First-order phase transitions in gauge theories

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The statistical mechanics of field theories with broken, global, and local SU(3) gauge symmetries is studied in the one-loop approximation. The presence of a cubic term in the potential, when allowed by symmetry, is to restore the symmetry by a first-order phase transition rather than by a second-order transition. The transition temperatures are estimated to be too low for a finite density of magnetic monopoles to be excited.

I. INTRODUCTION

Recently, Kirzhnits and Linde[1,2] showed that relativistic field theories with spontaneous symmetry breaking undergo a phase transition at high temperatures leading to a restoration of the gauge symmetry. Weinberg[3] and Dolan and Jackiw[4] made detailed calculations of this phenomenon by extending the theory to include local gauge symmetry. These authors confirmed the predictions of Ref. 1 and calculated the transition temperatures T_c in the one-loop approximation. Typically, they found a continuous second-order phase transition at T_c. They showed that the one-loop results are reliable in the high-temperature (or weak coupling) regime. Weinberg[3] also discussed the astrophysical implications of such phase transitions. In the present article a new aspect of the statistical mechanics of relativistic field theories is investigated in which one obtains a first-order discontinuous restoration of gauge symmetry. This happens in gauge theories which allow a cubic term in the potential energy. Thus, at the transition temperature T_0, two phases coexist, an ordered phase in which the equilibrium state breaks gauge invariance and a disordered phase. Above T_0 the system undergoes a discontinuous change into a disordered state.[11]

The finite-temperature functional formulation has been discussed by Bernard[5] and in the earlier references quoted above. We shall follow the notation of Ref. 5. In Sec. II we discuss the global SU(3) gauge theory in the one-loop level. The transition temperature and the nature of phase transitions are obtained by minimizing the effective (Helmholtz) potential. In Sec. III the calculations are modified to take into account a local SU(3) gauge theory. Apart from changing the value of T_0 the qualitative features of the global theory are unaltered. Finally, we give a qualitative argument to show that the magnetic monopoles which exist in such theories play no significant role in the statistical mechanics. In other words, it is found that the symmetry is restored at temperatures which are small compared to the masses of the monopoles.

II. STATISTICAL MECHANICS OF A GLOBAL SU(3)-SYMMETRIC THEORY

It is well known[6] that in the Landau theory of phase transition (i.e., a mean-field theory), whenever the symmetry of the model allows a cubic term in the order parameter expansion of the free energy, then one obtains a discontinuous first-order phase transition. Let us show that the same phenomenon occurs for a gauge theory. We illustrate this for an SU(3) global scalar gauge theory. The Lagrangian density is taken to be

$$\mathcal{L}=\tfrac{1}{4}\mathrm{Tr}(\partial_\mu\phi)(\partial^\mu\phi)-V(\phi)\ , \tag{2.1}$$

where

$$V(\phi)=-\frac{\mu^2}{4}(\mathrm{Tr}\,\phi^2)+\frac{\lambda_1}{16}(\mathrm{Tr}\,\phi^2)^2+\frac{\lambda_2}{6}(\mathrm{Tr}\,\phi^3) \tag{2.2}$$

and

$$\phi=\sum_{a=1}^{8}\phi^a\lambda_a\ . \tag{2.3}$$

The $\{\lambda_a\}$'s are the generators of SU(3). The scalar field ϕ takes values in the Lie algebra of SU(3). Under a gauge transformation by $\Omega\in\mathrm{SU}(3)$ we have $\phi\to\Omega\phi\Omega^{-1}$. We note that a cubic term in the potential energy is allowed by symmetry. We have neglected a term of the form $\mathrm{Tr}\,\phi^4$, but this does not change our results. The stability condition implies $\lambda_1>0$ and we take without loss of generality $\lambda_2>0$.

