Density fluctuations in the Feynman-Wilson fluid and asymptotic structure of multiparticle events

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(Received 14 March 1978; revised manuscript received 22 June 1978)

It is shown that in the framework of factorizable models with no transverse dimensions, simple considerations of the elastic cross section σ_{el} can provide much information on the asymptotic structure of multiparticle events. Effectively, we determine the stable equilibrium configurations of an analog Feynman-Wilson fluid with nearest neighbors interacting via the analog potential $V(y) = -\ln\sigma_{el}(y)$. Various interesting phenomena are traced if $V(y)$ exhibits spontaneous symmetry breaking. These are illustrated in the context of a specific model with Koba-Nielsen-Olesen scaling, which leads to a higher-order phase transition. In particular, we find that one-gap production is favored asymptotically, while two or more gaps (of equal size) are produced with quickly decreasing probability. At the "walls," or between two gaps, metastable structures of a definite mass spectrum are developed. Asymptotically, we have $\sigma_n \sim \sigma_{el}$ and the implications of this result on Koba-Nielsen-Olesen scaling are discussed.

I. INTRODUCTION

It has been shown[1] quite generally that the Koba-Nielsen-Olesen (KNO) scaling hypothesis[2] implies a critical behavior for the Feynman-Wilson (FW) fluid[3] at asymptotic rapidities Y, unless the Mellin transform $\tilde{Q}(z,E)$ of the generating function $Q(z,Y)$ suffers an accumulation of singularities with $\mathrm{Re}E\to\infty$ when the "fugacity" z lies near unity. In the latter case, the FW fluid analogy breaks down, since there is no thermodynamic limit in the classical sense.[4] Assuming an asymptotic behavior of the form

$$\psi(x)\equiv\langle n\rangle\sigma_n(Y)/\sigma_{tot}(Y)\sim e^{-ax^\kappa}, \quad x=n/\langle n\rangle,\ a>0,\ \kappa>1, \tag{1}$$

for the KNO scaling function, the following behaviors were found[1] for the average multiplicity, moments of the distribution, and analog pressure, respectively,

$$\langle n\rangle \underset{Y\to\infty}{\sim} Y^{1-\eta}, \quad C_q=\langle n^q\rangle/\langle n\rangle^q \underset{q\to\infty}{\sim} q^{\eta q}, \quad P\underset{z\to 1+}{\sim}(z-1)^{1/(1-\eta)}, \tag{2}$$

where $0<\eta=1/\kappa<1$. These agree, at least to first order in the ϵ expansion, with the results obtained with the absorptive-model (AM) cutting rules[5] in Reggeon field theory (RFT).[6] The latter result, giving the rightmost singularity of $\tilde{Q}(z,E)$ in the E plane ($E=j-1$), which is identified with the analog pressure, shows the precise nature of the z-plane singularity which is responsible for the critical behavior of the FW system at $Y\to\infty$. It is worth noting that the Abramovskii-Gribov-Kancheli (AGK) cutting rules[7] in RFT do not seem to allow for FW fluid analogy.

Since the FW fluid lives in one dimension, standard methods of classical statistical mechanics[8] allow the determination of the analog "potential interaction" $V(y)$ between the FW-fluid molecules, provided that only nearest neighbors interact, from a precise knowledge of the functional form $P=P(z)$. Hence, under the assumption that hadrons are produced from a factorizable multiperipheral chain, $V(y)$ has been determined,[9] and also the elastic cross section $\sigma_{el}(y)=\exp[-V(y)]$, whose multiperipheral iteration builds up the n-hadron production cross section σ_n. The asymptotic behavior (1) of the KNO scaling function leads[9] to the specific results

$$V(y_i,y_{i-1})=\frac{a}{(y_i-y_{i-1})^{\kappa-1}}+\frac{\kappa+1}{2}\ln(y_i-y_{i-1})+\text{const}, \tag{3}$$

$$\sigma_n(Y)=\text{const}\times(n+1)^{\kappa/2}Y^{-(\kappa+1)/2}\times\exp[-a(n+1)^\kappa/Y^{\kappa-1}], \tag{4}$$

which are exact for $\kappa=2$ only, but for our purpose are good approximations of the exact series representations obtained[9] for general values of $\kappa>1$. Note that the effect of the transverse dimensions, which is ignored, has left its trace into $\kappa\neq1$, and

$\exp[-V(y)]$ may be thought of as being the effective (t-integrated) propagator relevant to the multiperipheral model.

Let us stress at this point that the potential (3) is of infinite range, and in fact increases logarithmically with distance; hence the one-dimensional "lattice" with lattice points interacting via (3) can exhibit a phase transition. It is in fact clear that the series of calculations which led from (2) to (3) can be reversed,[9] hence we can explicitly see that the interaction (3) implies a singularity of the form $(z-1)^{1/(1-\eta)}$ in the fugacity plane.

