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Scattering of light in a strongly magnetized plasma

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The Thomson cross section in a strongly magnetized electron plasma is reexamined with a view to extending previous results of Canuto, Lodenquai, and Ruderman beyond the cold-plasma limit. To this end a formal relation is established between the differential photon cross section and the polarization currents induced in the medium by the incident photon. In the cold-plasma limit our formal approach leads to a substantially simplified differential cross section allowing an intuitive understanding of the scattering mechanism and the resulting anisotropy. For the integrated partial and total (summed over final polarizations) cross sections we recover previous results. The structure of this theory at and near the cyclotron resonance is examined including the necessary dissipative and broadening mechanisms. Extensions beyond the cold-plasma limit are discussed for the application to the problem of the accreting x-ray pulsars. Under conditions typical of the accretion column of some x-ray pulsars it is expected that the birefringent properties of the vacuum may significantly alter both the photon cross sections and the polarization properties of the radiation.

I. INTRODUCTION

Recent measurements in the hard x-rays from Her X-1 by Trümper *et al.*[1] seem to confirm the long-postulated existence of magnetic fields exceeding 10^{12} G at the surface of neutron stars. The presence of such fields can so severely confine the motion of electrons perpendicular to the field lines as to alter dramatically the macroscopic properties of the stellar surface and atmosphere.[2,3] In an influential paper Canuto, Lodenquai, and Ruderman[4] (hereafter to be referred as CLR) showed that this reduced mobility of the electron results in a dramatic drop (at low frequencies) of the Thomson opacity for those polarization modes having their electric vector perpendicular to the external field. Recent astrophysical work on the modeling of the radiation from the accreting x-ray pulsars[5] has been significantly affected by this and related results on the opacity.[6]

Attempts to model the newly measured Her X-1 cyclotron feature necessitate, however, an extension of these essentially classical results to the hard x-ray, high-temperature and quantizing magnetic field regimes, where the classical cold-plasma limit is known to fail.[7] Indeed, the limit of classical electrodynamics fails to reflect the intricate harmonic structure of the electron's quantum-mechanical motion at photon energies $\hbar\omega \gtrsim \hbar eB/m_e c \approx 50$ keV, where B is the external magnetic field.[8] Furthermore, the polarization of the vacuum itself ceases to be a negligible effect as was recently demonstrated by Novick *et al.*[9]

This paper is the first of a series of studies devoted to a thorough examination of the medium polarization effect on the radiative processes which determine the radiation and heat transport in the accretion column of magnetic neutron stars. At low frequencies our formal approach leads to results which, while equivalent to the CLR results, also allow for a simple interpretation of the anisotropic features in the total cross section obtained in that paper. A simple expression is also obtained for the differential cross section, which avoids much of the algebraic complexity of previous presentations. The numerical evaluation of our expression confirms recent results of Börner and Mészáros[10] obtained through the CLR formalism. It should finally be mentioned that collective plasma phenomena, which are completely ignored in our presentation, are expected to be unimportant for the regime of frequency and density visualized in the problem of the accreting x-ray pulsars, where typically the radiation frequency exceeds the plasma frequency by several orders of magnitude. Extensions of these ideas beyond the classical cold-plasma limit, while briefly discussed at the end of this paper, will be reported in detail in subsequent publications.

II. ROLE OF THE MEDIUM POLARIZABILITY

We first wish to underline that the hitherto published low-frequency CLR results for the differential cross section (their algebraic complexity notwithstanding) are consistent with and can be summarized by the following formal expression:

$$\frac{d\sigma}{d\Omega}\bigg|_{\vec{k}\to\vec{k}'} = \frac{k'}{k} r_0^2 \,|(\vec{e}', \vec{\Pi}\cdot\vec{e})|^2 / |\vec{e}_t'|^2 |\vec{e}_t|^2 , \tag{1}$$

$$\Pi_{\alpha\beta} = \frac{\omega^2}{\omega_p^2}(\delta_{\alpha\beta} - \epsilon_{\alpha\beta}), \tag{2}$$

where $\epsilon_{\alpha\beta}$ is the medium dielectric tensor, $\vec{e}$ and

$\vec{e}'$ are the polarizations of the incident and outgoing E-vectors, $\vec{e}_t = (\overleftrightarrow{1} - \vec{k}\vec{k}) \cdot \vec{e}$ is the transverse component of the polarization vector. $r_0 = e^2/mc^2$ is the electron classical radius and ω_p is the plasma frequency of the electron component, i.e., $\omega_p^2 = 4\pi e^2 n_e m^{-1}$. The implicit assumption for the validity of this expression is that the parameter $w = \omega_p^2/\omega^2 \ll 1$ so that only the electron component needs to be included. In an isotropic plasma $\Pi_{\alpha\beta} = \delta_{\alpha\beta}$ so that Eq. (1) reduces to the well-known result[11,12]

$$\frac{d\sigma}{d\Omega} = r_0^2 \left|(\hat{e}'^* \cdot \hat{e})\right|^2 , \tag{3}$$

where $\hat{e}$ denotes the normalized polarization vector.

A derivation of Eq. (1) is given in the Appendixes to show the generality of this expression, which is valid classically as well as within the framework of quantum electrodynamics in the limit of low density. What does change in the various limits is the expression for $\Pi_{\alpha\beta}$. We shall amplify on this point later. Equation (1) is indeed no surprise to anyone familiar with the mechanism of the scattering of light. Charged particles set up polarization currents in response to an incident electromagnetic wave, which in turn become secondary centers of radiation. In the presence of a strong magnetic field the medium response $\Pi_{\alpha\beta} e_\beta$ is highly anisotropic and, therefore, the resultant radiation is anisotropic as well. The details of the final results depend critically on the structure of the propagation modes $\vec{e}$ and $\vec{e}'$ allowed by the medium. In fact for the purposes of the ensuing discussion it will prove essential to keep separate track of the role of the polarization modes which often carry all the dependence on the angles of the incident and final photons, as distinguished from the effect of $\Pi_{\alpha\beta}$ which is independent of or has a weak dependence on direction.

