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Multiparticle production in the unitarization scheme complemented by duality

M. Fukugita
National Laboratory for High Energy Physics, Tsukuba-gun, Ibaraki-ken, 300-32 Japan

T. Inami
Institute of Physics, University of Tokyo, Komaba, Meguro-ku, Tokyo, 153 Japan

N. Sakai
Department of Physics, Tohoku University, Sendai, 980 Japan

S. Yazaki
Department of Physics, University of Tokyo, Tokyo, 113 Japan
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It is shown that the multiperipheral resonance production model, which was originally proposed to test symmetries and duality, can be made a realistic model for multiparticle production by incorporating production of pseudoscalar, axial-vector, and scalar mesons in addition to vector and tensor mesons. With some symmetry assumptions, we determine the coupling strengths of produced mesons to exchanged Reggeons from data of inclusive cross section of ρ^0 and f production. We also obtain the relative weight of resonance multiplets to the inclusive pion production cross section. The coupling strengths thus obtained are used to test the consistency relation between two-body scattering and multiparticle production reactions in the unitarization scheme combined with duality.

I. INTRODUCTION

Remarkable progress has recently been made in our understanding of high-energy two-body reactions by implementing duality ideas to multiparticle unitarity. In particular, the spectra and couplings of the Pomeron, Reggeons, and exotic exchanges[1-12] and the magnitude of Okubo-Zweig-Iizuka (OZI) rule violation[12-16] have been calculated.

This scheme based on unitarity and duality should make it possible to connect two-body reactions with multiparticle reactions as the intermediate states in the unitarity equation. However, few discussions have so far been made on multiparticle reactions from this point of view.[17] This is mainly because there is an ambiguity in identifying the $q\bar{q}$ state of duality diagrams with particles produced in multiparticle reactions.

Chan *et al.* have proposed to regard this $q\bar{q}$ state as a low-mass cluster.[2] Schmid *et al.* have pointed out that both even- and odd-charge-conjugation states should be produced and have suggested identifying the $q\bar{q}$ states as vector and tensor resonances.[3,4] The resonance production model in the dual unitarization scheme has been found to be quite successful in explaining quantitatively the size of ρ-A_2 exchange-degeneracy breaking and that of OZI-rule violation.[3,6,9,13]

Recent experiments on inclusive reactions have revealed the important role played by resonances in multiparticle production.[18] This observation suggests the possibility that the multiperipheral resonance production model is valid not only as a tool for symmetry test[3] but also as a realistic model for multiparticle production. For the model to be realistic, however, one should take account of the production of other important mesons in addition to vector and tensor mesons.

In this paper we present the multiperipheral resonance production model with pseudoscalar, axial-vector, and scalar mesons as well as vector and tensor mesons and make quantitative predictions on multiparticle reactions. The basic parameters of our model are the coupling strengths of produced mesons to exchanged Reggeons. It is shown that we can determine these coupling strengths from the available data on inclusive cross sections by assuming SU(2) (quartet) symmetry relations among produced mesons. The result can be used to test the relations derived previously in the unitarization scheme with duality. Furthermore, we obtain the relative contribution of various resonance multiplets to the inclusive cross section of pion production. We give predictions for cross-section ratios of particles belonging to the same multiplets, such as η/π and ω/ρ ratios.

We take vector and tensor trajectories for exchanged Reggeons in the input production amplitude, and in most of our calculation the input trajectory is assumed to have the intercept $\alpha^{\text{in}} \simeq \frac{1}{2}$. For *directly produced* resonance multiplets we consider four cases: (I) V, T and P, (II) V, T,

P and A, (III) V, T, and S, (IV) V, T, S, P, and A. Here V, T, S, P, and A stand for vector, tensor, scalar, pseudoscalar, and axial-vector ($J^{PC}=1^{+-}$) mesons, respectively. From the viewpoint of duality, the scalar mesons may be regarded as the daughter states of the tensor mesons. Axial-vector mesons with $J^{PC}=1^{+-}$ are the exchange-degenerate partners of the pseudoscalar mesons. Because kaons are produced much less copiously than pions, we mainly consider production of nonstrange mesons in the SU(2) framework. As for the pseudoscalar quartet, we take $\pi^{\pm,0}$ and $\eta_N = (p\bar{p}+n\bar{n})/\sqrt{2}$. Baryons are not treated.

The main results of our analysis can be summarized as follows:

(i) It is estimated that, at high energies, about 50–60% and 15–20% of produced pions are decay products of *directly produced* vector and tensor mesons, respectively, while the remaining 35–20% comes from pseudoscalar, axial-vector, and/or scalar mesons. The direct π production is less than about 20%.

(ii) The sum of the coupling strengths of the produced meson multiplets is determined from the logarithmic increase of the inclusive cross sections and is found to be just large enough to generate the Pomeron with $\alpha_P(0)\simeq 1$. In the scheme based on unitarity and duality, it is just the value required by the planar Reggeon bootstrap.[1,2,3,19]

(iii) The production cross section of mesons with odd charge conjugation is about 1.2–2 times larger than that with even charge conjugation, which is consistent with the requirement of two-body reactions.

(iv) The ratio V/P for direct production is found to be about 2–5. This is in accord with the quark-model value, i.e., $\sigma(V)/\sigma(P)=3$.

(v) We obtain $\sigma(\eta)/\sigma(\pi^0)\simeq 10\%$ for observed cross sections, even assuming $\sigma(\eta_N)=\sigma(\pi^0)$ for direct production. Thus experimentally observed strong suppression of η production compared with pion production is naturally explained by the fact that known resonances produce many pions but very few η mesons.

The effect of resonance production should also be reflected in a characteristic manner in correlation quantities among produced pions. The generating function for resonance production constructed in our scheme automatically satisfies the constraint of the quantum number conservation. We calculate two-pion correlation parameters f_2 and study the effect of various resonances.