We have already pointed out[9] that the "microscopic" origin of the singularity in the z plane must be searched in the spontaneous-symmetry-breaking (SSB) phenomenon exhibited by $V(y)$. Indeed, it is easy to see that considering three successive particles 1, 2, and 3 produced on the rapidity axis, if the distance $y_3 - y_1$ is small, only one symmetric minimum-potential-energy state exists for particle 2, at $y_2 = (y_3 + y_1)/2$. But a critical distance y_c exists, such that if $y_3 - y_1 > y_c$ the stationary point of $V(y_2 - y_1) + V(y_3 - y_2)$ at $(y_3 + y_1)/2$ changes character and becomes a local maximum, while two additional symmetric minima appear.

This property suggests that some sort of condensation phenomena must take place in the FW fluid before the phase transition, no matter how it manifests itself "macroscopically", actually takes place at $Y \to \infty$. It is precisely these phenomena that we want to investigate in this work.

The plan of this paper is as follows: In Sec. II we discuss the stable equilibrium configurations of a one-dimensional system, whose nearest-neighbors interact via $V(y)$; these configurations correspond to the most likely distributions of hadrons in rapidity space. In Sec. III we generalize our results to any case where $s^{-\gamma} < \sigma_{el} < \text{const}$ $(\gamma > 0)$ asymptotically, and compare the recent findings of RFT with ours. Finally, our conclusions are given in Sec. IV.

II. EQUILIBRIUM CONFIGURATIONS OF A ONE-DIMENSIONAL HADRONIC SYSTEM

Consider n hadrons produced via a factorizable mechanism, which strongly suppresses the transverse phase space, with rapidities y_i ($i = 1, 2, \ldots, n$; $y_{i-1} \leq y_i$). The total rapidity available is Y (see Fig. 1). The discussion is being made in terms of an analog one-dimensional FW system whose nearest neighbors interact through the potential (3). Hence, the stable equilibrium configurations of the system correspond to the most likely distributions of hadrons on the rapidity axis, and the classification of multiparticle events can be made in terms of these configurations. The end

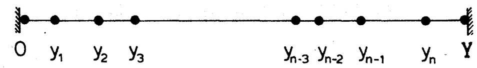

FIG. 1. Production of n hadrons in rapidity space. The target ($y=0$) and projectile ($y=Y$) form the walls of the "container" of the one-dimensional analog Feynman-Wilson fluid.

points 0 and $n+1$ (target and projectile) are rigidly placed at the positions $y=0$ and $y=Y$ and form the walls of the container of the system.

The total potential energy of the system is given by

$$V_{\text{tot}}(y_1, y_2, \ldots, y_n) = \sum_{i=1}^{n+1} V(y_i - y_{i-1}) \qquad (y_0 = 0,\ y_{n+1} = Y) \quad (5)$$

and the equilibrium of the λth particle is expressed by the conditions

$$\frac{\partial V_{\text{tot}}}{\partial y_\lambda} = V'(y_\lambda - y_{\lambda-1}) - V'(y_{\lambda+1} - y_\lambda) = 0\,, \quad (6)$$

$$\frac{\partial^2 V_{\text{tot}}}{\partial y_\lambda^2} = V''(y_\lambda - y_{\lambda-1}) + V''(y_{\lambda+1} - y_\lambda) > 0\,, \quad (7)$$

where primes denote differentiation with respect to the argument. The second condition guarantees the stability of the equilibrium (maximum on topological cross section). After the substitution of the expression (3) for the potential and of the inverse differences $(y_i - y_{i-1})^{-1} \equiv x_i$, conditions (6) and (7) read:

$$x_\lambda^\kappa - x_{\lambda+1}^\kappa = g(x_\lambda - x_{\lambda+1})\,, \qquad g = (\kappa+1)/[2(\kappa-1)a]\,, \quad (8)$$

$$\frac{\partial^2 V_{\text{tot}}}{\partial y_\lambda^2} = \kappa(\kappa-1)a[(x_\lambda^{\kappa-1} - g/\kappa)x_\lambda^2 + (x_{\lambda+1}^{\kappa-1} - g/\kappa)x_{\lambda+1}^2] > 0\,. \quad (9)$$

For simplicity, we restrict ourselves to κ integer; however, the same approach can be followed for κ rational. Condition (8) is now fulfilled if either of the following two conditions is fulfilled

$$x_\lambda = x_{\lambda+1}\,, \quad (10a)$$

$$\sum_{\rho=0}^{\kappa-1} x_\lambda^{\kappa-\rho-1} x_{\lambda+1}^\rho = g\,. \quad (10b)$$

In what follows, we distinguish between three regions of values that the inverse difference x_λ can take.

