Mass perturbation around the exact solution of a two-dimensional field-theoretical model

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The exact solution of a massless spinor field interacting in two dimensions via a derivative coupling with a massive pseudoscalar field is perturbed by a fermion mass operator. We resum the resulting infrared-singular perturbative series and show that for dimension $\dim(\bar{\psi}\psi) < 2$ the Green's functions can be made finite by the addition of one counterterm. Equations of motion and Ward identities are derived.

I. INTRODUCTION

In a previous paper[1] we discussed the two-dimensional model described by the Lagrangian density

$$\mathcal{L} = \frac{i}{2}\bar{\Psi}\overleftrightarrow{\partial\!\!\!/}\Psi - M\bar{\Psi}\Psi + \tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \tfrac{1}{2}m^2\varphi^2 - g\bar{\Psi}\gamma^\mu\gamma^5\Psi\partial_\mu\varphi \tag{1.1}$$

from the point of view of perturbation theory in the coupling constant g. In particular we have shown that Green's functions can be subtracted according to a modified Bogolubov-Parasiuk-Hepp-Zimmermann (BPHZ) subtraction scheme,[2] such that no new interactions are induced. These Green's functions reduce to the exactly soluble model of Rothe and Stamatescu,[3] when the fermion mass M goes to zero.

In this paper we want to investigate a different approach to the perturbative treatment of the model (1.1): Take $M=0$, and then treat the mass term $M\bar{\Psi}\Psi$ as a perturbation. In this way certain features, such as softly broken scale invariance, are taken into account from the very beginning. Perturbation theory in g never reveals scale invariance, unless one sums up logarithms.

The equivalence between the massive Thirring and the sine-Gordon model discovered by Coleman[4] is another feature illustrating the relevance of mass perturbation theory: In the sine-Gordon language it corresponds to perturbing about the free massless theory.

In lowest order of perturbation a convenient definition for the normal product associated with $[\bar{\Psi}\Psi](x)$ is, as we will see,

$$N[\bar{\Psi}\Psi](x) = :\exp[-2ig\gamma^5_{\alpha\beta}\varphi_0(x)]::\bar{\Psi}_{f\alpha}\Psi_{f\beta}:(x)\,, \tag{1.2}$$

where φ_0 is a free pseudoscalar field of mass m, and Ψ_f is a free zero-mass fermion field. Since the unpertrubed fermion field is massless, mass perturbation is highly singular in the infrared region. To circumvent this difficulty we partially resum the perturbative series, transfering a term proportional to $:\bar{\Psi}_f\Psi_f:$ to the unperturbed Lagrangian.

Our main results concern the ultraviolet behavior of the resulting Green's functions in the Euclidean region: The theory is renormalizable for $g^2 < \pi$, i.e., the Green's functions are made finite, in a sense to be made precise later, just by the addition of the counterterm $(:\cos 2\varphi:-1)$. If such a term is absorbed into the definition of the normal product $N[\bar{\Psi}\Psi](x)$ then the following results will emerge:

(a) For $0 \leq g^2 < \pi$ the Green's functions are finite.

(b) For $g^2 \geq \pi$ the theory is apparently meaningless. For $g^2 > \pi$ the theory is nonrenormalizable in the sense that there will be divergent graphs of arbitrarily high order. At $g^2 = \pi$ the propagator associated with the exponentiated field in (1.2) develops a nonintegrable singularity at short distances. It is not clear what modifications, if any, should be done to overcome this problem.

The above results correspond to what one would expect since the mass operator has dimension $\dim(\bar{\Psi}\Psi) = 1 + g^2/\pi$ and therefore the interaction becomes nonrenormalizable for $g^2 \geq \pi$ [i.e., $\dim(\bar{\Psi}\Psi) \geq 2$]. Extension of these results to the Thirring model are currently under investigation and we hope to obtain some results in the interesting region $2 > \dim(\bar{\Psi}\Psi) \geq 1$.

The equation of motion for the fermion field can be easily derived. For $g^2 < \pi$ is has the classical form, if all orders of perturbation are considered. We also derive Ward identities showing that in every order of perturbation γ^5 invariance is broken, as expected.