Let us briefly recall that for model (2.1) spontaneous symmetry breaking leaves the vacuum invariant under U(2). We show later that equilibrium states of the system at $T<T_0$ have U(2) symmetry. If, however, $\lambda_2=0$, the little group

H may be either U(2) or U(1)×U(1). This is seen readily by minimizing $V(\phi)$. The classical potential (2.2) is indeed the tree approximation to the effective potential $V_{\text{eff}}^{\beta}(\phi)$ and hence the problem of minimizing $V(\phi)$ will come up again later. Since ϕ is traceless and Hermitian we may work in its diagonal representation. Then minimizing $V(\phi)$ subject to $\text{Tr}\phi=0$ yields the minimum of $V(\phi)$ to be a solution of the form

$$\psi_1^0=\psi_2^0=\frac{-\psi_3^0}{2}, \tag{2.4}$$

and ψ_1^0 determined by

$$3\lambda_1(\psi_1^0)^2-\lambda_2\psi_1^0-\mu^2=0\,. \tag{2.5}$$

For $\lambda_2>0$, the minimum occurs for

$$\psi_1^0=\frac{\lambda_2+(\lambda_2{}^2+12\lambda_1\mu^2)^{1/2}}{6\lambda_1}\,. \tag{2.6}$$

Here we have denoted the eigenvalues of ϕ to be ψ_i, $(i=1,2,3)$. Since ψ is λ_8-like we see that $H=\text{U}(2)$.

Let us briefly outline the method of calculating the effective potential $V_{\text{eff}}^{\beta}(\phi)$ at finite temperatures.[2-5] It is given by

$$V_{\text{eff}}^{\beta}(\hat{\phi})=V_0(\hat{\phi})+V_1^{\beta}(\hat{\phi})+\left\langle\exp\left[\int d^4x\,\mathcal{L}_I(\hat{\phi}\;\psi(x))\right]\right\rangle. \tag{2.7}$$

$V_0(\hat{\phi})=V(\hat{\phi})$ is just the classical potential[1,2] with $\phi(x)$ replaced by constant fields $\hat{\phi}_a$. $V_1^{\beta}(\hat{\phi})$ is the one-loop correction and is temperature dependent $(\beta=1/kT)$. It is given by

$$V_1^{\beta}(\hat{\phi})=\lim_{\mathcal{V}\to\infty}\frac{-1}{\mathcal{V}\beta}\times\ln\left\{\int\prod_a[d\psi^a(x)]\exp\left[\int d^4x\,\mathcal{L}_0(\hat{\phi},\psi(x))\right]\right\}, \tag{2.8}$$

where $\mathcal{V}$ is the volume. $\mathcal{L}_0(\hat{\phi},\psi)$ is the quadratic part of the Lagrangian obtained from (1) by shifting the fields $\phi_a\to\hat{\phi}_a+\psi_a(x)$. The last term in (2.7) represents higher-loop corrections, and

$$\langle\cdots\rangle=\frac{\int\prod_a[d\psi^a(x)](\cdots)\exp[\int d^4x\,\mathcal{L}_0(\hat{\phi},\psi)]}{\int\prod_a[d\psi^a(x)]\exp[\int d^4x\,\mathcal{L}_0(\hat{\phi},\psi)]}\,. \tag{2.9}$$

We use the standard finite-temperature version of functional integrals. Furthermore,

$$\int d^4x\equiv\int d^3x\int_0^{\beta}d\tau\,. \tag{2.10}$$

Integrals over fields are subject to periodic boundary conditions,

$$\psi(x,\tau)=\pm\,\psi(x,\tau+\beta)\,. \tag{2.11}$$

The plus sign is for Bose fields and the minus sign for fermion fields, with the exception of the ghost fields where we use the plus sign.[5]

