–type model. However, the inclusion of higher-order corrections has reduced the overall size of the cut terms compared to our first simpler model.

We now turn to a detailed consideration of the results of our calculation which are embodied in Figs. 7–13. For fixed M^2/s we have included a plot with $M^2/s = 0.5$ and for fixed t the plots extend to $M^2/s = 0.5$, but we should mention that the model is only expected to be valid up to $M^2/s = 0.25$–0.30 (since above this level the $t_{ac}, \bar{t}_{ac}$ legs cannot be expected to Reggeize).

We first consider the differential-cross-section plots. In the reactions $\pi^{\pm}p \to \pi^0 X$ (Figs. 7 and 8), where wrong-signature nonsense zeros are incorporated into the signature factor, a dip can be expected in the pole only term at about $-t = 0.5$ (GeV/$c)^2$. Our previous absorption model merely moved this dip slightly towards $t = 0$, but left it as a pronounced effect. The present calculation eradicates the dip altogether for small M^2/s and the final curve is by no means dissimilar in shape to that of Pumplin[11] who has also studied the reaction $\pi^- p \to \pi^0 X$ although at a different energy. However, Pumplin uses a signature factor of $\exp(-i\pi\alpha/2)$ and would rely on absorption to introduce any dips. The normalizations of the two pole terms are slightly different which will account for the differing normalizations of the final curves.

For the reactions $\pi^{\pm}p \to \eta X$ (Figs. 9 and 10), the pole and final corrected graphs show much similarity, especially at small $|t|$, unlike the simpler model which shows some dip structure for small M^2/s at around $-t = 0.7$ (GeV/$c)^2$. There is of course a normalization change which at small $|t|$ is about a factor of 1.5 to 2.

Figure 11 shows the reaction $K^- p \to \bar{K}^0 X$ at a lower energy with the data taken from Ref. 12. The final curves represent the t variation at least as well as the pole graph and have better normalization, but the M^2/s variation is not well accounted for by either pole or final corrected graphs. This may possibly be explained by the inclusion of f exchange in the t_0 leg. This exchange dies away as M^2/s increases, thereby lessening the slope. However, since the ρp channel cannot properly be considered exotic, there is no reason to exclude this exchange on duality grounds. Further work on arriving at a fit using the final model is being considered. Figures 12 and 13 show the reactions $K^- p \to \bar{K}^0 X$ and $K^+ p \to K^0 X$ at the higher energy and the general trend is continued. Figure 13(a) especially shows where the final model remedies a major defect of the simpler model, which predicted a dip at $M^2/s = 0.05$ and $-t = 0.75$ (GeV/$c)^2$. This dip arose from the over-simple nature of the model.

The class of reactions we are studying has only two observables, the differential cross section and the target asymmetry. In an attempt at completeness, we have calculated the target asymmetry even though further approximations had to be made.

It is well known that either a factorizable Regge pole[15,17] or a fixed naturality exchange[16,20] for the t_0 leg will give zero target asymmetry. The angular kinematical behavior of any asymmetry present is well established[16,17,20,24]; most importantly, it will go to zero as $p_{c_\perp}$. Our calculation of the target asymmetry ensures that the kinematical constraints are automatically satisfied, but the kinematic suppression makes the target asymmetry small and so it is difficult to measure with any real significance experimentally.

The target asymmetries presented in Figs. 7–13 not only have the correct $p_{c_\perp}$ behavior but also are all small, the largest being less than 2%. Some dynamical structure can be seen, especially in $\pi^{\pm}p \to \pi^0 X$, but if experimental measurements were of the same order (about 2%) it would still be very hard either to confirm or refute this structure.

If large (~10%) target asymmetries are seen experimentally, then our simple prescription for an off forward, but factorizable, Regge-pole exchange for the t_0 leg will have to be abandoned in favor of

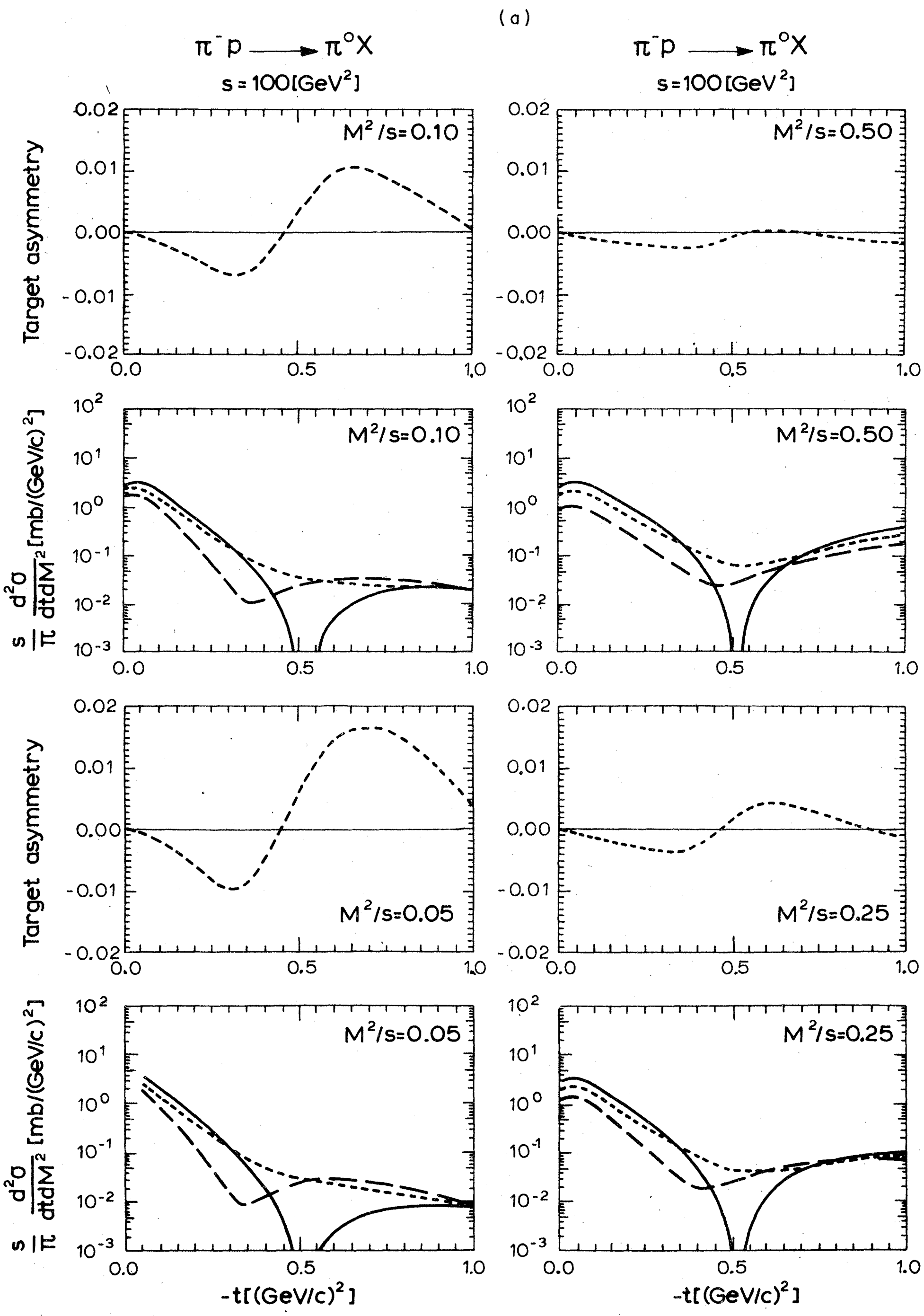

FIG. 7. Differential cross section and target asymmetry for the reaction $\pi^- + p \rightarrow \pi^0 + X$ for various fixed values of (a) M^2/s and (b) t. For all the differential-cross-section and target-asymmetry diagrams from Fig. 7 to Fig. 13, —— represents the pole-only contribution, – – – – represents the absorbed curve of the previous model (Ref. 9), and ----- represents the absorbed curve of the present model. The target-asymmetry curves are integrated over a t bin of width 0.2 $(\mathrm{GeV}/c)^2$ by eight-point Gauss-Legendre quadrature and averaged to yield the curves shown.