The paper is organized as follows: In Sec. II we collect some basic results of the $M=0$ theory. Section III contains a discussion of the ultraviolet behavior of the perturbed model in the Euclidean region. Finally, in Sec. IV equations of motion and Ward identities are derived.

II. THE UNPERTURBED MODEL

Consider a two-dimensional model of a massless spinor field interacting via a derivative coupling with a massive pseudoscalar field, so that the formal field equations are

$$i\partial\!\!\!/\Psi(x)=g[\gamma^{\mu}\gamma^{5}\partial_{\mu}\varphi_0\Psi](x)\,, \tag{2.1}$$

$$(\partial^2+m^2)\varphi_0(x)=g\partial^{\mu}[\overline{\Psi}\gamma_{\mu}\gamma_5\Psi](x)\,. \tag{2.2}$$

As discussed by Rothe and Stamatescu[3] there are various possibilities for defining the normal product associated with $[\overline{\Psi}\gamma^{\mu}\gamma^5\Psi](x)$. However, this ambiguity is in our case irrelevant since it manifests itself only by different mass and wave-function renormalizations of the φ_0 field. Thus we choose a definition producing a divergenceless axial current. From (2.2) it results then that φ_0 is a free field of mass m. Furthermore, it is easily seen that

$$\Psi=:e^{-ig\gamma^5\varphi_0}:\Psi_f\,, \tag{2.3}$$

where Ψ_f is a free massless spinor field, solves Eq. (2.1).

Using (2.3), normal products associated with formal products of the fields Ψ, φ_0, and their derivatives can be constructed. In particular, we define the current as

$$\begin{aligned}N[\overline{\Psi}\gamma^{\mu}\Psi](x)&=:\overline{\Psi}_f(x)e^{-ig\gamma^5\varphi_0(x)}\gamma^{\mu}e^{-ig\gamma^5\varphi_0(x)}\Psi_f(x):\\&=:\overline{\Psi}_f(x)\gamma^{\mu}\Psi_f(x):.\end{aligned} \tag{2.4}$$

The above definition implies immediately that $\partial_{\mu}N[\overline{\Psi}\gamma^{\mu}\gamma^5\Psi]=0$ as promised.

In the next sections the effect of a mass perturbation in this model will be examined. Since

$$\overline{\Psi}(x+\epsilon)\Psi(x)=e^{-g^2\Delta^-(\epsilon)}:e^{-ig\gamma^5[\varphi_0(x+\epsilon)+\varphi_0(x)]}:\times[:\overline{\Psi}_f(x+\epsilon)\Psi_f(x):+\langle 0|\overline{\Psi}_f(x+\epsilon)\Psi_f(x)|0\rangle]\,,$$

with $\Delta^-(x)=\langle 0|\varphi_0(x)\varphi_0(0)|0\rangle$, we see that a convenient definition for the mass operator is

$$\begin{aligned}N[\overline{\Psi}\Psi](x)&=\lim_{\epsilon\to 0}e^{+g^2\Delta^-(\epsilon)}\overline{\Psi}(x+\epsilon)\Psi(x)\\&=:e^{-2ig\gamma^5\varphi_0(x)}::\overline{\Psi}_f\Psi_f:(x)\,.\end{aligned} \tag{2.5}$$

III. MASS PERTURBATION

Perturbing the model of Sec. II by a fermion mass term will produce highly infrared-singular Green's functions. In order to prevent their occurrence, we rewrite the interaction $MN[\overline{\Psi}\Psi]$ as

$$\begin{aligned}MN[\overline{\Psi}\Psi]&=M(:e^{-2ig\gamma^5\varphi_0}:-1):\overline{\Psi}_f\Psi_f:\\&\quad+M:\overline{\Psi}_f\Psi_f:\end{aligned} \tag{3.1}$$

and consider the last term as being part of the unperturbed Lagrangian. The result of this step is to give a mass to the unperturbed fermion field which now becomes $:e^{-ig\gamma^5\varphi_0}:\Psi_0$ with Ψ_0 a free Dirac field of mass M.

