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Feynman rules of quantum chromodynamics inside a hadron

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We start from quantum chromodynamics in a finite volume of linear size L and examine its color-dielectric constant κ_L, especially the limit κ_∞ as $L \to \infty$. By choosing as our standard $\kappa_L = 1$ when L = some hadron size R, we conclude that κ_∞ must be <1; furthermore, from the fact that a free quark has not been observed we can estimate an upper bound: $\kappa_\infty < 1.3 \times 10^{-2}\alpha$ where α is the fine-structure constant of QCD inside the hadron. A permanent quark confinement corresponds to the limit $\kappa_\infty = 0$. The hadrons are viewed as small domain structures (with color-dielectric constant = 1) immersed in a perfect, or nearly perfect, color-dia-electric medium, which is the vacuum. The Feynman rules of QCD inside the hadron are derived; they are found to depend on the color-dielectric constant κ_∞ of the vacuum that lies outside. We show that, when $\kappa_\infty \to 0$, the mass of any color-nonsinglet state becomes ∞, but for color-singlet states their masses and scattering amplitudes remain finite. These new Feynman rules also depend on the hadron size R. Only at high energy and large four-momentum transfer can such R dependence be neglected and, for color-singlet states, these new rules be reduced to the usual ones.

I. INTRODUCTION

In a recent paper,[1] it was emphasized that in order to give quantum chromodynamics[2] (QCD) a well-defined meaning, a convenient method is to first contain the whole system within a volume of size L^3. At a finite L, there is the usual perturbation series which is finite to every order of the renormalized coupling constant g. Assuming that the limit $L \to \infty$ exists, one expects the existence of a long-range order in the vacuum for an infinite volume. Because of relativistic invariance, such a long-range order must be a Lorentz scalar. It is then suggested that this long-range order can be expressed in terms of the (color) dielectric constant κ of the vacuum. In this approach,[3] a permanent quark confinement is simply viewed as the vacuum of an infinite volume being a perfect "dia-electric" substance with its dielectric constant $\kappa \to 0$, while the "vacuum" inside a hadron is normal ($\kappa = 1$). As we shall see, such a description leads to a set of Feynman rules that is quite different from the usual ones given in the literature. The propagator of the vector gauge field inside the hadron has an explicit dependence on the dielectric constant κ of the vacuum that lies outside the hadron. In the limit $\kappa \to 0$, the mass of any color-nonsinglet state, such as a single quark or a single vector gauge particle, becomes infinite. On the other hand, as will be shown in this paper, for color-singlet states, their masses and scattering amplitudes remain finite when $\kappa \to 0$.

In the following, we begin in Sec. II with a brief summary of what is currently known about the (color) dielectric constant of the vacuum. In order to take into account the long-range order of the vacuum and its long-wavelength fluctuations, we adopt the standard soliton description by introducing a scalar field to represent the dynamics of such collective motion. The related phenomenological Lagrangian is given in Sec. III. This enables us in Sec. IV to write down the appropriate Feynman rules and to analyze their properties. These new Feynman rules are different from the usual ones because they are applicable to configurations near a *soliton* (or bag) solution, while the usual ones are for perturbations near a *spatially homogeneous* solution, such as a pure infinite vacuum without the soliton (or bag). [Since in our picture all hadrons are solitons (or bags), it is difficult to have small perturbations around such a pure vacuum.] As we shall see, the Feynman rules derived here have an explicit dependence on the hadron size R. Only at very high energy and four-momentum transfer can one neglect the R dependence and thereby reduce these new rules to the usual ones as an approximation.

In a relativistic theory, the dielectric constant κ of the vacuum is always the inverse of its magnetic susceptibility μ because the velocity of light $c/(\kappa\mu)^{1/2}$ must equal c itself. Thus, $\kappa \ll 1$ means $\mu \gg 1$, and a zero dielectric constant is the same as an infinite magnetic susceptibility.[4] It is interesting to ask whether there exists a critical temperature at which the infinite-volume (QCD) system can undergo a phase transition in its dielectric constant (or magnetic susceptibility). This and other questions related to the long-range order, the scale determination of the hadron mass (or radius), and the experimental limit of the (color) dielectric constant are briefly discussed in Sec. V.

II. DIELECTRIC CONSTANT

For simplicity, let us consider in this section a pure QCD system consisting of only color gauge fields V^a_μ ($a=1,2,\ldots,8$). We first contain the system within a finite volume L^3. So long as L is fixed, the (color) dielectric constant κ_L of the vacuum has no absolute meaning, since the transformation

$$V^a_\mu \to \kappa^{1/2} V^a_\mu\,,\qquad g\to \kappa^{-1/2} g \tag{1}$$

brings the covariant field derivative $V^a_{\mu\nu}\to \kappa^{1/2}V^a_{\mu\nu}$, and therefore the Lagrangian density

$$-\tfrac{1}{4}V^a_{\mu\nu}V^a_{\mu\nu}\to -\tfrac{1}{4}\kappa V^a_{\mu\nu}V^a_{\mu\nu}, \tag{2}$$

where g is the renormalized coupling constant, the subscripts μ,ν denote the space-time indices, and the superscript a is the color index. But when we consider two different volumes, say of sizes L and L', the ratio $\kappa_{L'}/\kappa_L$ is, of course, physically meaningful. Because of (1), this ratio is equal to the inverse of the corresponding ratio of the square of the renormalized coupling constants. By using the well-known properties of the β function,[5] one can readily derive

$$\kappa_{L'} < \kappa_L \text{ if } L' > L. \tag{3}$$

Furthermore, if

$$\beta(g)=0 \text{ only at } g=0, \tag{4}$$

then

$$\lim_{L\to\infty}\kappa_L = 0.$$

Throughout our discussions, we *assume* κ_L is a smooth function of L, and that the limit $L\to\infty$ exists. For completeness, a proof is given in Appendix A. [Although the mathematics given there is essentially identical to that used in deriving asymptotic freedom,[5] its application to the color dielectric constant focuses on a new aspect of the physics involved. As we shall see, this leads to the conclusion that domain structures (soliton solutions) should develop whenever there are quarks and antiquarks present; thereby one bypasses the usual difficulty of strong coupling in the so-called "infrared slavery."]

For small L, because g^2 is $\ll 1$, κ_L is clearly $\neq 0$. As a convention, we may define when $L=$ some hadronic radius R,

$$\kappa_R = 1. \tag{5}$$

Let

$$\kappa_\infty \equiv \lim_{L\to\infty}\kappa_L. \tag{6}$$

From (3) and (5), it follows that the dielectric constant of the vacuum for an infinite volume must be less than unity, i.e.,

$$\kappa_\infty < 1. \tag{7}$$

Furthermore, κ_∞ is zero if (4) holds. In the following, we assume

$$\kappa_\infty = 0 \tag{8a}$$

or at least

$$\kappa_\infty \ll 1. \tag{8b}$$

In the former, the vacuum for an infinite system is a perfect dia-electric, in the latter, a nearly perfect dia-electric. By following the arguments given in Sec. II of Ref. 1 (and as will also be shown in the following), one sees that the mass of any color-nonsinglet state diverges when $\kappa_\infty\to 0$. Thus, (8a) implies a permanent quark confinement, while (8b) implies a nearly permanent confinement.

At present, it is not known whether $\beta(g)=0$ has only a real root at $g=0$. From (7), we know that κ_∞ must be <1. But as yet, we are not able to decide by pure theoretical deduction how small κ_∞ actually is.[6] Now the fact that a single free quark has never been observed puts a lower limit on its mass m_q. As will be shown in Sec. V, if we set $m_q > 5$ GeV, which is a rather lenient lower bound of the free quark mass, then we can estimate an upper limit of κ_∞:

$$\kappa_\infty < 1.3\times 10^{-2}\alpha,$$

where $\alpha=(4\pi)^{-1}g_R{}^2$, with R chosen such that $\kappa_R=1$, in accordance with our convention (5). Thus, we can regard that as an *experimental* determination κ_∞ must be $\ll 1$. On esthetic grounds, one may conjecture $\kappa_\infty=0$. The otherwise puzzling phenomenon of quark confinement, or near confinement, now receives a natural "explanation".

III. SOLITON (OR BAG) MODEL

A. Phenomenological Lagrangian

In order to incorporate the long-range order of the vacuum and its long-wavelength fluctuations into the dynamics of the hadron, we adopt the soliton description[1] through the use of collective coordinates. The dielectric constant κ will now be represented by a scalar field σ. The simplest form is

$$\sigma \propto 1-\kappa. \tag{9}$$

If we assume (8a), then

$$\frac{\sigma}{\sigma_{\text{vac}}}=1-\kappa,$$

so that for an infinite volume, $\kappa=0$ and therefore $\sigma=\sigma_{\text{vac}}$. In the soliton picture, one has outside the hadron $\sigma=\sigma_{\text{vac}}$, but inside the hadron $\sigma\cong 0$ which gives $\kappa\cong 1$, consistent with our convention (5). The phenomenological Lagrangian density is assumed to be (see, however, the modification given in Sec. III D).

$$\mathcal{L}=-\tfrac{1}{4}\kappa V^a_{\mu\nu}V^a_{\mu\nu}-\psi^\dagger\gamma_4(\gamma_\mu D_\mu+f\sigma+m)\psi -\frac{1}{2}\left(\frac{\partial\sigma}{\partial x_\mu}\right)^2-U(\sigma)+\text{counterterms}, \tag{10}$$

where the superscript dagger denotes the Hermitian conjugation, $x_\mu=(\vec{r},it)$,

$$V^a_{\mu\nu}=\frac{\partial}{\partial x_\mu}V^a_\nu-\frac{\partial}{\partial x_\nu}V^a_\mu+gC^{abc}V^b_\mu V^c_\nu, \qquad D_\mu=\frac{\partial}{\partial x_\mu}-\tfrac{1}{2}ig\lambda^a V^a_\mu, \tag{11}$$

the λ^a's are the standard Gell-Mann matrices related to C^{abc} by the commutation relation

$$[\lambda^a,\lambda^b]=2iC^{abc}\lambda_c,$$

the γ_μ's are the usual Hermitian Dirac matrices, and the function $U(\sigma)$ has an absolute minimum at $\sigma=\sigma_{\text{vac}}$ and a local minimum at $\sigma=0$ with

$$U(\sigma_{\text{vac}})=0 \quad\text{and}\quad U(0)\equiv p>0. \tag{12}$$

As we shall see, the detailed form of $U(\sigma)$ is not important. If one wishes, one may assume $U(\sigma)$ to be simply a quartic function of σ. In (10) and (11), ψ denotes the quark field which, besides being a color triplet, also has F flavors, m is the mass matrix for quarks inside the hadron, and g and f are both renormalizable coupling constants. Since σ is only a phenomenological field, describing the long-range collective effects of QCD, its short-wavelength components do not exist in reality. The counterterms in $\mathcal{L}$ are for renormalization; they consist *only* of those due to loop diagrams of the vector gauge field V^a_μ and the quark field ψ. In the following, we shall ignore all σ loops, i.e., σ will be approximated by a classical field.

In our phenomenological Lagrangian density, the σ field is coupled to the quarks in two ways: One is through the quark-V^a_μ coupling g and $-\frac{1}{4}\kappa V^a_{\mu\nu}V^a_{\mu\nu}$ which, because of (9), couples V^a_μ with σ, the other is a direct quark-σ coupling f. So far as the quark confinement problem is concerned, there is no need to have the direct quark-σ coupling; it suffices to have the vacuum be a perfect dia-electric ($\kappa_\infty=0$). The origin of the f coupling lies in the dichotomy that only inside the hadron is the coupling between the quark and the vector gauge field really g, which is relatively small; outside the hadron, because of (1), it is actually $g/\kappa_\infty^{1/2}$. For $\kappa_\infty\ll 1$, $g/\kappa_\infty^{1/2}$ becomes very large; on the one hand, this has the desirable effect of preventing the quarks from moving outside the hadron, but on the other hand it also presents a technical difficulty for a diagram analysis in powers of g^2. In (10) the direct quark-σ coupling f with

$$f\sigma_{\text{vac}}\gg\text{hadron mass} \tag{13}$$

is introduced phenomenologically to give a convenient alternative formulation of the same physical effect, but bypassing the above difficulty. As we shall see, because of (13) the f coupling restricts the quarks to staying always inside the hadron, and that enables us to take full advantage of the relative smallness of the quark-V^a_μ coupling g. With the f coupling we may now expand any physical observable, say the hadron mass M, in a power series of $\alpha\equiv(4\pi)^{-1}g^2$:

$$M=M_0+\alpha M_1+\alpha^2M_2+\cdots. \tag{14}$$

If f were zero, it would be difficult to derive the zeroth-order term M_0, since when $\alpha=0$ both g and $g/\kappa_\infty^{1/2}$ should be zero; hence, without the f coupling quarks would be unconfined. (See Sec. III D for further discussions.)