Thus, after making the shift $\phi\to\hat{\phi}+\psi(x)$, we obtain for the Lagrangian density, after neglecting the constant and linear terms in $\psi(x)$, the following:

$$\mathcal{L}=\mathcal{L}_0(\hat{\phi},\psi(x))+\mathcal{L}_I(\hat{\phi},\psi(x)), \tag{2.12}$$

$$\mathcal{L}_0(\hat{\phi},\psi)=\tfrac{1}{2}(\partial_\mu\psi^a)(\partial^\mu\psi^a)-\tfrac{1}{2}\psi^a(M_s{}^2)_{ab}\psi^b, \tag{2.13}$$

$$\mathcal{L}_I(\hat{\phi},\psi)=\frac{1}{3!}\frac{\partial^3V(\hat{\phi})}{\partial\hat{\phi}^a\partial\hat{\phi}^b\partial\hat{\phi}^c}\psi^a\psi^b\psi^c+\frac{1}{4!}\frac{\partial^4V(\hat{\phi})}{\partial\hat{\phi}^a\partial\hat{\phi}^b\partial\hat{\phi}^c\partial\hat{\phi}^d}\psi^a\psi^b\psi^c\psi^d. \tag{2.14}$$

The mass matrix $(M_s{}^2)_{ab}$ is defined by

$$(M_s{}^2)_{ab}=\left[-\mu^2+\lambda_1\sum(\phi^c)^2\right]\delta_{ab}+2\lambda_1\,\hat{\phi}_a\hat{\phi}_b+2\lambda_2 d_{abc}\hat{\phi}_c\,, \tag{2.15}$$

where d_{abc}'s are defined by

$$\{\lambda_a,\lambda_b\}=\tfrac{4}{3}\delta_{ab}+2d_{abc}\lambda^c\,. \tag{2.16}$$

The integrals in the one-loop approximation are Gaussian and are easily evaluated. We find

$$V_1^{\beta}(\hat{\phi})=\frac{1}{2\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^3k}{(2\pi)^3}\ln\det[(\omega_n^2+\vec{k}^2)\delta_{ab}+(M_s{}^2)_{ab}]\,. \tag{2.17}$$

In arriving at (2.17) we have made the Fourier decomposition of $\psi(x,\tau)$,

$$\psi^a(\vec{x},\tau)=\frac{1}{\beta}\sum_n\int\frac{d^3k}{(2\pi)^3}\{\exp[i(\omega_n\tau+\vec{k}\cdot\vec{x})]\}\psi^a(\omega_n,\vec{k}), \tag{2.18}$$

where

$$\omega_n=\frac{2n\pi}{\beta}\,. \tag{2.19}$$

Integrals of the form (2.17) have been evaluated by Dolan and Jackiw.[4] They show that these contain two terms, one independent of temperature and the other temperature dependent. The temperature-independent term is removed by renormalization counterterms. The temperature-dependent term can be evaluated in the high-temperature regime. Thus, if the transition temperatures are high then we may use this procedure. Evaluating (2.17) by this technique, we find

$$V_1^\beta(\phi) = -\frac{4\pi^2}{45\beta^4} + \frac{(\mathrm{Tr}\, M_s{}^2)}{24\beta^2} + O\left(\frac{1}{\beta}\right)$$

$$= \frac{-4\pi^2}{45\beta^4} + \frac{-4\mu^2 + 5\lambda_1 \sum(\hat\phi_b)^2}{12\beta^2} + O\left(\frac{1}{\beta}\right). \tag{2.20}$$

Thus the effective potential in the one-loop level is given by (we ignore terms that do not depend on $\hat\phi$)

$$V_{\mathrm{eff}}^\beta(\hat\phi) = \frac{A(T)}{4}(\mathrm{Tr}\,\hat\phi^2) + \frac{\lambda_1}{16}(\mathrm{Tr}\,\hat\phi^2)^2 + \frac{\lambda_2}{6}(\mathrm{Tr}\,\hat\phi^3), \tag{2.21}$$

where

$$A(T) = -\mu^2 + \tfrac{5}{6}\lambda_1 (kT)^2. \tag{2.22}$$

Thus this expression for $V_{\mathrm{eff}}^\beta(\hat\phi)$ is just the mean-field result. The equilibrium state of the field theory at a temperature T is determined by the global minimum of $V_{\mathrm{eff}}(\hat\phi)$. As opposed to (2.2), the global minimum is temperature dependent. Again for simplicity let us work in a representation in which $\hat\phi$ is diagonal. Let us denote this by $\hat\psi$. Then we minimize $V_{\mathrm{eff}}(\psi)$ subject to $\mathrm{Tr}\,\psi = 0$.