In Sec. II we give a brief summary of the unitarization scheme with duality in the multiperipheral resonance production model and generalize the model to incorporate production of vector, tensor, scalar, pseudoscalar, and axial-vector mesons. In Sec. III we estimate the relative abundance of meson-resonance multiplets and their coupling strengths to exchanged Reggeons. In Sec. IV we give the generating function for resonance production and calculate correlations. Section V is devoted to a discussion.

II. UNITARIZATION SCHEME IN THE MULTIPERIPHERAL RESONANCE PRODUCTION MODEL

We give a brief summary of the unitarization scheme with duality in the multiperipheral resonance production model.[3,4,6]

A. Intercepts of the output poles

From unitarity the discontinuity of a two-body scattering amplitude is given by the overlap of the multiparticle production amplitudes. We identify produced particles in the intermediate state of the unitarity with resonances. The summation over the multiparticle states is carried out by using the multiperipheral model. By taking the Chew-Pignotti approximation, the amplitude F in the t-channel angular momentum space is given by

$$F = G + GS\frac{1}{j-\beta}F$$
$$= G + GS\frac{1}{j-\beta}G + GS\frac{1}{j-\beta}GS\frac{1}{j-\beta}G+\cdots, \tag{2.1}$$

where $\beta=2\alpha^{\text{in}}-1$ with the exchanged Reggeon trajectory α^{in}. Here S represents the product of the signature factors of the lower and upper Reggeon propagators (see Fig. 1) and G denotes the square of the coupling strength of the produced particle to exchanged Reggeons after transverse-momentum integration. For definiteness we first consider production of vector and tensor mesons. Then G is given in terms of the coupling strengths of the vector and tensor mesons,

$$G=g_T^{\,2}+g_V^{\,2}\,. \tag{2.2}$$

The basic parameters of our model, g^2 and ϵ, are

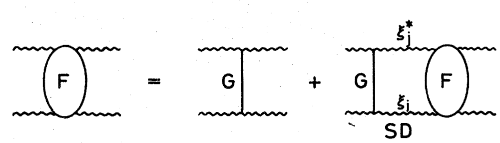


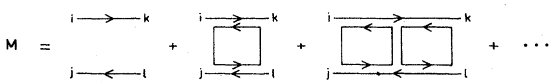
FIG. 1. The integral equation for the amplitude F in the multiperipheral model. Here D denotes the two-Reggeon propagator $1/(j-\beta)$ with $\beta=2\alpha^{\text{in}}-1$ and S is $\|\xi_j^*\xi_i\|$.

then defined by

$$g_T^2 = g^2(1+\epsilon)\,, \qquad g_V^2 = g^2(1-\epsilon)\,. \tag{2.3}$$

In this subsection we consider SU(N) symmetry.

Planar bootstrap. Summation of the planar diagrams in the unitarity leads to the Reggeon amplitude M,



$$= \sum_{n=0}^{\infty} \left(\frac{2g^2N}{j-\beta}\right)^n \delta_{ik}\delta^{jl}$$

$$= \frac{j-\beta}{j-\alpha^{\text{out}}}\delta_{ik}\delta^{jl} \tag{2.4}$$

with

$$\alpha^{\text{out}} = \beta + 2g^2N\,. \tag{2.5}$$

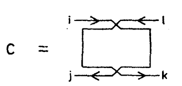
Imposing the bootstrap condition that the output Reggeons are equal to the input Reggeons, $\alpha^{\text{in}} = \alpha^{\text{out}} \equiv \alpha$, we obtain

$$\alpha = 1 - 2g^2N\,. \tag{2.6}$$

Taking $\alpha \simeq 0.5$ and $N=2$ [SU(2)], we obtain

$$g^2 = \frac{1-\alpha}{2N} \simeq 0.125\,. \tag{2.7}$$

Pomeron amplitude. The Pomeron is generated by the SU(N) singlet cylinder kernel,



$$= \pm\frac{2g^2}{j-\beta}\delta_i^j\delta_k^l \quad \text{for } C_t = \pm 1\,. \tag{2.8}$$

The Pomeron amplitude is written as

$$A = \sum_{n=0}^{\infty}(MC)^n M = \frac{1}{M^{-1}-C}$$

$$= \frac{j-\beta}{j-\beta-4g^2N}\delta_i^j\delta_k^l\,. \tag{2.9}$$

With the aid of Eq. (2.6), we obtain the leading Pomeron pole

$$\alpha_P = \beta + 4g^2N = 1\,.$$

Interference terms. In the case when $g_V^2 \neq g_T^2$ there remain diagrams with twists on produced particle lines. Such diagrams give interference terms. They give rise to three kinds of physical effects: (i) exotic exchanges, (ii) violation of the OZI rule, and (iii) breaking of exchange degeneracy. The magnitudes of these effects are estimated[12,6] to be

$$\text{(i)}\ \frac{F(\rho\rho \to \phi A_2)}{F(\rho\rho \to \omega A_2)} = \frac{\sqrt{2}}{N}\epsilon\,,$$

$$\text{(ii)}\ \frac{F(\rho^+\rho^- \to A_2^- A_2^+)}{F(\rho^+\rho^- \to A_2^+ A_2^-)} = 2\frac{\epsilon^2}{N}s^{\beta-\alpha}\,,$$

$$\text{(iii)}\ \alpha_\rho(0) - \alpha_{A_2}(0) = -8\epsilon g^2/N\,.$$

Both data of $\sigma(\pi^- p \to K^+\Sigma^-)/\sigma(\pi^+ p \to K^+\Sigma^+)$ and $\sigma(\pi^- p \to \phi n)/\sigma(\pi^- p \to \omega n)$ give $|\epsilon| \simeq 0.25$.[13] From the ρ-A_2 splitting[20] obtained from charge exchange data, we can determine the sign of ϵ and obtain[6,9]

$$\epsilon = (-0.2)-(-0.3)\,. \tag{2.10}$$

We can easily incorporate production of other meson multiplets into our calculation by replacing g_V^2 with Σg_i^2 $(C_i = -1)$ and g_T^2 with Σg_i^2 $(C_i = +1)$. Thus we have

$$g^2 = \tfrac{1}{2}(g_T^2 + g_V^2 + g_S^2 + g_P^2 + g_A^2)\,, \tag{2.11}$$

$$\epsilon = \frac{(g_T^2 + g_S^2 + g_P^2) - (g_V^2 + g_A^2)}{(g_T^2 + g_S^2 + g_P^2) + (g_V^2 + g_A^2)}\,. \tag{2.12}$$