III. COLD-PLASMA PROPAGATION MODES

In the cold-plasma limit the polarization tensor for a magnetized (gyrotropic) electron gas is[13]

$$\Pi = \frac{1}{1-u} \begin{pmatrix} 1 & -iu^{1/2} & 0 \\ iu^{1/2} & 1 & 0 \\ 0 & 0 & 1-u \end{pmatrix} , \tag{4}$$

with $u = \omega_B^2/\omega^2$, $\omega_B \equiv eB/mc$, and where the z axis has been assumed to coincide with the direction of $\vec{B}$. The eigenvalues of this tensor are

$$\pi_\pm = (1 \pm u^{1/2})^{-1}, \quad \pi_\parallel = 1. \tag{5}$$

The normal modes for propagation at an angle θ with respect to the external field B are elliptically polarized, and in general $\hat{e}$ is not perpendicular to the direction of propagation $\hat{k}$. In the coordinate frame (x', y', z') with the z' axis along $\hat{k}$, and y' aligned perpendicular to the Bk plane the polarization modes of the electric vector are[14]

$$\hat{e}^1 = c(1,\ i\alpha,\ \lambda_1), \quad \hat{e}^2 = c(i\alpha,\ 1,\ i\lambda_2), \tag{6a}$$

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \frac{wu^{1/2}\sin\theta}{\epsilon_l(1-u)} \begin{pmatrix} u^{1/2}\cos\theta + \alpha \\ \alpha u^{1/2}\cos\theta - 1 \end{pmatrix} , \tag{6b}$$

$$\alpha = \alpha(\theta) = -b[1 + (1+b^2)^{1/2}]^{-1} , \tag{6c}$$

$$b = 2u^{-1/2}(1-w)\frac{\cos\theta}{\sin^2\theta} \cong 2u^{-1/2}\frac{\cos\theta}{\sin^2\theta} , \tag{7}$$

where $\epsilon_l = \epsilon_l(\theta) = \hat{k}_\alpha \epsilon_{\alpha\beta} \hat{k}_\beta$ is the longitudinal polarizability, and we have adopted the transverse normalization $c = (1+\alpha^2)^{-1/2}$. The longitudinal component $\lambda_i = \lambda_i(\theta)$ is of the order of the parameter $w = \omega_p^2/\omega^2$ and will therefore be neglected in most of the ensuing discussion, since we shall only concern ourselves with high frequencies relative to the plasma frequency. This semitransverse approximation breaks down near the resonance $u = 1$, in which case $\lambda_i(\theta)$ can be of order 1, and must be included in the cross-section calculation. The degree of ellipticity $\alpha(\theta)$ of the (transverse) polarization ellipse is thus the same for the two modes, ordinary and extraordinary, and is a function of the propagation angle θ. For parallel propagation $\theta = 0$, $\alpha(\theta) = -1$, and the polarization is always circular, as expected. At $\theta = \frac{1}{2}\pi$ on the other hand, $\alpha = 0$ resulting in plane-polarized modes aligned $\perp$ and $\parallel$ to the Bk plane. The dependence of the parameter $\alpha(\theta)$ on θ is seen in Fig. 1 for several values of the parameter $u = \omega_B^2/\omega^2$ and for $w \ll 1$. For $\theta \neq 0$ the major axis of the polarization ellipse is always aligned $\perp$ to the Bk plane for the extraordinary mode, and $\parallel$ to the Bk plane for the ordinary mode.

The corresponding index of refraction may be obtained directly from the wave equation

$$[n_i^2(\overleftrightarrow{1} - \hat{k}\hat{k}) - \overleftrightarrow{\epsilon}] \cdot \hat{e}^i = 0, \tag{8}$$

which gives by projecting on the vector $\hat{e}^i{}_t$

$$n_i^2 = 1 - w(\hat{e}^i{}_t,\ \overleftrightarrow{\Pi} \cdot \hat{e}^i). \tag{9}$$

This expression generalizes for nontransverse propagation, a well-known relation between the refractive index and the forward scattering amplitude.[12] This expression also produces the usual dispersion relation found in the plasma literature[13] if $\hat{e}$ and $\hat{e}'$ are substituted by their explicit expressions in terms of the plasma parameters.

Of special interest for the following discussion will be the propagation properties at frequencies much below the cyclotron frequency, i.e., when $u \gg 1$. From Fig. 1 we see that except for a small range of angles $\theta \lesssim \theta_0$ ($\sin^2\theta_0 \equiv u^{-1/2}\cos\theta_0$) one has

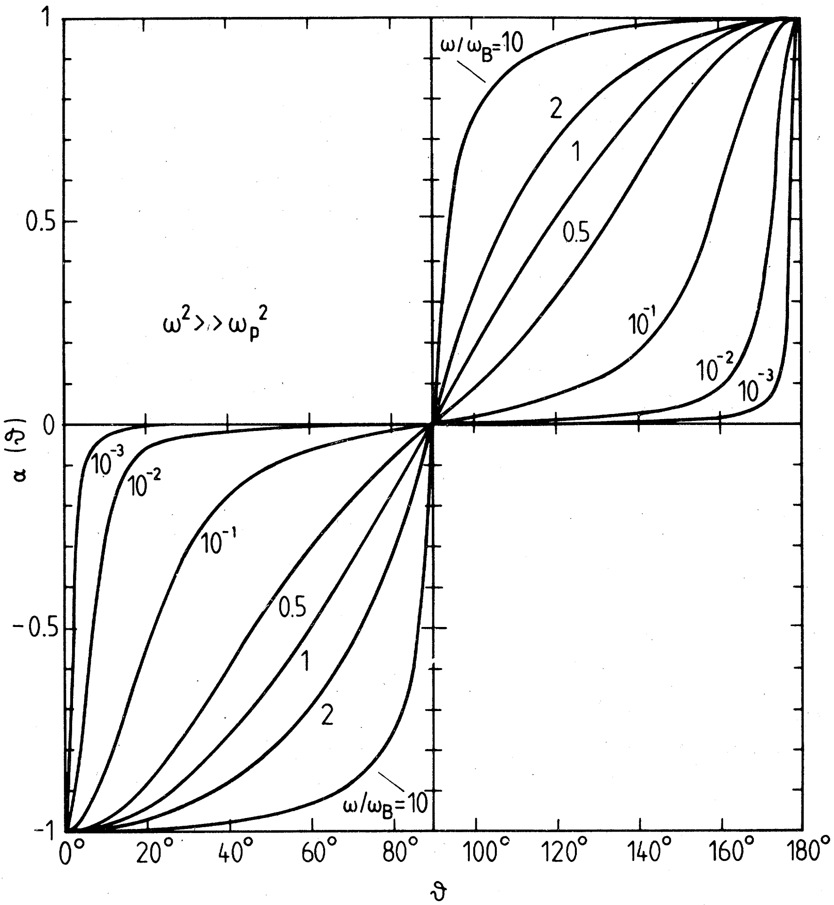

FIG. 1. The ellipticity parameter characterizing the cold-plasma polarization modes varies with the angle of propagation relative to B. At frequencies much less than the electron gyrofrequency ω_B the plasma modes tend to become plane polarized at almost all angles.