We find that the extrema of $V_{\mathrm{eff}}(\psi)$ are either

$$\psi_i = 0, \quad i = 1, 2, 3 \tag{2.23}$$

or

$$\psi_1 = \psi_2 = -\frac{\psi_3}{2}, \tag{2.24}$$

where

$$\psi_1(T) = \frac{\lambda_2 + [\lambda_2{}^2 - 12\lambda_1 A(T)]^{1/2}}{6\lambda_1}; \tag{2.25}$$

clearly $\psi_i = 0$ corresponds to a symmetry-restored state. We imagine lowering the temperature of the fields from a very high temperature T where $A(T) > (\lambda_2{}^2/12\lambda_1)$. Clearly, in this case the only minimum of V_{eff} is at $\psi = 0$. We have a disordered phase. Now as the temperature is lowered such that $A(T) < (\lambda_2{}^2/12\lambda_1)$ but $A(T) > 0$ (say) then an additional minimum occurs away from the origin. However, to be the true solution, the effective potential must be negative for this value of $\hat\phi$ [note that $V(\psi) = 0$ at $\psi = 0$].

Thus, look for roots of $V(\psi) = 0$. We find that $V(\psi) = 0$ (we have $\psi_1 = \psi_2 = -\frac{1}{2}\psi_3$), when $\psi_1 = 0$ (double root) and

$$\psi_1^{(\pm)} = \frac{2\{\lambda_2 \pm [\lambda_2{}^2 - \frac{27}{2}A(T)\lambda_1]^{1/2}\}}{9\lambda_1}. \tag{2.26}$$

Thus we find that in the range of temperatures determined by

$$\frac{\lambda_2{}^2}{12\lambda_1} > A(T) > \frac{2\lambda_2{}^2}{27\lambda_1},$$

the minimum at nonzero ψ_i is not a global minimum. Hence, one still has a symmetry-restored equilibrium state. Hence, at a temperature T_0 determined by

$$A(T_0) = \frac{2\lambda_2{}^2}{27\lambda_1}$$

or

$$(kT_0)^2 = \frac{6}{5\lambda_1}\left(\mu^2 + \frac{2\lambda_2{}^2}{27\lambda_1}\right), \tag{2.27}$$

there is a coexistence between two phases, one a broken-symmetry state $\psi_1 = \psi_2 = -\frac{1}{2}\psi_3 = 2\lambda_2/9\lambda_1$) and a symmetry-restored state with $\psi_1 = \psi_2 = \psi_3 = 0$. For $T < T_0$ one has only a broken-symmetry state with ψ_i's determined from (2.25). We see that all these states have U(2) symmetry. Figure 1 illus-

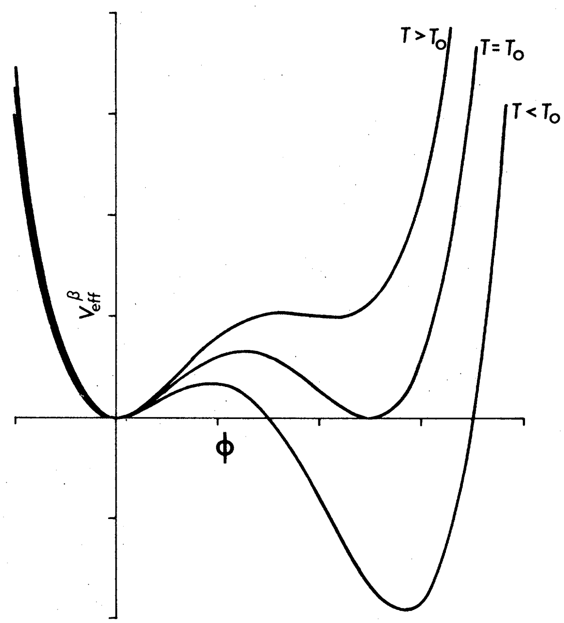

FIG. 1. Plot of effective potential against $\hat\phi$ for three different temperatures. At $T = T_0$ there are two equilibrium states. Below T_0 the equilibrium state is the broken-symmetry state, and above T_0 symmetry is restored discontinuously.

trates schematically the effective potential as a function of temperature. We thus have established the occurrence of a first-order phase transition.