B. Zeroth-order approximation

With the direct quark-σ coupling f, we can now in the zeroth-order calculation neglect the exchange of V^a_μ. The description of the hadron reduces to that of a simple soliton model, consisting of the scalar σ field and the quark ψ field. The relevant part of the Lagrangian density (10) now consists simply of

$$-\psi^\dagger\gamma_4\left(\gamma_\mu\frac{\partial}{\partial x_\mu}+f\sigma+m\right)\psi-\frac{1}{2}\left(\frac{\partial\sigma}{\partial x_\mu}\right)^2-U(\sigma). \tag{15}$$

As mentioned before, since σ is only a phenomenological field that has no short-wavelength components, we shall neglect all σ-loop diagrams. The remaining σ diagrams are all tree diagrams, which correspond to the quasiclassical approximation that has been extensively studied in the literature.[7] Here we give only a summary of the results for light quark hadrons, with $m=0$. In this case, ψ and σ can be reduced to c-number functions which satisfy

$$(-i\vec{\alpha}\cdot\vec{\nabla}+f\beta\sigma)\psi=\epsilon\psi$$

and (16)

$$-\nabla^2\sigma + U'(\sigma) = -fN\psi^\dagger\beta\psi,$$

where $U'(\sigma) = dU/d\sigma$, $\vec{\alpha}$ and β are the usual Dirac matrices, $\epsilon > 0$, ψ satisfies $\int \psi^\dagger\psi d^3r = 1$, and N is the total number of quarks and antiquarks,

$N = 2$ for mesons

and

$N = 3$ for baryons.

As shown in Ref. 7, for $f\sigma_{\text{vac}} \gg$ hadron mass and under very general assumptions, the hadron acquires a well-defined surface $\mathcal{S}$. One has

$$\psi = 0$$

and

$$\sigma = \sigma_{\text{vac}} \quad \text{outside } \mathcal{S}; \tag{17}$$

on the inner side of the surface $\mathcal{S}$,

$$-i\beta\vec{\alpha}\cdot\hat{n}\psi = \psi, \tag{18}$$

where $\hat{n}$ is the unit vector normal to $\mathcal{S}$, and inside $\mathcal{S}$ the field σ is $\cong 0$, with its deviation from zero proportional to $\psi^\dagger\beta\psi$. For the $s_{1/2}$ orbit, $\mathcal{S}$ is a spherical surface of radius R. Inside $\mathcal{S}$, (16) can be further reduced, through scaling, to a simple system of two coupled first-order differential equations:

$$\frac{du}{d\rho} = (-1 + u^2 - v^2)v$$

and (19)

$$\frac{dv}{d\rho} + \frac{2v}{\rho} = (1 + u^2 - v^2)u,$$

where ρ is related to the radius r by $\rho = \epsilon r$,

$$\psi \propto \begin{pmatrix} u \\ i(\vec{\sigma}\cdot\vec{r}/r)v \end{pmatrix}\zeta,$$

$$\zeta = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

$\vec{\sigma}$ is the usual Pauli spin matrix, and the variables ρ, u, and v are all dimensionless. Although (19) does not contain any explicit parameters, its solutions form a one-parameter family. As $\rho \to 0$, one has $v \to 0$ and $u \to u(0)$. For every $u(0)$ between 0 and a critical value $u_c = 1.7419$, there is a solution of (19). The solution can be obtained by direct integration from $\rho = 0$ to $\rho = \rho_0$. At $\rho = \rho_0$, one has $u(\rho_0) = v(\rho_0)$ and therefore the boundary condition (18) is satisfied. The radius of the hadron is given by

$$R = \rho_0/\epsilon.$$

A convenient parameter to label these solutions can be either $u(0)$ or the integral

$$n \equiv \int (u^2 + v^2) d^3\rho$$

over the region $\rho \le \rho_0$. As the initial value $u(0) \to 0$, one has $n \to 0$, but as $u(0) \to u_c = 1.7419$, $n \to \infty$.

The physical description of the soliton solution then resembles that of a gas bubble (i.e., the hadron) inside a medium (i.e., the vacuum). The hadron mass is determined by three parameters:

$$p,\ s,\ \text{and } n, \tag{20}$$

where p is defined by (12) which represents the pressure of the medium on the bubble, s is the surface tension which arises because σ changes from 0 to σ_{vac} across the soliton surface, and n determines the gas pressure inside the bubble which is due to both the kinetic energy of quarks and the excitation energy of σ. In either of the limits $n \to 0$ or $n \to \infty$ the hadron mass has the form, in the notation of (14),

$$M_0 = \frac{N\rho_0}{R} + \tfrac{1}{3}4\pi R^3 p + 4\pi R^2 s, \tag{21}$$

where

$\rho_0 = 2.0428$ when $n \to 0$

and (22)

$\rho_0 = 1$ when $n \to \infty$.

The double limit $n \to 0$ and $s \to 0$ gives the MIT bag,[8,9] and the double limit $n \to \infty$ and $p \to 0$ gives the SLAC bag.[10]

When $n \to 0$, the field σ assumes a simple form:

$\sigma = \sigma_{\text{vac}}$ outside $\mathcal{S}$

and (23)

$\sigma = 0$ inside $\mathcal{S}$.

For $n \ne 0$, though $\sigma = \sigma_{\text{vac}}$ outside $\mathcal{S}$, σ is only $\cong 0$ inside; as mentioned before, its deviation from 0 is proportional to $\psi^\dagger\beta\psi$.

C. Coulomb gauge

To simplify our discussions, in the following we shall regard $\sigma(x)$, and therefore also $\kappa(x)$, as a *given* function of $\vec{r}$. Let us consider a single hadron system so that its surface $\mathcal{S}$ is simply connected. We assume further that $\sigma(\vec{r})$ has the simple form given by (23); hence, because of (9)

$\kappa(\vec{r}) = \kappa_\infty \to 0$ outside $\mathcal{S}$

and (24)

$\kappa(\vec{r}) = 1$ inside $\mathcal{S}$,

where the precise form of the surface $\mathcal{S}$ is to be determined by minimizing the energy of the hadron state under consideration, as is done in the above zeroth-order calculation. [It is quite straightforward to extend our analysis to the more complicated case in which $\kappa(\vec{r})$ is only $\cong 1$ inside $\mathcal{S}$, as in the general $n \neq 0$ solutions examined in the preceding section.]

From (10), it follows that V_μ^a satisfies (neglecting the counterterms)

$$\frac{\partial}{\partial x_\mu}(\kappa V_{\mu\nu}^a) = -gJ_\nu^a\,, \tag{25}$$

where

$$J_\nu^a \equiv j_\nu^a + \kappa C^{abc} V_\mu^b V_{\mu\nu}^c \tag{26}$$

and

$$j_\nu^a = \tfrac{1}{2} i\psi^\dagger \gamma_4 \gamma_\nu \lambda^a \psi. \tag{27}$$

It is convenient to introduce the three-vectors $\vec{E}^a$, $\vec{D}^a$, $\vec{B}^a$, and $\vec{H}^a$, as in the case of the usual electromagnetic field in a medium:

$$V_{4k}^a = iE_k^a, \quad V_{ij}^a = \epsilon_{ijk} B_k^a\,,$$
$$\vec{D}^a = \kappa \vec{E}^a, \text{ and } \vec{H}^a = \kappa \vec{B}^a, \tag{28}$$

where the roman subscripts denote the space components, $\epsilon_{ijk} = +1$ if ijk is an even permutation of 123, -1 if it is an odd permutation, and 0 otherwise. Equation (25) becomes

$$\vec{\nabla}\cdot \vec{D}^a = gJ_0^a$$
and (29)
$$\vec{\nabla}\times\vec{H}^a - \dot{\vec{D}}^a = g\vec{J}^a,$$

where the dot denotes a time derivative, $J_\mu^a = (\vec{J}^a, iJ_0^a)$,

$$J_0^a = \tfrac{1}{2}\psi^\dagger\lambda^a\psi - \kappa C^{abc}\vec{V}^b\cdot\vec{E}^c$$
and (30)
$$\vec{J}^a = \tfrac{1}{2}\psi^\dagger\vec{\alpha}\lambda^a\psi - \kappa C^{abc}(V_0^b\vec{E}^c + \vec{V}^b\times\vec{B}^c).$$

In the Coulomb gauge, $\vec{V}^a$ is chosen to be divergence-free,

$$\vec{\nabla}\cdot\vec{V}^a = 0, \tag{31}$$

and it satisfies

$$\vec{V}^a\cdot\hat{n} = 0 \text{ on } \mathcal{S}, \tag{32}$$

where, as before, $\hat{n}$ is the normal of $\mathcal{S}$. Because the Lagrangian density (10) is locally gauge-invariant, one sees readily that (as will also be proved in Appendix B) both (31) and (32) can be satisfied. According to (24), $\kappa(\vec{r})$ has a finite discontinuity across $\mathcal{S}$, which together with (32) gives

$$\vec{V}^a\cdot\vec{\nabla}\kappa = 0. \tag{33}$$

The function V_0^a will now be regarded as a functional of $\vec{V}^a$, $\dot{\vec{V}}^a$, and

$$j_0^a = \tfrac{1}{2}\psi^\dagger\lambda^a\psi \tag{34}$$

through

$$-\vec{\nabla}\cdot(\kappa\vec{\nabla}V_0^a) = gI^a, \tag{35}$$

where on account of (28)–(33)

$$I^a = J_0^a + \kappa C^{abc}\dot{\vec{V}}^b\cdot\vec{\nabla}V_0^c. \tag{36}$$

Because $\kappa C^{abc}\dot{\vec{V}}^b\cdot\vec{\nabla}V_0^c = \vec{\nabla}\cdot(\kappa C^{abc}\dot{\vec{V}}^b V_0^c)$, the total "color" charge is

$$Q^a \equiv \int J_0^a d^3r = \int I^a d^3r; \tag{37}$$

furthermore, it is a constant of motion. Let $G(\vec{r},\vec{r}\,')$ be the Green's function defined by

$$-\vec{\nabla}\cdot[\kappa\vec{\nabla}G(\vec{r},\vec{r}\,')] = \delta^3(\vec{r}-\vec{r}\,'), \tag{38}$$

where $\vec{\nabla}$ operates on $\vec{r}$. The solution V_0^a of (35) satisfies

$$V_0^a(\vec{r}) = g\int G(\vec{r},\vec{r}\,')I^a(\vec{r}\,')d^3r'. \tag{39}$$

Since I^a depends on V_0^a, the functional dependence of

$$V_0^a = V_0^a(\vec{V}^b, \dot{\vec{V}}^b, j_0^b) \tag{40}$$

can be obtained from (36) and (39) by iteration.

From (24), one sees that as $\kappa_\infty \to 0$, the Green's function $G(\vec{r},\vec{r}\,') \to \infty$ everywhere. We may expand $G(\vec{r},\vec{r}\,')$ in powers of κ_∞:

$$G(\vec{r},\vec{r}\,') = \kappa_\infty^{-1}G_0(\vec{r},\vec{r}\,') + G_1(\vec{r},\vec{r}\,') + \kappa_\infty G_2(\vec{r},\vec{r}\,') + \cdots\ . \tag{41}$$

As shown in Appendix B, for arbitrary surface $\mathcal{S}$,

$$G_0(\vec{r},\vec{r}\,') = \text{constant} \tag{42}$$

when both $\vec{r}$ and $\vec{r}\,'$ are inside $\mathcal{S}$; furthermore, $G_0(\vec{r},\vec{r}\,')$ is independent of $\vec{r}$ when $\vec{r}$ is inside $\mathcal{S}$ (but $\vec{r}\,'$ may be outside). Likewise, owing to symmetry, $G_0(\vec{r},\vec{r}\,')$ is independent of $\vec{r}\,'$ when $\vec{r}\,'$ is inside $\mathcal{S}$. Hence for a color-singlet hadron, because $Q^a = 0$ and $j_0^a = \vec{V}^a = 0$ outside $\mathcal{S}$,

$$\lim_{\kappa_\infty\to 0} V_0^a(\vec{r}) = \text{finite}. \tag{43}$$

The Green's function $G(\vec{r},\vec{r}\,')$ for a spherical surface of radius R can be readily derived (and is also given in Appendix B), from which one sees that, e.g., when $\vec{r}$ and $\vec{r}\,'$ are both inside the surface $G_0(\vec{r},\vec{r}\,') = (4\pi R)^{-1}$ and

$$G_1(\vec{r},\vec{r}\,')=(4\pi)^{-1}\left\{\frac{1}{|\vec{r}-\vec{r}\,'|}+\frac{1}{R}\left[-1+\sum_{l=1}^{\infty}\frac{l+1}{l}\left(\frac{rr'}{R^2}\right)^l P_l(\cos\theta)\right]\right\}, \tag{44}$$

where the magnitudes of $\vec{r}$ and $\vec{r}\,'$ are r and r', respectively, the angle between them is θ, and the P_l's are the standard Legendre polynomials. [The sum in (44) can be easily carried out. See (B22) and (B23) of Appendix B.]