B. Inclusive cross sections for resonance production

The average multiplicity of resonances can be calculated in the leading order of ln$s$ by using the Mulller-Regger model.[6] We have

$$\begin{aligned} \langle n(\rho)\rangle &\simeq 3g_V^2 \ln s\,, \\ \langle n(\omega)\rangle &\simeq g_V^2(1-\cos^2\pi\alpha)\ln s\,, \\ \langle n(A_2)\rangle &\simeq 3g_T^2 \ln s\,, \\ \langle n(f)\rangle &\simeq g_T^2(1+\cos^2\pi\alpha)\ln s\,, \end{aligned} \tag{2.13}$$

and similar expressions for other meson multiplets. We note that the quartet symmetry holds for $\alpha = \frac{1}{2}$.

III. RELATIVE ABUNDANCE OF MESON-RESONANCE MULTIPLETS

Here we will estimate the relative abundance of directly produced resonance multiplets in *nonstrange mesonic systems* from data of inclusive cross sections of resonance production. At present experimental information is mainly confined to ρ^0, f, K^*, and Δ^{++}.[18,21-24] We will use the observed cross sections of ρ^0 and f (more precisely, the ratios ρ^0/π and f/π) and assume SU(2)-symmetry relations to determine the cross sections of other members of the vector and tensor resonances. Figure 2 shows a summary of data of inclusive ρ^0 production.

A. The ratios ρ^0/π and f/ρ of the nonstrange mesonic system

To determine the ratio ρ^0/π for the nonstrange mesonic system, we have to take account of the following effects: (i) the possible effect of leading

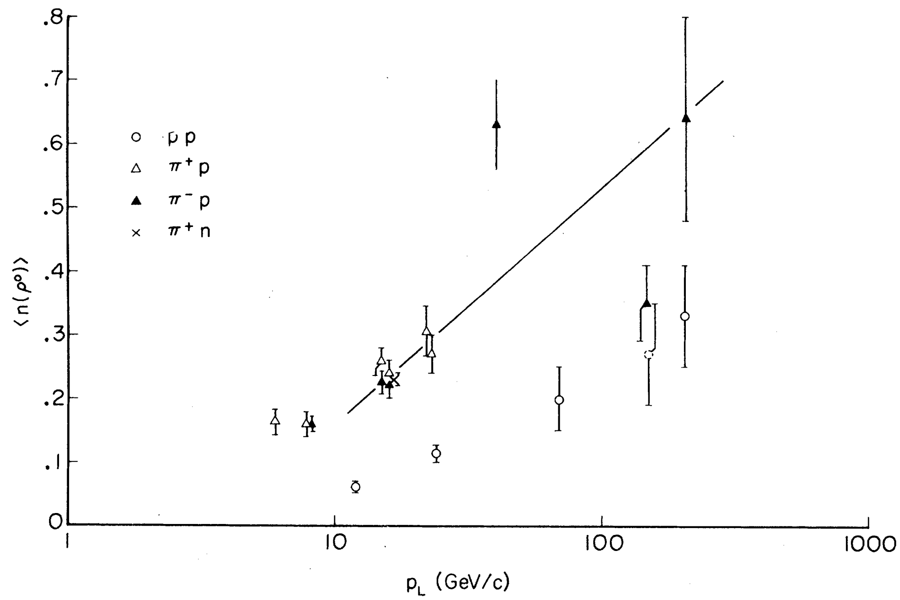


FIG. 2. Average multiplicity of ρ^0 production cross section as a function of energy. The line added in this figure corresponds to the slope $\langle n_{\rho^0}\rangle/\langle n_{\pi^-}\rangle = 0.25(\langle n_{\pi^-}\rangle \simeq 0.65 \ln s + \text{const})$. See Eqs. (3.6) and (3.11) and also text Sec. III A.

mesons, (ii) pions from decays of strange-meson resonances, and (iii) pions from decays of baryon resonances.

These three effects can be treated in the following way:

(i) The leading-meson effect can be largely reduced, if we compare resonance production with π^- production in π^+p and pp scattering and with π^+ production in π^-p scattering. We should also note that in our picture based on unitarity and duality the diffractively excited meson cluster is expected to have properties similar to those of nondiffractively produced meson clusters.

(ii) We subtract pions from the decays of K^* mesons and ignore other strange-meson resonances, which are expected to be produced much less copiously. We assume $\sigma(K^{*+}) \simeq \sigma(K^{*0})$ and $\sigma(K^{*-}) \simeq \sigma(\bar{K}^{*0})$.