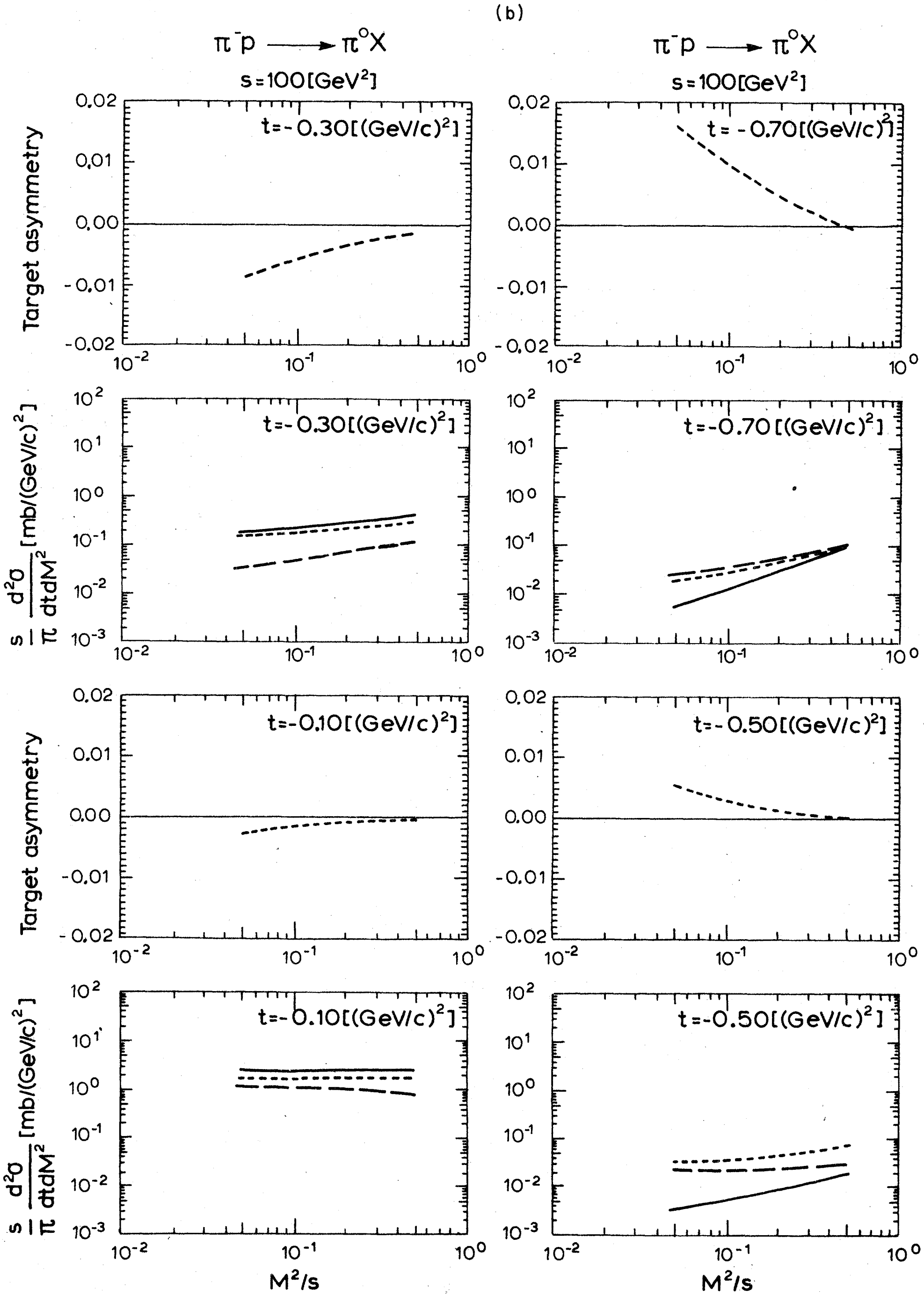

FIG. 7. (*Continued*)

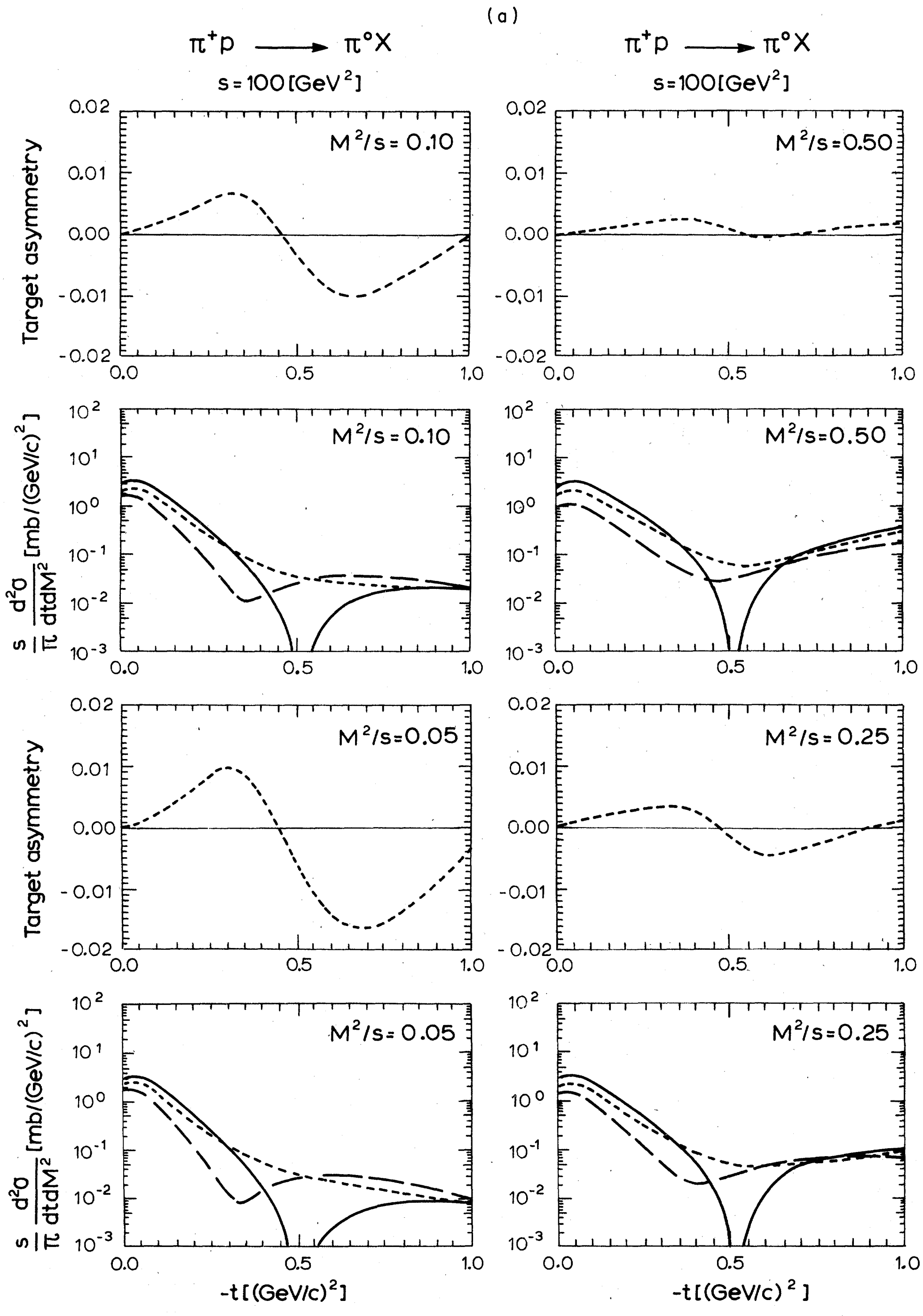

FIG. 8. Differential cross section and the target asymmetry for the reaction $\pi^+ + p \rightarrow \pi^0 + X$ for various fixed values of (a) M^2/s and (b) t. See caption of Fig. 7.