In the Coulomb gauge, ψ and $\vec{V}^a$ are regarded as independent generalized coordinates. Their conjugate momenta are, respectively, $i\psi^\dagger$ and

$$\Pi_i^a=-[\delta_{ij}-(\nabla_i\nabla_j/\nabla^2)]D_j^a\ . \tag{45}$$

It is useful to resolve $\vec{E}^a$ and $\vec{D}^a$ into their transverse and longitudinal components:

$$\vec{E}^a=\vec{E}^a_{tr}+\vec{E}^a_{long}$$

and (46)

$$\vec{D}^a=\vec{D}^a_{tr}+\vec{D}^a_{long}\ ,$$

where $\vec{\nabla}\cdot\vec{D}^a_{tr}=\vec{\nabla}\cdot\vec{E}^a_{tr}=0$ and $\vec{\nabla}\times\vec{D}^a_{long}=\vec{\nabla}\times\vec{E}^a_{long}=0$. Equation (45) becomes

$$\vec{\Pi}^a=-\vec{D}^a_{tr}. \tag{47}$$

By using (35) and (36), one can readily verify that

$$\int\vec{D}^a_{long}\cdot\vec{E}^a_{long}\,d^3r=g\int V_0^a(j_0^a+C^{abc}\vec{V}^b\cdot\vec{\Pi}^c)d^3r, \tag{48}$$

and therefore, through partial integrations, the Hamiltonian density can be set to be

$$\mathcal{H}=\tfrac{1}{2}\vec{D}^a_{tr}\cdot\vec{E}^a_{tr}+\tfrac{1}{2}\vec{D}^a_{long}\cdot\vec{E}^a_{long}+\tfrac{1}{2}\vec{B}^a\cdot\vec{H}^a$$
$$+\psi^\dagger\left(\vec{\alpha}\cdot\frac{1}{i}\vec{\nabla}+f\sigma+m\right)\psi-\vec{j}^a\cdot\vec{V}^a+\tfrac{1}{2}(\vec{\nabla}\sigma)^2+U(\sigma), \tag{49}$$

which is valid in the approximation that $\sigma(x)$ is a given time-independent classical field.

D. A modification

As mentioned in Sec. III A, the quark-V_μ^a coupling has the relatively small value g only inside the hadron; outside, it is $g/\kappa_\infty^{1/2}$ which $\to\infty$ when $\kappa_\infty\to 0$. In order to develop a series expansion in g, but not in $g/\kappa_\infty^{1/2}$, we introduce a direct quark-σ coupling f in the Lagrangian density (10). When $f\sigma_{vac}\gg$ hadron mass, the quark field ψ becomes 0 outside the hadron surface $\mathcal{S}$, in accordance with (17); thereby we avoid the very large coupling $g/\kappa_\infty^{1/2}$. The same problem also arises between the vector gauge fields themselves because of self-coupling. Likewise, we would also like to restrict

$$V_\mu^a=0\quad\text{outside } \mathcal{S}\ . \tag{50}$$

For simplicity, let us assume that, as in the preceding section, $\sigma(x)$ is given by (23). Within the context of a relativistic local theory, the above restriction can be achieved most simply by modifying the Lagrangian density from $\mathcal{L}$ to[11]

$$\mathcal{L}'=\mathcal{L}-\tfrac{1}{2}\kappa(f'\sigma)^2V_\mu^aV_\mu^a\ , \tag{51}$$

where, as before, κ and σ are related by (9) and $\mathcal{L}$ is given by (10). Like (13), this additional coupling f' satisfies

$$f'\sigma_{vac}\equiv\mu\gg\text{hadron mass.} \tag{52}$$

The modified Lagrangian density $\mathcal{L}'$ is no longer locally gauge invariant, though it remains globally invariant. Hence the total "color" charge Q^a is still conserved. Furthermore, as we shall see, since

$$f'\sigma=0\qquad\text{inside } \mathcal{S}, \tag{53}$$

we retain almost all the physical consequences of the locally gauge-invariant Lagrangian density $\mathcal{L}$.

[One may wonder: Why not directly set in the Lagrangian density (10) $f=0$, or in (51) $f=f'=0$, and simply impose "$\psi=0$ and $V_\mu^a=0$ outside $\mathcal{S}$" as constraints? Although such an approach will be discussed in Sec. IV, in principle there are several advantages in starting from a Lorentz-invariant Lagrangian density. (i) By using its soliton solution, one can then *derive* these constraints. This way it ensures the self-consistency of the constraints, especially since V_0^a is not an independent variable. For example, without the f' coupling, even if $\psi=\vec{V}^a=0$ outside $\mathcal{S}$, according to (39) there is still a tail of nonzero V_0^a outside $\mathcal{S}$, and that might lead to a Van der Waals-type force between hadrons which would be in violation of experimental observations. However, in the classical limit, one can show that when $\kappa_\infty\to 0$ the long-range force between hadrons must vanish. (ii) A covariant Lagrangian theory allows one to study the motion of the hadron surface $\mathcal{S}$ and to ensure relativistic covariance.]

Instead of (25), V_μ^a now satisfies

$$\frac{\partial}{\partial x_\mu}(\kappa V_{\mu\nu}^a)-\kappa(f'\sigma)^2V_\nu^a=-gJ_\nu^a\ , \tag{54}$$

and instead of (35), V_0^a satisfies

$$-\vec{\nabla}\cdot(\kappa\vec{\nabla}V_0^a)+\kappa(f'\sigma)^2V_0^a=gI^a, \tag{55}$$

where J_ν^a and I^a remain given by (26) and (36), respectively. Because the local gauge invariance is broken in $\mathcal{L}'$, it is not possible to impose the divergence condition (31); all three components of $\vec{V}^a$ are independent variables. However, since $\dot{V}_0^a$ is absent in $\mathcal{L}'$, only ψ and $\vec{V}^a$ can be regarded as independent generalized coordinates; V_0^a remains a dependent variable. Just as in (39), we may write

$$V_0^a(\vec{r})=g\int G(\vec{r},\vec{r}\,')I^a(\vec{r}\,')d^3r', \tag{56}$$

where, instead of (38), $G(\vec{r},\vec{r}\,')$ is now defined by

$$[-\vec{\nabla}\cdot(\kappa\vec{\nabla})+\kappa(f'\sigma)^2]G(\vec{r},\vec{r}\,')=\delta^3(\vec{r}-\vec{r}\,'). \tag{57}$$

At any fixed $\mu\equiv f'\sigma_{\text{vac}}$, when $\kappa_\infty\to 0$, this new Green's function $G(\vec{r},\vec{r}\,')\to\infty$ everywhere, as before. We may again expand $G(\vec{r},\vec{r}\,')$ in powers of κ_∞:

$$G(\vec{r},\vec{r}\,')=\kappa_\infty^{-1}G_0(\vec{r},\vec{r}\,')+G_1(\vec{r},\vec{r}\,')+\kappa_\infty G_2(\vec{r},\vec{r}\,')+\cdots\,. \tag{58}$$

It will be proved in Appendix C that (42) and (43) remain valid, i.e.,

$$G_0(\vec{r},\vec{r}\,')=\text{constant} \tag{59}$$

when $\vec{r}$ and $\vec{r}\,'$ are both inside $\mathcal{S}$, and for a color-singlet hadron, $V_0^a(\vec{r})=$ finite when $\kappa_\infty\to 0$. Furthermore, for a spherical surface of radius R and for $\vec{r},\vec{r}\,'$ inside the sphere, $G_1(\vec{r},\vec{r}\,')$ remains given by (44), apart from an additive constant. Since for a color singlet the additive constant in $G(\vec{r},\vec{r}\,')$, or $G_1(\vec{r},\vec{r}\,')$, is of no importance, we may rewrite (44) as

$$G_1(\vec{r},\vec{r}\,')=(4\pi)^{-1}\left[\frac{1}{|\vec{r}-\vec{r}\,'|}+\sum_{l=1}^{\infty}\frac{l+1}{lR}\left(\frac{rr'}{R^2}\right)^l P_l(\cos\theta)\right]. \tag{60}$$

Outside $\mathcal{S}$, this new G_1 decreases exponentially; hence, it is quite different from the old G_1 which has only a power dependence on $\vec{r}$ and/or $\vec{r}\,'$. [For nonspherical surfaces, the new G_1 can be quite different from the old G_1 even inside $\mathcal{S}$. See expression (C17) of Appendix C.]

Inside $\mathcal{S}$, since $\sigma=0$ we have $\kappa=1$ and $\mathcal{L}'=\mathcal{L}$; hence, $\vec{D}^a=\vec{E}^a$ and our Lagrangian density is locally gauge invariant. We may again adopt the Coulomb gauge and require (31) and (32), i.e., $\vec{\nabla}\cdot\vec{V}^a=0$ inside the surface and $\hat{n}\cdot\vec{V}^a=0$ on the surface. As in (46), $\vec{E}^a$ may be decomposed into $\vec{E}^a=\vec{E}^a_{\text{tr}}+\vec{E}^a_{\text{long}}$:

$$\vec{E}^a_{\text{long}}=-\vec{\nabla}V_0^a+\vec{\nabla}\phi^a,$$

$$\vec{E}^a_{\text{tr}}=-\dot{\vec{V}}^a-gC^{abc}\vec{V}^bV_0^c-\vec{\nabla}\phi^a, \tag{61}$$

and

$$\nabla^2\phi^a=-g\vec{\nabla}\cdot(C^{abc}\vec{V}^bV_0^c)\,.$$

As will be shown in Appendix C, one can always choose ϕ^a such that

$$\vec{E}^a_{\text{tr}}\cdot\hat{n}=0 \quad\text{on }\mathcal{S}. \tag{62}$$

Equation (47) now becomes

$$\vec{\Pi}^a=-\vec{E}^a_{\text{tr}} \quad\text{inside }\mathcal{S}. \tag{63}$$

Outside the surface, when $f\sigma_{\text{vac}}\to\infty$ and $f'\sigma_{\text{vac}}\to\infty$, we have

$$\psi=\vec{V}^a=0. \tag{64}$$

Therefore, the modified Lagrangian (51) provides a self-consistent device to exclude the quark and the gauge fields from the outside region.

IV. FEYNMAN DIAGRAMS

A. Reduced Hamiltonian

To simplify our discussions, we again ignore the surface motion,[12] and assume σ and κ as given by (23) and (24), respectively. Inside the surface $\mathcal{S}$, the equations of motion reduce to

$$\frac{\partial}{\partial x_\mu}V^a_{\mu\nu}=-gJ^a_\nu$$

and

$$(\gamma_\mu D_\mu+m)\psi=0,$$

where D_μ and J_ν^a are given by (11) and (26). Outside $\mathcal{S}$, we impose the constraints $\psi=\vec{V}^a=0$, in accordance with (64). We shall adopt the Coulomb gauge: $\vec{\nabla}\cdot\vec{V}^a=0$ inside $\mathcal{S}$. In addition, ψ and $\vec{V}^a$ satisfy the boundary conditions (18) and (32), respectively.