The perturbed Green's functions are formally defined by the Gell-Mann–Low (GML) formula

$$\begin{aligned}G^{(2N,L)}(x_1,\ldots,x_N;y_1,\ldots,y_N;z_1,\ldots,z_L)&\equiv\Big\langle T\prod_{i,j}^{N}\phi(x_i)\overline{\phi}(y_i)\prod_{k=1}^{L}\varphi(z_k)\Big\rangle\\&=\frac{{}^{(0)}\Big\langle T\prod_j^N:\exp[-ig\gamma^5_{x_i}\varphi_0(x_i)]:\Psi_0(x_i)\prod_j^N\overline{\Psi}_0(y_j):\exp[-ig\gamma^5_{y_j}\varphi_0(y_j)]:\prod_k^L\varphi_0(z_k)\exp[i\int d^2x\mathcal{L}_{\rm int}(\varphi_0,\Psi_0)]\Big\rangle^{(0)}}{{}^{(0)}\Big\langle T\exp[i\int d^2x\mathcal{L}_{\rm int}(\varphi_0,\Psi_0)]\Big\rangle^{(0)}}\end{aligned} \tag{3.2}$$

where $\mathcal{L}_{\rm int}(\varphi_0,\Psi_0)=-M(:e^{-2ig\gamma^5\varphi_0}:-1):\overline{\Psi}_0\Psi_0:$ is the effective Lagrangian density (i.e., the Lagrangian density after the resummation process indicated above). In what follows it is convenient to use

$$:e^{-i\lambda\gamma^5\varphi_0}:=\tfrac{1}{2}:e^{-i\lambda\varphi_0}+e^{i\lambda\varphi_0}:+\tfrac{1}{2}\gamma^5:e^{-i\lambda\varphi_0}-e^{i\lambda\varphi_0}: \tag{3.3}$$

and Wick's theorem for exponentials of free fields,

$${}^{(0)}\langle T:e^{i\lambda_1\varphi_0(x_1)}:\cdots:e^{i\lambda_m(x_m)}:\rangle^{(0)}=\exp\Big[-\sum_{k<l}\lambda_k\lambda_l\Delta(x_k-x_l)\Big]\,, \tag{3.4}$$

where $\Delta(x)={}^{(0)}\langle T\varphi_0(x)\varphi_0(0)\rangle^{(0)}$.

Equations (3.3) and (3.4) can be used to furnish a Feynman graph representation for the amplitudes contributing to a Green's function. A Feynman graph G consists of the following:

(a) External vertices: These are the vertices associated with the points $x_1,\ldots,x_N$; $y_1,\ldots,y_N$; $z_1,\ldots,z_L$ of the Green's function under consideration. The external vertices associated with fermion fields are of two types corresponding to the plus or minus signs in the exponential (3.3), with $\lambda=g$.

(b) Internal vertices: These are the vertices associated with the interaction Lagrangian. They are of three types corresponding to

$:\bar{\Psi}_0\Psi_0:$ or to $:e^{\pm 2ig\varphi_0}::\bar{\Psi}_0\Psi_0:$.

(c) A set of lines connecting different vertices. These lines are of the following types:

(1) fermion or scalar lines;

(2) lines associated with the contractions of the exponentiated fields. These will be called exponential lines. A given exponential line joining vertices x and y can be of one of the six types

$$e^{\pm g^2\Delta(x-y)},\quad e^{\pm 2g^2\Delta(x-y)},\quad \text{and}\quad e^{\pm 4g^2\Delta(x-y)}.$$

The Feynman amplitudes associated with Eq. (3.2) are not *a priori* well-defined distributions. If we restrict ourselves to the Euclidean region, we are able to apply Weinberg's power-counting theorem[5] to determine their possible singularities. The Feynman rules in the Euclidean region are essentially the same as above, but for the substitutions

$$\alpha^2 x \to -i\alpha^2 x ,$$

$$\Delta(x,m^2)\to\Delta(x,m^2)=\int\frac{\alpha^2 k}{(2\pi)^2}\frac{e^{-ik\cdot x}}{k^2+m^2}=\frac{1}{2\pi}K_0(m\rho),$$

$$S(x,M)\to S(x,M)=\int\frac{\alpha^2 k}{(2\pi)^2}\frac{\not k+M}{k^2+M^2}e^{-ik\cdot x},$$

with $\Delta(x,m^2)$ and $S(x,M)$ the pseudoscalar and fermion propagators, respectively, $k=(k_1{}^2+k_2{}^2)^{1/2}$, $\rho=(x_1{}^2+x_2{}^2)^{1/2}$, and $\{\gamma_i,\gamma_j\}=-2\delta_{ij}$.