(i) If $x_\lambda > g^{1/(\kappa-1)} = y_0^{-1}$, for some λ where y_0 is the minimum point of the two-particle potential, (10b) cannot be satisfied, since its left-hand side is clearly greater than $x_\lambda^{\kappa-1}$, and the only solution of (6) is (10a) (for every λ) for which (9) is obviously satisfied. Hence, if at least one distance $x_\lambda^{-1} = y_\lambda - y_{\lambda-1} = \alpha$ is less than y_0 we can only have one "symmetric" solution

$$x_\lambda = x_{\lambda+1} = \alpha^{-1} \quad (\alpha < y_0),$$

i.e., (11)

$$y_\lambda - y_{\lambda-1} = y_{\lambda+1} - y_\lambda = \alpha < y_0,$$

which corresponds to the only allowed stable equilibrium configuration. Since the condition $Y = (n+1)\alpha < (n+1)y_0$ for n fixed can only be fulfilled for Y finite, this configuration is not possible asymptotically. It involves a system of strongly correlated hadrons in rapidity space and the value of the potential (5) at equilibrium gives a measure of the production cross section:

$$\frac{d\sigma}{dy_1 dy_2 \cdots dy_n} = \exp(-V_{\text{tot}}) = \exp[-(n+1)V(\alpha)]. \qquad (12)$$

(ii) If $\tilde{y}_0^{-1} = (g/\kappa)^{1/(\kappa-1)} < x_\lambda < g^{1/(\kappa-1)} = y_0^{-1}$ for some λ, where $\tilde{y}_0 = y_c/2$ is the point of inflexion of the two-particle potential (3), it is easy to see that Eq. (10b) has one (and only one) real positive "asymmetric" solution, namely

$$x_\lambda = \beta^{-1}, \quad x_{\lambda+1} = \bar{\beta}^{-1},$$

i.e., (13)

$$y_\lambda - y_{\lambda-1} = \beta, \quad y_{\lambda+1} - y_\lambda = \bar{\beta},$$

where

$$\bar{\beta} > \tilde{y}_0 > \beta > y_0. \qquad (14)$$

This solution coexists with the symmetric one,

$$x_\lambda = x_{\lambda+1} = \beta^{-1},$$

i.e., (15)

$$y_\lambda - y_{\lambda-1} = y_{\lambda+1} - y_\lambda = \beta,$$

and it is easy to check that both solutions (13) and (15) satisfy condition (9), which means that they both correspond to stable equilibrium configurations. As discussed above, the symmetric solution ($x_\lambda = x_{\lambda+1}$ for every λ) is only accessible at finite total rapidity. This means that if $Y \to \infty$ for fixed n, the relation $(n+1)\beta = Y$ cannot be satisfied, and therefore at least one of the x_i's must equal $\bar{\beta}^{-1}$. If ρ distances are equal to $\bar{\beta}$ ($\rho \neq 0$), we have $(n-\rho+1)\beta + \rho\bar{\beta} = Y$ and for $Y \to \infty$ and n finite this relation is satisfied with $\bar{\beta} \to \infty$. It is obvious, however, that the corresponding potential,

$$V_{\text{tot}} = \frac{a(n-\rho+1)}{\beta^{\kappa-1}} + \frac{\kappa+1}{2}(n-\rho+1)\ln\beta + \frac{a\rho}{\bar{\beta}^{\kappa-1}} + \frac{\kappa+1}{2}\rho\ln\bar{\beta}, \qquad (16)$$

is an increasing function of ρ for large Y and takes its minimum value for $\rho = 1$. Moreover, since $\bar{\beta} \to \infty$ for $Y \to \infty$ we easily see from (10b) that $\beta \to y_0$ for $Y \to \infty$.

(iii) Finally, we consider the case where $x_\lambda < (g/\kappa)^{1/(\kappa-1)} = \tilde{y}_0^{-1}$ for some λ. It follows from (9) that the symmetric solution does not correspond to a stable configuration. We are brought back to case (ii) with the roles of β and $\bar{\beta}$ interchanged [$\bar{\beta} < \beta$ or $x_\lambda < x_{\lambda+1}$, as seen from (10b)].

From the above discussion we arrive at the following picture of multihadron production: For sufficiently small Y the most likely configuration involves hadrons equidistantly spaced in rapidity [see Fig. 2(a); there is only one "phase" of finite density present]. However, if the total rapidity available for the production of n hadrons becomes greater than $(n+1)y_0$, the stable asymmetric solution will manifest itself as at least one gap in the distribution of hadrons in rapidity space [see Fig. 2(b); we have coexistence of two "phases," one with finite and one with zero density].

To gain further insight into the structure of the multiparticle events one could calculate the moments $\langle r^p \rangle$ ($r = \Delta y$) of the gap distribution $\exp[-V(r)]$, as suggested by Pirilä and Thomas,[10] which are related to the coefficients of the cluster expansion in short-range-order models. However, in our model the potential is of infinite range, and these moments diverge (we have a system with long-range order). Instead, the analog-energy (U) distribution introduced by the same authors[11] is more appropriate for our system.

