Systems containing derivative coupling and/or vector fields in augmented quantum field theory

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Minimal equations for Green's functions for systems of spinor and (pseudo) scalar fields with derivative coupling are compared with those for systems with nonderivative couplings, and it is found that there remain terms with explicit coupling constant. For systems containing vector (or axial-vector) fields, one cannot derive Ward-Takahashi identities. Alternative choices (gauges) of two-point and three-point functions are discussed.

I. INTRODUCTION

In this paper we extend the Green's-function formalism of augmented quantum field theory, proposed by Klauder[1] and developed in our previous papers,[2] to systems of fields that involve derivative couplings and/or vector (or axial-vector) fields. Those systems are, as is well known, nonrenormalizable in the perturbative approach of canonical quantum field theory, except for quantum electrodynamics. As has been shown in our previous papers,[3,4] a bare vertex term does not appear in the equations for three-point (or four-point) functions of the augmented quantum field theory. Therefore, the situation concerning the divergences and renormalization in augmented quantum field theory (AQFT) is quite different from that of canonical quantum field theory (CQFT).

In Sect II, we consider scalar [with vector (derivative) coupling] and pseudoscalar [with axial-vector (derivative) coupling] models of a "nucleon-meson system," and show that the presence of derivative coupling alters essentially the structure of the minimal equations for many-point functions. In Sec. III, spinor and scalar QED are considered. Section IV is devoted to models involving spinor and massive vector (or axial-vector) fields. Section V contains concluding remarks and open questions.

II. "NUCLEON-MESON" SYSTEM WITH DERIVATIVE COUPLING

In this section, we consider systems consisting of a spinor field ψ ("nucleon field") and a spinless field φ with derivative coupling. To be definite, let us take a system with the action

$$\mathcal{S}=\int d^4x[\bar{\psi}(i\gamma_\mu\partial^\mu-M)\psi+\tfrac{1}{2}(\partial_\mu\varphi\partial^\mu\varphi-\kappa^2\varphi^2)-g\bar{\psi}\gamma_\mu\psi\partial^\mu\varphi]\,. \tag{2.1}$$

Then the field equations read

$$\varphi(\Box+\eta^2)\varphi+g\varphi\partial^\mu(\bar{\psi}\gamma_\mu\psi)=0\,, \tag{2.2}$$

$$\bar{\psi}(i\gamma_\mu\partial^\mu-M)\psi-g\bar{\psi}\gamma_\mu\psi\partial^\mu\varphi=0\,, \tag{2.3}$$

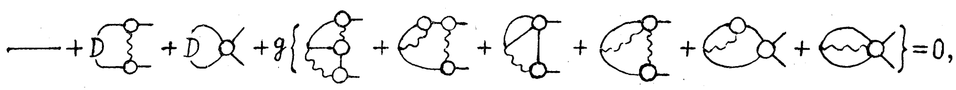
$$i(\partial^\mu\bar{\psi})\gamma_\mu\psi+M\bar{\psi}\psi+g\bar{\psi}\gamma_\mu\psi\partial^\mu\varphi=0\,. \tag{2.4}$$

[In Eqs. (2.3) and (2.4) spinor indices of $\bar{\psi}$ and ψ are not contracted, respectively.] The equations for the many-point functions read

$$\begin{aligned}\lim_{z'\to z}\,(\Box_z+\kappa^2)G_{2m,n+2}(x_1,\ldots,x_m;y_1,\ldots,y_m;z,z',z_1,\ldots,z_n)&\\+\sum_j G_{2m,n}(x_1,\ldots,x_m;y_1,\ldots,y_m;z,z_1,\ldots,\hat{z}_j,\ldots,z_n)\delta^4(z-z_j)&\\+g\lim_{x\to z}\gamma_\mu\partial^\mu_x G_{2m+2,n+1}(x,x_1,\ldots,x_m;x,y_1,\ldots y_m;z,z_1,\ldots,z_n)&\\+g\sum_j G_{2m,n+1}(x_1,\ldots,x_m;y_1,\ldots,y_m;\;z,z_1,\ldots,z_n)\delta^4(z-x_j)&\\+g\sum_j G_{2m,n+1}(x_1,\ldots,x_m;y_1,\ldots,y_m;z,z_1,\ldots,z_n)\delta^4(z-y_j)&=0\,,\end{aligned} \tag{2.5}$$