The Fourier transform $F_\pm^{(n)}(k)$ of $e^{\pm ng^2\Delta(x)}$ has the form

$$F_\pm^{(n)}(k)=\delta(k)+G_\pm^{(n)}(k)\,, \tag{3.5}$$

where asymptotically $G_\pm^{(n)}(k)\underset{k\to\infty}{\simeq}[(k^2)^{1\mp ng^2/4\pi}]^{-1}$, a result that will be extensively used.

In what follows we suppose that our Green's functions are in Euclidean form and our results are valid only in this region. The extension to the Minkowski region will require an analytic continuation.

We begin considering a special class of graphs called generalized. A subgraph $\gamma\subseteq G$ is called a generalized graph, if every line of G linking any pair of vertices of γ belongs to γ. Let γ be such a generalized graph and let r_1, s_1, r_2, and s_2 be the number of vertices of γ associated with the exponentiated fields $:e^{+ig\varphi_0}:$, $:e^{-ig\varphi_0}:$, $:e^{+2ig\varphi_0}:$, $:e^{-2ig\varphi_0}:$, respectively. Furthermore let n_f and n_b be the number of fermion and exponential lines in γ. Then, the ultraviolet degree of superficial divergence of γ is given by

$$\delta(\gamma)=2m-n_f-2n_b-\alpha(A^2-V_2-\tfrac{1}{4}V_1)\,, \tag{3.6}$$

where $\alpha=g^2/\pi$, $A=r_2-s_2+(r_1-s_1)/2$, $V_i=r_i+s_i$, and m is the number of loops of γ:

$$m=n_b+n_f-V+1\,,$$

$$V=\text{number of vertices of }\gamma. \tag{3.7}$$

Using (3.7), $V=V_0+V_1+V_2$, where $V_0=$ number of vertices of γ without exponentiated fields, and

$$2n_f+N_f=2V-V_1\,, \tag{3.8}$$

$N_f=$ number of external fermion lines of γ, formula (3.6) can be rewritten as

$$\delta(\gamma)=2-\deg V(\gamma)-\sum_{a=0}^{2}(2-\delta_a)V_a\,, \tag{3.9}$$

where $\deg V(\gamma)=N_f/2+\alpha A^2$ is the (ultraviolet) degree of the vertex obtained by reducing γ to a point and where δ_a denotes the degree of the vertex of type a:

$\delta_0=1$ for vertices without exponentiated fields,

$\delta_1=\frac{1}{2}+\alpha/4$ for vertices with either exponentiated field $e^{\pm ig\varphi_0}$,

$\delta_2=1+\alpha$ for vertices with either exponentiated field $e^{\pm 2ig\varphi_0}$.

With the aid of formula (3.9) for $\delta(\gamma)$ we now analyze the possible divergences of generalized Feynman graphs. Let $\alpha<1$. Then from (3.9) it follows that graphs with $N_f\geq 4$ are superficially convergent. Therefore we have to consider only the cases where $N_f=3$, 2, 1, and 0:

(a) $N_f=3$: Since N_f is odd there is at least one vertex of type 1, therefore

$$\delta(\gamma)\leq -1+\alpha/4-\alpha A^2-\sum_{a\neq 1}(2-\delta_a)V_a<0\,.$$

(b) $N_f=2$: If there is at least one vertex of type 0 (or one of type 1),

$$\delta(\gamma)<-\alpha A^2\leq 0\,,$$

whereas if there are only vertices of type 2,

$$\delta(\gamma)\leq -1+2\alpha-\alpha A^2\,.$$

Thus if $\alpha<1$ divergences can occur only if $A^2=0$, implying that V_2 is even [note, however, that $\delta(\gamma)<0$, if $\alpha<\frac{1}{2}$]. Since, in this case, γ contains an odd number of fermion lines, the graph is actually finite if a convenient regularization is employed. See Fig. 1 for the simplest example.