$\alpha(\theta) = -2u^{-1/2}\cos\theta/\sin^2\theta \ll 1$, so that the modes are almost completely linearly polarized ⊥ or ∥ to the Bk plane. This particular property of the low-frequency modes turns out to be a decisive factor in determining the remarkable anisotropy in the cross section reported in the CLR paper.

To evaluate the cross section we still need to express the plasma modes $\hat{e}^1$ and $\hat{e}^2$ in the coordinate frame with the z axis along $\vec{B}$, i.e., in the same frame as used in Eq. (4). For propagation along $\hat{k}(\theta, \varphi)$ we thus find after an appropriate rotation of coordinates ($\sqrt{2}\, e_\pm = e_x \pm i e_y$),

$$\hat{e}^1_\pm(\theta,\phi) = 2^{-1/2} c e^{\mp i\phi}[\cos\theta \mp \alpha(\theta) + \lambda_1(\theta)\sin\theta],$$

$$\hat{e}^2_\pm(\theta,\phi) = 2^{-1/2} c e^{\mp i\phi}[\alpha(\theta)\cos\theta \pm 1 + \lambda_2(\theta)\sin\theta], \quad (10)$$

$$\hat{e}^1_z(\theta,\phi) = -c[\sin\theta - \lambda_1(\theta)\cos\theta],$$

$$\hat{e}^2_z(\theta,\phi) = -c[\alpha(\theta)\sin\theta - \lambda_2(\theta)\cos\theta].$$

We adopt the transverse normalization $c^{-2} = 1 + \alpha^2(\theta)$.[14] Here we have used the rotating-coordinates notation since $\Pi_{\alpha\beta}$ is diagonal in these coordinates. We can therefore substitute in Eq. (1),

$$|(\hat{e}', \bar{\Pi}\cdot\hat{e})|^2 = \left| \hat{e}'^*_z\cdot\hat{e}_z + \frac{\hat{e}'^*_+\hat{e}_+}{1+u^{1/2}} + \frac{\hat{e}'^*_-\hat{e}_-}{1-u^{1/2}} \right|^2 . \quad (11)$$

According to Eqs. (1), (10), and (11) the differential cross section depends on the angles of the incident photon $\hat{k}(\theta, \phi=0)$ and of the final photon $\hat{k}'(\theta', \phi')$ through the polarization vectors.

From Eq. (11) we can now easily deduce the low-frequency behavior of $d\sigma/d\Omega$. Since $u \gg 1$ in this limit we see that the transverse components of the polarization current $e'^*_\alpha \Pi_{\alpha\beta} e_\beta$ are accordingly reduced so that we may write ($u \gg 1$)

$$\frac{d\sigma}{d\Omega}(\theta, \theta', \phi') \cong r_0^2[\,|\hat{e}'^*_z(\theta') e_z(\theta)|^2 + O(u^{-1/2})], \quad (12)$$

i.e., a result which is independent of the azimuthal angles ϕ and ϕ' of both the initial and final photon with respect to the Bk plane. It follows from this expression that those propagation modes, which have $\vec{e}$ perpendicular to the Bk plane will have their corresponding cross section drastically reduced with respect to the Thomson value. Before further analyzing the properties of $d\sigma/d\Omega$ it will prove useful to discuss the structure of the integrated cross section, as it allows a better insight into the final results.

IV. INTEGRATED CROSS SECTIONS

For an incident photon of polarization $\hat{e}^i(\theta, \phi)$ we obtain the integrated "partial" cross section into a final mode $\hat{e}^1$ or $\hat{e}^2$ by integrating Eqs. (1) and (11) over the angles θ', ϕ' of the final photon. The dependence on these angles is contained only in the vector $e^j(\theta', \phi')$ of the final mode. From Eq. (10) we see that the dependence of Eq. (11) on ϕ' is contained in terms proportional to $e^{\pm i\phi'}$, and to $e^{\pm 2i\phi'}$, while the absolute squares of the three terms in Eq. (11) are independent of ϕ'. Of these only the absolute squares contribute to the $d\phi'$ integral. The final result is thus $[\sigma_{\rm Th} = (8\pi/3)r_0^2]$

$$\frac{\sigma_{ij}}{\sigma_{\rm Th}} = A_z^j |\hat{e}_z^i|^2 + \frac{A_+^j |\hat{e}_+^i|^2}{|1+u^{1/2}|^2} + \frac{A_-^j |\hat{e}_-^i|^2}{|1-u^{1/2}|^2}, \qquad (13)$$

where the components of $\vec{e}^i = \vec{e}^i(\theta, \phi)$ are given in Eq. (10), and the constant vector $\vec{A}^j$ is defined in terms of angle integrals of the final polarization

$$A_\alpha^j \equiv \tfrac{3}{4} \int_{-1}^{1} d\cos\theta' \, |\hat{e}_\alpha^j(\theta', 0)|^2 \frac{n_j(\theta')}{n_i(\theta)} . \qquad (14)$$

From Eq. (10) we see that the transverse components of the polarization modes satisfy the following (completeness) property $[\hat{e}_t = \hat{e} - \hat{k}(\hat{k}\cdot\hat{e})]$:

$$\sum_{i=1}^{2} |\hat{e}_{t\pm}^i(\theta, \phi)|^2 = \tfrac{1}{2}(1+\cos^2\theta),$$
$$\sum_{i=1}^{2} |\hat{e}_{tz}^i(\theta, \phi)|^2 = \sin^2\theta, \qquad (15)$$

from which we conclude that in the semitransverse limit (i.e., $\hat{k}\cdot\hat{e} \cong 0$, $n_j^2 \cong n_i^2 \cong 1$),

$$\sum_{j=1}^{2} A_\pm^j = 1, \quad \sum_{j=1}^{2} A_z^j = 1. \qquad (16)$$

Numerical values of A_α^i as a function of frequency are given in Table I.