(iii) We estimate the inclusive pion cross section due to decays of baryon resonances, $\sigma_B(\pi)$, making the following approximation. We assume that baryon resonances of $I=\frac{1}{2}$ and those of $I=\frac{3}{2}$ are produced with equal amounts and that $I_t=0$ and $I_t=1$ exchanges contribute with equal weights to production of $I=\frac{1}{2}$ baryons. On the average, low-mass baryon resonances decay into πN and $\pi\pi N$, each with the branching ratio of about 50%. For these branching ratios, we obtain the average decay multiplicities of baryon resonances, $N_B(\pi)$,

$$N_B(\pi^+) \simeq 0.66, \quad N_B(\pi^0) \simeq 0.54, \quad N_B(\pi^-) \simeq 0.30. \tag{3.1}$$

Using the baryon-resonance production cross section $\sigma(B) \simeq 50$ mb in pp scattering and $\sigma(B) \simeq 16$ mb in $\pi^\pm p$ scattering,[25] we obtain

$$\sigma_B(\pi^+) \simeq 33 \text{ mb}, \sigma_B(\pi^0) \simeq 26 \text{ mb}, \quad \sigma_B(\pi^-) \simeq 15 \text{ mb} \tag{3.2}$$

for pp scattering and

$$\sigma_B(\pi^+) \simeq 11 \text{ mb}, \quad \sigma_B(\pi^0) \simeq 9 \text{ mb}, \quad \sigma_B(\pi^-) \simeq 5 \text{ mb} \tag{3.3}$$

for $\pi^\pm p$ scattering. We should note that baryon resonances contribute pion cross sections by a considerable amount at low energies.

The above estimate is based on crude approximations and may entail a large uncertainty. Furthermore, possible high-mass baryon resonances produced diffractively, which are ignored in the above estimate, may be important at high energies. To reduce these large uncertainties due to baryon resonances, we will use the data of $\pi^\pm p$ scattering to estimate the relative abundance of meson resonances in nonstrange mesonic systems.

The energy range of available ρ^0 data in π^+p scattering[21] is not large enough to see unambiguously the energy dependence of the ratio ρ^0/π^-. We have $\sigma(\rho^0)/\sigma(\pi^-) \simeq 0.22$ on the average; see Fig. 3. After making the corrections as described above, we obtain[28]

$$\sigma(\rho^0)/\sigma(\pi^-) \simeq 0.27 \tag{3.4}$$

for the nonstrange mesonic system. This value

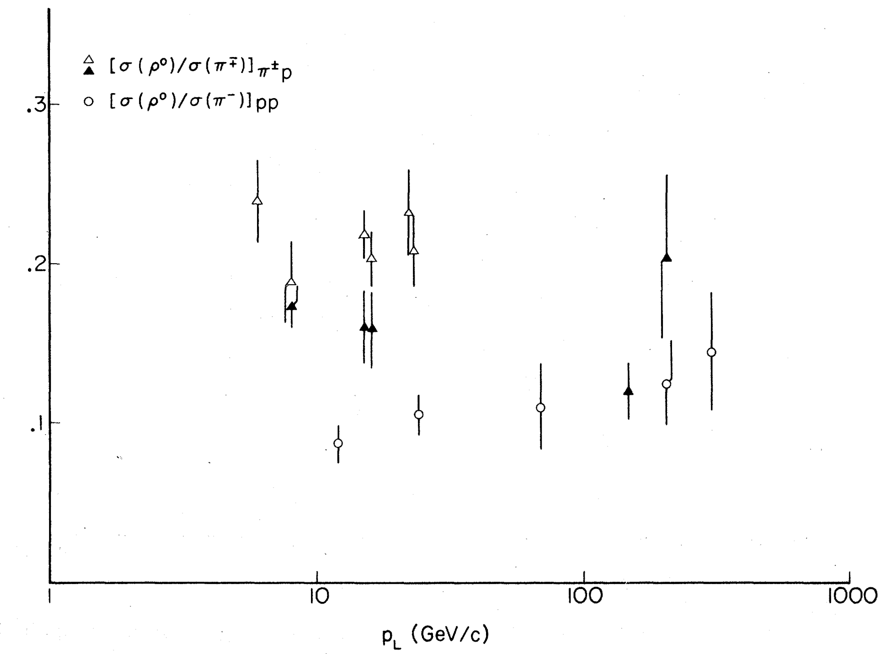


FIG. 3. The experimental ratios of $\rho^0/\pi^\pm$. The correction for the misidentification of K^- and Σ^- with π^- and that of p, K^+, and Σ^+ with π^+ has been made to the data, see Refs. 21–24.

turns out to be in accord with the ratio ρ^0/π^+ in the central region ($|Y^*|<1$) in π^-p scattering at 205 GeV/c.[22e]

The ratio ρ^0/π^0 is known only at an energy, 15 GeV/c.[21d] Making the corrections to the observed π^0 cross section, we obtain

$$\sigma(\rho^0)/\sigma(\pi^0) \simeq 0.23 \tag{3.5}$$

for the nonstrange mesonic system.

The difference between the above two estimates may be ascribed to the following: (i) The observed π^0 cross section gives an overestimate of the true π^0 cross section because of the misidentification of η with π^0. (ii) The constraint of charge conservation tends to suppress production of negatively charged mesons in π^+p scattering at low energies. Considering such uncertainties, we will take the ratio $\rho^0/\pi^\pm$ of the mesonic system

$$\sigma(\rho^0)/\sigma(\pi^\pm) \simeq 0.23\text{–}0.27\,, \tag{3.6}$$

which covers both values (3.4) and (3.5) (see Fig. 2).