(c) $N_f=1$: Again as N_f is odd there is at least one vertex of type 1. Thus

$$\delta(\gamma)\leq\alpha/4-\alpha A^2-\sum_{a\neq 1}(2-\delta_a)V_a<0\,,$$

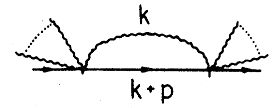

FIG. 1. Lowest-order graph with two external fermion lines. Solid and wavy lines represent fermion and exponential lines, respectively. For $p=0$ the term of the fermion propagator containing $\not k$ integrates to zero.

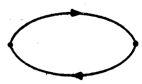

FIG. 2. The simplest vacuum diagram.

since in this case $A^2 \geq \frac{1}{4}$.

(d) $N_f = 0$: In this case $\delta(\gamma) = 2 - \alpha A^2 - \sum_a (2 - \delta_a) V_a$. We have either

(i) There are at least two vertices of type 1. We obtain

$$\delta(\gamma) \leq -1 + \alpha/2 < 0 .$$

(ii) There is no vertex of type 1. There are various subcases to be considered:

(1) There are two vertices of type 0 and other vertices. This corresponds to the logarithmically divergent vacuum graph shown in Fig. 2. It is clear that, due to the denominator in the GML formula, the contributions of this kind of graph are canceled.

(2) There are one vertex of type 0 and one vertex of type 2. We find a divergence associated with the graph shown in Fig. 3. Its divergence is α independent and considering the contributions of the denominator in the GML formula one sees that a counterterm proportional to

$$(:\cos 2g\varphi_0: - 1) \tag{3.10}$$

is necessary to remove it.

(3) There is no vertex of type 0 (i.e., all vertices are of type 2). We have

(a) V_2 is odd (i.e., $V_2 = 3, 5, \ldots$). Then $\delta(\gamma) \leq -1 + 2\alpha$ since $A^2 \geq 1$. In principle there could appear divergences. However, in this case the fermion lines form closed loops, and as V_2 is odd, the operation of taking the trace lowers $\delta(\gamma)$ by 1.

(b) V_2 is even (i.e., $V_2 = 2, 4, \ldots$). We have

$$\delta(\gamma) \leq -\alpha A^2 + 2\alpha$$

and divergences can occur only if $A^2 = 0$. Two such divergent graphs are shown in Fig. 4. We can proceed with the discussion as follows:

(i) If there is no external scalar line in γ (i.e., lines not of the exponential type, but associated with external scalar legs) then, as N_f and A^2 are both equal to zero, the reduced vertex $V(\gamma)$ has no lines. For the graphs shown in Fig. 4 the divergences are partially removed by combining these graphs with corresponding (disconnected) diagrams coming from the denominator in the GML formula. In Fig. 5 we show a graph, which becomes disconnected when the upper bubble is contracted to a point. For $\alpha < \frac{1}{2}$ this procedure is actually sufficient to remove all divergences of this type. If $\frac{1}{2} \leq \alpha < 1$ a logarithmic divergence remains which due to covariance however can be eliminated, by a convenient regularization (e.g., Pauli-Villars on the fermion lines of the graphs of this type).

(ii) If there are external scalar lines in γ, the most divergent part of the graphs obtained by the permutation of these lines cancel among themselves. For example, the most divergent part of the graphs in Fig. 6 cancels. For $\frac{1}{2} \leq \alpha < 1$, again as in the previous case, the remaining logarithmic divergence can be eliminated by a convenient regularization.