By following the discussions given in the preceding section (and as will also be shown in Appendix C), the Hamiltonian of the system can be reduced to the form

$$H_0=\int_\Omega \mathcal{H}\,d^3r+sA, \tag{65}$$

where Ω refers to the volume within $\mathcal{S}$,

$$\mathcal{H}=\tfrac{1}{2}\vec{\Pi}^a\cdot\vec{\Pi}^a+\tfrac{1}{2}gV_0^a(j_0^a+C^{abc}\vec{V}^b\cdot\vec{\Pi}^c)+\tfrac{1}{2}\vec{B}^a\cdot\vec{B}^a+\psi^\dagger\left(\vec{\alpha}\cdot\frac{1}{i}\vec{\nabla}+m\right)\psi-\vec{j}^a\cdot\vec{V}^a+p, \tag{66}$$

p is given by (12), s is the surface tension defined in (20), and A is the area of $\mathcal{S}$. As before, $\vec{\Pi}^a$

$=-\vec{E}^a_{tr}$ is the conjugate momentum of $\vec{V}^a$, $j^a_0 = \frac{1}{2}\psi^\dagger\lambda^a\psi$, and $\vec{j}^a = \frac{1}{2}\psi^\dagger\vec{\alpha}\lambda^a\psi$. Equations (37) and (56) may now be written as

$$Q^a = \int_\Omega I^a d^3r = \int_\Omega (j^a_0 + C^{abc}\vec{V}^b\cdot\vec{\Pi}^c)d^3r, \qquad (67)$$

$$V^a_0(\vec{r}) = g\int_\Omega G(\vec{r},\vec{r}\,')I^a(\vec{r}\,')d^3r', \qquad (68)$$

and

$$I^a = j^a_0 + C^{abc}\vec{V}^b\cdot(-\vec{E}^c + \vec{\nabla}V^c_0),$$

where $G(\vec{r},\vec{r}\,')$ is given by (57).

When $\kappa_\infty \to 0$, one has $G(\vec{r},\vec{r}\,')\to\infty$. As shown in Appendix C, for color nonsinglets, $Q^a \neq 0$,

$$H_0 \to (2\kappa_\infty)^{-1}g^2G_0Q^aQ^a \to \infty, \qquad (69)$$

where according to (59) G_0 is a constant. But for a color singlet, H_0 is *finite* and (65) becomes

$$H_0 = \int_\Omega \bar{\mathcal{H}}d^3r + sA, \qquad (70)$$

where $\bar{\mathcal{H}}$ is the same as $\mathcal{H}$, provided V^a_0 is replaced by

$$\bar{V}^a_0(\vec{r}) \equiv g\int G_1(\vec{r},\vec{r}\,')I^a(\vec{r}\,')d^3r', \qquad (71)$$

where G_1 is defined by (58). The explicit form of $\bar{\mathcal{H}}$ is

$$\bar{\mathcal{H}} = \tfrac{1}{2}\vec{\Pi}^a\cdot\vec{\Pi}^a + \tfrac{1}{2}g\bar{V}^a_0(j^a_0 + C^{abc}\vec{V}^b\cdot\vec{\Pi}^c) + \tfrac{1}{2}\vec{B}^a\cdot\vec{B}^a + \psi^\dagger\left(\vec{\alpha}\cdot\frac{1}{i}\vec{\nabla}+m\right)\psi + \vec{j}^a\cdot\vec{V}^a + p. \qquad (72)$$

From (69) and (70), we conclude that when $\kappa_\infty \to 0$, the Hamiltonian operator is always divergent for color nonsinglets, but *for color singlets, it is always finite*.

B. Feynman rules

In the Coulomb gauge, the derivation from the Lagrangian to the Hamiltonian follows the standard canonical procedure. However, because the Lagrangian contains nonlinear terms which are $\dot{\vec{V}}^a$ dependent, there must be some additional action[13] which gives rise to new loop diagrams. The simplest derivation is to follow the path-integration method of Faddeev and Popov.[14,15] The result can be expressed in terms of an additional Hamiltonian

$$H_1 = -\tfrac{1}{2}i\,\mathrm{tr}\ln(1-\Delta^{-1}M) + i\,\mathrm{tr}\ln(1-\Delta^{-1}N), \qquad (73)$$

where 1 is the unit matrix

$$\langle a,\vec{r}\,|1|\,b,\vec{\rho}\rangle = \delta^{ab}\delta^3(\vec{r}-\vec{\rho}),$$

and the matrices Δ, M, and N are given by

$$\langle a,\vec{r}\,|\Delta|\,b,\vec{\rho}\rangle = \delta^{ab}\vec{\nabla}^2\delta^3(\vec{r}-\vec{\rho}),$$

$$\langle a,\vec{r}\,|M|\,b,\vec{\rho}\rangle = gC^{abc}[\vec{V}^c(\vec{r})+\vec{V}^c(\vec{\rho})]\cdot\vec{\nabla}\delta^3(\vec{r}-\vec{\rho}) + g^2C^{ace}C^{bde}\vec{V}^c(\vec{r})\cdot\vec{V}^d(\vec{\rho})\delta^3(\vec{r}-\vec{\rho}), \qquad (74)$$

$$\langle a,\vec{r}\,|N|\,b,\vec{\rho}\rangle = gC^{abc}\vec{\nabla}\cdot[\vec{V}^c(\vec{r})\delta^3(\vec{r}-\vec{\rho})].$$

As before, $a,b,\ldots$ are the color indices, δ^{ab} is the usual Kronecker symbol, and $\vec{\nabla}$ acts on $\vec{r}$.

The Feynman diagrams can now be constructed by using the "effective Hamiltonian"

$$H_0 + H_1, \qquad (75)$$

where H_0 and H_1 are given by (65) and (73), respectively. In the Coulomb gauge, the propagator of the gauge field between two space-time points x and x' consists of a longitudinal part $D^{ab}_{\rm long}(x,x')$ and a transverse part $D^{ab}_{\rm tr}(x,x')$. (i) The longitudinal propagator is instantaneous in time, given by

$$D^{ab}_{\rm long}(x,x') = \delta(t-t')\delta^{ab}G(\vec{r},\vec{r}\,'), \qquad (76a)$$

where $G(\vec{r},\vec{r}\,')$ is defined by (57). As mentioned before, when $\kappa_\infty\to 0$, $G\to\infty$, and that leads to divergence whenever there are color-nonsinglet external lines. However, between color singlets all amplitudes remain finite when $\kappa_\infty \to 0$, and we may replace (76a) by

$$D^{ab}_{\rm long}(x,x') = \delta(t-t')\delta^{ab}G_1(\vec{r},\vec{r}\,'), \qquad (76b)$$

where $G_1(\vec{r},\vec{r}\,')$ is defined by (58) and it is *independent* of κ_∞. Both G_1 and G are symmetric in $\vec{r}$ and $\vec{r}\,'$. For a spherical surface, G_1 is given by (44) or (60). For a general surface, according to (C17) of Appendix C, G_1 is determined by

$$-\nabla^2G_1(\vec{r},\vec{r}\,') = \delta^3(\vec{r}-\vec{r}\,') \quad \text{inside } \mathcal{S}$$

and, when $\vec{r}$ is on the surface, the inhomogeneous Neumann boundary condition

$$-\hat{n}\cdot\vec{\nabla}G_1(\vec{r},\vec{r}\,') = A^{-1} \text{ on } \mathcal{S}, \qquad (76c)$$

where $\vec{r}\,'$ is inside $\mathcal{S}$, A is the area of $\mathcal{S}$, and, as before, $\hat{n}$ is the unit normal vector of $\mathcal{S}$. (G_1 is determined up to an additive constant, which is immaterial for color-singlet states.)

To derive the transverse part of the gauge field propagator $D^{ab}_{\rm tr}(x,x')$, we need the various radiation modes in a cavity of surface $\mathcal{S}$ (when $g=0$). The boundary condition of the radiation field can be derived most simply by integrating respectively the top equation of (29) over a small volume across $\mathcal{S}$ and the bottom equation over a small loop. This leads to the conclusion that the normal component of $\vec{D}$ and the tangential component of $\vec{H}$ should be both continuous across $\mathcal{S}$. Since both are zero outside $\mathcal{S}$, they must remain zero[16] on the inner side of $\mathcal{S}$. Now inside $\mathcal{S}$, $\kappa=1$; for the radiation field and when $g=0$, $\vec{D}^a = -\dot{\vec{V}}^a$ and $\vec{H}^a = \vec{\nabla}\times\vec{V}^a$. Therefore, we have the boundary condition

$$\hat{n}\cdot\dot{\vec{V}}^a=0$$

and (77)

$$\hat{n}\times(\vec{\nabla}\times\vec{V}^a)=0 \text{ on } S,$$

where, as before, $\hat{n}$ is the normal vector. (See Appendix C for further discussions.)

In the special case that S is a spherical surface of radius R, the usual TE and TB modes give the complete solution. These radiation modes can be expressed in terms of the scalar solution of

$$\nabla^2\phi+k^2\phi=0,$$

which may in turn be written as

$$\phi^{\lambda}_{k,l,m}(\vec{r})=\text{constant}\times j_l(kr)Y_{l,m}(\alpha,\beta), \tag{78}$$

where r, α, β are the spherical coordinates of $\vec{r}$, $Y_{l,m}$ is the usual spherical harmonics, j_l is the spherical Bessel function, $k>0$, and $\lambda=E$ or B depending on the TEor TB mode. For the TE mode

$$\vec{V}^E_{k,l,m}(\vec{r})=\vec{\nabla}\times(\vec{r}\phi^E_{k,l,m}) \tag{79}$$

and k is determined by

$$\frac{d}{dr}[rj_l(kr)]=0 \quad \text{at } r=R.$$

For the TB mode

$$\vec{V}^B_{k,l,m}(\vec{r})=\vec{\nabla}\times[\vec{\nabla}\times(\vec{r}\phi^B_{k,l,m})] \tag{80}$$

and k is determined by

$$j_l(kR)=0.$$

The constant in (78) is chosen so that the integral of $|\vec{V}^{\lambda}_{k,l,m}|^2$ over the volume $r\leq R$ is unity. At $r=R$, in either mode $\vec{V}^{\lambda}_{k,l,m}$ has only tangential components while $\vec{\nabla}\times\vec{V}^i_{k,l,m}$ has only a normal component, in accordance with (77).

The Feynman propagator $D^{ab}_{\text{tr}}(x,x')$ of the radiation field is given by

$$[D^{ab}_{\text{tr}}(x,x')]_{ij}=\delta^{ab}\sum_{\lambda,k,l,m}(2k)^{-1}[\vec{V}^{\lambda}_{k,l,m}(\vec{r})]_i \times[\vec{V}^{\lambda}_{k,l,m}(r')]^*_j e^{\mp ik(t-t')}, \tag{81}$$

where the minus sign in the exponent is for $t>t'$ and the plus sign for $t<t'$.

To derive the propagator of the quark field, we need the complete set of c-number solutions $(\chi_n)_\pm$ of the Dirac equation in a cavity:

$$\left(\vec{\alpha}\cdot\frac{1}{i}\vec{\nabla}+\beta m\right)(\chi_n)_\pm=\pm\epsilon_n(\chi_n)_\pm \ ,$$

where ϵ_n is >0 and $(\chi_n)_\pm$ satisfies the boundary condition (18). The Feynman propagator $S(x,x')$ of the quark field is given by, for $t>t'$,

$$S(x,x')=\sum_n(\chi_n(\vec{r}))_+(\chi^\dagger_n(\vec{r}')\beta)_+e^{-i\epsilon_n(t-t')}, \tag{82a}$$

and for $t<t'$,

$$S(x,x')=-\sum_n(\chi_n(\vec{r}))_-(\chi^\dagger_n(\vec{r}')\beta)_-e^{i\epsilon_n(t-t')}. \tag{82b}$$

The various g-dependent terms in H_0+H_1, defined by (75), give directly the great variety of vertices in this problem. These vertices, together with the propagators D^{ab}_{long}, D^{ab}_{tr}, and S given above, complete our discussion of Feynman rules.

We emphasize that the quark-vector and vector-vector interactions are local in character, while the color singletness of the hadron state is a global property. This is why in (76a) there is an explicit dependence on κ_∞ in the propagator $D^{ab}_{\text{long}}(x,x')$. When $\kappa_\infty\to 0$, our Feynman rules explicitly forbid the appearance of external color nonsinglets, such as free quarks or free vector gauge particles. The propagators D^{ab}_{long}, D^{ab}_{tr}, and S also depend on the linear size R of the hadron surface. Only at high energy and large four-momentum transfer can one neglect the effect of hadron surfaces; in that case, one recovers the "usual" Feynman rules used in the current literature on QCD.

V. REMARKS

A. Phase transition

In our picture, at low temperature, an infinite-volume QCD exists only in a *single* phase. A good analogy is to think of an infinite ferromagnet below the Curie temperature, which has a long-range order and also exists in a single phase. The hadrons are then analogous to some small-domain structures within the infinite ferromagnet. Another analogy that has been used frequently in the soliton (or bag) model is to regard the vacuum as a liquid and hadrons as bubbles. It seems reasonable to assume that there should exist a critical temperature at which the infinite-volume QCD system can undergo a phase transition in its dielectric constant κ (or magnetic susceptibility $\mu=1/\kappa$). The value of the critical temperature and the nature of the phase transition depend sensitively on the excitation curve of hadron spectroscopy. If one follows Chodos *et al.*[8] and approximates the excited hadron system as noninteracting bubbles of an ideal relativistic gas, then not only does there exist a critical temperature T_c, but T_c is also the maximum temperature that the system can attain.[8,17] However, as the size l of such bubbles grows, its dielectric constant κ_l should decrease, and therefore the effective coupling $g/\kappa_l^{1/2}$ would increase, which makes the quarks behave less and less like an ideal gas. Furthermore, as the bubbles increase in size, their mutual interactions should become more important. Hence, it remains an interesting open question whether the idea of a maximum temperature is really correct.