$$\lim_{x'\to x} \left[(i\gamma_\mu \partial_x^\mu - M) G_{2m+2,n}(x, x_1, \ldots, x_m; x', y_1, \ldots, y_m, z_1, \ldots, z_n) \right.$$
$$\left. - g\gamma_\mu \partial_x^\mu G_{2m+2,n+1}(x', x_1, \ldots, x_m; x', y_1, \ldots, y_m; x, z_1, \ldots, z_n) \right]$$
$$+ \sum_j G_{2m,n}(x_1, \ldots, x_m; x, y_1, \ldots, \hat{y}_j, \ldots, y_m; z_1, \ldots, z_n) \delta^4(x - y_j) = 0 . \quad (2.6)$$

[here, an equation adjoint to (2.6) is omitted]. In terms of irreducible many-point functions, one gets the following equations for G_{02}, G_{20}, and G_{21}: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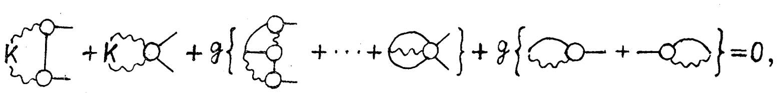
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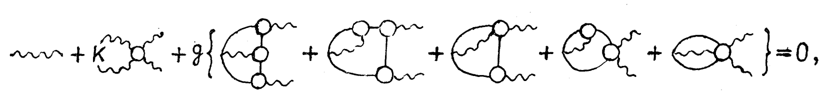
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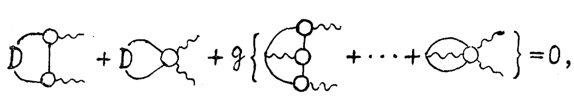
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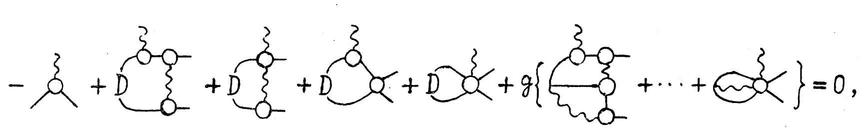
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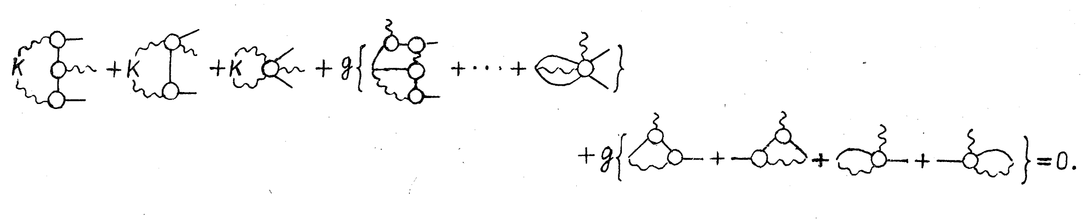
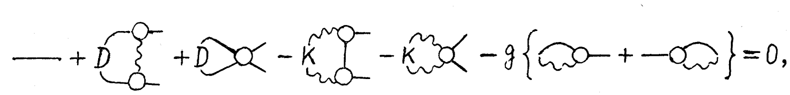
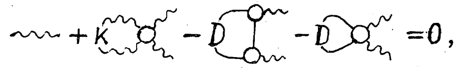
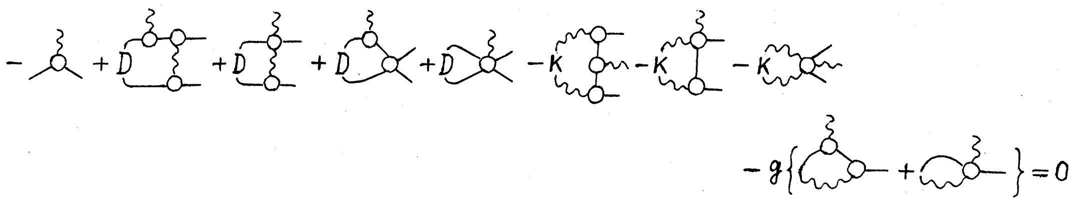

(2.12)

Taking appropriate linear combinations of these equations, one gets the minimal equations

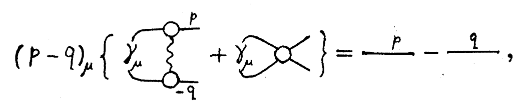

(2.13)