We now consider the case of graphs that are not generalized ones. Initially observe that for an arbitrary γ, $\delta(\gamma) = a + b\alpha$, with $a < 0$ except for the graphs containing one loop with two fermion lines (i.e., the graphs of Fig. 2, 3, and the first one of Fig. 4). One therefore realizes that nongeneralized graphs can be regularized by lowering conveniently the value of α. If

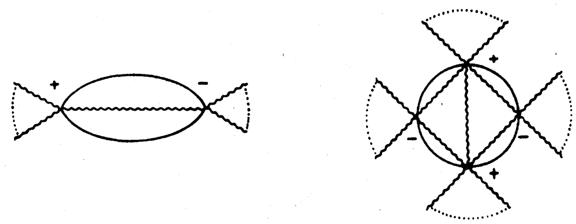

FIG. 4. Divergent diagrams without external fermion lines. The + (or −) sign at the vertices indicates the corresponding sign in the exponentiated field.

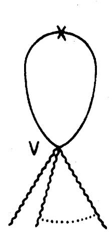

FIG. 3. The only surviving divergence in the regularized Green's function.

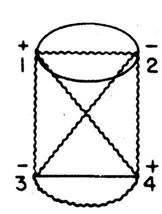

FIG. 5. The lines connecting the vertices 3 to 1 and 2 (and 4 to 1 and 2) cancel, when the bubble is contracted to a point.

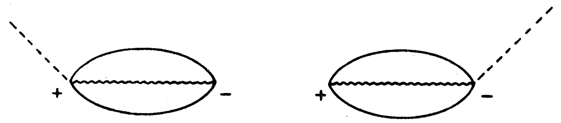

FIG. 6. Graphs with one external scalar (dashed) line. The most divergent part of these graphs cancels.

this is done then as in the usual BPH formalism[6] the divergence will not appear if the regularization is eliminated only after the loop integrations are made.

Note also that, due to the $\delta(k)$ part in (3.5), all remaining volume divergences from disconnected graphs are canceled in the GML formula.

Summing up, we conclude that all divergences of the regularized Green's functions can be eliminated by adding to the interaction Lagrangian a counterterm proportional to $(:\cos 2g\varphi_0:-1)$.

A point to be discussed is the arbitrariness introduced by the counterterm $:\cos 2g\varphi_0:$ [the minus one in (3.10) is actually irrelevant since it cancels in the GML formula]. The reason why this arbitrariness arises can be already seen in zeroth order of the resummed theory, if one considers in more detail the construction of the mass operator. In fact using $\Psi = :e^{-ig\gamma^5\varphi_0}:\Psi_0$ we have

$$\lim_{\epsilon\to 0} e^{g^2\Delta^-(\epsilon)}\overline{\Psi}(x+\epsilon)\Psi(x) = \lim_{\epsilon\to 0}[:e^{-2ig\gamma^5\varphi_0(x)}:\overline{\Psi}_0(x+\epsilon)\Psi_0(x)]$$
$$= \lim_{\epsilon\to 0} :e^{-2ig\gamma^5\varphi_0(x)}:[\Psi_0(x)\Psi_0(x): + \langle\overline{\Psi}_0(x+\epsilon)\Psi_0(x)\rangle]$$
$$= :e^{-2ig\gamma^5\varphi_0(x)}::\overline{\Psi}_0(x)\Psi_0(x): + \lim_{\epsilon\to 0} f(\epsilon^2):\cos 2g\varphi_0(x):,$$

where $f(\epsilon^2)$ is logarithmically divergent as $\epsilon^2 \to 0$.

From this one sees that the Wick ordering prescription is equivalent to the subtraction of the last term. In higher order $f(\epsilon^2)$ receives an additional divergent contribution coming from the graph of Fig. 3. The arbitrariness associated with the finite part of $f(\epsilon^2)$ can be eliminated once and for all if the value of the pseudoscalar physical mass is fixed.

IV. EQUATIONS OF MOTION AND WARD IDENTITIES

We now will derive equations of motion for the fermion and pseudoscalar fields, as well as Ward identities for the vector and axial-vector currents.

From the previous section, for $\alpha < 1$ the only divergent diagram is the one shown in the Fig. 3 which for definiteness we suppose has been eliminated by a convenient choice of the subtraction procedure.

We begin by considering the fermion's equation of motion. The graphs contributing to $\langle T\phi(x_1)X\rangle$ where

$$X = \prod_{i=2}^{N}\phi(x_i)\prod_{j=1}^{N}\overline{\phi}(y_i)\prod_{k=1}^{L}\varphi(z_k)$$

have the structure shown in Fig. 7 where V is the vertex at x_1.