B. Long-range order

Let us leave our problem for the moment and consider a ferromagnet of volume L^3 at a nonzero temperature below the Curie temperature. The magnetization per unit volume $M(H,L)$ under an external magnetic field has the following familiar properties: At any finite volume, $M(H,L)$ is an analytic function of H and $\lim_{H\to 0} M(H,L)=0$. Thus,

$$\lim_{L\to\infty}\lim_{H\to 0} M(H,L)=0. \tag{83}$$

On the other hand, if we take the infinite-volume limit first and then the $H\to 0$ limit, a finite nonzero magnetization results:

$$\lim_{H\to 0}\lim_{L\to\infty} M(H,L)\neq 0; \tag{84}$$

this is why we usually say the infinite ferromagnet carries a long-range order, which is characterized by its magnetization and given by the double limit (84).

In our problem, we may take the ratio of the (color) dielectric constant between two volumes of sizes l^3 and L^3 with $l<L$. By using (A2), (A5), (A7), and (A16) in Appendix A, we find, after neglecting $O(\alpha^2)$,

$$\kappa_L/\kappa_l=\left[1+\frac{11}{2\pi}\alpha\ln(L/l)\right]^{-1}, \tag{85}$$

where $\alpha=(4\pi)^{-1}g_l^{\,2}$. The approximation of neglecting $O(\alpha^2)$ is, of course, not a good one, but some insight into the long-range order in QCD may be obtained even in such a crude approximation. Let us keep the small size l always fixed. At any finite L, we have, according to (85), $\lim_{\alpha\to 0}(\kappa_L/\kappa_l)=1$ and therefore

$$\lim_{L\to\infty}\lim_{\alpha\to 0}[1-(\kappa_L/\kappa_l)]=0. \tag{86}$$

On the other hand, $\lim_{L\to\infty}(\kappa_L/\kappa_l)=0$ at any $\alpha\neq 0$. Hence,

$$\lim_{\alpha\to 0}\lim_{L\to\infty}[1-(\kappa_L/\kappa_l)]=1 \tag{87}$$

in analogy with (83) and (84). Hence, we may regard the long-range order in QCD to be characterized by the perfect dia-electric property of its vacuum when the volume becomes infinite. (See Ref. 1 for further discussions.)

C. Scale of hadron radius

Let us consider a QCD system in the limit of quark mass $=0$ inside the hadron. At first sight, since there seems to be no mass scale in the problem, it may be difficult to see how the physical hadron radius can possibly emerge. To explain this apparent contradiction, let us introduce, as in Appendix A, an ultraviolet momentum cutoff Λ and denote g_0 as the unrenormalized coupling constant. Inside a hadron of radius R, let α be the relevant fine-structure constant of the color gauge field. Clearly, α must be a function of $R\Lambda$ and g_0:

$$\alpha=f(R\Lambda,g_0).$$

When $\Lambda\to\infty$, one must vary g_0 accordingly so that the limiting function

$$\alpha(R)\equiv\lim_{\Lambda\to\infty} f(R\Lambda,g_0) \tag{88}$$

exists. This limiting function must contain a length scale since R has a dimension but α does not. (In a realistic theory, because the quark masses inside the hadron are not really zero, there are additional mass scales.)

In a quasiclassical soliton (or bag) model, the actual value of R is determined by minimizing the hadron energy spectrum, of which the lowest level is the pion. Its mass in a quasiclassical calculation is

$$M_\pi\cong\tfrac{1}{3}4\pi pR^3+4\pi sR^2+\frac{2\rho_0}{R}-\frac{\alpha(R)}{R}\eta_0, \tag{89}$$

where p, s, ρ_0 are given by (21) and (22), and η_0 is a positive number. As discussed in Ref. 1, since the minimum of $M_\pi^{\,2}=0$, $\alpha(R)$ should be near a critical value α_c; when $\alpha(R)=\alpha_c$, $M_\pi^{\,2}=0$, and that in turn determines the pion radius R.

D. Experimental upper limit of κ_∞

As mentioned in Sec. II, according to (7) κ_∞ must be <1; but as yet, no one has been able to determine through pure theoretical deduction just how small κ_∞ actually is. Within our picture, the fact that a single free quark has never been observed sets a lower limit on the free quark mass m_q. As shown in Appendix D,

$$\kappa_\infty\leq(0.11)\alpha(m_p/m_q)^{4/3}, \tag{90}$$

where m_p is the proton mass and $\alpha=g^2/4\pi$ is the fine structure for hadrons. Even without any detailed search of the literature, we may set $m_q>5$ GeV, from which we determine an *experimental* upper limit of κ_∞:

$$\kappa_\infty<1.3\times10^{-2}\alpha. \tag{91}$$

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APPENDIX A

The proof of properties (3) and (4) concerning the (color) dielectric constant of the vacuum, which are stated in Sec. II, will be given in this appendix. As in Sec. II, let us consider a pure QCD system consisting of only color gauge fields V_μ^a. For clarity, we introduce an *ultraviolet* momentum cutoff Λ, and denote g_0 as the unrenormalized coupling constant. The whole system is then enclosed in a finite volume L^3. Let κ_L and g_L be, respectively, the (color) dielectric constant of the vacuum and the renormalized coupling constant. Both κ_L and g_L are functions of g_0, L, and Λ. From dimensional considerations, they must depend only on g_0 and the product $L\Lambda$, i.e., $\kappa_L = F(L\Lambda, g_0)$ and

$$g_L = G(L\Lambda, g_0). \tag{A1}$$

Let us consider two different volumes l^3 and L^3, but with the same Λ and g_0. From (1), it follows that

$$\frac{\kappa_L}{\kappa_l} = \left(\frac{g_l}{g_L}\right)^2, \tag{A2}$$

where, as in (A1),

$$g_l = G(l\Lambda, g_0) \tag{A3}$$

or its inverse $g_0 = g_0(l\Lambda, g_l)$. Eliminating g_0 between (A1) and (A3), we may express g_L in terms of $L\Lambda$, $l\Lambda$, and g_l:

$$g_L = G(L\Lambda, g_0(l\Lambda, g_l)). \tag{A4}$$

Since the theory is a renormalizable one, the limit $\Lambda \to \infty$ of (A4) should exist; in this limit,

$$g_L = g\left(\frac{l}{L}, g_l\right) \equiv \lim_{\Lambda\to\infty} G(L\Lambda, g_0(l\Lambda, g_l)),$$

which may be written as

$$g_L = g(\lambda, g_l), \tag{A5}$$

where

$$\lambda = l/L. \tag{A6}$$

At $\lambda = 1$, (A5) becomes

$$g_l = g(1, g_l). \tag{A7}$$

Because $g_L = G(L\Lambda, g_0)$ is independent of l, one has $(\partial g_L/\partial l)_{L,\Lambda,g_0} = 0$; hence, $g(\lambda, g_l)$ satisfies the familiar renormalization-group equation[18] (but without the usual γ and δ functions)

$$[-(\partial/\partial \ln\lambda) + \beta(\partial/\partial g_l)]g(\lambda, g_l) = 0, \tag{A8}$$

where

$$\beta = -\partial g_l/\partial \ln l. \tag{A9}$$

So far, the finite volume is just a device for the infrared cutoff. Presumably, our physical result should not be sensitive to the precise form in which this infrared cutoff is introduced. In order to derive the dependence of β on g_l, we assume l^{-1} = the usual momentum value M chosen for renormalization.[19] The β function as a power series of g_l is then given by[19,20]

$$\beta(g_l) = -11(16\pi^2)^{-1}g_l^3 - 102(16\pi^2)^{-2}g_l^5 + O(g_l^7) \tag{A10}$$

for a pure SU_3 gauge field system. The solution of (A8) has the standard form

$$g(\lambda, g_l) = f(z), \tag{A11}$$

where

$$z = \ln\lambda + \int_{g_1}^{g_l} \frac{dg'}{\beta(g')} \tag{A12}$$

and g_1 is an arbitrary constant, which will be chosen so that

$$0 < g_1 \ll 1$$

and

$$\beta(g') < 0 \quad \text{for} \quad 0 < g' \le g_1. \tag{A13}$$

Let $f^{-1}(z)$ be the inverse of $f(z)$, i.e., $f^{-1}(f(z)) = z$. From (A11), one has $z = f^{-1}(g(\lambda, g_l))$, which at $\lambda = 1$, because of (A7), becomes $z = f^{-1}(g_l)$. Now z is also given by (A12). By setting $\lambda = 1$, one derives

$$f^{-1}(g_l) = \int_{g_1}^{g_l} \frac{dg'}{\beta(g')}. \tag{A14}$$

For g_l sufficiently small, the $O(g_l^5)$ term in (A10) may be neglected. Hence,

$$f^{-1}(g_l) \cong \tfrac{8}{11}\pi^2(g_l^{-2} - g_1^{-2}); \tag{A15}$$

its inverse function is

$$f(z) \cong [(8\pi^2)^{-1}11z + g_1^{-2}]^{-1/2},$$

and therefore

$$g(\lambda, g_l) \cong \left[\frac{(8\pi^2/11)g_l^2}{g_l^2\ln\lambda + (8\pi^2/11)}\right]^{1/2}. \tag{A16}$$

Since $g(\lambda, g_l)$ is an even function of g_l, we need only consider positive values of g_l. From (A14), we see that (i) the physical value of g_l can vary from 0 to ∞ if $\beta(g') = 0$ only at $g' = 0$, otherwise (ii) g_l can only vary between 0 and $\bar{g}$ where $\bar{g}$ is the smallest positive-definitive root of $\beta(\bar{g}) = 0$. When $g_l = \bar{g}$, the integral in (A14) diverges. In either case, setting $g_l = x$, one has in the physical region $df^{-1}(x)/dx = 1/\beta(x) < 0$; hence, the inverse function $f(z)$ satisfies

$$\frac{df(z)}{dz} < 0. \tag{A17}$$

By using (A11) and (A12), one finds

$$\frac{\partial}{\partial\lambda} g(\lambda, g_l) < 0. \tag{A18}$$

Since $\lambda = l/L$, this means for $L > l$, $g_L > g_l$ and therefore, because of (A2), which is $\kappa_L < \kappa_l$, the inequality (3) of Sec. II.

According to (A14) and (A15), as g_l varies from 0 to g_1, $f^{-1}(g_l)$ decreases from ∞ to 0. When g_l increases to $> g_1$, $f^{-1}(g_l)$ becomes negative. In the above case (i), as $g_l \to \infty$ we have either $f^{-1}(\infty) = -A$ = finite or $f^{-1}(\infty) = -\infty$; in the former the inverse function $f(z) \to \infty$ as $z \to -A+$, which means that because of (A11) and (A12), at any fixed g_l, $g_L = g(\lambda, g_l)$ becomes singular at a finite $\lambda = l/L$, in violation of our assumption that κ_L (therefore, also g_L) is a smooth function of L when $L \to \infty$. Consequently, we are left only with the latter: $f^{-1}(\infty) = -\infty$ or $f(-\infty) = \infty$. Since according to (A12), $z \to -\infty$ means $\lambda = l/L \to 0$, we conclude that in case (i) as $L \to \infty$, $g_L \to \infty$, and therefore $\kappa_L \to 0$. Statement (4) of Sec. II is then established.

In case (ii), $f^{-1}(g_l) \to -\infty$ as $g_l \to \bar{g}-$. Thus, $f(z) \to \bar{g}$ as $z \to -\infty$, and that gives $\lim_{L\to\infty} g_L = \bar{g}$. Adopting convention (5) of Sec. II, we find

$$\kappa_\infty = (g_R/\bar{g})^2, \tag{A19}$$

where, as before, R is some hadron radius chosen to set the scale $\kappa_R = 1$, and $\bar{g}$ is the smallest positive-definite root of $\beta(\bar{g}) = 0$.

APPENDIX B

In this appendix, we shall prove a number of technical points mentioned in Sec. III C concerning the Coulomb gauge. As we shall see, all of these proofs are quite elementary.