The U distribution helps to link theory (and especially factorizable models) with experiment,[11] and is defined by[11]

$$Q_n(U) = \frac{1}{n!}\int d\vec{y}\, P_n(\vec{y})\,\delta(U + \ln P_n(\vec{y})), \qquad (17)$$

where

$$P_n(\vec{y}) = \frac{d\sigma}{dy_1 dy_2 \cdots dy_n} = \exp[-V_{\text{tot}}(y_1, y_2, \cdots, y_n)]. \qquad (18)$$

Hence, in a factorizable model one has:

$$Q_n(U) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-i\lambda U} Z_n(Y;\lambda)\,d\lambda, \qquad (19)$$

where

$$Z_n(Y;\lambda) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{sY}\tilde{K}^{n+1}(s;\lambda)\,ds \qquad (20)$$

and

$$\tilde{K}(s;\lambda)=\int_0^\infty e^{-sy}K(y;\lambda)dy\,, \tag{21}$$

with

$$K(y;\lambda)=\exp[-(1-i\lambda)V(y)]. \tag{22}$$

In our model, the essential results do not depend on the details of the hard core, but stem basically from the long-range tail $\frac{1}{2}(\kappa+1)\ln y$ of the potential. Therefore, we can greatly simplify the algebra working with a kernel of the form

$$K(y;\lambda)=\begin{cases}0 & \text{if } y<A\,,\\ y^{-\gamma} & \text{if } y\geq A\,,\end{cases}$$
$$A>0\,,\quad \gamma=\frac{\kappa+1}{2}(1-i\lambda)\,, \tag{23}$$

where we replaced the hard core of (3) with a wall of infinite height at $y=A\approx y_0$. Hence, we readily obtain

$$\tilde{K}(s;\lambda)=s^{\gamma-1}\Gamma(1-\gamma,sA)\,. \tag{24}$$

Since the previous analysis has led to completely different dominant configurations for small and large Y (hadrons equidistant in rapidity and gap production, respectively), we now consider the U distribution in both small and large Y limits. In the former case most of the contribution to $Z_n(Y;\lambda)$ comes from the large-s behavior of (24), i.e.,

$$\tilde{K}^{n+1}(s;\lambda)=A^{-(n+1)\gamma}\frac{e^{-(n+1)As}}{s^{n+1}}\times\left[1-\frac{(n+1)\gamma}{sA}+O(s^{-2})\right]. \tag{25}$$

Assuming that Y is near the kinematic limit, $Y=(n+1)A$, we find

$$Z_n(Y;\lambda)=\frac{[Y-(n+1)A]^n}{n!}e^{-B\gamma}\,, \tag{26}$$

where

$$B=(n+1)\ln A+\frac{Y-(n+1)A}{A}\approx(n+1)\ln\frac{Y}{n+1}\,. \tag{27}$$

This gives only a δ function in the U distribution, namely

$$Q_n(U)=\left(\frac{Y}{n+1}\right)^{-[(\kappa+1)/2](n+1)}\frac{[Y-(n+1)A]^n}{n!}\times\delta\left(U-(n+1)\frac{\kappa+1}{2}\ln\frac{Y}{n+1}\right). \tag{28}$$

Therefore, the only point of the U space which contributes to the distribution is

$$U=(n+1)\frac{\kappa+1}{2}\ln\frac{Y}{n+1}=(n+1)V(\alpha)\,, \tag{29}$$

which corresponds to the total energy of the perfectly ordered system with equidistantly spaced hadrons at $\Delta y=\alpha=Y/(n+1)$, in agreement with our previous result (12).

On the other hand, for large Y the small s behavior of (24) gives the dominant contribution to $Z_n(Y;\lambda)$. We have

$$\tilde{K}^{n+1}(s;\lambda)=\sum_{\rho=1}^{n+1}(-1)^{n+1-\rho}\binom{n+1}{\rho}s^{\rho(\gamma-1)}\times\Gamma^\rho(1-\gamma)\Sigma^{n+1-\rho}\,, \tag{30}$$

where

$$\Sigma=\sum_{l=0}^{\infty}\frac{(-1)^lA^{1-\gamma+l}s^l}{l!(1-\gamma+l)}\,. \tag{31}$$

The $\rho=0$ term in (30) is omitted since it does not contribute to the inverse Laplace transform (20). Hence, for $Y\to\infty$ the leading contribution to $Z_n(Y;\lambda)$ comes from the $\rho=1$ term of (30), with the $l=0$ term of Σ. On the other hand, for $\kappa<(\rho_0+1)/(\rho_0-1)$, $(\rho_0\leq n+1)$, the "corrections" at finite Y to this contribution come from the $\rho=2,3,\ldots,\rho_0$ terms of (30) with the $l=0$ term of Σ only, i.e., for κ sufficiently small we can write, for some ρ_0,

$$\tilde{K}^{n+1}(s;\lambda)\approx\sum_{\rho=1}^{\rho_0}(-1)^{n+1-\rho}\binom{n+1}{\rho}A^{(n+1-\rho)(1-\gamma)}\frac{\Gamma^\rho(1-\gamma)s^{\rho(\gamma-1)}}{(1-\gamma)^{n+1-\rho}}\,. \tag{32}$$

Therefore, we now easily obtain

$$Q_n(U)\approx\frac{2}{(\kappa+1)Y\Gamma(n)}\sum_{\rho=1}^{\rho_0}(-1)^{\rho-1}\rho\binom{n+1}{\rho}\exp\left(-\frac{\kappa-1}{\kappa+1}U\right)\left[\frac{2U}{\kappa+1}-(n+1-\rho)\ln A-\rho\ln Y\right]^{n-1}\times\theta\left(\frac{2U}{\kappa+1}-(n+1-\rho)\ln A-\rho\ln Y\right). \tag{33}$$