The field $\phi(x)$ can be written alternatively as

$$\phi(x) = :e^{-ig\gamma^5\varphi(x)}:\Psi(x) = \lim_{\epsilon\to 0}:e^{-ig\gamma^5\varphi(x)}:\Psi(x+\epsilon)\,, \tag{4.1}$$

where in zeroth order φ and Ψ are free fields.

Applying $i\partial\!\!\!/$ to the Green's function under consideration and using Eq. (4.1), we get

$$i\partial\!\!\!/_{x_1}\langle T\phi(x_1)X\rangle = \lim_{\epsilon\to 0} i\partial\!\!\!/_{x_1}\langle T:e^{-ig\gamma^5\varphi(x_1)}:\Psi(x_1+\epsilon)X\rangle$$
$$= \lim_{\epsilon\to 0}\langle T[g:\partial\!\!\!/\varphi(x_1)\gamma^5 e^{-ig\gamma^5\varphi(x_1)}:\varphi(x_1+\epsilon) + :e^{+ig\gamma^5\varphi(x_1)}:i\partial\!\!\!/\Psi(x_1+\epsilon)]X\rangle\,.$$

Now

$$\lim_{\epsilon\to 0}\langle T:e^{+ig\gamma^5\varphi(x_1)}:[i\partial\!\!\!/\Psi(x_1+\epsilon)]X\rangle = \lim_{\epsilon\to 0}\langle T:e^{+ig\gamma^5\varphi(x_1)}:[(i\partial\!\!\!/-M)\Psi(x_1+\epsilon)+M\Psi(x_1+\epsilon)]X\rangle\,. \tag{4.2}$$

We denote the vertex at x_1 with the positive sign in the exponent by $\overline{V}$. The right-hand side of expression (4.2) is shown graphically in Fig. 8, where the operator $(i\partial\!\!\!/ - M)$ has been used to amputate the fermion

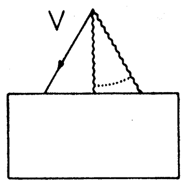

FIG. 7. General structure of a graph contributing to a Green's function. The vertex V corresponds to the field $\phi(x_1)$.

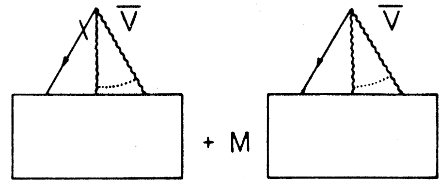

FIG. 8. Graphical representation of the right-hand side of Eq. (4.2).

line, which may link $\overline{V}$ to either an external vertex $\overline{\phi}(y_i)$ or to an interaction vertex $(:e^{-2ig\gamma^5\varphi}::\overline{\Psi}_0\Psi_0:$ or $M:\overline{\Psi}_0\Psi_0:)$.

Now, if the amputated line links $\overline{V}$ to an interaction vertex of the type $M:\overline{\Psi}_0\Psi_0:$ one gets a contribution that cancels the second graph in Fig. 8. Of course this cancelation is among terms of different orders in the perturbation scheme developed in Sec. III and the equation of motion to be derived is expected to be valid in the summed-up theory, i.e., if all orders of perturbation are considered. We therefore obtain

$$(i\partial\!\!\!/_{x_1} - M)\langle T\phi(x_1)X\rangle = g\langle TN[\gamma^\mu\gamma^5\partial_\mu\varphi\phi](x_1)X\rangle - \sum_{j=1}^{N}(-1)^{N+j}\langle TX_{\hat{y}_j}\rangle\delta(x_1 - y_j)\,, \tag{4.3}$$

where $X_{\hat{y}_j}$ is X with the field whose argument is y_j deleted. The first term on the right-hand side of Eq. (4.3) is defined by

$$\langle TN[\gamma^\mu\gamma^5\partial_\mu\varphi\phi](x_1)X\rangle = \lim_{\epsilon\to 0}\left[\langle T:(\partial\!\!\!/ e^{-ig\gamma^5\varphi(x_1)}):\Psi(x_1+\epsilon)X\rangle - (e^{g^2\Delta(\epsilon)}-1)\sum_{j=1}^{N}\delta(x_1-y_j)\langle TX_{\hat{y}_j}\rangle + M(e^{2g^2\Delta(\epsilon)}-1)\langle T\phi(x_1)X\rangle\right], \tag{4.4}$$

