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Anomalous components of the photon structure functions

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The leading contributions to the deep-inelastic structure functions of a real or slightly virtual photon are calculated using a generalization of the Altarelli-Parisi equations for quantum-chromodynamic scale breaking. These techniques provide a simple alternative to the more conventional operator-product–moment methods and lead to integral equations which are easily solved numerically.

I. INTRODUCTION

According to quantum chromodynamics (QCD), the structure functions of a real or slightly virtual photon contain anomalous pieces in addition to the hadroniclike terms which are expected from general arguments (e.g., vector-meson dominance). These anomalous pieces derive from the pointlike coupling of the photon to quarks and ultimately dominate at sufficiently large Q^2 in the Bjorken limit. They, moreover, can be calculated directly from QCD without using any experimental information. This remarkable fact was first demonstrated by Witten,[1] who built upon the earlier discovery by Walsh and Zerwas[2] and Kingsley[3] that the free-quark model predicts anomalous, nonscaling contributions to the photon structure functions.

The photon structure functions are accessible experimentally, as first emphasized by Walsh[4] and by Brodsky *et al.*,[4] in the process $e+e \to e+e+$ hadrons. A more recent and comprehensive discussion of two-photon processes and QCD, including the special consequences for experiment of the anomalous components of the photon structure functions, is given in Ref. 5. An attractive and complementary approach to study the photon structure functions is provided by the Drell-Yan process initiated by a real or slightly virtual photon, $\gamma N \to \mu^+\mu^- +$ hadrons.[6,7]

In this note we show how Witten's results for the anomalous terms in the transverse and longitudinal structure functions of the photon follow immediately from a simple generalization of the Altarelli-Parisi[8] equations for scale breaking. We display results for the anomalous quark and gluon contributions of the photon in the two- (u,d), three- (u,d,s), and four-flavor (u,d,s,c) cases. Although the findings reported herein are completely equivalent to those of Witten,[1] who used the conventional operator-product-moment formalism, our approach, in common with the Altarelli-Parisi method for conventional targets, has the virtues of simplicity and ease of physical interpretation. It leads directly to integral equations in x (momentum fraction) space, which are easily solved numerically.

Alternatively, one may Mellin transform the integral equations, converting them to the moment equations of Witten. In this note we focus on the numerical solution in x space since it has not been discussed previously. A derivation of Witten's results from a diagrammatic point of view has been given by Llewellyn Smith.[6] This diagrammatic approach, our approach using the Altarelli-Parisi evolution equations, and Witten's approach using the operator-product expansion, represent for the photon the three currently available techniques for calculating QCD renormalization effects. Although all three methods are physically equivalent and give identical answers for hadronic and photonic targets, each has unique advantages (and disadvantages) and deserves consideration.

II. BORN TERMS

In the *absence* of strong-interaction corrections, the photon structure functions to lowest order in e^2 are given by the box and crossed box diagrams of Figs. 1(a) and 1(b) respectively. Both diagrams contribute to leading order and are required for gauge invariance.

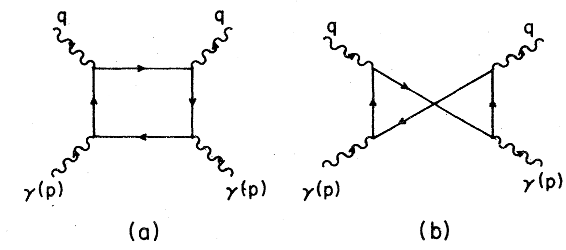

FIG. 1. Box diagrams which give the deep-inelastic structure functions of a photon target $\gamma(p)$ in the absence of strong-interaction corrections. The target photon is real or slightly virtual; the upper photon is highly virtual in the limit of interest.

If we define photon structure functions by

$$\frac{1}{\pi}p_0 \int d^4x\, e^{-iq\cdot x}\langle\gamma(p)|\,[j^\mu(x),j^\nu(0)]\,|\gamma(p)\rangle_{\text{spin avg}}$$
$$= W_1\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)+W_2\left(p^\mu-q^\mu\frac{p\cdot q}{q^2}\right)\left(p^\nu-q^\nu\frac{p\cdot q}{q^2}\right) \tag{1}$$

and take the Bjorken limit

$$Q^2\equiv -q^2\to\infty,\quad \nu\equiv p\cdot q\to\infty,\quad \text{with } x\equiv Q^2/2\nu \text{ fixed,}$$

the contributions from the box diagrams for a particular quark flavor (i) and three colors are[1-3]