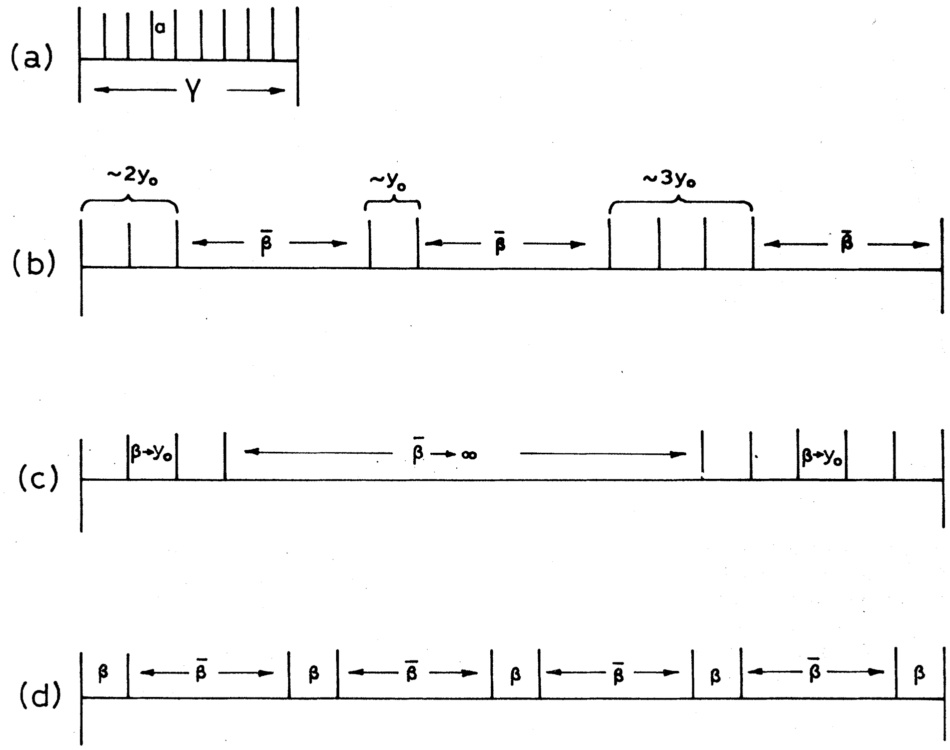

FIG. 2. Some stable equilibrium configurations of the one-dimensional Feynman-Wilson fluid (corresponding to $n=8$ hadrons). (a) The only allowed stable equilibrium configuration for $Y \leqq (n+1)y_0 = 9y_0$. (b) A typical configuration which can result if $Y > (n+1)y_0 = 9y_0$. Note the structures of fixed mass, $M_{n_h} \simeq \cosh(n_h y_0)$ which involve n_h+1 hadrons. (c) The most likely configuration at asymptotic energies involves the production of one large gap; hence the asymptotic result $\sigma_n \sim \sigma_{el}$. (d) The least probable stable equilibrium configuration involves pairing of hadrons into minimal structures of mass $\cosh y_0$ [the configuration (a) is not stable asymptotically].

The first term of this series represents the leading contribution to $Q_n(U)$ at truly asymptotic energies; it has a maximum at

$$U=(n-1)\frac{\kappa+1}{\kappa-1}+\frac{\kappa+1}{2}(n\ln A+\ln Y)\,, \tag{34}$$

which obviously corresponds to the analog energy of the one-gap configuration [Fig. 2(c)], which was found previously to dominate asymptotically. The other terms in (33) represent nonasymptotic fluctuations relative to the first one, at least for κ sufficiently small. Each of them produces a maximal effect for

$$U=(n-1)\frac{\kappa+1}{\kappa-1}+\frac{\kappa+1}{2}[(n+1-\rho)\ln A+\rho\ln Y]\,. \tag{35}$$

These points can obviously be identified with the analog energies of the two, three, etc., gap production configurations found previously to be present at finite energies [see Eq. (16), where asymptotically $\beta\approx y_0\approx A$ and $\ln\bar{\beta}\to\ln Y$; of course the terms related to the details of the hard core, are absent here]. Figure 3 shows the truly asymptotic contribution (one gap) to $Q_4(U)$, together with fluctuations owing to two and three gap configurations.

Finally, one can calculate the moments

$$\langle U^p\rangle=\int U^p Q_n(U)dU\Big/\int Q_n(U)dU \tag{36}$$

of the U distribution. In particular, at truly asymptotic energies [$\rho=1$ term in (33)] we find

$$\langle U\rangle=\frac{\kappa+1}{\kappa-1}n+\frac{\kappa+1}{2}(n\ln A+\ln Y) \tag{37}$$

and

$$\Delta U\equiv(\langle U^2\rangle-\langle U\rangle^2)^{1/2}=\frac{k+1}{k-1}\sqrt{n} \tag{38}$$

independent of Y. This is a particularly reasonable result in statistical systems.