The inclusive cross section of f-meson production has been measured only at relatively low energies.[21(a),21(c),22(c)] The data are consistent with no energy dependence of the ratio f/ρ^0 (Fig. 4). We obtain on the average

$$\sigma(f)/\sigma(\rho^0) \simeq 0.22\,. \tag{3.7}$$

B. Coupling strengths of produced resonance multiplets

Let us now estimate the contributions of various resonance multiplets to the inclusive pion cross section at high energies. We assume the cross-section ratio ρ^0/π of the mesonic system to be energy independent. Then this ratio gives that of the coefficients of logarithmic increase of the

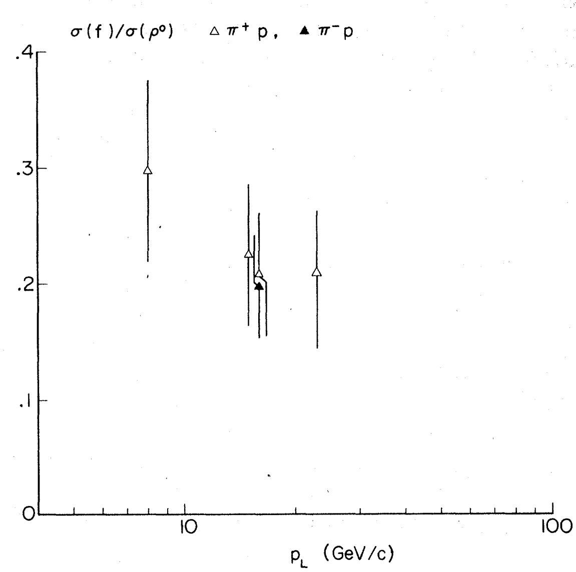


FIG. 4. The experimental ratio of f/ρ^0; see Refs. 21 and 22.

TABLE I. The fraction of produced pions coming from the decay of each of the directly produced meson multiplets V, T, S, P (direct and $\eta+\eta'$), and A estimated in the four cases.

	I $(V+T+P)$	II $(V+T+P+A)$	III $(V+T+S)$	IV $(V+T+S+P+A)$
V	47–61%	49–62%	56–66%	50–63%
T	18–21%	18–21%	18–21%	18–21%
S			26–13%	~3%
P (direct)	22–11%	8– 4%		7– 3%
P $(\eta+\eta')$	13– 7%	5– 3%		4– 2%
A		20–10%		18– 8%

average multiplicities.

As will be seen soon, vector and tensor resonances turn out to give the dominant part (about 65–80%) of the produced pions. The rest of the pions come from other resonances or are produced directly. We consider several cases of direct production of meson multiplets: (I) V, T, and P, (II) V, T, P, and A, (III) V, T, and S, (IV) V, T, S, P, and A, where V, T, S, P, and A denote vector, tensor, scalar,[29] pseudoscalar, and axial-vector $(J^{PC}=1^{+-})$ mesons, respectively. Here the 1^{+-} mesons are the exchange degenerate partner of the 0^{-+} mesons.[30]

We assume that at high energies the SU(2) quartet symmetry is satisfied for direct production of meson multiplets. Namely, we have in the leading order of ln$s$

$$\begin{aligned}\sigma_{\text{direct}}(\rho^+) &\simeq \sigma_{\text{direct}}(\rho^-) \simeq \sigma_{\text{direct}}(\rho^0) \\ &\simeq \sigma_{\text{direct}}(\omega)\,, \\ \sigma_{\text{direct}}(A_2^+) &\simeq \sigma_{\text{direct}}(A_2^-) \simeq \sigma_{\text{direct}}(A_2^0) \\ &\simeq \sigma_{\text{direct}}(f)\,,\end{aligned} \tag{3.8}$$

and so forth. Furthermore, we assume the relation[31]

$$\frac{\sigma_{\text{direct}}(P)}{\sigma_{\text{direct}}(V)/3} \simeq \frac{\sigma_{\text{direct}}(A)/3}{\sigma_{\text{direct}}(T)/5} \tag{3.9}$$

for axial-vector-meson production in the cases (II) and (IV), and

$$\sigma_{\text{direct}}(T)/5 \simeq \sigma_{\text{direct}}(S) \tag{3.10}$$

for scalar meson production in the case (IV). For the isosinglet member of the pseudoscalar meson quartet we use the state $\eta_N \equiv (\mathcal{P}\bar{\mathcal{P}}+\mathfrak{N}\bar{\mathfrak{N}})/\sqrt{2} \simeq (\eta+\eta')/\sqrt{2}$.

We take account of cascade decays such as $A_2 \to \rho\pi \to 3\pi$ and $B \to \omega\pi \to 4\pi$ using the known branching ratios.[33] For the decays of the scalar mesons, we take $\delta \to \pi\eta$ and $\epsilon \to \pi\pi$ for simplicity.

The fraction of pions coming from the decay of each of the directly produced meson multiplets in the four models is calculated using the ratios $\rho^0/\pi^\pm$ and f/ρ^0 estimated in the preceding subsection [Eqs. (3.6) and (3.7) as inputs]. The result is given in Table I. In particular, we see that the vector meson contributes about 50–60% to produced pions, and the tensor meson about 15–20%.

Combining the experimental value of the logarithmic increase of the average multiplicity of pions,

$$\langle n(\pi^-)\rangle_{\pi^+p} \simeq 0.65 \ln s + \text{const}\,, \tag{3.11}$$

with the above result, we can calculate the basic parameters of our model, g^2 and ϵ, as well as the couplings of various resonances. As seen from Table II, the weight ϵ of the twisted kernel

TABLE II. The couplings of produced meson multiplets to exchanged Reggeons and the parameter g^2 and ϵ estimated in the four cases.

	I	II	III	IV
g_V^2	0.105–0.137	0.109–0.139	0.126–0.148	0.111–0.142
g_T^2	0.033–0.039	0.033–0.039	0.033–0.039	0.033–0.039
g_S^2			0.066–0.033	0.007–0.008
g_P^2	0.143–0.071	0.052–0.027		0.048–0.021
g_A^2		0.028–0.014		0.025–0.010
g^2	0.140–0.124	0.111–0.110	0.113–0.110	0.112–0.110
ϵ	+0.25– –0.11	–0.24– –0.40	–0.12– –0.35	–0.22– –0.38

TABLE III. The predicted ratios of ω/ρ^0 and $\eta/\pi^\pm$ in the four cases.