$$F_2^{(i)}(x,Q^2)_B\equiv \nu W_2^{(i)}$$
$$=3\left(\tfrac{1}{2}\right)\frac{e^2\epsilon_i^{\,4}}{4\pi^2}\,x\left\{a(x)\ln\left(\frac{Q^2(1-x)}{xm_i^{\,2}-x^2(1-x)p^2}\right)-2[1-3x(1-x)]+\frac{[1+2x(1-x)]}{[1-(p^2/m_i^{\,2})x(1-x)]}+O\left(\frac{m_i^{\,2}}{Q^2},\frac{p^2}{Q^2}\right)\right\} \tag{2}$$

and

$$F_L^{(i)}(x,Q^2)_B\equiv \nu W_2^{(i)}-2xW_1^{(i)}$$
$$=3\left(\tfrac{1}{2}\right)\frac{e^2\epsilon_i^{\,4}}{4\pi^2}\,x\left[c(x)+O\left(\frac{m_i^{\,2}}{Q^2},\frac{p^2}{Q^2}\right)\right], \tag{3}$$

where ϵ_i is the quark (antiquark) charge in units of e,

$$a(x)=x^2+(1-x)^2, \tag{4}$$

and

$$c(x)=4x(1-x)\ . \tag{5}$$

The overall factor of $(\frac{1}{2})$ in Eqs. (2) and (3) comes from dividing the results of Figs. 1(a) and 1(b) equally into a quark and an antiquark contribution. The important features of these formulas for our purposes are the presence of a nonscaling $\ln(Q^2)$ term in F_2 and a nonzero, Q^2-independent contribution in F_L.

When the strong interactions are turned on, F_2 and F_L are renormalized and have the forms ($\alpha_\gamma\equiv e^2/4\pi$)

$$F_2(x,Q^2)=3\left(\frac{\alpha_\gamma}{2\pi}\right)x\left[h(x)\ln\left(\frac{Q^2}{\Lambda^2}\right)+s(x)\right]+F_2(x,Q^2)_H \tag{6}$$

and

$$F_L(x,Q^2)=3\left(\frac{\alpha_\gamma}{2\pi}\right)xl(x)+F_L(x,Q^2)_H\ , \tag{7}$$

where Λ^2 is a hadronic mass scale and the subscript H denotes contributions[9] which behave in the same manner as the structure functions of a composite hadronic target, i.e.,

$$\lim_{Q^2\to\infty}[F_{2,L}(x,Q^2)_H]\to 0,\quad x\neq 0. \tag{8}$$

We first calculate $h(x)$ and a corresponding term in the gluon distribution. Once these are known, the longitudinal function $l(x)$ is given by quadrature. In principle, the function $s(x)$ in Eq. (6) is also calculable without experimental information, except for its lowest moment (area); however, the task is much more complicated since it requires knowledge of two-loop renormalization effects.[1,10] If such additional effects are considered, Eq. (6) will contain an extra piece proportional to $h(x)\ln[\ln(Q^2/\Lambda^2)]$. (See Refs. 1 and 10.) In this paper we ignore these refinements.

III. TRANSVERSE STRUCTURE FUNCTION

For fully composite hadronic targets the one-loop Altarelli-Parisi equations for the quark and gluon distributions are[8]

$$\frac{dq^i(x,t)}{dt}=\frac{\alpha_s(t)}{2\pi}\int_x^1\frac{dy}{y}\left[P_{qq}\left(\frac{x}{y}\right)q^i(y,t)+P_{qG}\left(\frac{x}{y}\right)G(y,t)\right],\quad i=1,\ldots,2f \tag{9}$$

and

$$\frac{dG(x,t)}{dt}=\frac{\alpha_s(t)}{2\pi}\int_x^1\frac{dy}{y}\left[P_{Gq}\left(\frac{x}{y}\right)\sum_{j=1}^{2f}q^j(y,t)+P_{GG}\left(\frac{x}{y}\right)G(y,t)\right], \tag{10}$$

where

$$\alpha_s(t)=\frac{\alpha_s(0)}{1+\alpha_s(0)bt}\underset{t\to\infty}{\to}\frac{1}{bt} \tag{11}$$

is the running coupling constant, $4\pi b=(11-2f/3)$, f is the number of flavors, $t\equiv\ln(Q^2/\Lambda^2)$, and P_{qq}, P_{qG}, P_{Gq}, P_{GG} are the fragmentation functions defined by Altarelli and Parisi [SU(3) color]:

$$P_{qq}(z)=\frac{4}{3}\left[\frac{1+z^2}{(1-z)_+}+\tfrac{3}{2}\delta(z-1)\right], \tag{12}$$