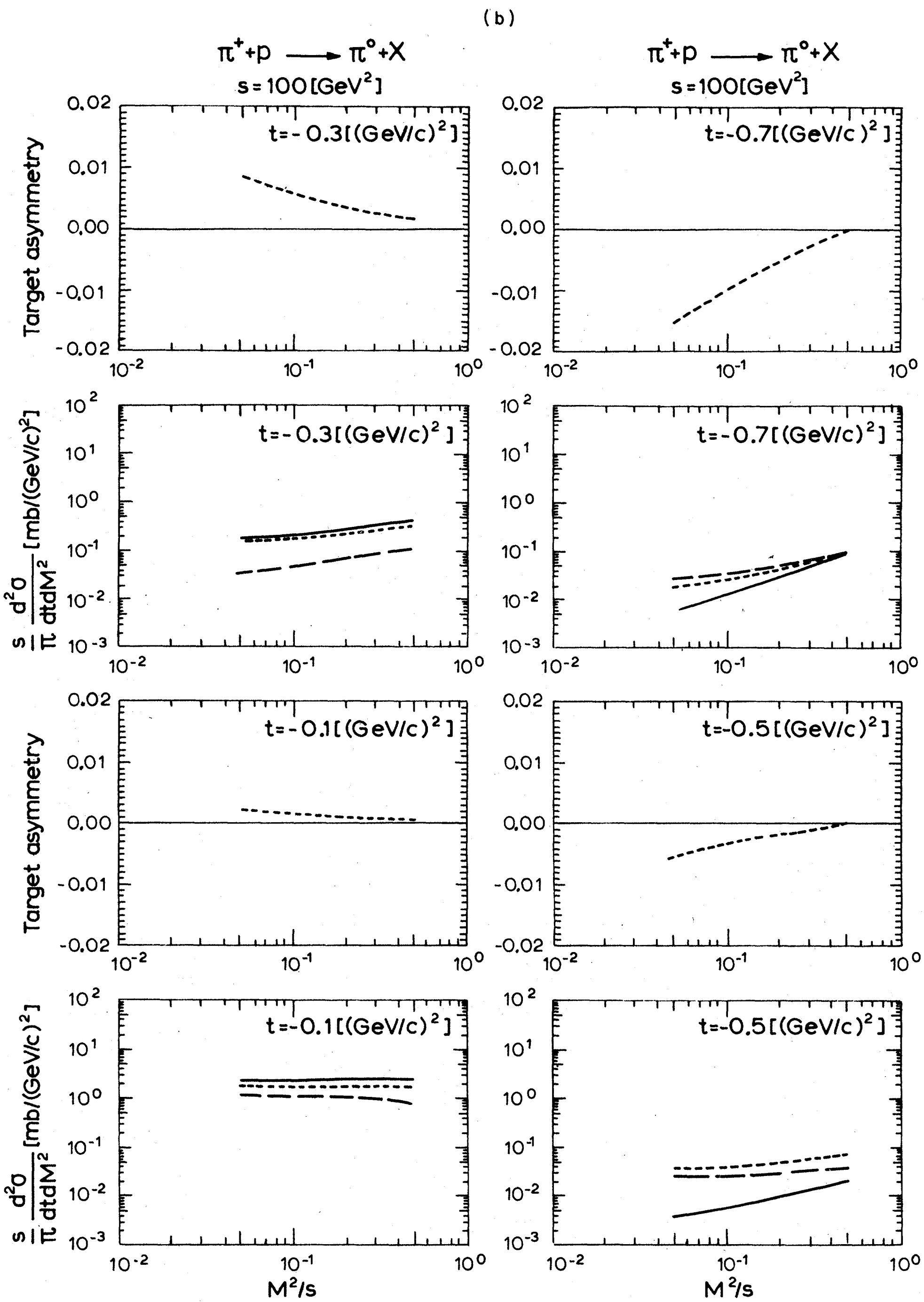

FIG. 8. *(Continued)*

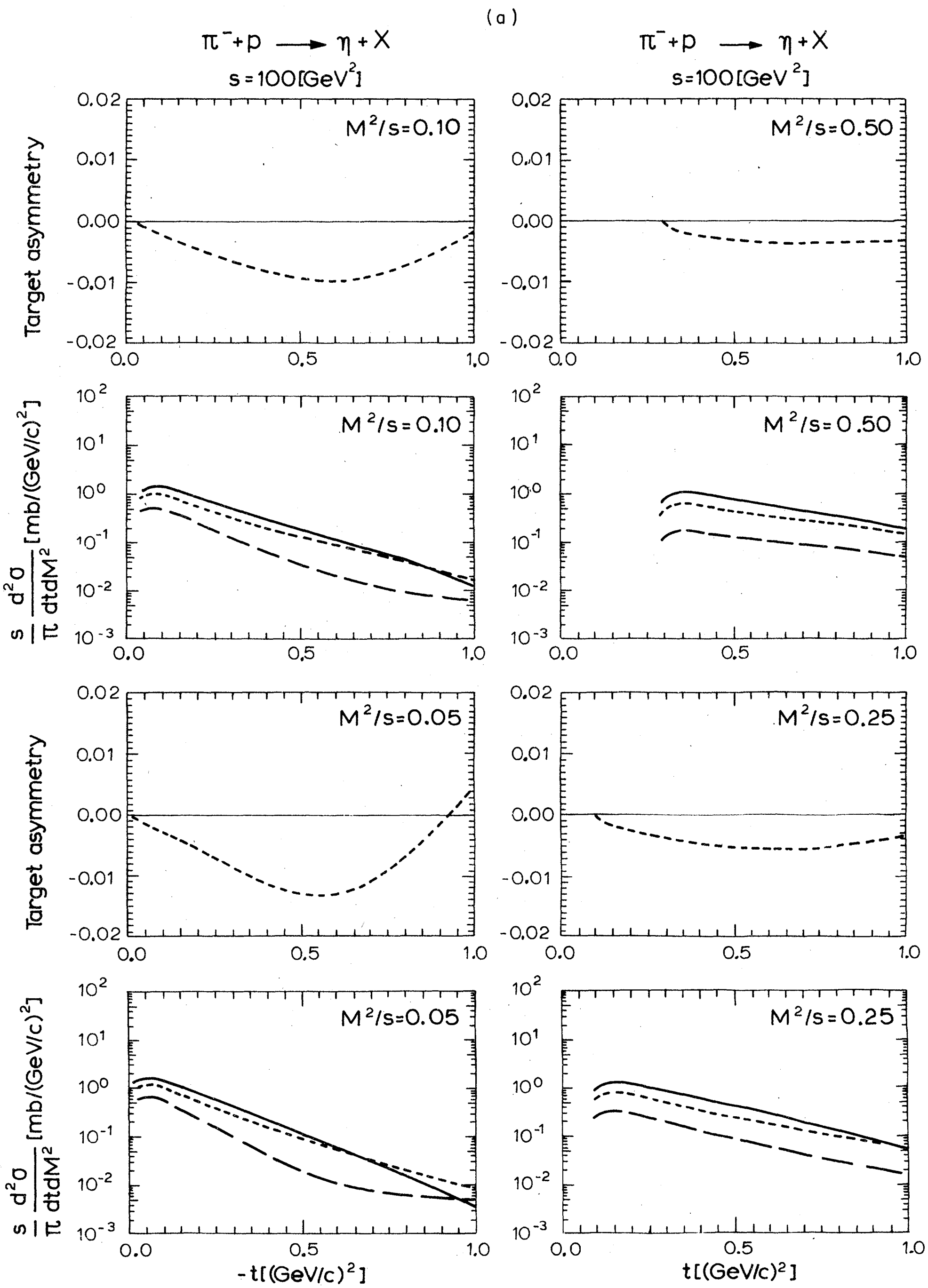

FIG. 9. Differential cross section and the target asymmetry for the reaction $\pi^- + p \to \eta + X$ at various fixed values of (a) M^2/s and (b) t. See caption of Fig. 7.

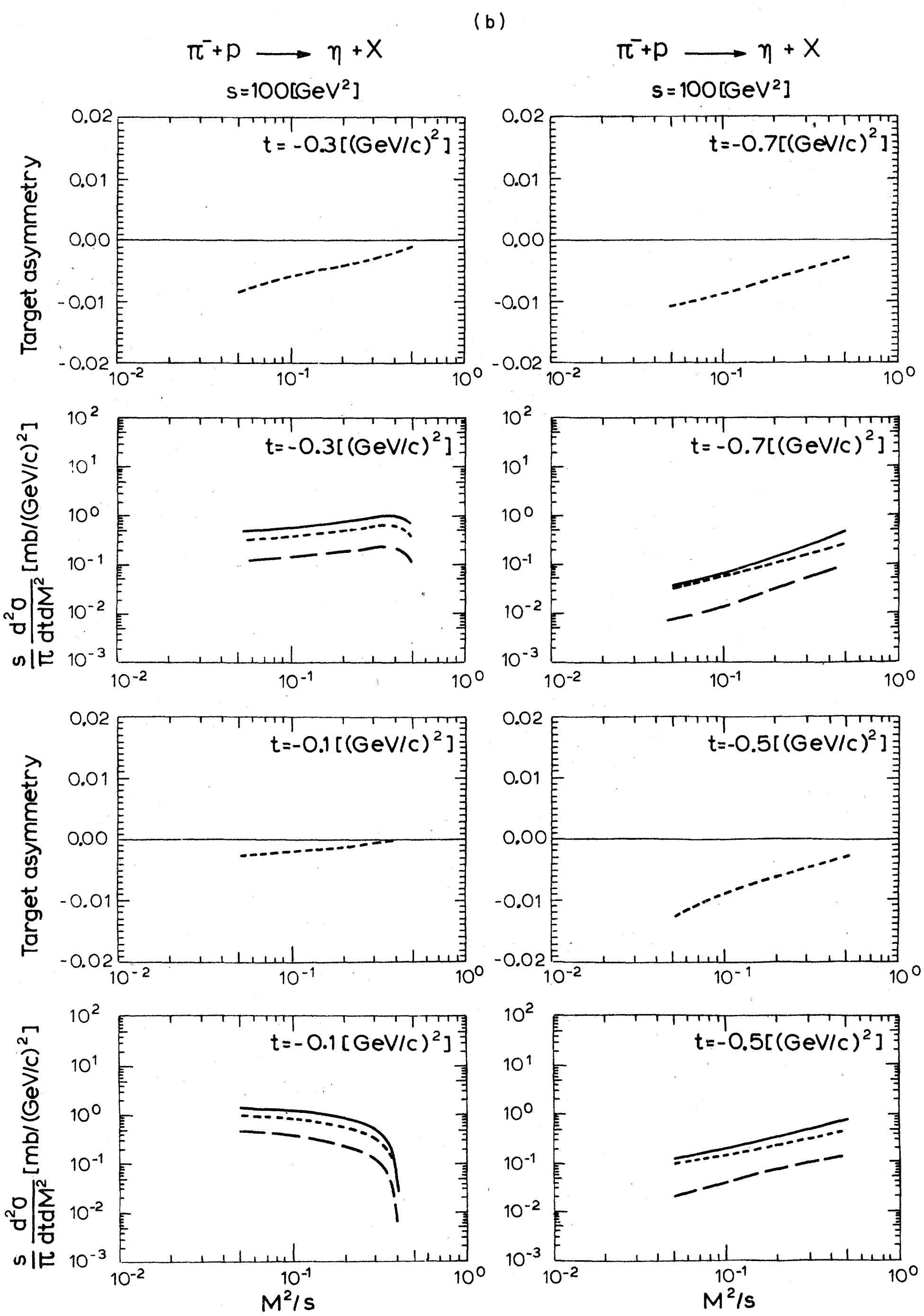

FIG. 9. *(Continued)*

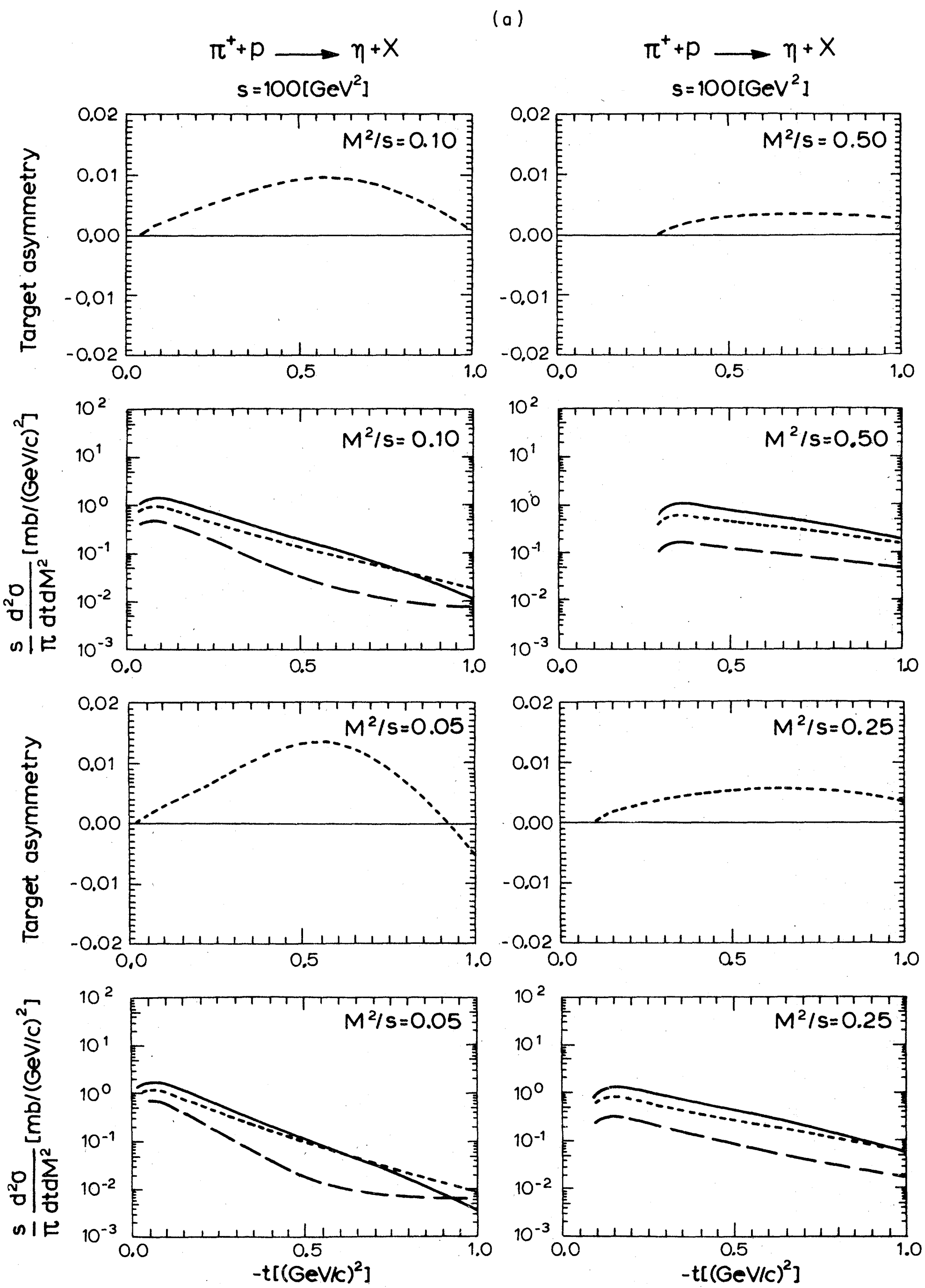

FIG. 10. Differential cross section and the target asymmetry for the reaction $\pi^+ + p \rightarrow \eta + X$ at various fixed values of (a) M^2/s and (b) t. See caption of Fig. 7.