With the help of Eq. (16) we are led to an even simpler expression for the total cross section obtained from Eq. (13) as the sum over final polarizations. After substitution of $\hat{e}^i$ from Eq. (10) we find for the two (initial) modes the following total cross sections (semitransverse approximation):

$$\frac{\sigma_1(\theta)}{\sigma_{\rm Th}} = [1+\alpha^2(\theta)]^{-1}\left[\sin^2\theta + \frac{1}{2}\left|\frac{\cos\theta - \alpha(\theta)}{1+u^{1/2}}\right|^2 + \frac{1}{2}\left|\frac{\cos\theta + \alpha(\theta)}{1-u^{1/2}}\right|^2\right], \qquad (17)$$

$$\frac{\sigma_2(\theta)}{\sigma_{\rm Th}} = [1+\alpha^2(\theta)]^{-1}\left[\alpha^2(\theta)\sin^2\theta + \frac{1}{2}\left|\frac{1+\alpha(\theta)\cos\theta}{1+u^{1/2}}\right|^2 + \frac{1}{2}\left|\frac{1-\alpha(\theta)\cos\theta}{1-u^{1/2}}\right|^2\right].$$

These results as well as Eq. (13) are appropriately independent of the azimuthal angle ϕ. The generalization of this result for the case $\hat{e}\cdot\hat{k} \neq 0$ is given in Appendix B. Equations (17) are in complete agreement with the corresponding cross section of CLR when account is taken of the different notations adopted for the plasma-polarization modes.

Equations (17) may be conveniently summarized by the formal expression

$$\sigma_i(\theta) = \sigma_{\rm Th} |\overleftrightarrow{\Pi}\cdot\hat{e}^i(\theta,\phi)|^2, \qquad (18)$$

where $\Pi_{\alpha\beta}$ is the polarization tensor defined in Eq. (2). The product $\Pi_{\alpha\beta}e_\beta^i$ is proportional to the polarization current induced in the medium by the incident wave, thus suggesting an intuitively appealing interpretation of our results. It should further be underlined that while Eqs. (17) represent the cold-plasma limit adopted in the last section, Eq. (18) is valid more generally. It is

TABLE I. Coefficients A_α^i for the ordinary mode, $i = 1$, as a function of the frequency. The corresponding values for the extraordinary mode are $A_\alpha^2 = 1 - A_\alpha^1$. Note also that the relation $\sum_\alpha A_\alpha^1 = 1.5$ is satisfied reflecting the normalization of the $\hat{e}$ vector in Eq. (14).

ω/ω_B	A_z^1	A_+^1	A_-^1
10^{-3}	1	0.2536	0.2464
10^{-2}	0.9997	0.2806	0.2197
10^{-1}	0.9894	0.3951	0.1155
0.2	0.9726	0.4609	0.0664
0.5	0.9219	0.5644	0.0136
1	0.8562	0.6438	0
2	0.7749	0.7136	0.0115
5	0.6676	0.7849	0.0474
10	0.5990	0.8243	0.0767
100	0.5107	0.8707	0.1185

therefore worthwhile to point out that Eq. (18) may be directly obtained from Eq. (1) making use of some matrix algebra in the same limit of semi-transverse waves,

$$\sigma_i(\theta) = r_0^2 \int d\Omega_{k'} \sum_j (\overline{\Pi}\cdot \hat{e}^i, \hat{e}_t^j)(\hat{e}_t^j, \overline{\Pi}\cdot \hat{e}^i)$$

$$= \sigma_{\rm Th} \,|\overline{\Pi}\cdot \hat{e}^i(\theta,\varphi)|^2, \qquad (19)$$

where we have performed the angular integration with the help of the following matrix identities[15]:

$$\sum_{j=1}^{2} \hat{e}_t^j \hat{e}_t^{j*} = \overline{1} - \hat{k}'\hat{k}', \qquad (20)$$

$$\int d\Omega_{k'} \hat{k}'_\alpha \hat{k}'_\beta = \frac{4\pi}{3}\delta_{\alpha\beta}\ , \qquad (21)$$

Equation (20) is a completeness statement asserting that the polarization modes span completely the two-dimensional space transverse to $\hat{k}'(\theta', \varphi')$.

V. ANISOTROPY FEATURES

We are now ready to discuss systematically the anisotropy features characterizing the integrated as well as the differential cross sections. A very general feature is already apparent from Eq. (11) by simple inspection. Because of the medium effect the different polarization components carry a different weight and their relative importance varies with the frequency.

At low frequencies relative to ω_B the polarization components perpendicular to B are reduced since $u \gg 1$ leading to results that depend very dramatically on the polarization modes. Those modes with a significant component e_z scatter at roughly the normal Thomson cross section while the cross section is drastically reduced if $|\hat{e}_z| \ll 1$ [cf. Eq. (10)]. At frequencies near the cyclotron frequency one circular polarization becomes resonant (i.e., the one with the electric vector rotating in the same sense as the electron's gyration). As a function of frequency the cross section of the extraordinary mode obtains a sharp maximum at $u = 1$ (Fig. 2). This maximum value is not infinite, as predicted by Eq. (11), but turns out to be roughly equal to the cyclotron absorption cross section after correcting the cold-plasma polarization tensor to properly account for absorption and dissipative effects. [In this theory the ordinary mode remains unaffected by the resonance since at $u = 1$, $e_-^1(\theta) = \alpha(\theta) + \cos\theta = 0$.] This aspect of the theory will be further discussed in Sec. VI. Finally, at high frequencies ($u \ll 1$), Eq. (11) predicts an iso-