$$P_{Gq}(z)=\frac{4}{3}\frac{1+(1-z)^2}{z}\ , \tag{13}$$

$$P_{qG}(z)=\tfrac{1}{2}[z^2+(1-z)^2]\ , \tag{14}$$

$$P_{GG}(z)=6\left[\frac{1-z}{z}+\frac{z}{(1-z)_+}+z(1-z)+\left(\tfrac{11}{12}-\tfrac{1}{18}f\right)\delta(z-1)\right]\ , \tag{15}$$

with

$$\int_x^1 \frac{dy}{y} f(y)/(1-x/y)_+ \equiv f(x)\ln\left(\frac{1-x}{x}\right) + \int_x^1 \frac{dy}{y}\frac{f(y)-f(x)}{(1-x/y)} . \tag{16}$$

If we set $\alpha_s(t)=0$, Eqs. (9) and (10) express the naive parton-model result

$$\frac{dq^i(x,t)}{dt}=0 \quad (\alpha_s=0) \tag{17}$$

and

$$\frac{dG(x,t)}{dt}=0 \quad (\alpha_s=0) . \tag{18}$$

For a photon target, however, we see from Eq. (2) that this naive result is wrong. Instead we find, using the parton-model formula

$$F_2(x,t)=\sum_{i=1}^{2f}\epsilon_i^2 x q^i(x,t), \tag{19}$$

that

$$\frac{dq^i(x,t)}{dt}=3\epsilon_i^2\left(\frac{\alpha_\gamma}{2\pi}\right)a(x) \quad (\alpha_s=0). \tag{20}$$

In the absence of strong interactions, gluons do not couple to the photon; hence the companion equation (18) remains valid.

The proper generalization for a photon target of the Altarelli-Parisi equations is therefore

$$\frac{dq^i(x,t)}{dt}=\epsilon_i^2\frac{\alpha_\gamma}{2\pi}a(x)+\frac{\alpha_s(t)}{2\pi}\int_x^1\frac{dy}{y}\left[P_{qq}\left(\frac{x}{y}\right)q^i(y,t) + P_{qG}\left(\frac{x}{y}\right)G(y,t)\right] \tag{21}$$

and

$$\frac{dG(x,t)}{dt}=0+\frac{\alpha_s(t)}{2\pi}\int_x^1\frac{dy}{y}\left[P_{Gq}\left(\frac{x}{y}\right)\sum_{j=1}^{2f}q^j(y,t) + P_{GG}\left(\frac{x}{y}\right)G(y,t)\right] . \tag{22}$$

The above derivation is somewhat heuristic. An alternative way to see that Eqs. (21) and (22) are valid is to first write down Altarelli-Parisi equations which describe one-loop scale-breaking/renormalization to all orders in α_s *and* α_γ. This is obvious, given the original Altarelli-Parisi equations, since the photon and gluon are now on an equal footing, except that the photon only couples to charged fields (quarks).

If we denote by Γ the probability distribution for finding an elementary photon in the physical photon, then

$$\frac{d\tilde{q}^i}{dt}(x,t)=\frac{1}{2\pi}\int_x^1\frac{dy}{y}\left[\alpha_s(t)P_{qq}\left(\frac{x}{y}\right)\tilde{q}^i(y,t)+\alpha_s(t)P_{qG}\left(\frac{x}{y}\right)\tilde{G}(y,t)+\alpha_\gamma(t)P_{q\gamma}\left(\frac{x}{y}\right)\epsilon_i^2\tilde{\Gamma}(x,t)\right] , \tag{23}$$

$$\frac{d\tilde{G}}{dt}(x,t)=\frac{1}{2\pi}\int_x^1\frac{dy}{y}\left[\alpha_s(t)P_{Gq}\left(\frac{x}{y}\right)\sum_{j=1}^{2f}\tilde{q}^j(y,t)+\alpha_s(t)P_{GG}\left(\frac{x}{y}\right)\tilde{G}(y,t)\right] , \tag{24}$$

and

$$\frac{d\tilde{\Gamma}}{dt}(x,t)=\frac{1}{2\pi}\int_x^1\frac{dy}{y}\left[\alpha_\gamma(t)P_{\gamma q}\left(\frac{x}{y}\right)\sum_{i=1}^{2f}\epsilon_j^2\tilde{q}^j(y,t)\right], \tag{25}$$

where $\tilde{}$ denotes distributions valid to all orders in α_γ. The new fragmentation functions $P_{q\gamma}$ and $P_{\gamma q}$ appearing in the above equations are simply related to P_{qG} and P_{Gq}, namely,