III. DISCUSSION AND GENERALIZATION

Having traced the regions of multiparticle phase space which are favored by the potential interac-

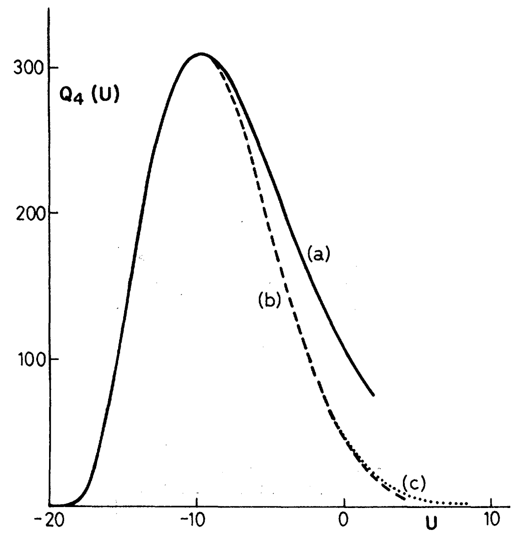

FIG. 3. Typical U distribution for $n=4$ and $\kappa=2$ ($Y=10$, $A=0.025$). (a) One-gap production [$\rho=1$ term in Eq. (33)]. (b) Modification of the one-gap U distribution owing to the production of a second gap [$\rho=1$ plus $\rho=2$ term in Eq. (33)]. (c) Modification of the two-gap U distribution, due to a third gap [$\rho=1$ plus $\rho=2$ plus $\rho=3$ terms in Eq. (33)].

tion (3), we now proceed to discuss in more detail the physical content of our model, and to show that in fact, most of our results are quite independent of the "details" of this potential.

From (16) we see that one gap [see Fig. 2(c)] is produced with maximal probability, whereas two, three, etc., gaps are produced with quickly decreasing probabilities. Since the symmetric solution is not stable asymptotically, the least likely stable configuration at asymptotic energies involves the production of pairs with spacing $\Delta y = y_0$ [see Fig. 2(d)]. In general, as $Y\to\infty$ localized structures in rapidity are produced, with fixed size $\Delta y = n_h y_0$ which contain n_h+1 hadrons [see Fig. 2(b)]. It is remarkable, that this phenomenon has also been discovered in a theory of strong interactions with spontaneously broken internal symmetry, recently suggested by Arnold,[12] which leads to condensation in rapidity and charge space. We may identify the structures found in our model with metastable hadrons H_{n_h} of very high spin ($p_L \gg p_T$), which decay into n_h+1 ordinary mesons, e.g., $H_{n_h} \to (n_h+1)\pi$, and have a mass spectrum $M_{n_h} \sim \cosh(n_h y_0)$. This spectrum seems to be uncorrelated with the ordinary hadron spectrum, described in this model by the quantized intercepts of "Regge trajectories"[9]

$$\mathrm{Re}\,\alpha_m(0) = 1 + \left(\frac{2\pi m}{b}\right)^{\mu} \cos\frac{\mu\pi}{2} \quad (m=0,1,2,\ldots), \tag{39}$$

where $b=\mu(\kappa a/\mu)^{\eta}$, $\mu=(1-\eta)^{-1}$, which result[9] from the potential (3). The dynamical origin of the heavy metastable hadrons H_{n_h} must be searched in the critical behavior of the hadronic matter.[12]

Since the one-gap configuration gives asymptotically the dominant contribution to the n-particle production cross section, for fixed n and $Y\to\infty$ we have

$$\sigma_n \sim \sigma_{el}, \tag{40}$$

which may be directly checked from the explicit calculations of Ref. (9). Similar behavior of σ_n has been found by Bartels and Rabinovici[13] in the framework of RFT. Although we are working with a factorizable model, we arrived at a "gap expansion" of σ_n, of the same nature as obtained by Pomeron-interaction corrections which are highly nonfactorizable. Of course, our factorizable model based on the iteration of $\sigma_{el} = \exp[-V(y)]$ is free of the Finkelstein-Kajantie disease[14]; also the sum of σ_n relevant to the configurations discussed in the previous sections, saturates σ_{tot} (Ref. 9) whereas this is not true in RFT.

In view of the fact that KNO scaling and (40) are common properties of both RFT and the present model, we now discuss the implications of (40) for the small x behavior of the KNO scaling function. Notice that for fixed n, we achieve $x\to 0$ by letting $Y\to\infty$, and $\psi(x)\sim\langle n\rangle\sigma_{el}/\sigma_{tot}$. In RFT with AGK cutting rules it is found[6] that $\langle n\rangle \sim Y^{1+\eta}$, $\sigma_{el}\sim Y^{2\eta-\nu}$, $\sigma_{tot}\sim Y^{\eta}$ hence

$$\psi(x) \underset{x\to 0}{\sim} x^{-(1+2\eta-\nu)/(1+\eta)}. \tag{41}$$

On the other hand, in RFT with AM cutting rules, the asymptotic behavior of the average multiplicity takes the form[5] $\langle n\rangle \sim Y^{1-\eta}$ while σ_{el} and σ_{tot} remain unchanged. Hence