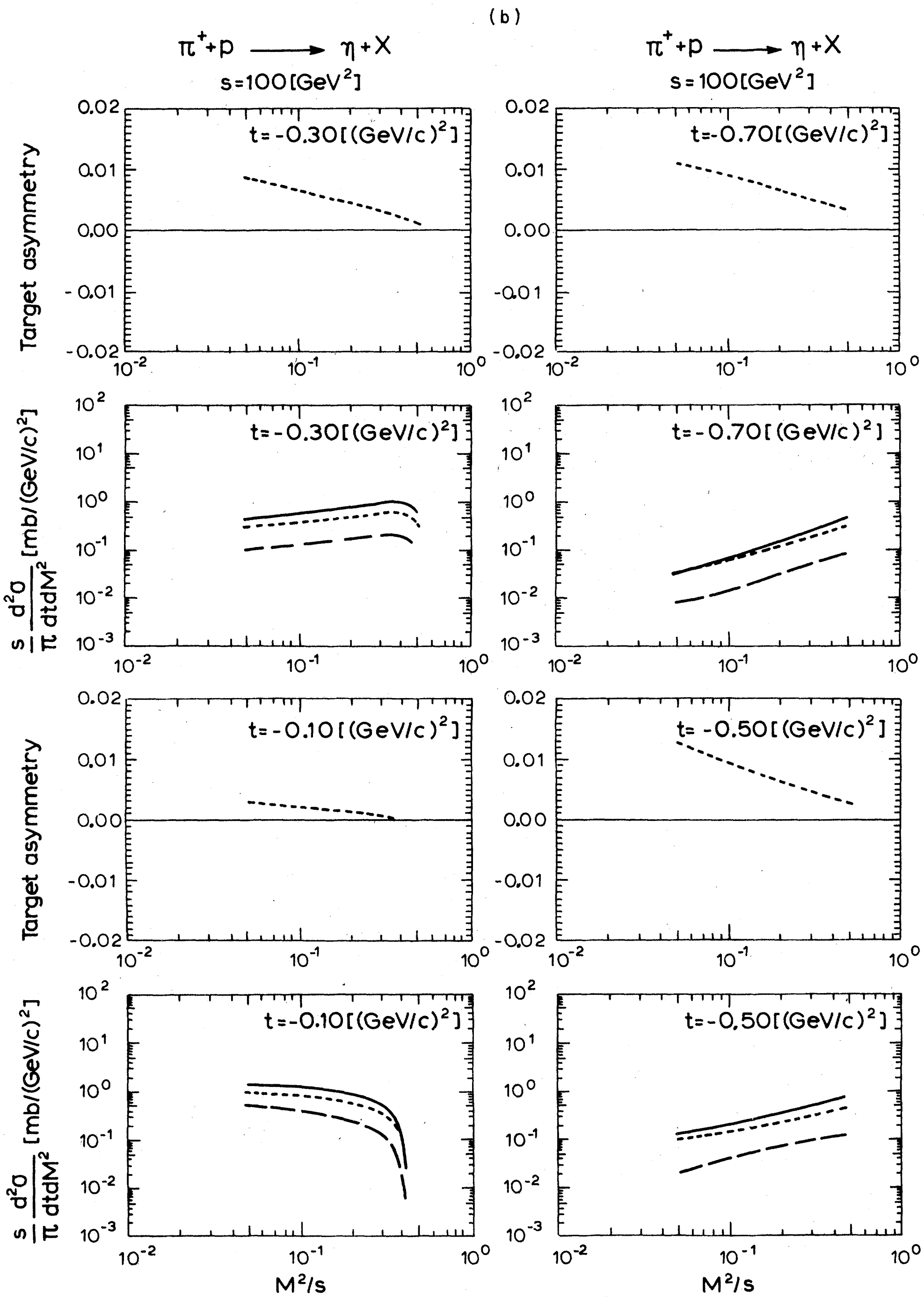

FIG. 10. *(Continued)*

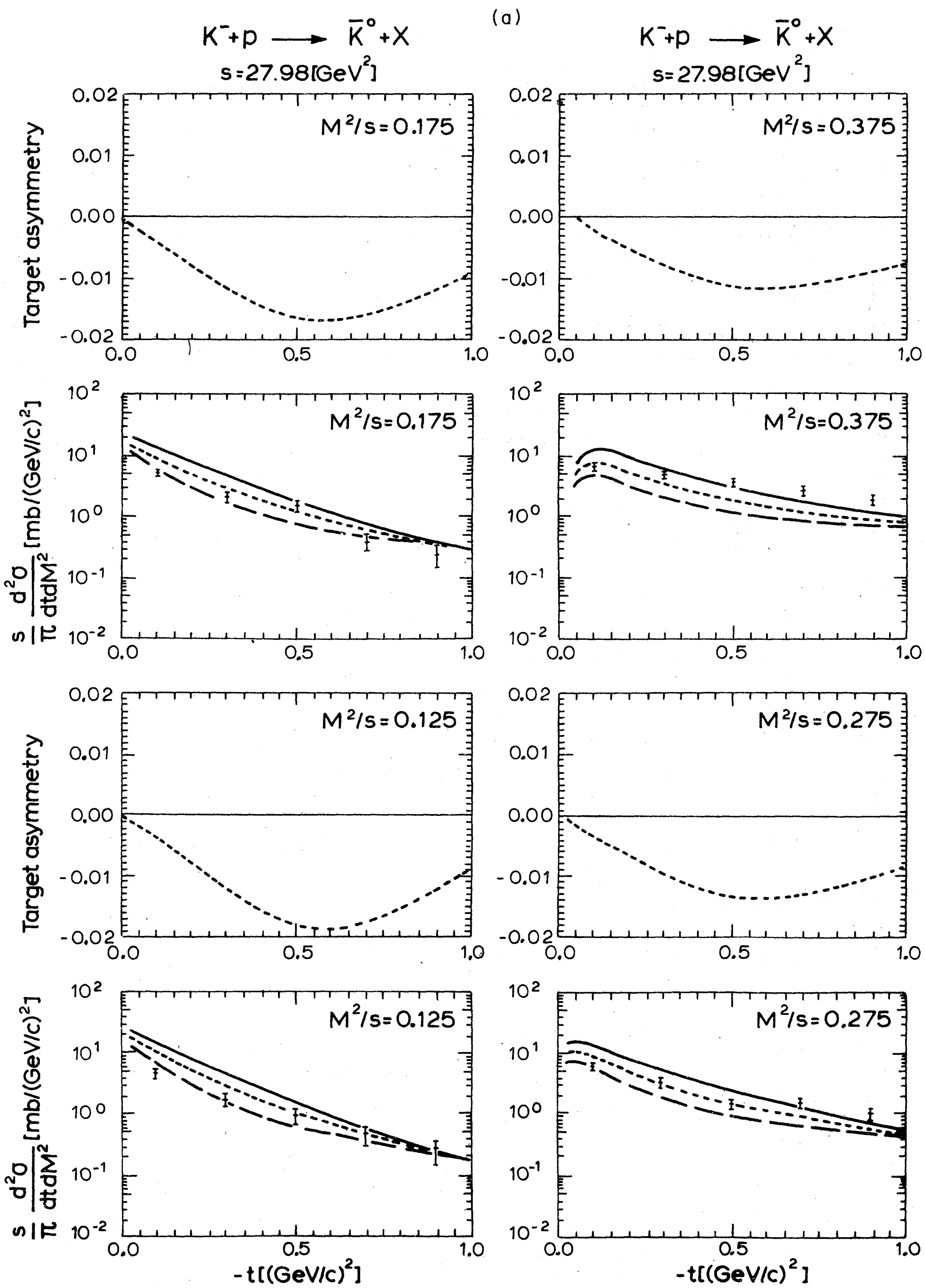

FIG. 11. Differential cross section and the target asymmetry at low energy for the reaction $K^- + p \to \bar{K}^0 + X$ at various fixed values of (a) M^2/s and (b) t. The data are from Ref. 12. See caption of Fig. 7.