$$P_{q\gamma}(z)=2P_{qG}(z) \tag{26}$$

and

$$P_{\gamma q}(z)=\tfrac{3}{4}P_{Gq}(z) . \tag{27}$$

If we now specialize to lowest order in the electromagnetic interactions then

$$\alpha_\gamma(t)=\alpha_\gamma+O(\alpha_\gamma^2), \tag{28}$$

$$\tilde{\Gamma}(x,t)=\delta(x-1)+O(\alpha_\gamma), \tag{29}$$

$$\tilde{q}^i(x,t)=q^i(x,t)+O(\alpha_\gamma^2), \tag{30}$$

$$\tilde{G}^i(x,t)=G^i(x,t)+O(\alpha_\gamma^2). \tag{31}$$

Substituting Eqs. (28)–(31) into (23) and (24) we obtain (21) and (22).

The solution of these equations in the asymptotic limit is straightforward. Let

$$q^i(x,t)=\frac{\alpha_\gamma}{2\pi}h^i(x)t\left[1+O\left(\frac{1}{t}\right)\right] \tag{32}$$

and

$$G(x,t)=\frac{\alpha_\gamma}{2\pi}h^G(x)t\left[1+O\left(\frac{1}{t}\right)\right]. \tag{33}$$

Substituting into Eqs. (21) and (22) we find the t-independent integral equations:

$$h^i(x)=\epsilon_i^2a(x)+\frac{1}{2\pi b}\int_x^1\frac{dy}{y}\left[P_{qq}\left(\frac{x}{y}\right)h^i(y) + P_{qG}\left(\frac{x}{y}\right)h^G(y)\right] \tag{34}$$

and

$$h^G(x) = 0 + \frac{1}{2\pi b}\int_x^1 \frac{dy}{y}\left[P_{Gq}\left(\frac{x}{y}\right)\sum_{j=1}^{2f} h^j(y,t) + P_{GG}\left(\frac{x}{y}\right)h^G(y)\right] \quad (35)$$

[b was defined in Eq. (11)].

As usual we can simplify the calculation by defining singlet and nonsinglet quark combinations. The form of Eqs. (34) and (35) ensures that quarks (antiquarks) with equal squared electric charges will have equal h^i. Thus, we need only consider h^u $(=h^{\bar{u}})$ and h^d $(=h^{\bar{d}})$. Define

$$h^{NS} = \tfrac{1}{2}(h^u - h^d), \quad (36)$$

$$h^S = \frac{1}{2f}\sum_{i=1}^{2f} h^i \quad . \quad (37)$$

In terms of these combinations one can cast Eqs. (34) and (35) into the form

$$h^{NS}(x) = \tfrac{1}{6}a(x) + \frac{1}{2\pi b}\int_x^1 \frac{dy}{y}P_{qq}\left(\frac{x}{y}\right)h^{NS}(y) , \quad (38)$$

$$h^S(x) = a_S a(x) + \frac{1}{2\pi b}\int_x^1 \frac{dy}{y}\left[P_{qq}\left(\frac{x}{y}\right)h^S(y) + P_{qG}\left(\frac{x}{y}\right)h^G(y)\right] , \quad (39)$$

$$h^G(x) = 0 + \frac{1}{2\pi b}\int_x^1 \frac{dy}{y}\left[2fP_{Gq}\left(\frac{x}{y}\right)h^S(y) + P_{GG}\left(\frac{x}{y}\right)h^G(y)\right], \quad (40)$$

where $a_S = \frac{5}{18}, \frac{2}{9}, \frac{5}{18}$ for $f = 4, 3, 2$ respectively.

In terms of these quantities, the leading term of the full F_2 structure function defined in Eq. (6) is

$$h(x) = \tfrac{20}{9}h^S(x) + \tfrac{4}{3}h^{NS}(x), \quad f=4 \quad (41)$$

$$h(x) = \tfrac{4}{3}h^S(x) + \tfrac{8}{9}h^{NS}(x), \quad f=3 \quad (42)$$

and

$$h(x) = \tfrac{10}{9}h^S(x) + \tfrac{2}{3}h^{NS}(x), \quad f=2 . \quad (43)$$

(Note that due to changes in b and a_S the values of h^S and h^{NS} also change when the flavor symmetry is varied.)