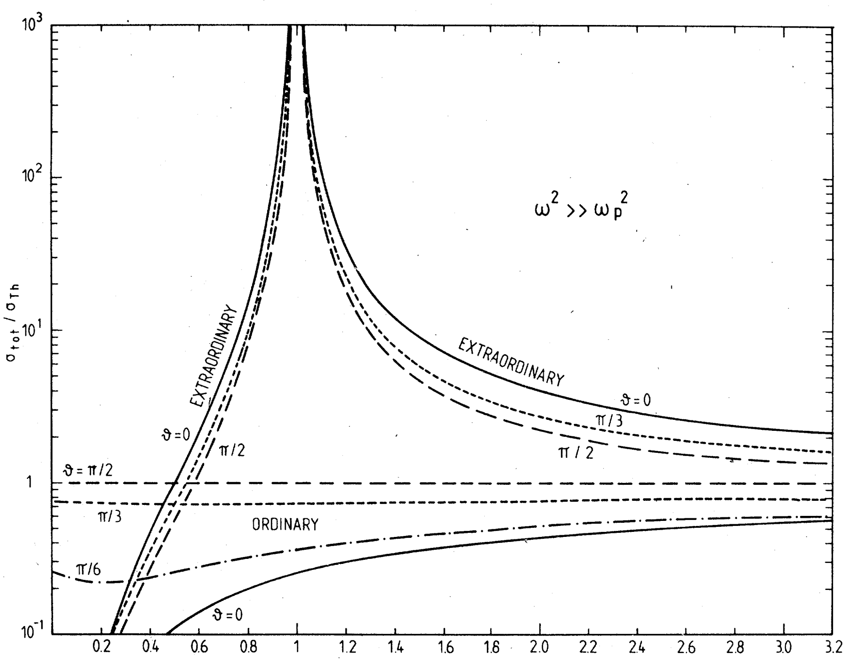

FIG. 2. Frequency dependence of the cross sections near the cyclotron resonance at various angles of propagation. The ordinary mode remains unaffected by the resonance at all angles of propagation. The absence of "peaks" at the higher harmonics $\omega = n\,\omega_B$ ($n = 2, 3, \cdots$) and $\theta \neq 0$ is inherent in the cold-plasma limit adopted for the medium polarization tensor. No temperature broadening was included.

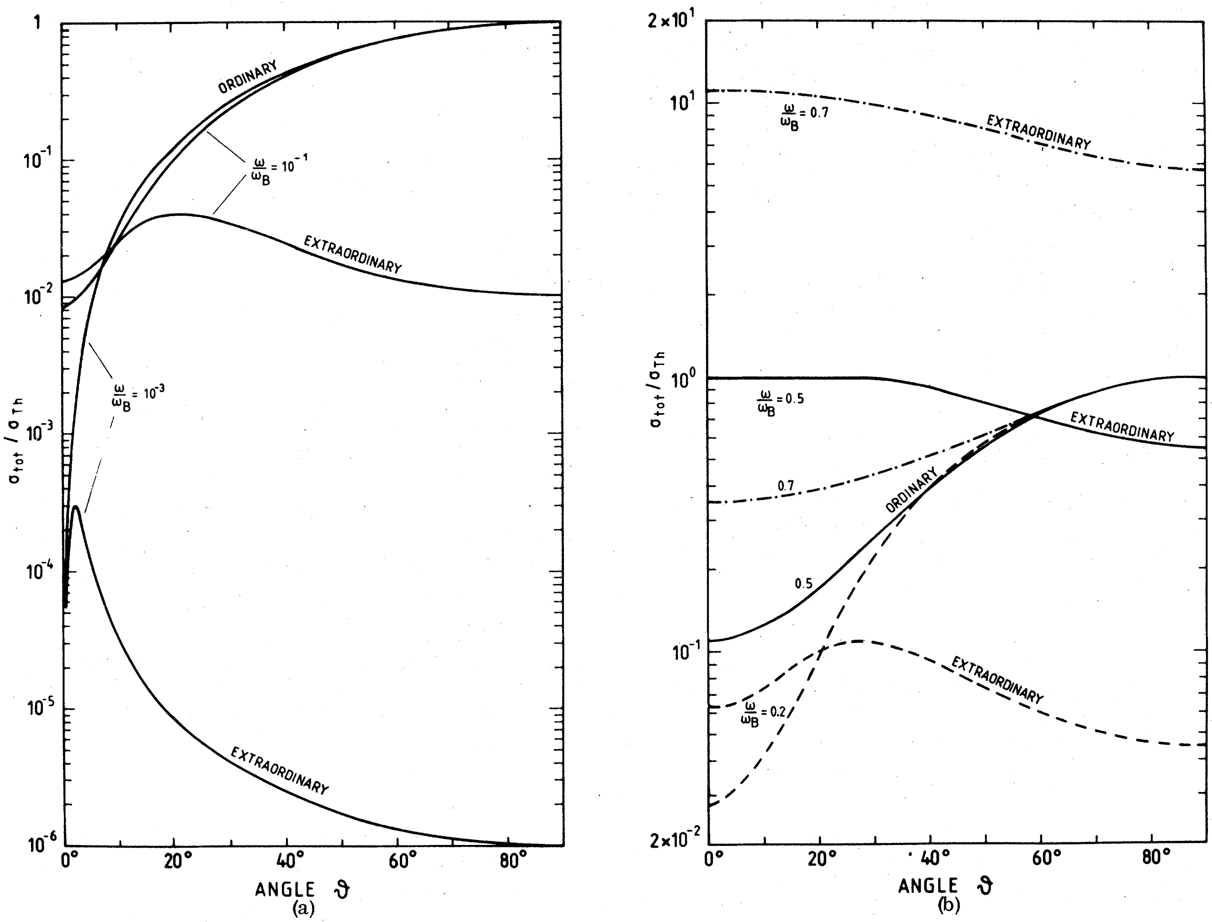

FIG. 3. Total photon cross section as a function of the incident photon's direction relative to the B field. (a) At low frequencies the characteristic anisotropy is largely determined by the magnitude of the polarization component e_z parallel to B, cf. Eq. (22). (b) At intermediate frequencies the behavior of $\sigma_{tot}(\theta)$ becomes more complex as the polarization components $\perp$ to B begin to play a role.

tropic cross section approaching the Thomson limit. In the high-frequency limit, however, one should keep in mind that the classical theory itself breaks down, and that both relativistic and quantum-mechanical effects may have to be included as will be further discussed in Sec. VII.