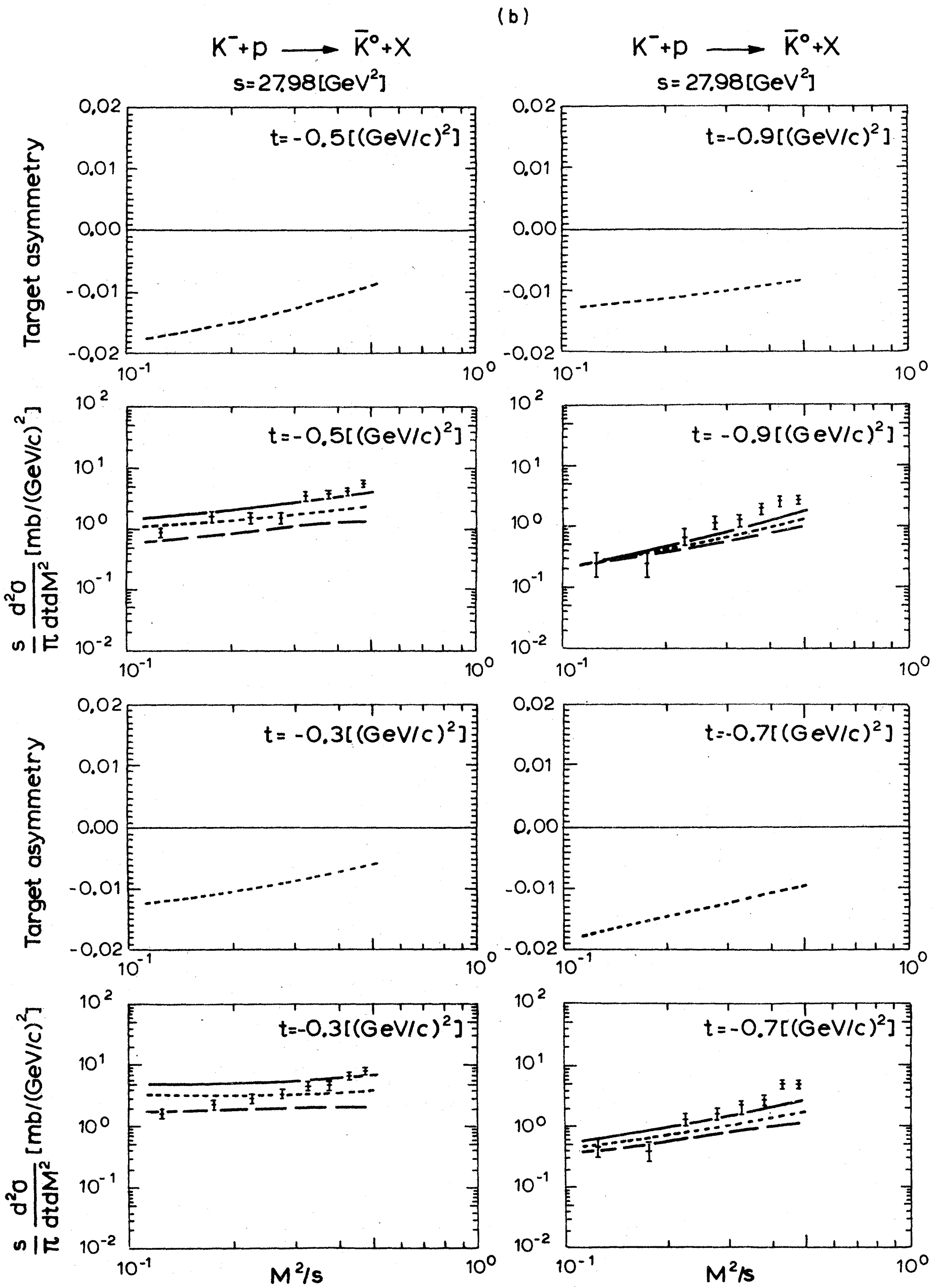

FIG. 11. *(Continued)*

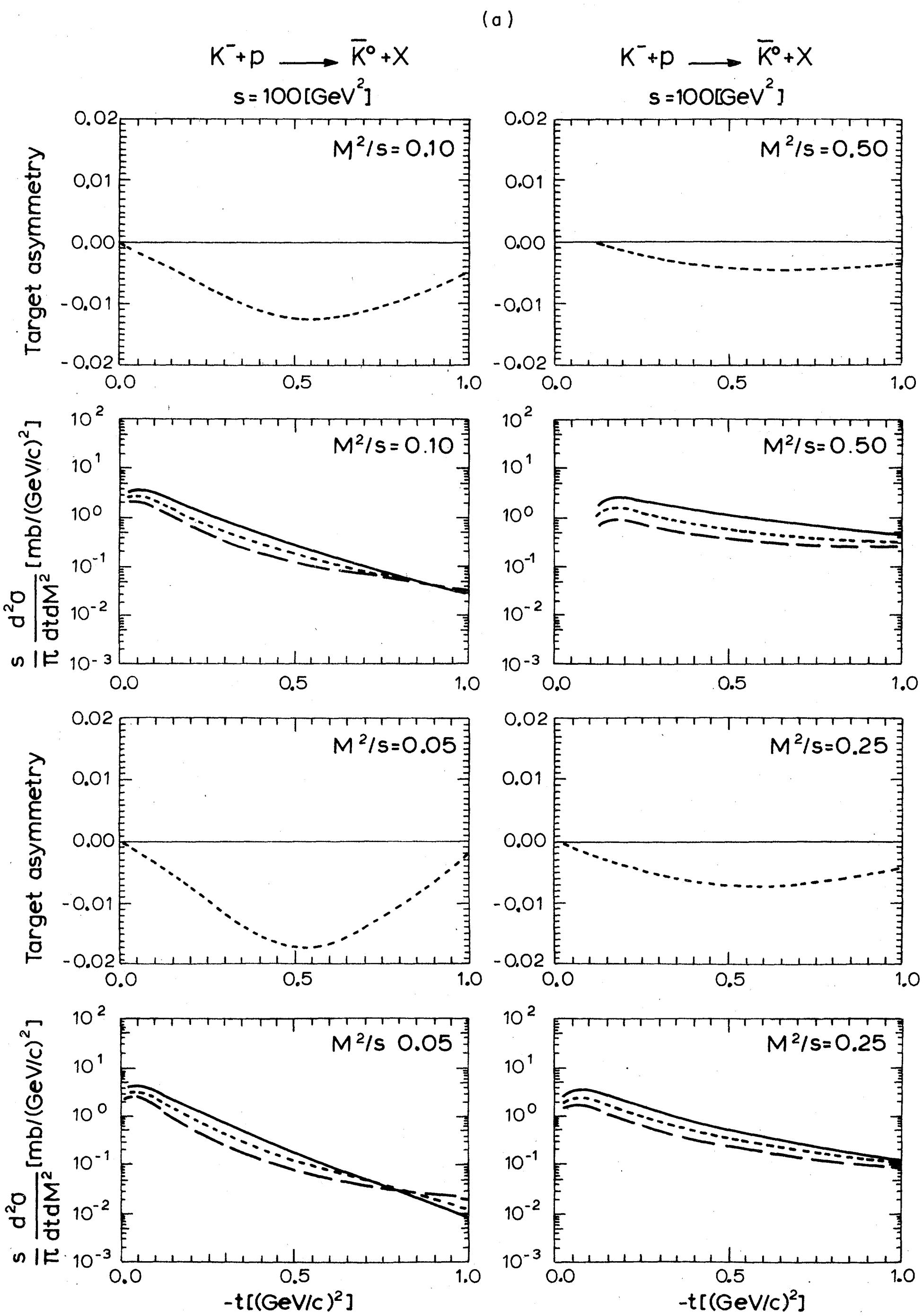

FIG. 12. Differential cross section and the target asymmetry for the reaction $K^- + p \to \bar{K}^0 + X$ at various fixed values of (a) M^2/s and (b) t. See caption of Fig. 7.