The masses of the scalar fields may be readily evaluated from $(M_s^2)_{ab}$ for the equilibrium value of $\hat{\phi}$ which is λ_8-like for temperatures $T < T_0$. Since our calculations are valid in the high-temperature range, the following formulas are valid for $T \lesssim T_0$. We find that $(M^2)_{ab} = \delta_{ab}(M^2)_{aa}$ with

$$(M_s^2)_{aa} = \sqrt{3}\,\lambda_2 \phi^8(T), \quad a = 1,2,3$$

$$(M_s^2)_{aa} = 0, \quad a = 4,5,6,7 \tag{2.28}$$

$$(M_s^2)_{aa} = \lambda_1[\phi^8(T)]^2 - A(T), \quad a = 8$$

where

$$\phi_8 = \frac{\lambda_2 + [\lambda_2^2 - 12\lambda_1 A(T)]^{1/2}}{2\sqrt{3}\,\lambda_1} \ . \tag{2.29}$$

We can now estimate the limits of validity of our results. From (2.27) we see that $(kT_0)^2 > 6\mu^2/5\lambda_1 = (kT_c)^2$ where T_c is the critical temperature for a theory without the cubic term where the phase transition is of second order and the masses of the scalar fields vanish continuously at $T = T_c$. Typically $\mu^2/\lambda_1 \sim O(1/G_F) \simeq 10^5 M_p^2$ where G_F = Fermi coupling constant and M_p is the proton mass. Thus $(kT_0) \simeq 300$ GeV. Thus, this is indeed a high temperature.

III. LOCAL SU(3) GAUGE THEORY

The calculations for a local gauge theory are more involved because of the need to take into account the gauge-fixing and ghost terms in the Lagrangian. The Lagrangian density is now given by

$$\mathcal{L} = -\tfrac{1}{2}\,\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{4}\mathrm{Tr}(D_\mu\phi)(D^\mu\phi) - V(\phi) \ . \tag{3.1}$$

$V(\phi)$ is given by (2.2):

$$D_\mu\phi = \partial_\mu\phi + g[A_\mu, \phi], \tag{3.2}$$

$$A_\mu = \frac{A_\mu^a \lambda_a}{2i} \ , \tag{3.3}$$

$$\phi = \phi^a \lambda_a \ , \tag{3.4}$$

$$F_{\mu\nu} = \frac{F^a_{\mu\nu}\lambda_a}{2i} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\mu]. \tag{3.5}$$

Under SU(3) the fields transform as follows:

$$\phi \to \Omega\phi\Omega^{-1}, \quad \Omega \in \mathrm{SU}(3),$$

$$A_\mu \to \Omega A_\mu \Omega^{-1} + \frac{1}{g}(\partial_\mu\Omega)\Omega^{-1} \ . \tag{3.6}$$

As in Sec. II we shift the scalar fields $\phi \to \hat{\phi} + \psi(x)$ in order to study the symmetry-breaking pattern. Again, apart from constant and linear terms in ψ, the Lagrangian density may be written as

$$\mathcal{L} = \mathcal{L}_0(\hat{\phi}; \psi, A_\mu) + \mathcal{L}_I(\hat{\phi}; \psi, A_\mu) \ , \tag{3.7}$$

$$\mathcal{L}_0(\hat{\phi}; \psi, A_\mu) = -\tfrac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 + \tfrac{1}{2}(\partial_\mu\psi^a)(\partial^\mu\psi^a) + \tfrac{1}{2}(M_w^2)_{ab}A_\mu^a A^{bu} - \tfrac{1}{2}(M_s^2)_{ab}\psi^a\psi^b + gf_{abc}\hat{\phi}^a\partial_\mu\psi^b A^{c\mu} \ , \tag{3.8}$$

where

$$(M_w^2)_{ab} = g^2 f_{acd} f_{bce}\,\hat{\phi}^d\hat{\phi}^e \ . \tag{3.9}$$

$\mathcal{L}_I$ may be written down readily, but since it does not contribute to the effective potential in the one-loop level, we ignore it in what follows.