Our equations are easily compared with those of Witten in Mellin transform space. In this space Eqs. (38)–(40) may be solved algebraically, and h^S and h^{NS} used to obtain the moments of $h(x)$,

$$h^n = \int_0^1 dx\, x^{n-1}h(x), \quad (44)$$

$$h^n = h^n_{\text{box}}\left\{\frac{9}{34}\frac{1}{[1-(1/2\pi b)P^n_{qq}]} + \frac{25}{34}\frac{[1-(1/2\pi b)P^n_{GG}]}{[1-(1/2\pi b)(P^n_{qq}+P^n_{GG}) + (1/2\pi b)^2(P^n_{qq}P^n_{GG} - 2fP^n_{qG}P^n_{Gq})]}\right\}, \quad (45)$$

where

$$h^n_{\text{box}} = \sum_{i=1}^{2f}\epsilon_i^4\left[\frac{n^2+n+2}{(n+2)n(n+1)}\right].$$

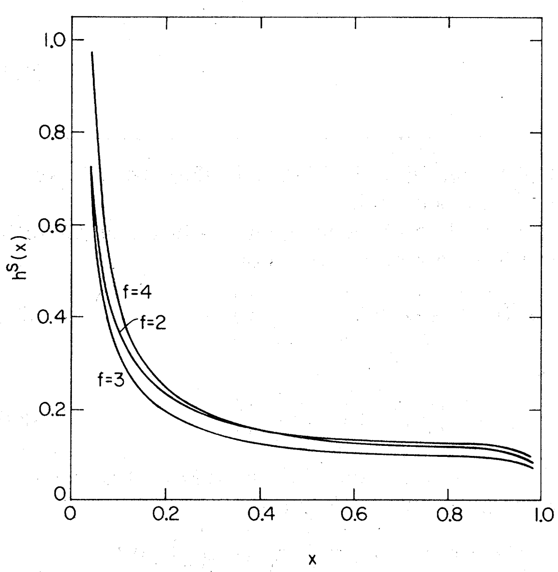

FIG. 2. The singlet function $h^S(x)$ for flavor groups SU(f), f = 4, 3, 2. See Eqs. (37), (39), and (40).

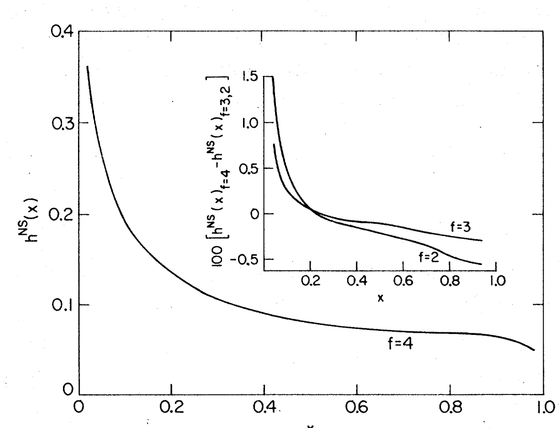

FIG. 3. The nonsinglet function $h^{NS}(x)$ for flavor groups SU(f), f = 4, 3, 2. See Eqs. (36) and (38).