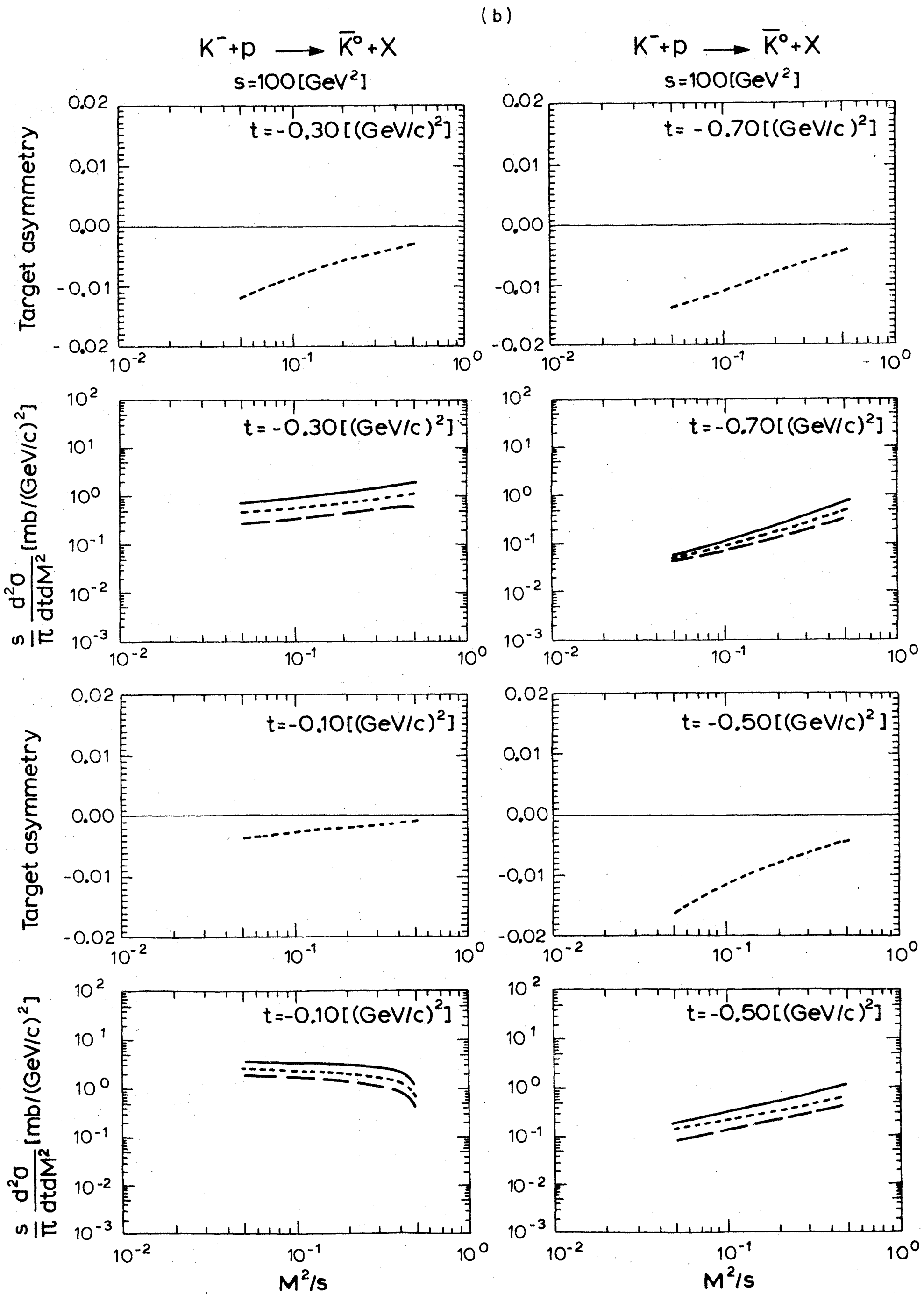

FIG. 12. *(Continued)*

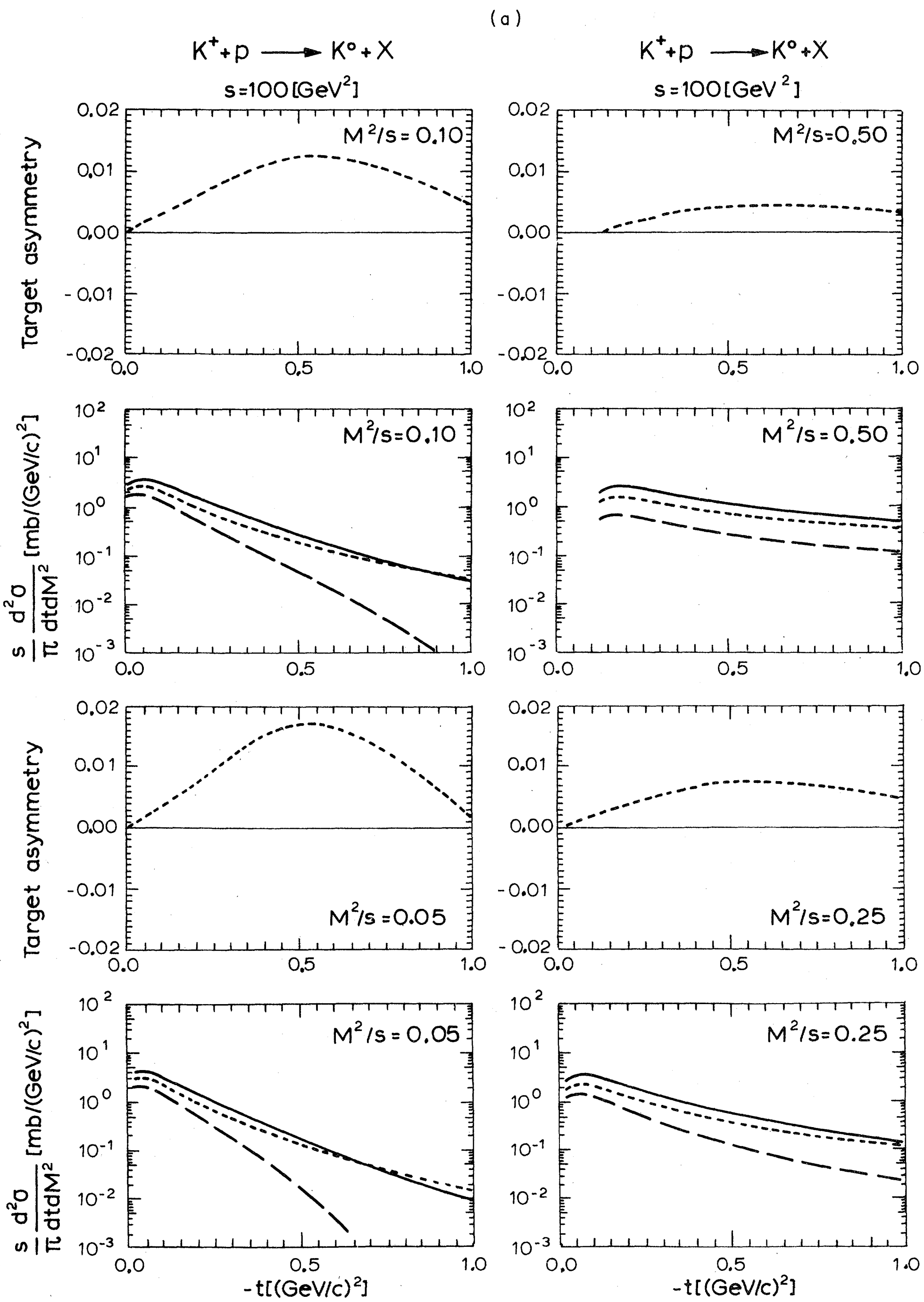

FIG. 13. Differential cross section and the target asymmetry for the reaction $K^+ + p \to K^0 + X$ at various fixed values of (a) M^2/s and (b) t. See caption of Fig. 7.

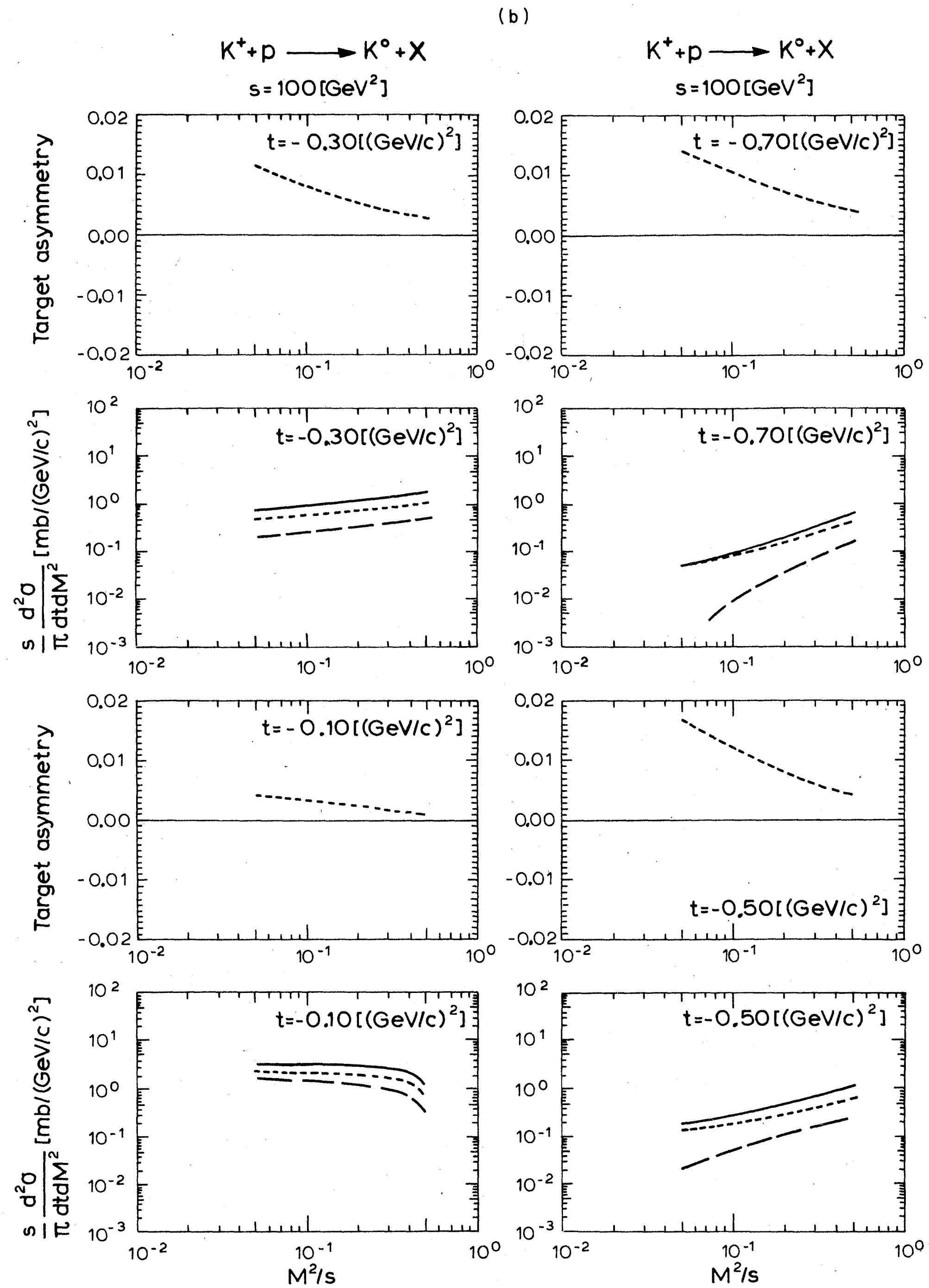

FIG. 13. (*Continued*)

some other mechanism. One such mechanism is the exchange of a ρP cut down the t_0 leg. This is not a pure naturality exchange and so is not constrained to give a zero flip contribution for $t_0=0$ and therefore could be considered without any absorption-type corrections of the type considered here. Soffer and Wray[25] have considered just such a mechanism, but they were obliged to insert the kinematical factor $\sqrt{-t}$ by hand. Their model is compared with data by Dick *et al.*[26] The data were obtained in a $\pi^{\pm}p\to\pi^{\pm}X$ experiment at $p_{1ab}=8$ (GeV/c). For $0.5<x<0.8$, the model of Soffer and Wray gives a target asymmetry of about 7% rising to 10% at $x=0.95$. Alternatively, for elastic data the asymmetry is about 17% and when M_X has risen to 2.0 GeV ($x=0.75$) the target asymmetry cannot be said to be significantly different from zero. This could be explained if, when Regge-pole exchange is expected to dominate (i.e., $M_X{}^2>4.0$ GeV^2), the target asymmetry does become small, as predicted by a factorizable pole and our model.