$$\psi(x) \underset{x\to 0}{\sim} x^{(\nu-1)/(1-\eta)}. \tag{42}$$

Finally, in the framework of the critical FW fluid it is found[1,9]

$$\langle n\rangle \sim Y^{1-\eta}, \quad \sigma_{el}\sim Y^{\eta-2}, \quad \sigma_{tot}\sim Y^{-\eta},$$

hence,

$$\psi(x) \underset{x\to 0}{\sim} x \tag{43}$$

independent of any critical exponents. With all model estimates[15] of the critical exponents η and ν in RFT, we see that (41) tends to infinity for $x\to 0$, while (42) tends to zero [which is always the case with the FW prediction (43)]. Present data seem to be compatible with the latter case. This result is not in favor of the particular assumptions hidden in the AGK cutting rules.[7] Future high-energy experiments probing into $x\ll 0.1$ will certainly

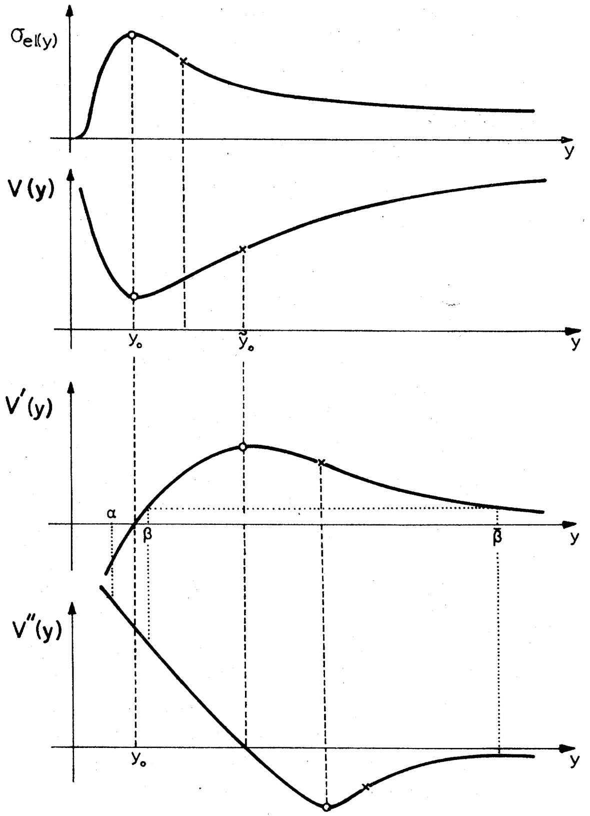

FIG. 4. General characteristics of the analog potential $V(y)$ and of its first and second derivatives, when the elastic cross section $\sigma_{el} = \exp[-V(y)]$ decreases asymptotically slower than any power of the C.M. energy. It is clear that if $y_\lambda - y_{\lambda-1} > y_0$, Eq. (6) has *two* stable solutions, β and $\bar{\beta}$. Moreover, $\beta \to y_0$ if $\bar{\beta} \to \infty$.

be crucial to test the validity of these models.

Although we obtained all of the above results with explicit reference to the specific potential (3), which has the virtues of s-channel unitarity and KNO scaling, it is easy to see (see Fig. 4) that the results concerning the asymptotic structure of the multiparticle events and the gap expansion of σ_n, are in fact independent of the particular form of the potential provided that it obeys the following two conditions:

(a) At large y, $V(y)$ increases slower than const $\times y^1$. This is equivalent to requiring that the elastic cross section decreases slower than any power of the energy for large energy ($\sigma_{el} > s^{-\gamma}$, $\gamma > 0$).

(b) The potential $V(y)$ has a minimum at $y = y_0$, i.e., σ_{el} has a maximum which will, in general, be of kinematic origin since σ_{el} will have a threshold and will be rather quickly decreasing at intermediate energies.

These conditions mean that $V(y)$ will have a point of inflexion at $\tilde{y}_0 > y_0$, i.e., σ_{el}, as can be easily seen, will have a point of inflexion between y_0 and $\tilde{y}_0$. It is worth noting that this is precisely the condition in order to have an SSB phenomenon of the minimum potential energy state of the λth particle in the potential well formed by the $(\lambda - 1)$th and the $(\lambda + 1)$th particles, as can be easily seen from (7) and discussed in Sec. I.

In Fig. 4 we show a possible behavior of σ_{el} and the corresponding potential $V(y)$ which satisfy the conditions (a) and (b). The first and second derivatives $V'(y)$ and $V''(y)$ of the potential are also shown. Recalling (6) and (7) and inspecting this figure, we easily convince ourselves that the stable equilibrium configurations found in Sec. II [in the cases (i), (ii), and (iii) considered there] hold true in any case $V(y)$ obeys conditions (a) and (b). Perhaps we should explicitly note that the stability of the asymmetric solution in case (ii), now requires $V''(\beta) > -V''(\bar{\beta})$ which is true, at least asymptotically, since $\bar{\beta} \to \infty$, $\beta \to y_0$, and $V''(y) \to 0_-$ for $y \to \infty$.