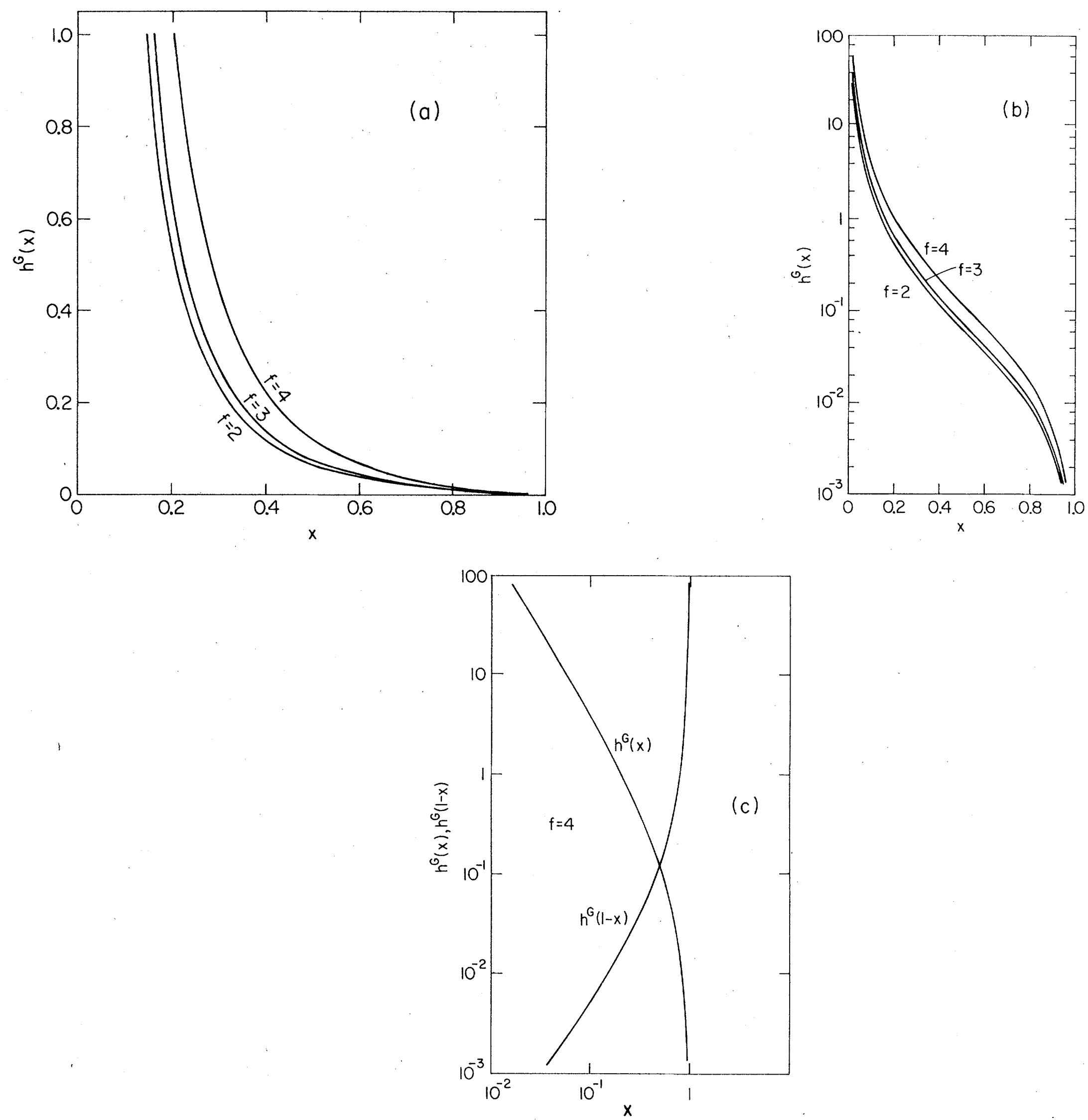

FIG. 4. The gluon function $h^G(x)$ for flavor groups SU(f), f = 4, 3, 2. See Eqs. (39) and (40). (a) linear scale, (b) semilog scale, and (c) log-log scale showing details near $x = 0$ and $x = 1$ of (a).

Using the results given in Ref. 8 for the P^n's one can easily establish that Eq. (45) is identical to Witten's result [Eq. (12) in Ref. 1] after multiplication by 4 to adjust to our normalization convention.

We have solved Eqs. (34) and (35) by a straightforward iteration procedure. These equations can be written symbolically as

$$g(x) = b(x) + \frac{1}{2\pi b}\int_x^1 \frac{dy}{y}\,[P(x/y)g(y)] \qquad (46)$$

where $g(x)$ denotes the functions $h^1(x), \ldots, h^{2f}(x)$, $h^G(x)$; $b(x)$ denotes the respective Born terms, and $P(x/y)$ is a matrix containing the kernel functions of the Altarelli-Parisi equations. The iteration then proceeds as follows: Taking $g^{(0)}(x) = b(x)$, the kth approximation to $g(x)$ is given by

$$g^{(k)}(x) = (1-\beta)g^{(k-1)}(x) + \beta\left\{ b(x) + \frac{1}{2\pi b}\int_x^1 \frac{dy}{y}\left[P\left(\frac{x}{y}\right)g^{(k-1)}(x)\right]\right\}. \qquad (47)$$

Here β is a convergence factor that keeps $g^{(k)}(x)$ from fluctuating too wildly. If β is chosen too large, the procedure does not converge. We use $\beta = 0.1$.