The equation of motion for the pseudoscalar field is derived similarly. The result is

$$(\partial_z{}^2 - m^2)\langle T\varphi(z)\phi(x_1)X\rangle = -2igM\langle TN[\overline{\phi}\gamma^5\phi](z)\phi(x_1)X\rangle + ig\sum_{j=1}^{N}[\delta(z-x_j)\gamma^5_{x_j} + \delta(z-y_j)\gamma^{5T}_{y_j}]\langle T\phi(x_1)X\rangle$$
$$-\sum_{k=1}^{L}\delta(z-z_k)\langle T\phi(x_1)X_{\hat{z}_k}\rangle\,, \tag{4.5}$$

where the Green's function containing $N[\overline{\phi}\gamma^5\phi](z)$ is defined by

$$\langle TN[\overline{\phi}\gamma^5\phi](z)\phi(x_1)X\rangle = \text{F.P.}\ \frac{{}^{(0)}\langle:\overline{\Psi}_0 e^{-2ig\gamma^5\varphi_0}\gamma^5\Psi_0:(z)\phi^{(0)}(x_1)X^{(0)}\exp[\int d^2x\,\mathfrak{L}_{\text{int}}(\varphi_0,\Psi_0)]\rangle^{(0)}}{{}^{(0)}\langle T\exp[\int d^2x\,\mathfrak{L}_{\text{int}}(\varphi_0,\Psi_0)]\rangle^{(0)}} \tag{4.6}$$

where F. P. is the prescription to delete graphs containing diagrams of Fig. 3 (where V stands for the special vertex) type as subgraphs. Equations (4.5) is valid order by order in the perturbation.

Ward identities associated with the vector and axial-vector currents can be derived in the usual way. The Green's function $\langle TN[\overline{\phi}\gamma^\mu\phi](x)\phi(x_1)X\rangle$ is defined analogously to the expression (4.6) replacing in it the special vertex by: $\overline{\Psi}_0\gamma^\mu\Psi_0$:. There is no F. P. prescription necessary in the whole region $0 \leq \alpha < 1$. This is so, because we calculate graphs containing $\overline{\phi}\gamma^\mu\phi$ first in the region $0 < \alpha < \frac{1}{2}$, where no additional divergences occur and then continue to $\alpha \geq \frac{1}{2}$. We obtain

$$\partial^\mu\langle TN[\overline{\phi}\gamma_\mu\phi](x)\phi(x_1)X\rangle = \sum_{i=1}^{N}[\delta(x-x_i)+\delta(x-y_i)]\langle T\phi(x_1)X\rangle\,, \tag{4.7}$$

whereas for the axial-vector current defined similarly results

$$\partial^\mu\langle TN[\overline{\phi}\gamma_\mu\gamma_5\phi](x)\phi(x_1)X\rangle = 2iM\langle TN^*[\phi\gamma^5\phi](x)\phi(x_1)X\rangle + \sum_{i=1}^{N}[\delta(x-x_i)\gamma^5_{x_i} + \delta(x-y_i)\gamma^{5T}_{y_i}]\langle T\phi(x_1)X\rangle\,, \tag{4.8}$$

where the Green's function containing $N^*[\overline{\phi}\gamma^5\phi]$ is defined by Eq. (4.6) omitting the F. P. prescription and making the replacement

$$:\overline{\Psi}_0\gamma^5 e^{-2ig\gamma^5\varphi_0}\Psi_0: \to \overline{\Psi}_0 :e^{-2ig\gamma^5\varphi_0}:\gamma^5\Psi_0.$$

Equation (4.7) is valid in every order of perturbation, whereas to obtain (4.8) we must combine different orders as in the case of the fermion equation of motion. Equation (4.8) shows explicitly the expected broken γ^5 invariance.

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