	I	II	III	IV
$\frac{\sigma(\omega)}{\sigma(\rho^0)}$	0.76–0.84	1.36–1.09	0.91	1.25–0.99
$\frac{\sigma(\eta)}{\sigma(\pi^\pm)}$	0.21–0.12	0.09–0.06	0.33–0.18	0.11–0.09

turns out to be

$$-0.4 \lesssim \epsilon \lesssim 0\ , \tag{3.12}$$

which agrees with the value from two-body reactions,[34] Eq. (2.10), as mentioned in Sec. II A. The coupling constant g^2 is also in good agreement with the Reggeon bootstrap condition (2.6), i.e., $g^2 \simeq 0.125$, and hence it can generate the Pomeron with $\alpha_P(0) \simeq 8g^2 \simeq 1$.

Other couplings, g_i^2, in Table II are the measure of the strength of direct production of various meson multiplets. Predictions for the observed production ratios ω/ρ^0 and $\eta/\pi^\pm$ are given in Table III. The following features are of interest to note:

(i) Even after taking account of resonance decays, the inclusive cross section of the ω meson is nearly equal to that of the ρ meson.[35] A more detailed inspection shows that the ratio ω/ρ^0 is appreciably larger than unity in the models (II) and (IV). This is ascribed to the decays of the axial-vector mesons ($B\to\pi\omega$ and $H\to\pi\rho$).

(ii) It is remarkable that production of the η meson is strongly suppressed compared to pion production, in accord with recent exclusive experiments[37,38]; even the quartet symmetry has been assumed for π and η_N production.

(iii) The ratio ρ/π for direct production is predicted to be about 2–5, which is in accord with the spin weight of the SU(6) quark model, $\sigma(\rho)/\sigma(\pi)=3$.[32]

Among the four cases, models (II) and (IV) seem to be favored; they give reasonable values of the weight ϵ and the ratio $\eta/\pi^\pm$. If we take this result literally, we should expect that B mesons are produced rather copiously[39] (perhaps as much as about a half of A_2 mesons).

C. Strange-meson-resonance production

We have so far considered nonstrange-meson production in the framework of SU(2) symmetry. Here we discuss briefly the possible consequence of our model on production of strange meson resonances. It seems reasonable in our picture to assume that the suppression of strange-meson production is independent of the spin of produced mesons.[4] Then we have

$$\frac{\sigma_{\text{direct}}(K^{*\pm})}{\sigma_{\text{direct}}(\rho^\pm)}=\frac{\sigma_{\text{direct}}(K^{**\pm})}{\sigma_{\text{direct}}(A_2^\pm)} = \cdots = r. \tag{3.13}$$

We calculate the production ratios of various strange mesons by taking model (II). We have from Table II

$$\begin{aligned}\sigma_{\text{direct}}(K^*):\ \sigma_{\text{direct}}(K^{**}):\ \sigma_{\text{direct}}(K):\ \sigma_{\text{direct}}(Q_B) &= g_V^2:\ g_T^2:\ g_P^2:\ g_A^2\\ &\simeq 1:0.30\text{–}0.28:0.48\text{–}0.19:0.26\text{–}0.10\end{aligned} \tag{3.14}$$

and obtain

$$\sigma_{\text{direct}}(K^\pm)/\sigma(K^\pm)\simeq 0.24\text{–}0.12\ . \tag{3.15}$$

Recalling that the corresponding fraction for $\pi^\pm$ is 0.08–0.04 (see Table II), we obtain

$$\begin{aligned} r &= \sigma_{\text{direct}}(K^\pm)/\sigma_{\text{direct}}(\pi^\pm)\\ &\simeq 3\sigma(K^\pm)/\sigma(\pi^\pm)\ . \end{aligned} \tag{3.16}$$

With the experimental K/π ratio, $\sigma(K^\pm)/\sigma(\pi^\pm) \simeq 0.07$,[26] we obtain

$$r \simeq 0.20\ . \tag{3.17}$$

Taking account of the decays $K^{**}\to K^*\pi$ and $A_2 \to K\bar{K}$, etc., we predict the ratios among K, K^*, and K^{**} production

$$\sigma(K):\sigma(K^*):\sigma(K^{**}) \simeq 1:0.53\text{–}0.62:0.13\text{–}0.15\ , \tag{3.18}$$

and those among ρ, K^*, and ϕ production

$$\sigma(\rho^0):\sigma(K^{*\pm}):\sigma(\phi)\simeq 1:0.18:0.03\ , \tag{3.19}$$

using the factorization relation $K^{*\pm}/\rho^0=\phi/K^{*\pm}$ for direct production.

The experimental ratio of K^*/K increases with energy, and it is 0.34 at 15 GeV/c [Ref. 21(c)] and about 0.5 at 69 GeV/c.[24b] The experimental ratio of K^*/ρ increases with energy due to the threshold effect. The ratio $K^{*\pm}/\rho$ is about 0.15–0.40 at 69 GeV/c.[24b] The ratio ϕ/ρ^0 measured at 150 GeV/c is about 0.08,[40] which is appreciably larger than the predicted value (3.19). This large ratio of ϕ/ρ^0 should presumably be ascribed to large violation of the OZI rule in the central region.[41]

IV. CORRELATIONS

Previous attempts to calculate correlations based on the cluster model are obliged to be supplemented by some *ad hoc* assumptions on the conservation constraints. In the present scheme based on the quark line flow of duality diagrams we can obtain correlations due to the quantum number conservation constraints as well as to resonance decays. We calculate the correlation parameters $f_n = a_n \ln s + \text{const}$ from the generating function given in the Appendix. For numerical evaluation we take model II of Sec. III C in which production of V, T, P, and A is taken into account.