All the calculations of differential cross sections and target asymmetries were carried out numerically.[27] All the figures were, for accuracy, plotted by computer.[28]

Note added in proof. After completion of this work we received notice of papers[29,30] containing experimental results for the reactions $\pi^{\pm}p\to\pi^0X$ and $\pi^{\pm}p\to\eta X$, which amply confirm the predictions of the present paper. In particular the reported experiment confirms the following:

(1) Our assumption of helicity-nonflip dominance at the inclusive vertex at small momentum transfers for the exchanged Reggeon is indeed correct. This is shown by the forward peak in the t distribution for fixed M^2/s of the inclusive differential cross section. (See Fig. 11 of Ref. 30.)

(2) Our assumption of the presence of wrong-signature nonsense zeros in the triple-Regge-pole amplitudes, which would correspond to the Reggeized Born term model of Appendix A of Irving and Worden,[31] is correct. This is shown by the dip in the inclusive differential cross section at $-t\simeq 0.5$ (GeV/c)^2. (See Fig. 11 of Ref. 30.)

(3) Our assumption of absorption corrections to the pure triple-Regge-pole graph is correct. This is shown by the fact that the dip in the inclusive differential cross section is, in fact, a dip and not an exact zero. (See Fig. 11 of Ref. 30.)

All these facts, taken together with the evidence mentioned in the Introduction to this paper, provide powerful motivation for the further study of the contribution of Regge cuts to single-particle inclusive reactions and point undeniably to their existence.

ACKNOWLEDGMENTS

We wish to thank Professor H. G. Eggleston for encouragement in this work. Two of the authors (J. H. T. and H. N. T.) wish to thank the Science Research Council of Great Britain for Research Studentships.

APPENDIX A

The technique of Abarbanel and Itzykson[14] is used in Ref. 13 to derive the relativistic eikonal approximation via Schwinger's use of functional derivatives.[32]

In this case we consider [Figs. 3(a) and 3(b)]

$$\tau(B)=\lim_{\epsilon\to 0^+}\lim_{p_b{}^2\to m^2}\lim_{p_b'{}^2\to m^2}(p_b{}^2-m^2)(p_b'{}^2-m^2)\langle p_b'\lambda_b'|G(B)|p_b\lambda_b\rangle\,, \tag{A1}$$

where λ_b,λ_b' are the particle helicities. We have

$$G(B)=\frac{1}{\not P-m-gB(x)+i\epsilon}=\frac{\not P+m}{P^2-m^2}+\frac{(\not P+m)\,gB(x)(\not P+m)}{P^2-m^2-(\not P+m)[gB(x)-i\epsilon](P^2-m^2)}\,. \tag{A2}$$

We use a formal integral representation and the operator identity

$$\exp(A+B)=(\exp A)\,T\exp\left(\int_0^1 dt\exp(-At)B\exp(At)\right) \tag{A3}$$

to see that, sandwiched between the states,

$$G(B)\simeq\int_0^\infty dt\exp\{i[P^2-m^2+i\epsilon(\not P+m)]t\}\,T\exp\left[-i\int_0^t d\tau\exp[-i(P^2+i\epsilon\not P)\tau]B(x)\exp[i(P^2+i\epsilon\not P)\tau]\right]\,. \tag{A4}$$

Inserting this in (A1) yields

$$\tau(B)=\left\langle p_b'\right|T\exp\left(-i\int_0^\infty d\tau(\not P+m)gB(x-2P\tau)\right)(\not P+m)gB(x)(\not P+m)\left|p_b\right\rangle\,. \tag{A5}$$

We now make the eikonal approximation by replacing the operator P by $\bar{p}_b = \frac{1}{2}(p_b + p_b')$, a c-number vector, and ignoring the time ordering. Using plane-wave representations of the states, we get

$$\tau_E(B) = \bar{u}^{\lambda_b'}(p_b')\left\{\int \frac{d^4x}{(2\pi)^4} \exp[i(p_b - p_b')\cdot x]\frac{\partial}{\partial\alpha_b}\exp\left[-ig\int_{\alpha_b}^{\infty}(\bar{\not p}_b + m)B(x - 2\bar{p}_b\tau)\,d\tau\right]\Bigg|_{\alpha_b=0}\right\}u^{\lambda}(p_b)2m\,. \tag{A6}$$

Expanding $\exp(\bar{\not p}_b + m)$, we note that[33]

$$\bar{u}^{\lambda_b'}(p_b')(\bar{\not p}_b + m)^n u^{\lambda_b}(p_b) \simeq (2m)^n \bar{u}^{\lambda_b'}(p_b')u^{\lambda_b}(p_b)\,, \tag{A7}$$

and obtain

$$\tau_E(B) = \bar{u}^{\lambda_b'}(p_b')\left\{\int \frac{d^4x}{(2\pi)^4}\exp[i(p_b - p_b')\cdot x]\frac{\partial}{\partial\alpha_b}\exp\left[-ig2m\int_{\alpha_b}^{\infty}B(x - 2\bar{p}_b\tau)d\tau\right]\Bigg|_{\alpha_b=0}\right\}2mu^{\lambda_b}(p_b)\,. \tag{A8}$$

This has the same form as was used in Ref. 13, except for the addition of spinors. Thus, we must make the following replacement in (2.8):

$$Y(t_{ac}, \bar{t}_{ac}, t_0, s, M_X^2) = \bar{u}^{\lambda_{\bar{b}}}(p_{\bar{b}})(\not p_{\bar{b}}' + m)\tilde{Y}(t_{ac}, \bar{t}_{ac}, t_0, s, M_X^2)(\not p_b' + m)u^{\lambda_b}(p_b)\,.$$

APPENDIX B

The off-forward triple-Regge expression for $\pi\pi p \to \pi\pi p$ has the general form

$$Y_R(t_{ac}, \bar{t}_{ac}, t_0, s, M_X^2) = \beta_{\pi\pi\rho}(t_{ac})\xi_\rho(t_{ac})\left(\frac{s}{M_X^2}\right)^{\alpha_\rho(t_{ac})}\beta_{\pi\pi\rho}(\bar{t}_{ac})\,\xi_\rho(\bar{t}_{ac})\left(\frac{s}{M_X^2}\right)^{\alpha_\rho(\bar{t}_{ac})}$$
$$\times G_{\rho\rho R}(t_{ac}, \bar{t}_{ac}, t_0)\xi_R(t_0)(M_X^2)^{\alpha_R(t_0)}\beta_{pp R}^{\lambda\lambda'}(t_0)\,, \tag{B1}$$

where R stands for the Pomeron, the f, or the ρ Reggeon, the λ's are the proton helicities, and ξ is the signature factor.

We first consider forward scattering, exchanging an elementary ρ meson, and then generalize the result. The current for the $\rho\pi^-\pi^0$ vertex is

$$\frac{g_{\rho\pi\pi}}{2}P_\mu(\bar{\phi}_{\pi^0}\phi_{\pi^-})_F\,,$$

where

$$P_\mu = (p_{\pi^-} + p_{\pi^0})_\mu\,,$$

and

$$(\bar{\phi}_{\pi^0}\phi_{\pi^-})_F = \mathrm{SU}(3)\text{ factor}\,.$$

The elementary ρ propagator is

$$\frac{-g^{\mu\nu} + Q^\mu Q^\nu/m_\rho^2}{t - m_\rho^2}\,,$$

where

$$Q^\mu = (p_{\pi^-} - p_{\pi^0})^\mu\,,$$

and a factor of Γ_ν^X arises from the inclusive vertex.

Squaring and summing over final states we get a second-rank tensor of the form

$$\mathrm{Disc}(V_1)g_{\mu\nu} + \mathrm{Disc}(V_2)p_\mu p_\nu = \sum_X \Gamma_\mu^X\Gamma_\nu^{x}\,,$$

where p_μ is the momentum of the incoming proton.

By considering mass-shell ρ-p scattering we find that (neglecting V_1)

$$\mathrm{Disc}(V_2) = 8m_\rho^2\sigma_{\text{tot}}\frac{1}{\Delta(M_X^2, m_\rho^2, m_p^2)}\,,$$

where Δ is the Kibble function.

We now consider γ-p total cross sections which are given, in the vector-meson-dominance (VMD) model[34] by

$$\sigma_{\text{tot}}^{\gamma p} = \left(98.6 + \frac{64.9}{(M_X^2)^{1/2}}\right)\sum_V \frac{e_V}{(1 - t/m_V^2)^2}\ \mu\text{b}\,,$$

where $\sum_V$ denotes the sum over possible vector mesons which couple directly to the photon (Fig. 14).

We extract the ρ-only contribution and obtain

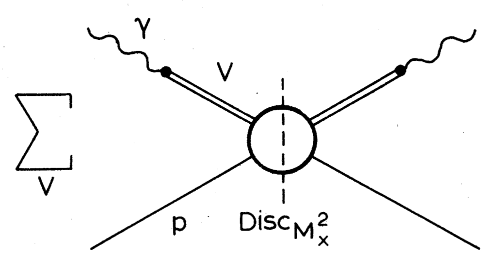

FIG. 14. A diagram symbolizing the vector-meson dominance model for the elastic scattering of photons and protons.

(factoring off the γ-ρ coupling constant)

$$\sigma_{\rm tot}^{\rho p} = 0.65 \times \frac{270}{1000}\left(98.6 + \frac{64.9}{(M_X{}^2)^{1/2}}\right)(m_\rho{}^2)^2 \text{ mb}.$$

Thus, we obtain

$$Y'(t,s,M_X{}^2) = \left(\frac{g_{\rho\pi\pi}}{2}\right)^2 (P\cdot p)^2 (\bar{\phi}_{\pi^0}\phi_{\pi^-})_F{}^2 \times \left(\frac{1}{t-m_\rho{}^2}\right)^2 \frac{1.404(m_\rho{}^2)^3}{\Delta(M_X{}^2, m_\rho{}^2, m_p{}^2)} \times \left(98.6 + \frac{64.9}{(M_X{}^2)^{1/2}}\right) \text{ mb}. \tag{B2}$$

We Reggeize via[35]

$$\frac{1}{t-m_\rho{}^2} \to \alpha_\rho'\Gamma(1-\alpha_\rho(t)) \times \left(\frac{1+\tau e^{-i\pi\alpha_\rho(t)}}{2}\right)\left(\frac{s}{M_X{}^2}\right)^{\alpha_\rho(t)-1} \tag{B3}$$

and note that $P\cdot p \sim s$ and $\Delta^{1/2}(M_X{}^2, m_\rho{}^2, m_p{}^2) \sim M_X{}^2$ in the triple-Regge region. We assume that the two terms in (B2) correspond to Pomeron and f-Reggeon exchanges, which gives the correct $M_X{}^2$ dependence when compared with the forward form of (B1).