1. Conditions (31) and (32)

Suppose $\vec{V}^a$ satisfies neither (31) nor (32). Under an infinitesimal gauge transformation, $\vec{V}^a$ becomes $\vec{V}^a(\theta)$ where

$$\vec{V}^a(\theta) = \vec{V}^a + C^{abc}\theta^b \vec{V}^c - g^{-1}\vec{\nabla}\theta^a \tag{B1}$$

and θ^a is an infinitesimal. We first show that it is always possible to choose θ^a such that

$$\nabla^2\theta^a - g\vec{\nabla}\cdot(C^{abc}\theta^b\vec{V}^c) = g\epsilon\vec{\nabla}\cdot\vec{V}^a \tag{B2}$$

and therefore

$$\vec{\nabla}\cdot\vec{V}^a(\theta) = (1-\epsilon)\vec{\nabla}\cdot\vec{V}^a\ , \tag{B3}$$

where $\epsilon = 0+$. This can be readily established by expanding

$$\theta^a = g\theta_0^a + g^2\theta_1^a + g^3\theta_2^a + \cdots \tag{B4}$$

and noting that the inhomogeneous Laplace equations

$$\nabla^2\theta_0{}^2 = \epsilon\vec{\nabla}\cdot\vec{V}^a \tag{B5}$$

and

$$\nabla^2\theta_i^a = C^{abc}\vec{\nabla}\cdot(\theta_{i-1}^b\vec{V}^c) \text{ for } i \geq 1 \tag{B6}$$

all have solutions. By using (B3) and varying ϵ continuously, we can transform $\vec{\nabla}\cdot\vec{V}^a$ from any value to 0. Hence, (31) is satisfied.

Next, we assume $\vec{V}^a$ to satisfy (31) but not (32), i.e., $\vec{\nabla}\cdot\vec{V}^a = 0$, but $\hat{n}\cdot\vec{V}^a \neq 0$ on $\mathcal{S}$. Again, we consider an infinitesimal gauge transformation $\vec{V}^a \to \vec{V}^a(\theta)$, but now we want to choose θ^a such that (31) remains satisfied,

$$\vec{\nabla}\cdot\vec{V}^a(\theta) = 0\ ,$$

and, in addition (B7)

$$\hat{n}\cdot\vec{V}^a(\theta) = (1-\epsilon)\hat{n}\cdot\vec{V}^a \text{ on } \mathcal{S},$$

where $\epsilon = 0+$. By using (B4), we note that in order to achieve (B7) we must have

$$\nabla^2\theta_0^a = 0 \tag{B8}$$

and

$$\hat{n}\cdot\vec{\nabla}\theta_0^a = \epsilon\hat{n}\cdot\vec{V}^a \text{ on } \mathcal{S}. \tag{B9}$$

Furthermore, besides (B6), we also need (for $i \geq 1$)

$$\hat{n}\cdot\vec{\nabla}\theta_i^a = C^{abc}\theta_{i-1}^b(\hat{n}\cdot\vec{V}^c) \text{ on } \mathcal{S}. \tag{B10}$$

For simplicity, let us first assume $\mathcal{S}$ to be a spherical surface of radius R. The general solution of (B8) can be written in terms of the spherical coordinates (r, α, β)

$$\theta_0^a = \sum A_{lm}^a r^l Y_{lm}(\alpha, \beta),$$

where Y_{lm} is the usual spherical harmonics. Let

$$\hat{n}\cdot\vec{V}^a = \sum B_{lm}^a Y_{lm}(\alpha,\beta) \text{ at } r = R,$$

where

$$B_{0,0}^a \propto \int_{r=R} \hat{n}\cdot\vec{V}^a R^2 \sin\alpha\, d\alpha\, d\beta$$

$$= \int_{r\leq R} \vec{\nabla}\cdot\vec{V}^a d^3r = 0.$$

By choosing

$$A_{lm}^a = \epsilon B_{lm}^a/(lR^{l-1}) \text{ for } l \geq 1$$

and leaving $A_{0,0}^a$ arbitrary, the above θ_0^a satisfies both (B8) and (B9). Let $\bar{\theta}_1^a$ by any particular solution of

$$\nabla^2\bar{\theta}_1^a = C^{abc}\vec{\nabla}\cdot(\theta_0^b\vec{V}^c)$$

and

$\alpha_1^a \equiv \theta_1^a - \bar{\theta}_1^a$.

In order to have (B6) and (B10) for $i=1$, α_1^a must satisfy

$$\nabla^2 \alpha_1^a = 0$$

and

$$\hat{n}\cdot\vec{\nabla}\alpha_1^a = \hat{n}\cdot(C^{abc}\theta_0^b \vec{V}^c - \vec{\nabla}\bar{\theta}_1^a),$$

which have exactly the same form as (B8) and (B9). Consequently, α_1^a exists and so does θ_1^a. Likewise, we can construct $\theta_2^a, \theta_3^a, \ldots$, and therefore (B7) is satisfied. By varying ϵ continuously, we can transform $\hat{n}\cdot\vec{V}^a$ from any value to 0 on $\mathcal{S}$. (When $\mathcal{S}$ is not a spherical surface, by choosing the appropriate curvilinear coordinates, we can apply identical arguments as those given above.) Conditions (31) and (32) are then established.

2. Equations (42)-(44)

Consider a simple problem in classical electrostatics: A unit point charge is placed at $\vec{r}=\vec{r}_0$ inside a simply connected surface $\mathcal{S}$. The dielectric constant is $\kappa = 1$ inside $\mathcal{S}$, but $\kappa = \kappa_\infty \to 0$ outside. The electrostatic potential $V(\vec{r})$ in this problem is the Green's function $G(\vec{r}, \vec{r}_0)$ defined by (38). Let $V(\vec{r}) = V_{\rm in}(\vec{r})$ or $V_{\rm out}(\vec{r})$ depending on whether $\vec{r}$ is inside or outside $\mathcal{S}$. Hence,

$$-\nabla^2 V_{\rm out}(\vec{r}) = 0$$

and (B11)

$$-\nabla^2 V_{\rm in}(\vec{r}) = \delta^3(\vec{r} - \vec{r}_0).$$

The boundary conditions at the surface $\mathcal{S}$ are

$$V_{\rm in}(\vec{r}) = V_{\rm out}(\vec{r})$$

and (B12)

$$\hat{n}\cdot\vec{\nabla}V_{\rm in}(\vec{r}) = \kappa_\infty \hat{n}\cdot\vec{\nabla}V_{\rm out}(\vec{r}),$$

where, as before, $\hat{n}$ is the normal vector of $\mathcal{S}$. We now expand V_α (α = in or out) in powers of κ_∞:

$$V_\alpha = \kappa_\infty^{-1} V_\alpha^{(0)} + V_\alpha^{(1)} + \kappa_\infty V_\alpha^{(2)} + \cdots . \qquad \text{(B13)}$$

Equation (B11) becomes

$$-\nabla^2 V_{\rm in}^{(0)} = 0, \quad -\nabla^2 V_{\rm in}^{(1)} = \delta^3(\vec{r} - \vec{r}_0),$$

$$-\nabla^2 V_{\rm in}^{(l)} = 0 \text{ for } l \geq 2$$

and (B14)

$$-\nabla^2 V_{\rm out}^{(m)} = 0 \text{ for all } m.$$

The boundary condition (B12) requires that at the surface $\mathcal{S}$

$$V_{\rm in}^{(l)} = V_{\rm out}^{(l)} \text{ for all } l,$$

$$\hat{n}\cdot\vec{\nabla}V_{\rm in}^{(0)} = 0, \qquad \text{(B15)}$$

and

$$\hat{n}\cdot\vec{\nabla}V_{\rm in}^{(m)} = \hat{n}\cdot\vec{\nabla}V_{\rm out}^{(m-1)} \text{ for } m \geq 1.$$

Let $\vec{E}_\alpha^{(n)}$ be the electrostatic field associated with $V_\alpha^{(n)}$:

$$\vec{E}_\alpha^{(n)} \equiv -\vec{\nabla}V_\alpha^{(n)}. \qquad \text{(B16)}$$

Now $\vec{E}_{\rm in}^{(0)}$ is, by definition, irrotational; from (B14) it is also divergence-free, and from (B15) its normal component is zero on $\mathcal{S}$. Hence, $\vec{E}_{\rm in}^{(0)} = 0$ and therefore

$$V_{\rm in}^{(0)}(\vec{r}) = \text{constant}. \qquad \text{(B17)}$$

So far, $\vec{r}_0$ is assumed to be inside $\mathcal{S}$. When $\vec{r}$ is also inside $\mathcal{S}$, the Green's function $G(\vec{r}, \vec{r}_0) = V_{\rm in}(\vec{r})$ and $G_0(\vec{r}, \vec{r}_0) = V_{\rm in}^{(0)}(\vec{r})$ where G_0 is defined by (41). Thus, (B17) shows that when $\vec{r}$ and $\vec{r}_0$ are both inside $\mathcal{S}$, $G_0(\vec{r}, \vec{r}_0)$ is independent of $\vec{r}$; by symmetry $G_0(\vec{r}, \vec{r}_0)$ is also independent of $\vec{r}_0$, and therefore (42) follows. By following exactly the same argument, but for $\vec{r}_0$ outside $\mathcal{S}$, we find $G_0(\vec{r}, \vec{r}_0)$ to be independent of $\vec{r}$, so long as $\vec{r}$ is inside $\mathcal{S}$. These properties may be summarized as follows:

$$G_0(\vec{r}, \vec{r}\,') \text{ is independent of } \vec{r} \text{ (or } \vec{r}\,'\text{) whenever } \vec{r} \text{ (or } \vec{r}\,'\text{) is inside } \mathcal{S}. \qquad \text{(B18)}$$

To determine the constant in (B17), we have to examine $V_{\rm out}^{(0)}$. Since according to (B14) and (B15), $-\nabla^2 V_{\rm out}^{(0)} = 0$ outside $\mathcal{S}$ and $V_{\rm out}^{(0)} = V_{\rm in}^{(0)}$ = constant on $\mathcal{S}$, one sees that $V_{\rm out}^{(0)}(\vec{r})$ is the same as the electrostatic potential in free space outside a *perfect conductor* of surface $\mathcal{S}$. Let $\sigma(\vec{r})$ be the surface charge density of that problem. We have $-\vec{\nabla}V_{\rm out}^{(0)} = \hat{n}\sigma$ on $\mathcal{S}$. From (B14) and (B15) we see that

$$-\nabla^2 V_{\rm in}^{(1)} = \delta^3(\vec{r} - \vec{r}_0)$$

and (B19)

$$-\hat{n}\cdot\vec{\nabla}V_{\rm in}^{(1)} = \sigma \text{ on } \mathcal{S}.$$

Hence,

$$\int \sigma\, d^2S = -\int_\Omega \nabla^2 V_{\rm in}^{(1)} d^3r = 1$$

where d^2S is the surface element of $\mathcal{S}$ and Ω is the volume inside $\mathcal{S}$. From (41), it follows that $G_1(\vec{r}, \vec{r}_0) = V_{\rm in}^{(1)}(\vec{r})$ when $\vec{r}$ is inside $\mathcal{S}$. Consequently, inside $\mathcal{S}$, $G_1(\vec{r}, \vec{r}_0)$ is the electrostatic potential generated by a positive unit charge at $\vec{r} = \vec{r}_0$ with the boundary condition (B19) on $\mathcal{S}$.

If $\psi = \vec{V}^a = 0$ outside $\mathcal{S}$, then $j_0^a = J_0^a = I^a = 0$ also outside $\mathcal{S}$. Hence, according to (37), (39), (41), and (B18), as $\kappa_\infty \to 0$,

$$V_0^a(\vec{r}) = \kappa_\infty^{-1} g G_0(\vec{r}, \vec{R}) Q^a + g\int G_1(\vec{r}, \vec{r}\,') I^a(\vec{r}\,') d^3r' + O(\kappa_\infty), \tag{B20}$$

where $\vec{R}$ is any position vector on $\mathcal{S}$. According to (B18), $G_0(\vec{r}, \vec{r}\,') = G_0(\vec{r}, \vec{R})$ whenever $\vec{r}\,'$ is inside $\mathcal{S}$. Now, for a color singlet $Q^a = 0$, $V_0^a(\vec{r})$ is finite as $\kappa_\infty \to 0$. Equation (43) then follows.