The generating function in the Pomeron sector with positive G parity is given in the leading power of s by

$$W(\ln s) \sim \exp[(g_0^2 + g_1^2)\ln s] \times \cosh[2(g_+^2 g_-^2)^{1/2} \ln s], \tag{4.1}$$

where g_0^2 and $(g_1^2, g_\pm^2)$ are the sums of coupling strengths of the $I=0$ and $I=1$ (neutral and charge $=\pm1$) meson production, respectively [see (A9)]. Charge conservation is ensured by the fact that the couplings g_+^2 and g_-^2 appear as a product.[42,43] The average multiplicity of the resonance R_i is

$$\frac{\langle n(R_i)\rangle}{\ln s} \simeq g_i^2 \frac{\partial}{\partial(g_i^2)} \ln W(\ln s) \tag{4.2}$$

and agrees with that given in Sec. II [Eq. (2.13)].

The two-particle correlation parameter f_2^{ab} for pions with charge states a and b is given by the sum of two contributions, one due to the decay of a single resonance and the other representing the correlation between two resonances,

$$f_2^{ab}/\ln s \simeq f^{(1)} + f^{(2)} . \tag{4.3}$$

The single-resonance contribution is given by

$$f^{(1)} = \sum_i g_i^2 N(R_i \to \pi^a \pi^b X) , \tag{4.4}$$

where g_i is the coupling of the resonance R_i and $N(R_i \to \pi^a\pi^b X)$ is the sum of the branching ratios multiplied by the decay multiplicities N_a and N_b of the pions a and b. The two-resonance contribution is

$$f^{(2)} = -\frac{\sum_l F_l^a \sum_m F_m^b}{2(g_T^2 + g_V^2 + g_P^2 + g_A^2)} \tag{4.5}$$

with

$$F_l^a = (g_{l^+})^2 N(R_{l^+} \to \pi^a X) - (g_{l^-})^2 N(R_{l^-} \to \pi^a X) , \tag{4.6}$$

where l and m run ρ, A_2, B, and π. It satisfies the relation

$$f^{(2)}(+,+) = f^{(2)}(-,-) = -f^{(2)}(+,-) < 0 , \tag{4.7}$$

which is a consequence of the absence of the f pole in the Mueller-Regge diagram for the two-resonance production in the central region.

Using the known branching ratios of the produced resonances, we obtain

$$\begin{aligned} f_2^{--}/\ln s &\simeq (f_2^{+-} - \langle n_-\rangle)/\ln s \\ &\simeq 1.19 g_T^2 + 1.82 g_A^2 + 0.12 g_P^2 - \frac{(0.95 g_T^2 + g_V^2 + g_P^2 + g_A^2)^2}{2(g_T^2 + g_V^2 + g_P^2 + g_A^2)} , \\ f_2^{-0}/\ln s &\simeq 2.55 g_T^2 + 1.90 g_V^2 + 0.52 g_P^2 + 5.60 g_A^2 , \\ f_2^{00}/\ln s &\simeq 3.05 g_T^2 + 1.97 g_A^2 + 2.30 g_P^2 , \end{aligned} \tag{4.8}$$

and

$$\langle n_-\rangle/\ln s \simeq 3.50 g_T^2 + 2.91 g_V^2 + 1.62 g_P^2 + 4.74 g_A^2 \simeq 0.65 .$$

With the coupling strengths g_T^2, g_V^2, g_P^2, and g_A^2 given in Table II we have

$$\begin{aligned} f_2^{--}/\ln s &\simeq (f_2^{+-} - \langle n_-\rangle)/\ln s \simeq -0.01 , \\ f_2^{-0}/\ln s &\simeq 0.48\text{–}0.46 , \\ f_2^{00}/\ln s &\simeq 0.28\text{–}0.21 . \end{aligned} \tag{4.9}$$

Correlations are considered to be affected rather strongly by the existence of the small diffractive components.[44,27] To test the prediction for correlations in usual hadron collisions, we need to calculate the diffractive component which appears as a higher-order effect in the topological expansion. Alternatively one can look at annihilation processes, where the diffractive excitation is absent. We have the same values of $f_2/\langle n\rangle$ for annihilation processes as for the Pomeron sector.

We have also calculated the generating function and the correlation parameter in the broken-SU(3) case in a similar manner and have found quite similar numerical results.[45]

V. DISCUSSION

In this paper we have shown that final-state pions are produced predominantly through resonances. In the multiperipheral model a serious difficulty has been shown to exist between the magnitude of the average multiplicity and the intercept of the generated Pomeron.[46,1] The possibility of cluster (or resonance) production has been suggested to solve this difficulty.[2,3] Our quantitative analysis has shown that the magnitudes of the average multiplicities of resonances are just large enough to generate the Pomeron with the intercept $\alpha_P(0) \simeq 1$.

We have shown that the ratio of the vector meson to the pseudoscalar meson for direct production is in accord with the spin weight of the quark model. This fact suggests that the quark model usually used to represent the quantum-number flow may be extended so that it describes the spin content of produced particles.

Since the contribution from baryon resonances to pion cross sections is quite large at low energies, we had to subtract it before estimating the relative abundance of resonances in the mesonic system. Our estimate of this baryon-resonance effect is crude and may entail a rather large uncertainty. In order to make a more complete analysis one has to extend the present scheme so that baryons can be treated on the same footing as mesons.

To calculate the relative contributions of various meson multiplets to pion production we have used the ratios ρ^0/π and f/ρ^0 as inputs. These ratios are estimated by using data at relatively low energies. Measurements of the ratios ρ^0/π and f/ρ^0 at higher energies will be most useful in estimating more precisely the fraction of indirect production of pions.

The unitarization scheme so far developed includes natural-parity Reggeons both in the multiperipheral chain and as output poles. In the present paper we have considered unnatural-parity mesons as well as natural-parity mesons in the produced multiparticle states. To make the scheme fully consistent, we should include unnatural-parity Reggeons in the multiperipheral chain as well.[47]

One may extend the present picture to hard processes, e.g., e^+e^- annihilation and deep-inelastic lepton-hadron scattering. Then one expects that final-state hadrons of such processes, at least the sea component, will have properties similar to those of multiparticle production in hadron-hadron reactions.