If σ_{el} has more than one local minima at intermediate energies, our picture remains basically unchanged and the stable asymmetric solution is manifested as a large gap at asymptotic energies, after some possible transition phenomena and local instabilities have settled. We conclude that it is the SSB mechanism, and not the details of the particu-

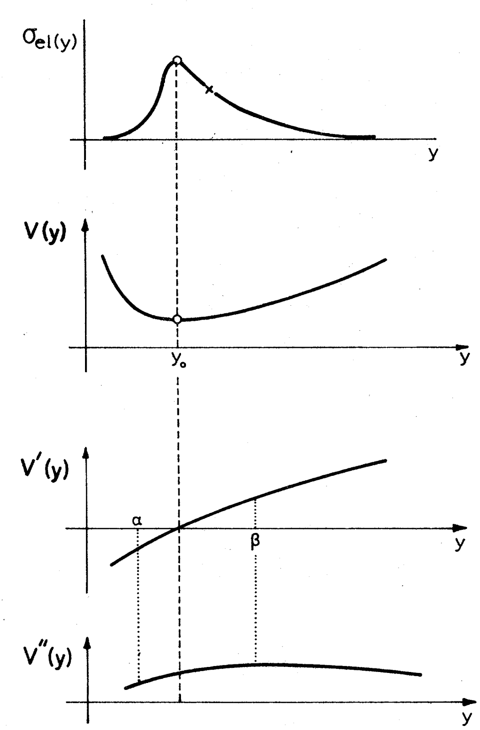

FIG. 5. Same as Fig. 3, but in a case where the elastic cross section decreases faster than any power of the energy. Now, Eq. (6) has always *one* solution.

lar potential (3), which is responsible for the density fluctuations manifested as gap production at asymptotic energies. Of course, we have to be careful in interpreting this phenomenon as a "phase transition" (compared with the behavior of a plastic cord near the strength limit). One has to check the analytic structure of $Q(z, Y)$ in z, for each specific potential; e.g., Eq. (3) clearly leads to phase transition.

To complete this discussion, we now consider the case where the elastic cross section decreases faster than any power of the energy (while it still has a maximum at $y=y_0$). This means that the potential $V(y)=-\ln\sigma_{el}(y)$ increases faster than (const) $\times y^1$ and does not have the point of inflexion at $y=\bar{y}_0$, i.e., the spontaneous symmetry breaking is lost. By inspection of Fig. 5, which shows the same quantities as Fig. 4 but corresponds to the case $V(y)>y^1$ we easily see that there is only the symmetric solution allowed and leads to a stable configuration. Hence, in this rather unphysical case fluctuations do not appear. Finally, we notice that an increasing σ_{el} as Y^γ, $\gamma>0$, lies outside the framework of factorizable models, since it results in violation of s-channel unitarity.[16]

IV. CONCLUSIONS

We have investigated the implications of a factorizable mechanism for hadron production at asymptotic energies, which we introduced in previous work, on the structure of multiparticle events. The advantage of this specific mechanism is that although it involves a singularity at $j=1$, it does not spoil s-channel unitarity; it also allows for KNO scaling. We have shown that most of our results are in fact independent of its "details," but derive mostly from the property of SSB that the corresponding potential, relevant to the analog FW fluid, possesses.

In particular, we studied the density fluctuations in rapidity space, motivated by the critical behavior of the analog FW fluid, traced previously in the framework of this model. For this purpose we studied the stable equilibrium configurations of the equivalent one-dimensional classical system.

It is found that at sufficiently low energies, hadrons are equidistantly produced in rapidity. At high energies [more precisely, for $Y>(n+1)y_0$ where n is the number of hadrons produced, and y_0 is a constant] gap production occurs with probability decreasing as the number of gaps increases. Asymptotically, the one-gap configuration is dominant. This hierarchy of gap configurations, shown in Fig. 2, is similar to the corresponding result found in the framework of RFT[13] and shows the specific structure of density fluctuations connected to the underlying critical behavior which at least our model potential (3) exhibits.

The structures produced near the walls or between two large gaps for $Y\to\infty$, have fixed size when the number of hadrons they involve is fixed. This property allows their interpretation as heavy hadrons with high spin and well-defined mass spectrum. Similar results have been found by Arnold in Ref. 12.

An immediate consequence of this picture is that $\sigma_n \sim \sigma_{el}$ for fixed n and $Y\to\infty$, which is also found in Ref. 13. This property, if combined with KNO scaling, implies a behavior of the form $\psi(x)\sim x^\alpha$ near $x=0$ for the KNO scaling function. The exponent α depends crucially on the particular theory. The AGK cutting rules together with the RFT estimates of the critical exponents give $\alpha<0$ while the AM cutting rules, or our model based on the critical FW fluid, predict $\alpha>0$.

To conclude, we note that three seemingly completely different approaches to the high energy hadron physics, namely RFT, the stochastic-field approach of Arnold, and the critical-FW-fluid model with the specific "potential" (3), lead to qualitatively similar results, i.e., production of gaps, together with heavy metastable hadrons. This may not be too surprising since in all three models, a critical behavior is exhibited by the hadronic system they described. Hence, the similarities may be ascribed to an underlying universality principle.

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