To derive (44), we assume $\mathcal{S}$ to be a spherical surface of radius R. As before, a unit charge is placed at $\vec{r} = \vec{r}_0$ inside $\mathcal{S}$. It is straightforward to show that for $r < R$

$$G(\vec{r}, \vec{r}_0) = V_{\rm in}(\vec{r}) = (4\pi)^{-1}\left[\frac{1}{|\vec{r} - \vec{r}_0|} + \sum_{l=0}^{\infty} \frac{(l+1)(1-\kappa_\infty)}{l + (l+1)\kappa_\infty} \frac{(rr_0)^l}{R^{2l+1}} P_l(\cos\theta)\right] \tag{B21}$$

and for $r > R$

$$G(\vec{r}, \vec{r}_0) = V_{\rm out}(\vec{r}) = (4\pi r)^{-1} \sum_{l=0}^{\infty} \frac{2l+1}{l + (l+1)\kappa_\infty}\left(\frac{r_0}{r}\right)^l P_l(\cos\theta),$$

where r and r_0 are, respectively, the magnitudes of $\vec{r}$ and $\vec{r}_0$, θ is the angle between them, and the P_l's are Legendre polynomials. Taking the limit $\kappa_\infty \to 0$ and by using (41), we obtain (44).

As $\kappa_\infty \to 0$, an alternative form of (44) can be given in terms of image charges. We observe that for $r < R$, (B21) becomes

$$G(\vec{r}, \vec{r}_0) = V_{\rm in}(\vec{r}) = (4\pi)^{-1}\left[\frac{1 - 2\kappa_\infty}{\kappa_\infty R} + \frac{1}{|\vec{r} - \vec{r}_0|} + \frac{Rr_0}{|r_0^2\vec{r} - R^2\vec{r}_0|} + \frac{r_0}{R}\int_{(R/r_0)^2}^{\infty} dx\left(\frac{1}{|\vec{r} - x\vec{r}_0|} - \frac{1}{xr_0}\right)\right] + O(\kappa_\infty) \tag{B22}$$

and for $r > R$

$$G(\vec{r}, \vec{r}_0) = V_{\rm out}(\vec{r}) = (4\pi)^{-1}\left[\frac{1 - 2\kappa_\infty}{\kappa_\infty r} + \frac{2}{|\vec{r} - \vec{r}_0|} + \int_0^1 \frac{dx}{x}\left(\frac{1}{|\vec{r} - x\vec{r}_0|} - \frac{1}{r}\right)\right] + O(\kappa_\infty).$$

Thus, as $\kappa_\infty \to 0$, $V_{\rm in}(\vec{r})$, apart from an additive constant, is the electrostatic potential due to a unit charge at $\vec{r} = \vec{r}_0$ inside the sphere, an image point charge of magnitude (R/r_0) at $\vec{r} = R^2\vec{r}_0/r_0^2$ outside the sphere, and a continuous line distribution, also outside the sphere, along the $\vec{r}_0$ direction from $\vec{r} = R^2\vec{r}_0/r_0^2$ to ∞; $V_{\rm out}(\vec{r})$ is that due to two point charges, one of magnitude $1 - 2\kappa_\infty$ at $\vec{r} = 0$ and the other of magnitude $2\kappa_\infty$ at $\vec{r} = \vec{r}_0$, plus a continuous line distribution of image charges along $\vec{r}_0$ from $\vec{r} = 0$ to $\vec{r}_0$, all inside the sphere. The integrals in (B22) can be readily integrated. We find for $r < R$

$$G(\vec{r}, \vec{r}_0) = V_{\rm in}(\vec{r}) = (4\pi)^{-1}\left\{\frac{1 - 2\kappa_\infty}{\kappa_\infty R} + \frac{1}{|\vec{r} - \vec{r}_0|} + \frac{Rr_0}{|r_0^2\vec{r} - R^2\vec{r}_0|} - R^{-1}\ln\left[\frac{1}{2} - \frac{rr_0}{2R^2}\cos\theta + \frac{1}{2R^2}(R^4 - 2R^2rr_0\cos\theta + r^2r_0^2)^{1/2}\right]\right\} + O(\kappa_\infty) \tag{B23}$$

and for $r > R$

$$G(\vec{r}, \vec{r}_0) = V_{\rm out}(\vec{r}) = (4\pi)^{-1}\left\{\frac{1 - 2\kappa_\infty}{\kappa_\infty r} + \frac{2}{|\vec{r} - \vec{r}_0|} - r^{-1}\ln\left[\frac{1}{2} - \frac{r_0}{2r}\cos\theta + \frac{1}{2r}(r^2 - 2rr_0\cos\theta + r_0^2)^{1/2}\right]\right\} + O(\kappa_\infty).$$

APPENDIX C

Several properties of the modified Lagrangian density $\mathcal{L}'$ and the related Green's function $G(\vec{r}, \vec{r}\,')$ are stated in Sec. III D and Sec. IV; these will be established in this appendix.

1. Equations (59) and (60). The proof is essentially identical to that given in Appendix B 2. Let

$V(\vec{r})$ be the solution of

$$[-\vec{\nabla}\cdot(\kappa\vec{\nabla})+\kappa(f'\sigma)^2]V(\vec{r})=\delta^3(\vec{r}-\vec{r}_0) \qquad (C1)$$

that equals the Green's function $G(\vec{r},\vec{r}_0)$, defined by (57),

$$G(\vec{r},\vec{r}_0)=V(\vec{r}). \qquad (C2)$$

Denote $V(\vec{r})=V_{in}(\vec{r})$ or $V_{out}(\vec{r})$ depending on whether $\vec{r}$ is inside or outside the surface S. For $\vec{r}_0$ inside S, because of (23), (24), and (52), (C1) becomes

$$(\nabla^2-\mu^2)V_{out}(\vec{r})=0$$

and (C3)

$$-\nabla^2 V_{in}(\vec{r})=\delta^3(\vec{r}-\vec{r}_0)$$

with the boundary conditions at the surface S:

$$V_{in}(\vec{r})=V_{out}(\vec{r})$$

and (C4)

$$\hat{n}\cdot\vec{\nabla}V_{in}(\vec{r})=\kappa_\infty\hat{n}\cdot\vec{\nabla}V_{out}(\vec{r}),$$

where, as in (B12), $\hat{n}$ is the normal vector of S. Just as in (B13), we may expand V_α (α = in or out) in powers of κ_∞:

$$V_\alpha=\kappa_\infty^{-1}V_\alpha^{(0)}+V_\alpha^{(1)}+\kappa_\infty V_\alpha^{(2)}+\cdots, \qquad (C5)$$

where because of (C3)

$$-\nabla^2 V_{in}^{(0)}=0,\quad -\nabla^2 V_{in}^{(1)}=\delta^3(\vec{r}-\vec{r}_0),$$

$$-\nabla^2 V_{in}^{(l)}=0,\ \text{for } l\geq 2 \qquad (C6)$$

and

$$(\nabla^2-\mu^2)V_{out}^{(m)}=0,\ \text{for all } m.$$

The boundary condition (C4) becomes

$$V_{in}^{(l)}=V_{out}^{(l)},\quad \text{for all } l$$

$$\hat{n}\cdot\vec{\nabla}V_{in}^{(0)}=0, \qquad (C7)$$

$$\hat{n}\cdot\vec{\nabla}V_{in}^{(m)}=\hat{n}\cdot\vec{\nabla}V_{out}^{(m-1)},\ \text{for } m\geq 1$$

at the surface S.

Since $V_{in}^{(0)}$ satisfies exactly the same equations here as in Appendix B 2, by following identical arguments as those given after (B15), one establishes, as in (B17),

$$V_{in}^{(0)}(\vec{r})=\text{constant}, \qquad (C8)$$

and, as in (B18),

$G_0(\vec{r},\vec{r}\,')$ is independent of $\vec{r}$ (or $\vec{r}\,'$) whenever

$\vec{r}$ (or $\vec{r}\,'$) is inside S. (C9)

Hence, (59) follows.

If $\psi=\vec{V}^a=0$ outside S, then $j_0^a=J_0^a=I^a=0$ also outside S. Hence, (56) reduces to (B20), i.e.,

$$V_0^a(\vec{r})=\kappa_\infty^{-1}gG_0(\vec{r},\vec{R})Q^a$$

$$+g\int G_1(\vec{r},\vec{r}\,')I^a(r')d^3r'+O(\kappa_\infty), \qquad (C10)$$

where, as before, $\vec{R}$ is any position vector on S. According to (C9), $G_0(\vec{r},\vec{r}\,')=G_0(\vec{r},\vec{R})$ whenever $\vec{r}\,'$ is inside S. For a color singlet, $Q^a=0$; hence (C10) becomes

$$V_0^a(\vec{r})=g\int G_1(\vec{r},\vec{r}\,')I^a(\vec{r}\,')d^3r'+O(\kappa_\infty), \qquad (C11)$$

which is finite when $\kappa_\infty\to 0$.

To derive (60), let us assume S to be a spherical surface of radius R. By using (C6)–(C8), one sees that $V_{out}^{(0)}$ is given by

$$V_{out}^{(0)}=qe^{-\mu r}/r, \qquad (C12)$$

where q is a constant. The function $V_{in}^{(1)}$ satisfies

$$-\nabla^2V_{in}^{(1)}=\delta^3(\vec{r}-\vec{r}_0)$$

and (C13)

$$-\hat{n}\cdot\vec{\nabla}V_{in}^{(1)}\equiv\sigma=\text{constant on } S.$$

Because of the Gauss theorem, $\sigma=(4\pi R^2)^{-1}$ when S is a spherical surface of radius R. Since $\hat{n}\cdot\vec{\nabla}V_{in}^{(1)}=\hat{n}\cdot\vec{\nabla}V_{out}^{(0)}$ on S, we determine the constant in (C12): $q=(4\pi)^{-1}(1+\mu R)^{-1}\exp(\mu R)$. For $r<R$,

$$G_0(\vec{r},\vec{r}_0)=V_{in}^{(0)}(\vec{r})=(4\pi R)^{-1}(1+\mu R)^{-1} \qquad (C14)$$

and for $r>R$

$$G_0(\vec{r},\vec{r}_0)=V_{out}^{(0)}(\vec{r})=(4\pi r)^{-1}(1+\mu R)^{-1}e^{-\mu(r-R)}. \qquad (C15)$$

By setting $\mu=0$, we recover the previous form of $G_0(\vec{r},\vec{r}_0)$ discussed in Appendix B.

In the present case of a spherical surface, (C13) is identical to (B19); the solution $V_{in}^{(1)}$ must also be the same, apart from an additive constant which is not determined by (C13). Thus, (60) is established.

For an arbitrary surface and assuming $\mu\gg$ hadron mass, one has

$$\hat{n}\cdot\vec{\nabla}V_{out}^{(0)}\cong-\mu V_{out}^{(0)}\quad \text{on } S. \qquad (C16)$$

In this approximation, since $\hat{n}\cdot\vec{\nabla}V_{out}^{(0)}=\hat{n}\cdot\vec{\nabla}V_{in}^{(1)}$ and $V_{out}^{(0)}=V_{in}^{(0)}$ = constant on S, the normal electric field defined by (C13) is a constant for any surface. Hence for $\vec{r}$ and $\vec{r}_0$ both inside S, $G_1(\vec{r},\vec{r}_0)=V_{in}^{(1)}(\vec{r})$ is the electrostatic potential generated by

a positive unit charge at $\vec{r}=\vec{r}_0$, with the normal component of its electric field being a constant on the surface. (C17)

We note that for nonspherical surfaces, even inside $\mathcal{S}$ the Green's function $G_1(\vec{r}, \vec{r}_0)$ determined by (C17) can be quite different from that derived in Appendix B; however, their difference must be of the form $f(\vec{r}) + f(\vec{r}_0)$ due to the reciprocity relation. [Compare (C17) with the derivation following (B19) in Appendix B.] As mentioned earlier, outside $\mathcal{S}$ the $G_1(\vec{r}, \vec{r}_0)$ function of Appendix B has a long tail, which leads to a Van der Waals-type force between hadrons, in violation of experimental results.

2. Equation (62). Inside $\mathcal{S}$, since $\sigma = 0$ and $\kappa = 1$, the Lagrangian density is locally gauge invariant. In the Coulomb gauge, according to (31) and (32), $\vec{\nabla}\cdot\vec{V}^a = 0$ inside $\mathcal{S}$ and $\hat{n}\cdot\vec{V}^a = 0$ on $\mathcal{S}$.