The anisotropic features in the total cross section at $\omega \ll \omega_B$ can be seen in Fig. 3, which gives the numerical evaluation of Eq. (17). While our algebraic expression is exactly equivalent to the CLR result our graph deviates significantly from theirs. At this point we are in agreement with results of Börner and Mészáros[10] obtained strictly within the CLR formalism. The difference is thus probably due to a faulty numerical evaluation in CLR.

It is recalled that in our notation always $|\alpha(\theta)| \leq 1$, so that the order of magnitude of the various terms can be immediately read off Eq. (17) without need for further reductions. The low-frequency limit ($u \gg 1$) is thus seen to be entirely related to polarization component along $\vec{B}$ giving

$$\frac{\sigma_1(\theta)}{\sigma_{\mathrm{Th}}} = [1+\alpha^2(\theta)]^{-1}\sin^2\theta + O(u^{-1}),$$
$$\frac{\sigma_2(\theta)}{\sigma_{\mathrm{Th}}} = [1+\alpha^2(\theta)]^{-1}\alpha^2(\theta)\sin^2\theta + O(u^{-1}). \tag{22}$$

These expressions indeed contain most of the features displayed in Fig. 3(a). It is thus seen that the cross section for the ordinary mode (polarization vector almost parallel to the Bk plane) is modulated by a simple $\sin^2\theta$ factor. The modulation of the extraordinary mode is further reduced by the ellipticity parameter $\alpha^2(\theta)$ thus accounting for the more complex shape of $\sigma_2(\theta)$. The maximum occurs at $\theta = \theta_0$, such that $\sin^2\theta_0/\cos\theta_0 = u^{-1/2}$, the value at the maximum being $\sigma_2(\theta_0) = u^{-1/2}\sigma_{\mathrm{Th}}$. Figures 4(a) and 4(b) give the integrated "partial" cross sections according to Eqs. (13) and display

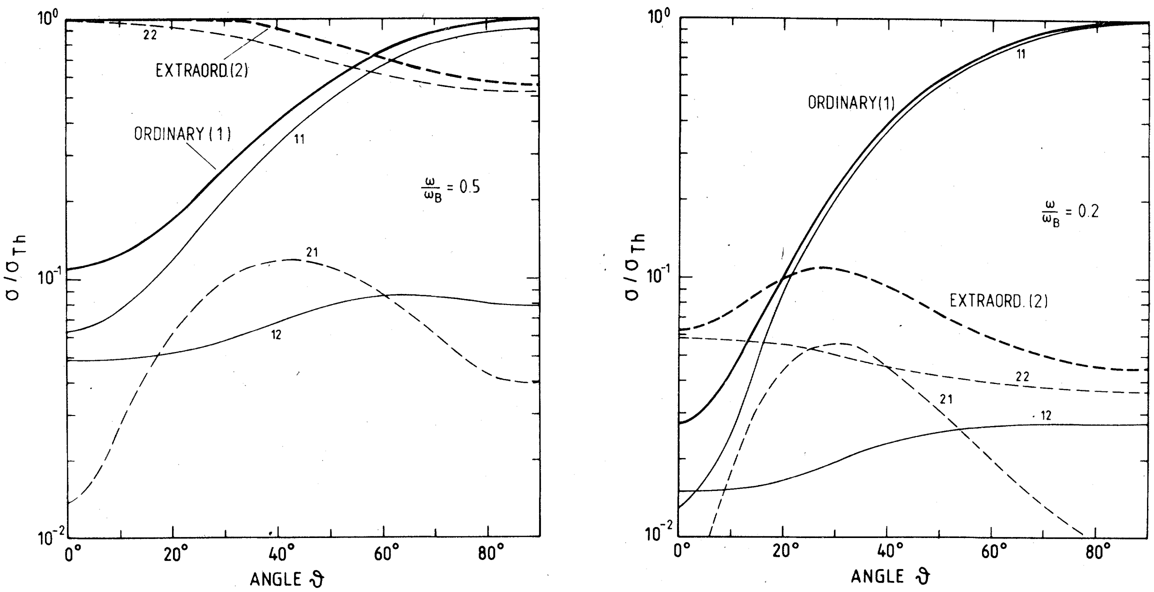

FIG. 4. Total cross sections σ_1, σ_2 (heavy line) viewed as the sum of partial cross sections showing the probability of scattering into each of the two (final) normal modes. They satisfy the relation $\sigma_i = \sigma_{i1} + \sigma_{i2}$ $(i = 1, 2)$. Typically a process involving change in polarization occurs with a cross section which is less than either of the total cross sections.

anisotropic features, which may again be interpreted in a similar fashion.

Color of the magnetic sky: It is instructive to speculate on the possible "color" of the ionized magnetic "sky." It is well known that the blue color of the terrestrial sky is related to the frequency-dependent Rayleigh cross section $\sigma_R = \sigma_{\mathrm{Th}}\omega^4(\omega^2 - \omega_0^2)^{-2}$, which ensures that the *scattered* light will have an altered spectral constitution with respect to the incident white. Here ω_0 characterizes the atomic binding of the electron and results in an isotropic (with respect to the direction of the incident photon) but frequency-dependent effect for $\omega \lesssim \omega_0$. This effect is contrary to the white color we would expect of an illuminated ionized gas, since in this case the scattering from free electrons occurs at a frequency-independent cross section $\sigma_{\mathrm{Th}} = (8\pi/3)(e^2/mc^2)^2$. For the case of the magnetic sky Eq. (11) shows that both of these effects are at play. The electron appears "bound" to photons polarized transverse to the magnetic field, while it also responds as a free electron to radiation with its E vector parallel to the external field. Due to this mixed Thomson-Rayleigh behavior the color of the magnetic sky would depend on the angle of illumination as well as on the angle of scattering. The "color" as well as the atmospheric transparency would further depend on the polarizations of illumination and observation. The atmosphere might be optically thick to one mode of polarization and optically thin to the other. The evaluation of $d\sigma/d\Omega$ according to Eqs. (1) and (11) reveals a significant dependence on the angle θ' of the final photon but a weaker dependence on ϕ', cf., Figs. 5 and 6. At low frequencies the simplified dependence