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APPENDIX: GENERATING FUNCTION OF MULTIPLICITY DISTRIBUTIONS

The generating function in the Pomeron sector with positive G parity is represented by a 12×12 matrix. The rows and columns are labeled by exchanged Reggeon pairs: (ρ^+,ρ^-), (ρ^0,ρ^0), (ρ^-,ρ^+), (f,f), (f,ρ^0), (ρ^0,f), (A_2^+,A_2^-), (A_2^0,A_2^0), (A_2^-,A_2^+), (ω,ω), (A_2^0,ω), (ω,A_2^0). Reggeons of the first six pairs have even G parity and those of the last six odd G parity. The coupling matrix G can be decomposed into even- and odd-G-parity pairs

$$G=\begin{pmatrix} G_{\text{even, even}} & G_{\text{even, odd}} \\ G_{\text{odd, even}} & G_{\text{odd, odd}} \end{pmatrix}, \tag{A1}$$

with the symmetry

$$\begin{aligned} G_{\text{even, even}} &= G_{\text{odd,odd}}\,, \\ G_{\text{even,odd}} &= G_{\text{odd, even}}\,. \end{aligned} \tag{A2}$$

The multiperipheral equation for the generating function $W(J)$ becomes

$$W(J)=G+G\frac{1}{J-\beta}SW(J)\ , \tag{A3}$$

where the signature matrix S has only diagonal elements and becomes identical for even- and odd-G-parity pairs

$$S=\begin{pmatrix} \bar{S} & 0 \\ 0 & \bar{S} \end{pmatrix}, \tag{A4}$$

$$\bar{S}=\begin{bmatrix} 1 & & & & & 0 \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & e^{i\pi\alpha} & \\ 0 & & & & & e^{-i\pi\alpha} \end{bmatrix} \tag{A5}$$

With the aid of Eqs. (4.1) and (4.4), we can separate the multiperipheral equation (4.3) into two. By writing

$$W(J=W_+(J)\frac{1+\rho_1}{2}+W_-(J)\frac{1-\rho_1}{2} \tag{A6}$$

with $\rho_1=\left(\begin{smallmatrix}0&1\\1&0\end{smallmatrix}\right)$, we have

$$W_\pm(J)=G_\pm+G_\pm\frac{1}{J-\beta}\bar{S}W_\pm(J)\ , \tag{A7}$$

where the coupling matrices $G_\pm$ are defined similarly to Eq. (4.6). The coupling G_+ is given by

$$G_+=\begin{bmatrix} g_0^{\,2}+g_1^{\,2} & g_-^{\,2} & 0 & g_-^{\,2} & g_-^{\,2} & g_-^{\,2} \\ g_+^{\,2} & g_0^{\,2} & g_-^{\,2} & g_1^{\,2} & 0 & 0 \\ 0 & g_+^{\,2} & g_0^{\,2}+g_1^{\,2} & g_+^{\,2} & -g_+^{\,2} & -g_+^{\,2} \\ g_+^{\,2} & g_1^{\,2} & g_-^{\,2} & g_2^{\,0} & 0 & 0 \\ g_+^{\,2} & 0 & -g_-^{\,2} & 0 & g_2^{\,2} & g_3^{\,2} \\ g_+^{\,2} & 0 & -g_-^{\,2} & 0 & g_3^{\,2} & g_2^{\,2} \end{bmatrix} \tag{A8}$$

with

$$
\begin{aligned}
g_0{}^2 &= (g_f)^2 + (g_\omega)^2 + (g_\eta)^2 + (g_H)^2 , \\
g_1{}^2 &= (g_{A_2^0})^2 + (g_{\rho^0})^2 + (g_{\pi^0})^2 + (g_{B^0})^2 , \\
g_2{}^2 &= (g_f)^2 + (g_{A_2^0})^2 + (g_\eta)^2 + (g_{\pi^0})^2 , \\
g_3{}^2 &= (g_\omega)^2 + (g_{\rho^0})^2 + (g_H)^2 + (g_{B^0})^2 , \\
g_\pm{}^2 &= (g_{A_2^\pm})^2 + (g_{\rho^\pm})^2 + (g_{\pi^\pm})^2 + (g_{B^\pm})^2 .
\end{aligned}
\tag{A9}
$$

Here the suffixes correspond to produced mesons. One should understand $(g_\omega)^2$, etc. as the product of the coupling constant squared and the parameter of the generating function.

The leading poles are contained in the solution $W_+(J)$,

$$W_+(J) = \frac{1}{J - \beta - G_+\bar{S}} G_+ , \tag{A10}$$

and they are determined by the condition

$$\det \| G_+ - (J - \beta)\bar{S}^{-1} \| = 0 . \tag{A11}$$

Taking $\alpha = \frac{1}{2}$, we obtain from Eq. (4.11)

$$(g_0{}^2 - g_1{}^2 - \lambda)[(g_0{}^2 + g_1{}^2 - \lambda)^2 - 4g_+{}^2 g_-{}^2]\{[\lambda^2 + (g_2{}^2 - g_3{}^2)(g_2{}^2 + g_3{}^2)](g_0{}^2 + g_1{}^2 - \lambda) - 4g_+{}^2 g_-{}^2 (g_2{}^2 - g_3{}^2)\} = 0 \tag{A12}$$

with $\lambda \equiv J - \beta$. We find the leading poles at

$$J = \alpha^{\text{out}} = \beta + g_0{}^2 + g_1{}^2 \pm 2(g_+{}^2 g_-{}^2)^{1/2} . \tag{A13}$$

Both poles should be taken to ensure the absence of wrong prong numbers. It is then straightforward to obtain the generating function Eq. (4.1) given in the text.

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