To derive (62), we first show that it is always possible to choose a solution ϕ^a of the last equation of (61),

$$\nabla^2\phi^a = -g\vec{\nabla}\cdot(C^{abc}\vec{V}^b V_0^c), \tag{C18}$$

such that

$$\hat{n}\cdot\vec{\nabla}\phi^a = 0 \quad \text{on } \mathcal{S}. \tag{C19}$$

To see this, let us assume that $\phi^a = \phi_0^a$ is a solution of (C18) inside $\mathcal{S}$, i.e.,

$$\nabla^2\phi_0^a = -g\vec{\nabla}\cdot(C^{abc}\vec{V}^b V_0^c) \tag{C20}$$

but $\hat{n}\cdot\vec{\nabla}\phi_0^a \neq 0$ on $\mathcal{S}$. For simplicity, let $\mathcal{S}$ be a spherical surface of radius R and let (r, α, β) be the spherical coordinates of $\vec{r}$. Expand

$$\hat{n}\cdot\vec{\nabla}\phi_0^a = \sum_{l,m} A_{lm}^a Y_{lm}(\alpha,\beta) \quad \text{at } r = R, \tag{C21}$$

where the Y_{lm}'s are the spherical harmonics. Because of (C20) and (32),

$$A_{00}^a \propto \int_\Omega \nabla^2\phi_0^a d^3r = 0, \tag{C22}$$

where Ω denotes the volume inside $\mathcal{S}$. Now choose ϕ_1^a to be a solution of $\nabla^2\phi_1^a = 0$ inside $\mathcal{S}$. Hence

$$\phi_1^a = \sum_{l,m} B_{lm}^a r^l Y_{lm}(\alpha,\beta). \tag{C23}$$

We shall set

$$B_{lm}^a = -A_{lm}^a / lR^{l-1} \text{ for all } l \neq 0; \tag{C24}$$

therefore, for arbitrary B_{00}^a, $\hat{n}\cdot\vec{\nabla}(\phi_0^a + \phi_1^a) = 0$ on $\mathcal{S}$. Thus, $\phi^a \equiv \phi_0^a + \phi_1^a$ satisfies (C18) and (C19). Equation (62) now follows.

3. Reduced Hamiltonian (65). In this section, we assume

$$\psi = \vec{V}^a = 0 \quad \text{outside } \mathcal{S}, \tag{C25}$$

in accordance with (64). By using (10) and (51), we find the total Lagrangian $L \equiv \int \mathcal{L}' d^3r$ can be decomposed into three parts:

$$L = \int_\Omega \mathcal{L}_{\text{in}} d^3r - sA + \int_{\text{out}} \mathcal{L}_{\text{out}} d^3r, \tag{C26}$$

in which we omit the counterterms in (10), A is the area of $\mathcal{S}$, s is the surface tension defined in (20) [due to the integration of $\frac{1}{2}(\nabla\sigma)^2 + U(\sigma)$ over the surface], Ω refers to the volume inside $\mathcal{S}$ and "out" the volume outside,

$$\mathcal{L}_{\text{in}} = -\tfrac{1}{4}V_{\mu\nu}^a V_{\mu\nu}^a - \psi^\dagger\gamma_4(\gamma_\mu D_\mu + m)\psi - p, \tag{C27}$$

where p is given by (12), and

$$\mathcal{L}_{\text{out}} = \tfrac{1}{2}\kappa_\infty[(\vec{\nabla}V_0^a)^2 + \mu^2(V_0^a)^2]. \tag{C28}$$

In the Coulomb gauge, the conjugate momentum of $\vec{V}^a$ is

$$\vec{\Pi}^a = -\vec{E}_{\text{tr}}^a, \tag{C29}$$

where $\vec{E}_{\text{tr}}^a$ is given by (61) and it satisfies (62). The Hamiltonian is given by

$$H = \int_\Omega (\vec{\Pi}^a\cdot\dot{\vec{V}}^a + i\psi^\dagger\dot{\psi})d^3r - L. \tag{C30}$$

Through partial integrations, we find

$$\int_\Omega \vec{E}_{\text{long}}^a\cdot\vec{E}_{\text{long}}^a d^3r = \int V_0^a\hat{n}\cdot\vec{\nabla}V_0^a d^2S + g\int_\Omega V_0^a(j_0^a + C^{abc}\vec{V}^b\cdot\vec{\Pi}^c)d^3r \tag{C31}$$

and

$$\int_\Omega(\vec{\Pi}^a\cdot\dot{\vec{V}}^a + i\psi^\dagger\dot{\psi} - \mathcal{L}_{\text{in}})d^3r = \int_\Omega \mathcal{H}\, d^3r - \tfrac{1}{2}\int (V_0^a\hat{n}\cdot\vec{\nabla}V_0^a)_{\text{in}} d^2S, \tag{C32}$$

where $\mathcal{H}$ is given by (66), d^2S is the surface element of $\mathcal{S}$, and $(\;)_{\text{in}}$ refers to the value of the inner side of $\mathcal{S}$. Now, outside $\mathcal{S}$, because of (C25), Eq. (55) becomes $(\nabla^2 - \mu^2)V_0^a = 0$. Hence,

$$\int_{\text{out}} \mathcal{L}_{\text{out}} d^3r = -\tfrac{1}{2}\kappa_\infty \int (V_0^a\hat{n}\cdot\vec{\nabla}V_0^a)_{\text{out}} d^2S. \tag{C33}$$

Furthermore, at the surface $(V_0^a)_{\text{out}} = (V_0^a)_{\text{in}}$ and $\kappa_\infty(\hat{n}\cdot\vec{\nabla}V_0^a)_{\text{out}} = (\hat{n}\cdot\vec{\nabla}V_0^a)_{\text{in}}$. Hence, H is the same as H_0 given by (65).

4. "Radiation field" inside the hadron. The Lagrangian density $\mathcal{L}'$ given by (51) is not locally gauge invariant. As mentioned before, inside the hadron, $\sigma=0$ and therefore $\mathcal{L}'=\mathcal{L}$ given by (10), which is locally gauge invariant. Outside, $\sigma=\sigma_{\text{vac}} \neq 0$. Hence, local gauge invariance is broken. So long as we restrict ourselves to the inside region, we can perform local gauge transformations. However, in order to make a connection with the outside solution, it is sometimes necessary to transform the inside solution to a specific gauge; only then can one satisfy the continuity condition at the surface.

As an example, let us consider the free radiation fields $\vec{V}^E_{k,l,m}$ and $\vec{V}^B_{k,l,m}$ of the TE and TB modes discussed in Sec. IV B. For these free field solutions, we may set $g=0$. Equation (54) becomes

$$\frac{\partial}{\partial x_\alpha}V_{\alpha\beta}=0 \quad \text{inside } S$$

and (C34)

$$\frac{\partial}{\partial x_\alpha}V_{\alpha\beta}-\mu^2V_\beta=0 \quad \text{outside } S,$$

where for simplicity the color index is omitted and since $g=0$, $V_{\alpha\beta}=(\partial V_\beta/\partial x_\alpha)-(\partial V_\alpha/\partial x_\beta)$. Assume S to be a spherical surface of radius R. For $r<R$, the solution for the TE, or TB, mode may be written as ($\lambda=E$ or B)

$$\vec{V}^\lambda(x)=\vec{V}^\lambda_{k,l,m}(\vec{r})e^{\pm ikt}$$

and (C35)

$$V_0^\lambda(x)=0,$$

where $\vec{V}^\lambda_{k,l,m}(\vec{r})$ is given by (79) or (80). In spherical coordinates $\vec{r}=(r,\theta,\phi)$, since $\vec{\nabla}\cdot\vec{V}^\lambda=0$ we have

$$\frac{\partial V_r^\lambda}{\partial r}+\frac{1}{r}\frac{\partial V_\theta^\lambda}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial V_\phi^\lambda}{\partial\phi}=0 \tag{C36}$$

and at $r=R-$, because of (77),

$$V_r^\lambda=0. \tag{C37}$$

In the outside region, the lower equation in (C34) gives

$$\frac{\partial V_\beta}{\partial x_\beta}=0; \tag{C38}$$

for $\mu \gg R^{-1}$, its solution near the surface ($r=R+$) must be of the form

$$V_\alpha(r,\theta,\phi,t)=(R/r)V_\alpha(R,\theta,\phi,t)e^{-M(r-R)}, \tag{C39}$$

where

$$M=\mu\{1+O[(\mu R)^{-2}]\}.$$

In order to connect the outside solution with the inside solution, we set the outside solution V_α at $r=R+$,

$$V_\theta=V_\theta^\lambda, \quad V_\phi=V_\phi^\lambda,$$
$$V_0=V_0^\lambda=0,$$

and (C40)

$$V_r=-\frac{1}{\mu}\left(\frac{\partial V_r^\lambda}{\partial r}\right)_R,$$

where V_α^λ is the inside solution given by (C35), and $(\partial V_r^\lambda/\partial r)_R$ refers to its value at $r=R$. The last term in (C40) is needed so that at $r=R+$, neglecting $O(1/\mu R)$, $\partial V_\alpha/\partial x_\alpha=0$. To the zeroth order in $(\mu R)^{-1}$, (C40) and (C35) give the same field at $r=R$. Hence, to that order the continuity condition at the surface is satisfied.

It is of interest to examine the first-order correction. To the first order in $(\mu R)^{-1}$, we must modify the inside solution (C35) by a local gauge transformation (which is allowed since the inside Lagrangian density is locally gauge invariant):

$$\vec{V}=\vec{V}^\lambda+\mu^{-1}\vec{\nabla}\chi$$

and

$$V_0=-\mu^{-1}\dot{\chi}=\mp\mu^{-1}(ik)\chi, \tag{C41}$$

where χ is analytic inside the sphere ($r\leq R$) and it should satisfy the following boundary conditions at the surface:

$$\chi(R,\theta,\phi,t)=0$$

and

$$\left(\frac{\partial\chi}{\partial r}\right)_R=-\left(\frac{\partial V_r^\lambda}{\partial r}\right)_R. \tag{C42}$$

Consequently, (C40) and (C41) satisfy the continuity condition at $r=R$. [That such an χ exists can be easily seen by assuming, e.g., $\chi\propto r^l(r^2-R^2)\times Y_{lm}(\theta,\phi)$ for the appropriate TE or TB solution.]

We note further that as $\kappa_\infty\to 0$ the integral $\kappa_\infty(f'\sigma)^2\vec{V}^a\cdot\vec{V}^a$ over the outside region becomes zero. Hence, the constraint $\vec{V}^a=0$ outside S is derived by using the Lagrangian (51) provided that we take the limit $\kappa_\infty\to 0$ first and then $f'\sigma_{\text{vac}}\equiv\mu\to\infty$.

APPENDIX D

In this appendix, we give the derivation of (90), which leads to the experimental upper limit of κ_∞ discussed in Sec. V D. From (69), the mass of a free quark is given approximately by

$$m_q\cong\frac{2g^2G_0}{3\kappa_\infty}+\frac{4\pi}{3}pR^3, \tag{D1}$$

where p is defined in (12). Here, for simplicity, we neglect the surface tension s. According to (C14)

$$G_0 = (4\pi R)^{-1}(1+\mu R)^{-1}. \quad \text{(D2)}$$

We discuss the two limiting cases: $\mu \ll (4\pi p/m_q)^{1/3}$ and $\mu \gg (4\pi p/m_q)^{1/3}$.

1. $\mu \ll (4\pi p/m_q)^{1/3}$. In this case, $G_0 \cong (4\pi R)^{-1}$. By setting $dm_q/dR = 0$, we find

$$m_q = \frac{4}{3}\left(\frac{2\alpha}{3\kappa_\infty}\right)^{3/4}(4\pi p)^{1/4}, \quad \text{(D3)}$$

where $\alpha = (4\pi)^{-1}g^2$. Likewise, the proton mass is

$$m_p = \tfrac{4}{3}(3\rho_0)^{3/4}(4\pi p)^{1/4}, \quad \text{(D4)}$$

where ρ_0 is given by (22). Hence,

$$\kappa_\infty = \frac{2\alpha}{9\rho_0}\left(\frac{m_p}{m_q}\right)^{4/3}. \quad \text{(D5)}$$

Taking $\rho_0 = 2.0428$, we obtain

$$\kappa_\infty = (0.109)\alpha(m_p/m_q)^{4/3}. \quad \text{(D6)}$$

2. $\mu \gg (4\pi p/m_q)^{1/3}$. In this case, $G_0 \cong (4\pi\mu R^2)^{-1}$ and that gives

$$m_q = \frac{5}{6}\left(\frac{4\alpha}{3\kappa_\infty \mu}\right)^{3/5}(4\pi p)^{2/5}. \quad \text{(D7)}$$

By setting $\rho_0 = 2.0428$, we find, instead of (D6),

$$\kappa_\infty = (0.012)\alpha(m_p/m_q)^{5/3}(m_p/\mu). \quad \text{(D8)}$$

Since (D8) always gives a smaller value than (D6), we establish (90).

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