$$d\sigma_{ij}/d\Omega \cong 2\pi r_0^2 |e_z^i|^2 |e_z^j|^2$$

is applicable, thus the $1 \to 1$ transition peaks at $\theta' = \theta = 90^0$, as expected, displaying the $\sigma_{11} \propto \sin^2\theta \sin^2\theta'$ dependence. For $\omega/\omega_B \ll 1$ the cross section in the extraordinary mode is much smaller than that of the ordinary mode, while this inequality is reversed in the resonance region, $(\omega - \omega_B)/\omega_B \ll 1$. The angular dependence near the resonance is dominated by the e_-^2 polarization component, i.e., for $\omega \cong \omega_H$,

$$\sigma_{22}/\sigma_{\mathrm{Th}} \cong \tfrac{1}{4}\,\frac{(1+\cos^2\theta)(1+\cos^2\theta')}{(1-\omega_B/\omega)^2+\gamma^2}\ ,$$

where γ is the damping constant to be specified in the next section. Physically one may argue, that each time a resonance (extraordinary) photon is absorbed—an event of propability distribution $\propto \frac{1}{2}(1+\cos^2\theta)$-it is reemitted with the typical angular distribution characterizing cyclotron emission.

On similar grounds one would expect an approximate isotropy in the angle ϕ' near the resonance, an effect which is immediately verified from Eq. (11) whenever it is dominated by the resonant

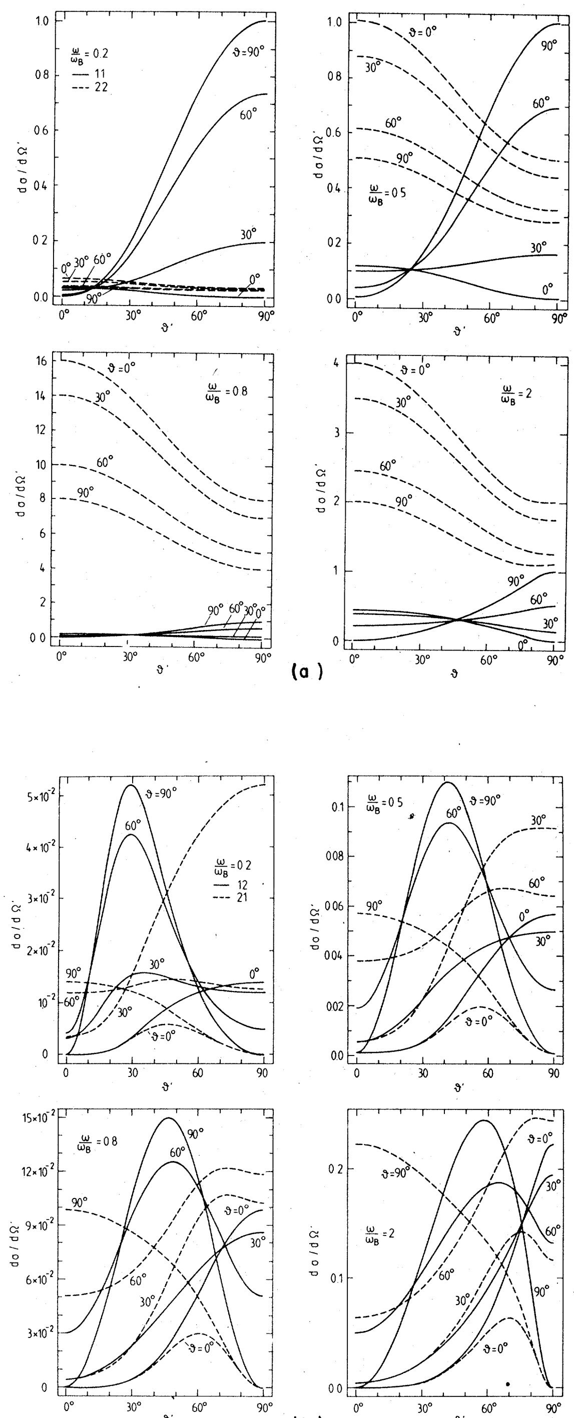

FIG. 5. Differential cross section $d\sigma_{ij}(\theta, \theta')/d\Omega'$ averaged over the azimuthal angle φ'. Angular dependence at several frequencies for (a) the polarization conserving transitions $1 \rightarrow 1$ (full line), $2 \rightarrow 2$ (broken), and (b) the polarization changing transitions $1 \rightarrow 2$ (full line), $2 \rightarrow 1$ (broken). The units for the cross section are $2\pi r_0^2 = 2\pi(e^2/mc^2)^2$.