For the off-forward case, we assume

$$\beta_{pp f}^{++}(t_0) = g_{ppf}\alpha_f'\Gamma(1-\alpha_f(t_0))$$

(similarly for the ρ) and

$$\beta_{ppP}^{++}(t_0) = \beta_{ppP}^{++}\exp(\tilde{a}t_0),$$

where $\tilde{a}$ is found by comparing πp and pp elastic scattering.

There remains $G_{\rho\rho R}(t_{ac}, \bar{t}_{ac}, t_0)$ which could in general be expected to exhibit a complicated behavior.[36] However, at this level of approximation we take the behavior of $G_{\rho\rho R}$ to be constant.

Enforcing consistency with the forward normalization, we are now able to do the calculations for differential cross sections and target asymmetries. Γ-function approximations are given in Table IV.

APPENDIX C

In our approximation, the off-forward triple-Regge expression [Eq. (B1)] is given by a sum of terms of the form

$$T = \exp(Et_{ac} + F\bar{t}_{ac} + Gt_0),$$

where E, F, and G can be complex and ReE, ReF, and Re$G>0$. This holds for all three exchanges considered, but for ρ exchange at the t_0 leg this term will be multiplied by the factor given in Eq. (3.5).

We will give all the integrals used in the calculation in terms of this general term [notation equations (2.3) and (2.7)], with $0<C<1$ and $0<C'<1$ used as the absorption parameters for a-b and c-b Pomeron exchange.

A large number of integrals are given, but they can all be evaluated by repeated application of the two integrals given below[37]

$$\int_{-\infty}^{\infty} dx \exp(-\alpha x^2 - \beta x) = \left(\frac{\pi}{\alpha}\right)^{1/2}\exp\left(\frac{\beta^2}{4\alpha}\right),$$

$$\int_{-\infty}^{\infty} dx\, x \exp(-\alpha x^2 - \beta x) = \frac{-\beta}{2\alpha}\left(\frac{\pi}{\alpha}\right)^{1/2}\exp\left(\frac{\beta^2}{4\alpha}\right).$$

All the integrals are formed by choosing separate terms from the general expression (2.8). Each integral is denoted by I or I', with a suffix to indicate which term is being considered. I integrals are for terms which arise from Pomeron or f exchange in the t_0 leg and I' integrals are for ρ exchange. Thus, the I integrals contribute to the differential cross section and the I' integrals to the target asymmetry.

Extra Pomerons exchanged in one channel only

$$I_{a,n} = \left[\frac{(-Ca)^n}{nn!(A)^{n-1}}\right]\frac{1}{A/n+Ex+G} \times \exp\left[(E+F)t + \frac{p_{c_\perp}{}^2E^2}{A/n+Ex+G}\right],$$

$$I_{c,n} = \left[\frac{(-C'a')^n}{nn!(A')^{n-1}}\right]\frac{1}{A'/n+E/x+G} \times \exp\left[(E+F)t + \frac{p_{c_\perp}{}^2E^2}{A/n+E/x+G}\right],$$

$$I_{\bar{a},n} = \left[\frac{(-Ca)^n}{nn!(A^*)^{n-1}}\right]\frac{1}{A^*/n+Fx+G} \times \exp\left[(E+F)t + \frac{p_{c_\perp}{}^2F^2}{A^*/n+Fx+G}\right],$$

TABLE IV. Exponential approximation to gamma functions.

Function	Approximation valid for $0<\lvert t\rvert<1$ [(GeV/c)^2]
$\Gamma(1-\alpha_\rho(t))$	$1.475605\exp(0.6118244t)$
$\Gamma(1-\alpha_f(t))$	$1.307707\exp(0.4856229t)$

$$I_{\bar{c},n}=\left[\frac{(-C'a')^n}{nn!(A'^*)^{n-1}}\right]\frac{1}{A'^*/n+F/x+G}\times\exp\left[(E+F)t+\frac{p_{c_\perp}{}^2F^2}{A'^*/n+F/x+G}\right],$$

$$I'_{a,n}=I_{a,n}\left(\frac{-p_{c_\perp}Ee^{i\phi}}{A/n+Ex+G}\right),$$

$$I'_{c,n}=I_{c,n}\left[\frac{-p_{c_\perp}Ee^{i\phi}}{x(A/n+E/x+G)}\right],$$

$$I'_{\bar{a},n}=I_{\bar{a},n}\left(\frac{p_{c_\perp}Fe^{i\phi}}{A/n+Fx+G}\right),$$

$$I'_{\bar{c},n}=I_{\bar{c},n}\left[\frac{p_{c_\perp}Fe^{i\phi}}{x(A/n+F/x+G)}\right].$$

Extra Pomerons exchanged in both channels on either incoming or outgoing side

$$I_{a,c,n,m}=\frac{(-Ca)^n(-C'a')^m e^{(E+F)t}}{nn!(A)^{n-1}mm!(A')^{m-1}[(A/n+Ex+G)(A'/m+E/x+G)-(E+G)^2]}\times\exp\left\{\frac{p_{c_\perp}{}^2E^2[(A/n+Ex+G)+x^2(A'/m+E/x+G)-2x(E+G)]}{x^2[(A/n+Ex+G)(A'/m+E/x+G)-(E+G)^2]}\right\},$$

$$I_{\bar{a},\bar{c},n,m}=\frac{(-Ca)^n(-C'a')^m e^{(E+F)t}}{nn!(A^*)^{n-1}mm!(A'^*)^{m-1}[(A^*/n+Ex+G)(A'^*/m+E/x+G)-(E+G)^2]}\times\exp\left\{\frac{p_{c_\perp}{}^2F^2[(A^*/n+Fx+G)+x^2(A'^*/m+F/x+G)-2x(E+G)]}{x^2[(A^*/n+Fx+G)(A'^*/m+F/x+G)-(F+G)^2]}\right\},$$

$$I'_{a,c,n,m}=I_{a,c,n,m}\left[\frac{-p_{c_\perp}E(A'/m+A/nx)e^{i\phi}}{(A/n+Ex+G)(A'/m+E/x+G)-(E+G)^2}\right],$$

$$I'_{\bar{a},\bar{c},n,m}=I_{\bar{a},\bar{c},n,m}\left[\frac{p_{c_\perp}F(A'^*/m+A^*/nx)e^{i\phi}}{(A^*/n+Ex+G)(A'^*/m+E/x+G)-(E+G)^2}\right].$$

Extra Pomerons exchanged in two channels, one on the incoming and one on the outgoing side

$$I_{a,\bar{a},n,m}=\frac{(-Ca)^n(-Ca)^m e^{(E+F)t}}{nn!(A)^{n-1}mm!(A^*)^{m-1}[(A/n+Ex+G)(A^*/m+Fx+G)-G^2]}\times\exp\left\{\frac{p_{c_\perp}{}^2[F^2(A/n+Ex+G)+E^2(A^*/m+Fx+G)-2EFG]}{(A/n+Ex+G)(A^*/m+Fx+G)-G^2}\right\},$$

$$I_{c,\bar{a},n,m}=\frac{(-C'a')^n(-Ca)^m e^{(E+F)t}}{nn!(A')^{n-1}mm!(A^*)^{m-1}[(A'/n+E/x+G)(A^*/m+Fx+G)-G^2]}\times\exp\left\{\frac{p_{c_\perp}{}^2[F^2(A'/n+E/x+G)+(E^2/x^2)(A^*/m+Fx+G)-2EFG/x]}{(A'/n+E/x+G)(A^*/m+Fx+G)-G^2}\right\},$$

$$I_{c,\bar{c},n,m}=\frac{(-C'a')^n(-C'a')^m e^{(E+F)t}}{nn!(A')^{n-1}mm!(A'^*)^{m-1}[(A'/n+E/x+G)(A'^*/m+F/x+G)-G^2]}\times\exp\left\{\frac{p_{c_\perp}{}^2[(F^2/x^2)(A'/n+E/x+G)+(E^2/x^2)(A'^*/m+F/x+G)-2EFG/x^2]}{(A'/n+E/x+G)(A'^*/m+F/x+G)-G^2}\right\},$$

$$I_{a,\bar{c},n,m}=\frac{(-Ca)^n(-C'a')^m e^{(E+F)t}}{nn!(A)^{n-1}mm!(A'^*)^{m-1}[(A/n+Ex+G)(A'^*/m+F/x+G)-G^2]}\times\exp\left\{\frac{p_{c_\perp}{}^2[(F^2/x^2)(A/n+Ex+G)+E^2(A'^*/m+E/x+G)-2EFG/x]}{(A/n+Ex+G)(A'^*/m+F/x+G)-G^2}\right\},$$

$$I'_{a,\bar{a},n,m}=I_{a,\bar{a},n,m}\left[\frac{p_{c_\perp}e^{i\phi}(FA/n-EA^*/m)}{(A/n+Ex+G)(A^*/m+Fx+G)-G^2}\right],$$

$$I'_{c,\bar{a},n,m}=I_{c,\bar{a},n,m}\left[\frac{p_{c_\perp}e^{i\phi}(FA'/n-EA^*/mx)}{(A'/n+E/x+G)(A^*/m+Fx+G)-G^2}\right],$$

$$I'_{c,\bar{c},n,m}=I_{c,\bar{c},n,m}\left[\frac{p_{c_\perp}e^{i\phi}(FA'/xn-EA'^*/mx)}{(A'/n+E/x+G)(A'^*/m+F/x+G)-G^2}\right],$$

$$I'_{a,\bar{c},n,m}=I_{a,\bar{c},n,m}\left[\frac{p_{c_\perp}e^{i\phi}(FA/nx-EA'^*/m)}{(A/n+Ex+G)(A'^*/m+F/x+G)-G^2}\right].$$

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