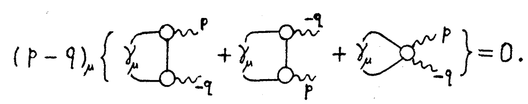

(2.14)


(2.15)

[In (2.15) transposed diagrams are omitted]. Similarly, one gets the following constraints:

$$k_\mu \int d^4p \, \mathrm{Tr}[\gamma_\mu G_{21}(p-k; -p; k)] = 0 , \tag{2.16}$$


(2.17)


(2.18)

Some comments are in order. In this case, contrary to the case without derivative coupling, one cannot eliminate terms with an explicit coupling constant from the minimal equations for $G_{2m,n}$ ($m \neq 0$). If one interprets the terms in { } on the left-hand side of (2.13) as subtracted, they do not have a pole at $p^2 = M^2$, but cannot be ignored off the mass shell. On the other hand, if one interprets them as they stand, they contribute to the residue of G_{20} even if G_{21} has nice asymptotic behavior. (Note that these terms resemble the self-energy part in CQFT, but differ by a factor G_{20}.) Constraint (2.16) implies that G_{21} must be chosen subject to this constraint if one treats Eqs. (2.13) and (2.14) as a descending problem,[3,4,5] while other constraints do not affect the input to this descending problem, because they are constraint on higher many-point functions.

One can repeat the same procedure for the pseudoscalar [with axial-vector (derivative) coupling] model with the action

$$\mathcal{S}_{(p)} = \int d^4x [\bar\psi(i\gamma_\mu \partial^\mu - M)\psi + \tfrac{1}{2}(\partial_\mu\chi\partial^\mu\chi - \kappa^2\chi^2) - ig\bar\psi\gamma_5\gamma_\mu\psi\partial^\mu\chi] . \tag{2.19}$$

The only essential difference is that the constraint (2.16) is automatically satisfied if parity is conserved.

III. QUANTUM ELECTRODYNAMICS

In this section, we first consider the electron-photon system and later on proceed to deal with charged scalar field. As we shall see, there are some important differences.

Let us take the action

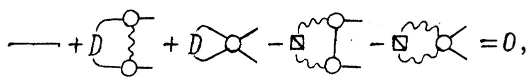
$$\mathcal{S}_{(\mathrm{QED})} = \int d^4x [\bar\psi(i\gamma_\mu\partial^\mu - M)\psi - \partial_\mu A_\nu(\partial^\mu A^\nu - \partial^\nu A^\mu) - ie\bar\psi\gamma_\mu\psi A^\mu] . \tag{3.1}$$

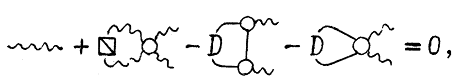
Then the equations for $G_{m,n}$ read

$$\begin{aligned} &\lim_{x'\to x} (i\gamma_\mu\partial^\mu - M) G_{2m+2,n,\mu_1\ldots\mu_n}(x, x_1, \ldots, x_m; x'; y_1, \ldots, y_m; z_1, \ldots, z_n) \\ &+ \sum_j G_{2m,n,\mu_1\ldots\mu_n}(x_1, \ldots, x_m; x, y_1, \ldots, \hat y_j, \ldots, y_m; z_1, \ldots, z_n)\delta^4(x - y_j) \\ &+ ie\gamma_\lambda G_{2m+2,n+1,\lambda\mu_1\ldots\mu_n}(x, x_1, \ldots, x_m; x, y_1, \ldots, y_m; x, z_1, \ldots, z_n) = 0 , \end{aligned} \tag{3.2}$$

$$\lim_{z'\to z}(\delta_{\lambda\nu}\Box_z - \partial_z^\lambda\partial_z^\nu)G_{2m,n+2,\nu\rho\mu_1\ldots\mu_n}(x_1,\ldots,x_m;y_1,\ldots,y_m;z,z',z_1,\ldots,z_n)$$
$$+\sum_j G_{2m,n,\rho\mu_1\ldots\hat{\mu}_j\ldots\mu_n}(x_1,\ldots,x_m;y_1,\ldots,y_m;z,z_1,\ldots,\hat{z}_j,\ldots,z_n)\delta^4(z-z_j)\delta_{\lambda\mu_j}$$
$$+ie\gamma_\lambda G_{2m+2,n+1,\rho\mu_1\ldots\mu_n}(z,x_1,\ldots,x_m;z,y_1,\ldots,y_m;z,z_1,\ldots,z_n)=0 \qquad (3.3)$$