The functions $h^S(x)$, $h^{NS}(x)$, $h^G(x)$, and $h(x)$, calculated from the functions generated in (47), are

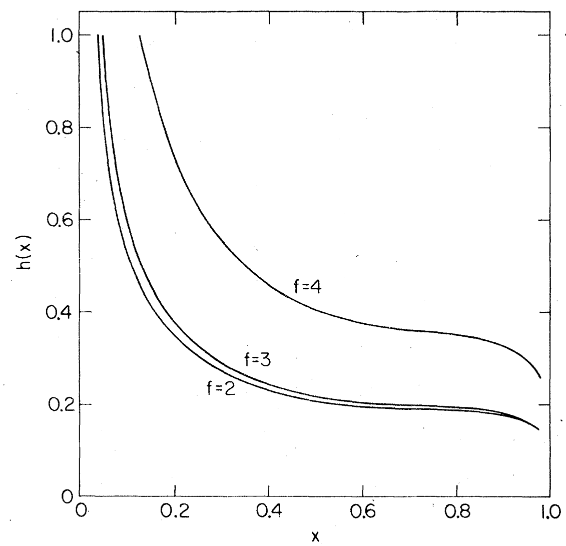

FIG. 5. The combination $h(x)$ of quark functions which enters the transverse structure function of the photon for flavor groups SU(f), $f=4,3,2$. See Eqs. (6) and (41)–(43).

shown in Figs. 2–5, respectively, for flavors $f=4,3,2$. The curve in Fig. 3 shows $h^{\rm NS}(x)$ for $f=4$; the insert gives $[h^{\rm NS}(x)_{f=4}-h^{\rm NS}(x)_{f=3,2}]\times 100$. Changes in the curves caused by increasing the number of flavors are dictated primarily by the charges of the additional quarks. Addition of quarks with charge $\pm\frac{2}{3}$ has a substantially greater effect than addition of quarks with charge $\pm\frac{1}{3}$. Plots of $h^G(x)$ and $h^G(1-x)$ on semilog and log-log scales are included in Fig. 4 to show details near $x=0$ and $x=1$. We note that direct numerical solution of the integral equations by the above technique has the potential for very accurate determination of the behavior near the end points. We do not pursue this here, but we note that this may offer advantages over the moment-inversion technique (where supplementary considerations must be invoked to determine end-point behavior).

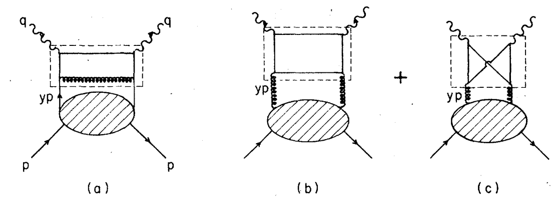

FIG. 6. The $O(\alpha_s)$ contributions to the longitudinal structure functions for a composite hadronic target, Eq. (48). The parts of the diagrams enclosed in dotted lines represent the induced longitudinal coupling of (a) a quark, and (b) plus (c) a gluon, to the electromagnetic current.

IV. LONGITUDINAL STRUCTURE FUNCTION

Once the anomalous quark and gluon contributions of the photon are known, the leading contribution to the longitudinal structure function follows straightforwardly. All that is needed is the longitudinal analog of Eq. (19). Since in the naive parton model F_L vanishes identically, the first nonzero contributions occur in $O(\alpha_s)$. For a composite *hadron* the result is[11]

$$F_L(x,t)_H=\frac{\alpha_s(t)}{2\pi}x\int_x^1\frac{dy}{y}\sum_{i=1}^{2f}\epsilon_i^2\left[\frac{8}{3}\left(\frac{x}{y}\right)q^i(y,t)+2\frac{x}{y}\left(1-\frac{x}{y}\right)G(y,t)\right]. \tag{48}$$

The physical interpretation of Eq. (48) is straightforward. The quark term comes from the process shown in Fig. 6(a) in which a quark carrying a fraction y of the target momentum interacts with the longitudinal current after gluon emission. Similarly, the gluon term in Eq. (48) comes from the processes shown in Figs. 6(b) and 6(c) in which a gluon with momentum fraction y interacts with the longitudinal current after pair creation.

We have seen above, however, that for a photon target there is a contribution to F_L even in the absence of strong interactions. The generalization of Eq. (46) which takes this into account is

$$F_L(x,t)=\sum_{i=1}^{2f}\epsilon_i^4\frac{\alpha_\gamma}{2\pi}xc(x)+\frac{\alpha_s(t)}{2\pi}x\int_x^1\frac{dy}{y}\sum_{i=1}^{2f}\epsilon_i^2\left[\frac{8}{3}\left(\frac{x}{y}\right)q^i(x,t)+2\frac{x}{y}\left(1-\frac{x}{y}\right)G(y,t)\right], \tag{49}$$

where $c(x)$ is defined in Eq. (5).