We must now add to $\mathcal{L}_0$ the gauge-fixing and ghost terms. Weinberg[7] has demonstrated that the computation of the effective potential is reliable only in the Landau gauge; otherwise one must suitably modify the R_ξ gauge.[8] Here we therefore compute V_1^β in the Landau gauge. We do this by starting with the R_ξ gauge and taking the $\xi \to \infty$ limit. The gauge-fixing contribution to the Lagrangian is given by[9]

$$-\tfrac{1}{2}F_a F_a = -\tfrac{1}{2}\left[\xi(\partial_\mu A^{a\mu})^2 + \frac{1}{\xi}(gf_{abc}\hat{\phi}^b\psi^c)^2 - 2gf_{abc}\hat{\phi}^b\psi^c\partial_\mu A^{a\mu}\right], \tag{3.10}$$

where

$$F_a = \sqrt{\xi}\left(\partial_\mu A^{a\mu} - \frac{1}{\xi}\,gf_{abc}\hat{\phi}^b\psi^c\right) . \tag{3.11}$$

The Landau gauge corresponds to the $\xi \to \infty$ limit. The quadratic part of the action including (3.10) is

$$\int(\mathcal{L}_0(\hat{\phi}, \psi, A_\mu) - \tfrac{1}{2}F_a F_a)d^4x = -\frac{1}{2\beta}\sum_n\sum_k\{A_\mu^a(k_n)(\Delta_F^{-1}(k_n;\hat{\phi}))^{\mu\nu}_{ab}A_\nu^b(k_n) + \psi^a(\vec{k}_n)(\Delta_F^{-1}(k_n;\hat{\phi}))_{ab}\psi^b(k_n)\},$$

where

$$k_n^\mu = (\omega_n, \vec{k}) \ , \tag{3.13}$$

and where

$$(\Delta_F^{-1})^{\mu\nu}_{ab} = \{[(\omega_n^2 + \vec{k}^2)\delta_{ab} + (M_w^2)_{ab}]\delta_{\mu\nu} + (\xi - 1)k_n^\mu k_n^\nu\delta_{ab}\}, \tag{3.14}$$

$$(\Delta_F^{-1})_{ab}=(\omega_n^2+\vec{k}^2)\delta_{ab}+\left(M_s^2+\frac{1}{\xi}M_w^2\right)_{ab} . \quad (3.15)$$

The ghost contribution can be evaluated by standard techniques.[9] This contribution can be written as an effective Lagrangian for anticommuting ghost fields. Since we are interested in one-loop calculations, we only need to take into account the quadratic part of the ghost Lagrangian. This yields to the effective potential a contribution of the form

$$\frac{1}{2\beta}\sum_n\int\frac{d^3k}{(2\pi)^3}\ \ln\det\left[(\omega_n^2+\vec{k}^2)\delta_{ab}+\frac{(\mu^2)_{ab}}{\xi}\right] , \quad (3.16)$$

where $(\mu^2)_{ab}$ is the mass matrix. On evaluating this integral in the same approximation as for the other two-point functions, we find the significant term to be $\mathrm{Tr}\,\mu^2/24\beta^2\xi$ which in the $\xi\to\infty$ limit vanishes. Hence, we may neglect the ghost contribution in the one-loop level. We can now write down the effective potential in the one-loop level:

$$V_{\mathrm{eff}}^{\beta}=V_0(\hat{\phi})+V_1^{\beta}(\hat{\phi}) , \quad (3.17)$$

where

$$V_1^{\beta}(\hat{\phi})=\frac{1}{2\beta}\sum_n\int\frac{d^3k}{(2\pi)^3}\,[\ln\det(\Delta_F^{-1})_{ab}+\ln\det(\Delta_F^{-1})_{ab}^{\mu\nu}] . \quad (3.18)$$