[here, the equation adjoint to (3.2) is not written explicitly]. From these equations, one gets the minimal equations for G_{02}, G_{20}, and G_{21} as follows:


(3.4)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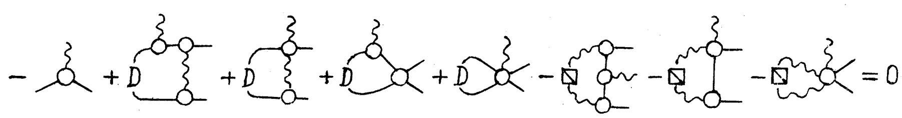

(3.6)

(here transposed diagrams are omitted).

Though one can derive a constraint of the form (2.17) and a constraint

$$k_\mu \int d^4p\,\mathrm{Tr}[\gamma_\mu G_{2,1,2}(p;-p-k;k)]=0\,, \qquad (3.7)$$

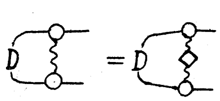
one cannot derive the Ward-Takahashi identity

$$p_\lambda K_{\lambda\nu}(p)G_{21\nu}(q;-p-q;p)=G_{20}(q)-G_{20}(p+q)\,. \qquad (3.8)$$

If one assumes that $G_{m,n}$ $(n\neq 0)$ have the form

$$G_{2m,n,\mu_1\ldots\mu_n}(p_1,\ldots,p_m;q_1,\ldots,q_m;k_1,\ldots,k_n)$$
$$=\prod_{j=1}^n(\delta_{\mu_j\lambda_j}-k_{j\lambda_j}k_{j\mu_j}/k_j^2)$$
$$\times G_{2m,n,\lambda_1\ldots\lambda_n}(p_1,\ldots,p_m;q_1,\ldots,q_m;p_1,\ldots,p_n)\,, \qquad (3.9)$$

constraint (3.7) is automatically satisfied, and the second term on the left-hand side of Eq. (3.4) can be interpreted as follows:


(3.10)

where

$$\Diamond=(G_{02}^{\blacktriangle})^{-1}\,, \qquad (3.11)$$

$$G_{02\mu\nu}(k)=(\delta_{\mu\nu}-k_\mu k_\nu/k^2)G_{02}^{\blacktriangle}(k^2)\,. \qquad (3.12)$$

Situations are different in the Feynman gauge and the Källén gauge, but we do not go into details here.

It can be easily seen that if $G_{2,1\mu}$ behaves like

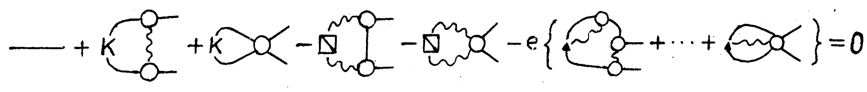
$$G_{2,1\mu}(p;-p-k;k)\sim e\frac{\gamma\cdot p+M}{p^2-M^2}\gamma_\nu\frac{\gamma\cdot(p+k)+M}{(p+k)^2-M^2}$$
$$\times\frac{\delta_{\mu\nu}-k_\mu k_\nu/k^2}{k^2} \qquad (3.13)$$

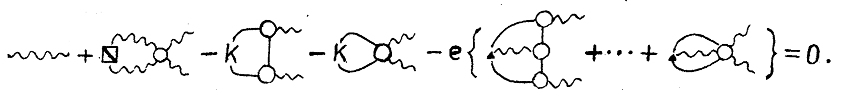
in the low-energy region, Eq. (3.6) gives the same anomalous magnetic moment of electron up to a small correction due to the contribution of the high-energy region and the terms containing irreducible higher many-point functions. This indicates that AQED is not much different from the conventional QED in the low-energy region.