To calculate the renormalization of $c(x)$, we need only substitute into the right-hand side of Eq. (49) the leading asymptotic forms [Eqs. (32) and (33)] for q^i and G calculated in Sec. III. We find that F_L has the form specified by Eq. (7) and that the renormalized function $l(x)$ is given by

$$l(x)=a_Bc(x)+\frac{1}{2\pi b}\int_x^1\frac{dy}{y}\left[\frac{8}{3}\left(\frac{x}{y}\right)h(y)+2a_G\left(\frac{x}{y}\right)\left(1-\frac{x}{y}\right)h^G(y)\right], \tag{50}$$

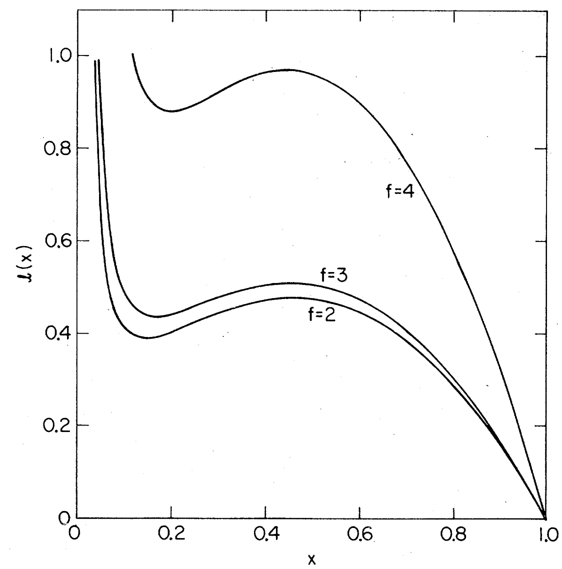

FIG. 7. The function $l\ (x)$ which determines the longitudinal structure function of the photon for flavor groups SU(f), $f = 4, 3, 2$. See Eqs. (7) and (50).

where $a_G = \frac{20}{9}, \frac{4}{3}, \frac{10}{9}$, and $a_B = \frac{68}{81}, \frac{4}{9}, \frac{34}{81}$, for $f = 4, 3, 2$ respectively. Numerical results for $l(x)$ are shown in Fig. 7.

V. SUMMARY

We have seen that the Altarelli-Parisi approach to scale breaking is easily extended to photon targets. When this is done, the strong-interaction renormalizations of the direct photon-parton diagrams are easily computed from a Q^2 independent integral equation. The renormalized direct photon-parton contributions will dominate in the Bjorken limit and will provide an important test of QCD.

In this paper we have calculated only the leading (one-loop renormalization) contributions to F_2 and F_L. We believe, however, that our method would also be useful in computing next-to-leading (two-loop renormalization) contributions as well as quark- and target-mass effects.

Note added. Subsequent to and independent of our work, Koller, Walsh, and Zerwas[12] use a completely analogous method to calculate the QCD renormalization of the functions which describe the inclusive fragmentation of quarks and gluons to photons. They suggest that the process $e^+e^- \rightarrow \gamma +$ hadrons be used as a test of these predictions.

ACKNOWLEDGMENTS

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[1]E. Witten, Nucl. Phys. B120, 189 (1977). Note that the structure functions as defined by Witten are a factor of 4 smaller than the more conventional normalization used here.

[2]T. F. Walsh and P. Zerwas, Phys. Lett. 44B, 195 (1973).

[3]R. L. Kingsley, Nucl. Phys. B60, 45 (1973).

[4]T. F. Walsh, Phys. Lett. 36B, 121 (1971); S. J. Brodsky, T. Kinoshita, and H. Terazawa, Phys. Rev. Lett. 27, 280 (1971).

[5]S. J. Brodsky, T. DeGrand, J. Gunion, and J. Weis, Phys. Rev. D 19, 1418 (1979).

[6]C. H. Llewellyn Smith, Phys. Lett. 79B, 83 (1978); see also W. R. Frazer and J. F. Gunion, Univ. of California, Davis, Report No. UCD-78-5, 1978 (unpublished).

[7]R. J. DeWitt, L. M. Jones, J. D. Sullivan, D. E. Willen, and H. W. Wyld, Jr. (unpublished).

[8]G. Altarelli and G. Parisi, Nucl. Phys. B126, 298 (1977). We have assumed that the color group is SU(3).

[9]M. A. Ahmed and G. G. Ross, Phys. Lett. 59B, 369 (1975).

[10]A. J. Buras and W. A. Bardeen, Report No. Fermilab-Pub-78/91-THY, 1978 (unpublished).

[11]G. Altarelli, R. K. Ellis, and G. Martinelli, Nucl. Phys. B143, 521 (1978).

[12]K. Koller, T. F. Walsh, and P. M. Zerwas, DESY Report No. 78/77, 1978 (unpublished).