The determinant over the $(\mu\nu)$ indices is evaluated using the result

$$\det(a\delta_{\mu\nu}+bk_\mu k_\nu)=a^4\left(1+\frac{b}{a}k^2\right) . \quad (3.19)$$

The remaining integrals are evaluated as before. We find after letting $\xi\to\infty$ that

$$V_1^{\beta}(\hat{\phi})=3\left[\frac{-\pi^2}{9\beta^4}+\frac{3}{24}\frac{g^2}{\beta^2}\sum(\phi^a)^2+O\left(\frac{1}{\beta}\right)\right] . \quad (3.20)$$

Thus the effective potential is given by

$$V_{\mathrm{eff}}^{\beta}(\hat{\phi})=\frac{1}{2}\left(-\mu^2+\frac{5\lambda_1}{6\beta^2}+\frac{3}{4}\frac{g^2}{\beta^2}\right)\sum(\hat{\phi}^c)^2+\frac{\lambda_1}{4}\left[\sum(\hat{\phi}^a)^2\right]^2+\frac{\lambda_2}{3}\sum d_{abc}\hat{\phi}^a\hat{\phi}^b\hat{\phi}^c . \quad (3.21)$$

Thus the problem of finding the equilibrium state of the field theory is identical to the previous case, with the obvious modification

$$A(T)\to A'(T)=-\mu^2+\left(\frac{5\lambda_1}{6}+\frac{3g^2}{4}\right)(kT)^2 .$$

Then the local gauge theory undergoes a first-order phase transition at a temperature T_0 determined by

$$(kT_0)^2=\frac{2\lambda_2/27\lambda_1+\mu^2}{5\lambda_1/6+3g^2/4} . \quad (3.22)$$

The masses of the vector bosons as a function of temperature $T\lesssim T_0$ are evaluated by substituting the equilibrium value of $\hat{\phi}=\lambda_8\phi_8$ determined by minimizing $V_{\mathrm{eff}}(\hat{\phi})$ into the expression for the mass matrix for the vector bosons. We notice that the symmetry is restored in the gauge theory at lower temperatures than for the global theory.

IV. HIGHER-LOOP CORRECTIONS AND MAGNETIC MONOPOLES

If the phase transition were of second kind ($\lambda_2=0$), then near T_c fluctuations in the field would have been uncontrollable and higher-loop corrections would have been significant. Kirzhnits and Linde[2] show that the one-loop calculations are reliable provided $(T-T_c)/T_c>\lambda_1$. In the present case we have a first-order phase transition which may be thought of as an interrupted second-order transition in which the order parameter goes to zero discontinuously at T_0. Hence we expect that the one-loop level is reliable for $T\sim T_0$, as fluctuations are unimportant near T_0 for sufficiently large λ_2.

The theory that we have considered admits magnetic-monopole solutions. It would be useful to inquire if these objects will change the nature of symmetry restoration. We recall that the monopole mass in these theories[10] is of the order

$$M_m=O\left(\frac{\mu}{\sqrt{\lambda_1}\,g}\right) . \quad (4.1)$$

On the other hand, the transition temperature (kT_0) is

$$(kT_0)=O\left(\frac{\mu}{(\frac{5}{6}\lambda_1+3g^2/4)^{1/2}}\right) . \quad (4.2)$$

If $\lambda_1\ll g^2$ then $kT_0\sim O(\mu/g)$. Hence $(M_m/kT_0)\sim O(1/\sqrt{\lambda_1})\gg 1$.

Hence near T_0 the density of magnetic monopoles will be very small and we might ignore them. If $g^2\ll\lambda_1$ then

$$(kT_0)\sim O\left(\frac{\mu}{\sqrt{\lambda_1}}\right)\text{and }\frac{M_m}{kT_0}\simeq\left(\frac{1}{g}\right)\gg 1 .$$

The conclusions are unaltered. Hence it would seem that temperatures at which symmetry of the gauge field theory is restored are small compared to the masses of the monopoles, and thus the effects of these objects on the statistical properties of the fields may be ignored.

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