Curtailing further discussion of spinor QED, let us consider scalar QED with the action

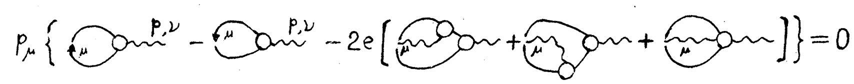
$$\mathcal{S}_{\mathrm{SQED}}=\int d^4x[\partial_\mu\phi^*\partial^\mu\phi-\eta^2\phi^*\phi-\tfrac{1}{4}(\partial_{[\mu}A_{\nu]})^2$$
$$-(ie\phi^*\overleftrightarrow{\partial}_\mu\phi+2e^2\phi^*\phi A_\mu)A^\mu]\,. \qquad (3.14)$$

Then the minimal equations for G_{20} and G_{02} read


(3.15)


(3.16)

The constraint corresponding to (3.7) is


(3.17)

An important difference from spinor QED is the presence of terms with explicit e in Eqs. (3.15) and (3.16), which is a consequence of derivative coupling. If one assumes that $G_{m,n}$ $(n\neq 0)$ have the form (3.9) with respect to photon momenta, constraint (3.17) is automatically satisfied.

IV. SYSTEM INVOLVING THE VECTOR OR AXIAL-VECTOR FIELD

In this section, we consider a massive vector and axial-vector field interacting with a spinor field. In the perturbative approach to CQFT, such an interaction is unrenormalizable and unrenormalizable interactions, as far as divergences are concerned.

The actions to be considered are

$$\mathcal{S}_{V(A)}=\int d^4x[\bar\psi(i\gamma_n\partial^\mu-M)\psi+\tfrac{1}{4}(\partial_{[\mu}B_{\nu]})^2 -\tfrac{1}{2}\eta^2B_\mu{}^2-ig\bar\psi\Gamma\gamma_\mu\psi B_\mu], \qquad (4.1)$$

with $\Gamma=1$ and $\Gamma=\gamma_5$ for the vector and axial-vector cases, respectively. Here we do not write equations explicitly, but it should be noticed that we have a constraint of the form (3.17). In AQFT, one can assume that $G_{m,n}$ $(n\neq 0)$ has the form (3.9) without affecting the integral equations, which guarantees constraint (3.17). On the other hand, in the axial-vector case constraint (3.17) is automatically satisfied if parity is conserved, so that one can choose arbitrary $G_{2,1}$ as input to the minimal equations for G_{02} and G_{20}.

V. CONCLUDING REMARKS

As we have seen above, if the interaction involves derivatives of fields, the minimal equations are explicitly different from those of nonderivative interactions, and retain terms explicitly depending on the coupling constant.

In AQED, one cannot derive the Ward-Takahashi identities from field equations, but constraint (3.7), which is a consequence of conservation of $\bar\psi\gamma_\mu\psi$, can be satisfied if $G_{2,1,\mu}(\,;k)=\tilde G_{2,1,\nu}(\,;k)(\delta_{\mu\nu}-k_\mu k_\nu/k^2)$. Of course, it is not necessary that $G_{2,1}$ assumes this form, because the constraint can be satisfied if $G_{2,1,\nu}(\,;p)=\tilde G_{2,1,\mu}(\,;p)(p_\mu p_\nu-\delta_{\mu\nu}p^2)+\hat G_{2,1,\mu}(\,;p)$ with $\int d^4kp_\mu\,\mathrm{Tr}[\gamma_\mu\hat G_{2,1,\nu}(k,-k-p;p)]=0$. This, however, means that it is not easy to find a $G_{2,1}$ in other gauges. Whether $G_{0,2}$ has a longitudinal component depends on behaviors of $G_{0,4}$ and $G_{2,2}$, but one can choose those of the form (3.9) without conflicts, so that $G_{0,2,\mu\nu}(k)=\hat G_{02}(k^2)(\delta_{\mu\nu}-k_\mu k_\nu/k^2)$. Whether or not the lack of the Ward-Takahashi identities is a serious deficiency of AQED is a matter of opinion, but it is nice that $G_{m,n}$ $(n\neq 0)$ can have a better asymptotic behavior so that the integrals in the equations for many-point functions converge.

On the other hand, it is an interesting feature of augmented massive vector field theory that one can choose $G_{0,2,\mu\nu}(p)=G_{02}(p^2)(\delta_{\mu\nu}-p_\mu p_\nu/p^2)$ which is a generalization of the massive gauge field propagator $G^{(\mathrm{gauge})}_{\mu\nu}(p)\equiv(\delta_{\mu\nu}-p_\mu p_\nu/p^2)(p^2-m^2-i\epsilon)^{-1}$, without creating troubles in the Schwinger-Dyson equations.

It seems that one can construct drastically inequivalent theories starting from the same system of equations for Green's functions AQFT.

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