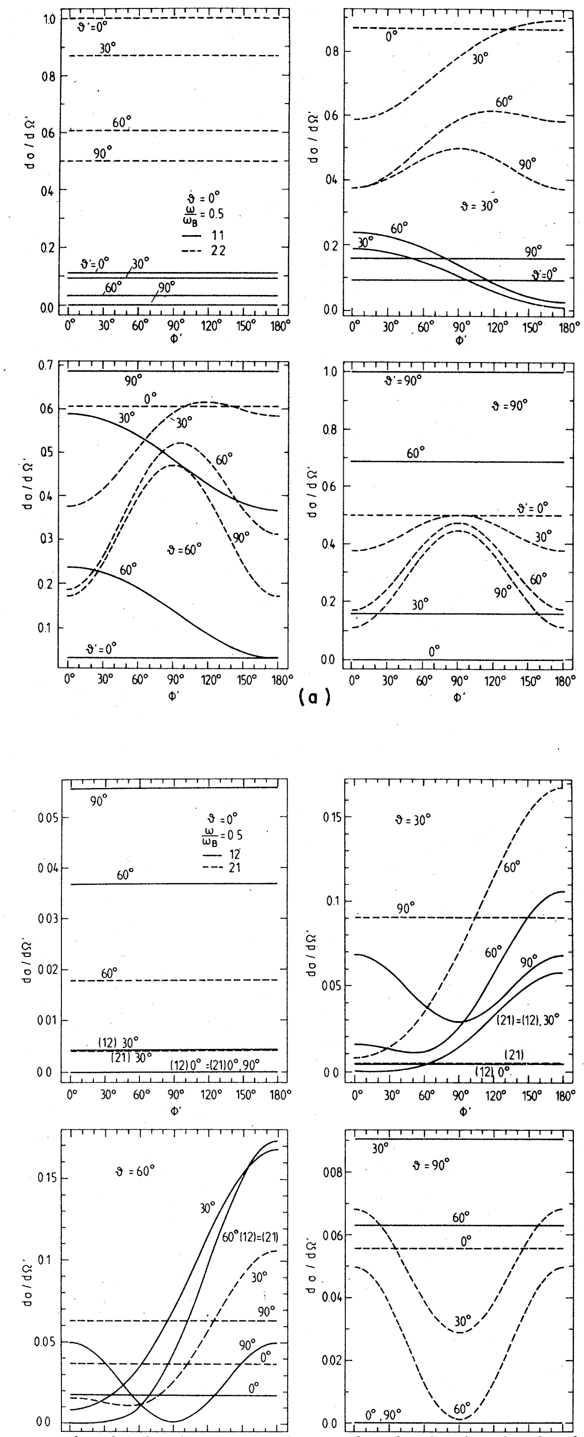

FIG. 6. Same as Fig. 5 for the φ'-dependent cross sections $d\sigma_{ij}(\theta, \theta'\varphi')/d\Omega'$, with the frequency fixed at $\omega = 0.5\omega_B$. The units for $d\sigma/d\Omega$ are $r_0^2 = (e^2/mc^2)^2$. The number identifying the curves is the final photon angle θ'.

term. An isotropy in ϕ' is also predicted at very low frequencies, $\omega \ll \omega_B$, from Eq. (11). A detailed study of the ϕ' dependence is given in Fig. 6 for the frequency $\omega = 0.5\omega_B$, where we expect to see the strongest anisotropic effects. When the incident photon is directed along the magnetic field ($\theta = 0$), the scattered radiation is isotropic in ϕ', while anisotropy sets in at angles $\theta \neq 0$. As expected the differential cross section satisfies the microreversibility property, which dictates that $d\sigma_{ij}(\hat{k} \to \hat{k}') = d\sigma_{ji}(\hat{k}' \to \hat{k})$. This explains the coincidence of the 12 and 21 lines in the $\theta' = 30°$, $\theta = 30°$ graph of Fig. 6(b) and a similar feature in the $\theta = 60°$ graph ($\theta' = 60°$), Fig. 6(b). The isotropy observed in the $\theta' = 0°$, $\theta \neq 0$ lines in Fig. 6 can be related to the isotropic $\theta = 0°$ graphs by a similar argument. For illumination angle $\theta = 90°$ in the ordinary mode we also find a ϕ' isotropy, as expected since the polarization vector of this mode is aligned along the magnetic axis and therefore "sees" free electrons.

Another fundamental property reflected in Fig. 6 is the absence of polarization-change scattering in the forward direction $\theta' = \theta, \phi' = 0$, i.e., $d\sigma_{ij}(\theta' = \theta, \phi' = 0) = 0$ for $i \neq j$. It is quite natural that there should be no mixing of modes in this case, since the forward scattering amplitude is related directly with the definition of the refractive index, and of the normal modes.

VI. CROSS SECTION AT THE RESONANCE

It is well known that at photon frequency $\omega = \omega_B = eB/mc$ a normal mode of the medium can be excited, and, in fact, this particular frequency is used experimentally, since it provides an efficient means for the resonant heating of the plasma. The cold-plasma polarization tensor becomes infinite at this frequency indicating the failure of this theory to provide a dissipation mechanism for the rising transverse energy of the electron. Cyclotron emission is actually such a mechanism and gives rise to the "natural" width of the line. Classically, this broadening effect is obtained by including the radiation-damping force in the electron's equation of motion, and contributes an imaginary part to the polarization tensor[16] whose eigenvalues are now

$$\pi_\pm = (1 \pm u^{1/2} + i\gamma)^{-1}, \quad \pi_\parallel = (1 + i\gamma)^{-1} \tag{23}$$

with $\gamma = \frac{2}{3}(e^2/m_e c^3)$. Quantum mechanically the same value is recovered up to relativistic corrections. It can in fact be verified that at $\omega = \omega_B$ the so corrected Thomson cross section reproduces the well-known cyclotron absorption cross section ($i = 1, 2$)

$$\sigma^i_{\rm abs} = 4\pi^2 \frac{e^2}{mc} |\hat{e}^i_-|^2 \delta(\omega - \omega_B), \tag{24}$$

if we make the usual substitution

$$\pi\delta(\omega - \omega_B) = \frac{\Gamma/2}{(\omega - \omega_B)^2 + \Gamma^2/4}, \tag{25}$$

$$\Gamma = \tfrac{4}{3}(e^2\omega^2/m_e c^3), \tag{26}$$

to account for the broadening effect, cf. Appendix B. Here e^i_- represents the polarization component which can be cyclotron absorbed (in the nonrelativistic limit[2]).

In ordinary laboratory plasmas the broadening due to thermal motion or to Coulomb collisions usually dominates over the natural broadening. In the latter case the dissipation mechanism is Coulomb bremsstrahlung resulting in $\gamma_c = \nu_c/\omega$, where the electron-ion collision frequency ν_c in the limit of quantizing magnetic fields is given by $\nu_c \cong 3 \times 10^8 n_{20} B_{12}^{-3/2}$ sec^{-1}.[17]

The thermal broadening effect is related to the fact that each velocity component [within the velocity distribution $f(v_\parallel)$ of the electrons] absorbs a Doppler-shifted frequency $\omega(\theta) = \omega_B(1 + v_\parallel \cos\theta/c)$. This results (nonrelativistically) in a thermal spread,

$$\delta\omega/\omega = c^{-1}\delta v_{\rm th}\cos\theta,$$

which is distinctly anisotropic reflecting the one-dimensional character of the electron's mobility. For $kT \ll \hbar\omega_B$ a rough expression for the cross section is found as

$$\frac{\sigma_i}{\sigma_{\rm Th}} = \pi\gamma^{-1}\omega \int dv f(v) |e^i_-|^2 \delta(\omega - \omega_B)$$

$$\cong \frac{3\pi mc^3}{2e^2\omega_B} \frac{cf(v)|e^i_-(\omega)|^2}{|\cos\theta - v/c|}, \tag{27}$$

where $v = v_\parallel(\omega)$ is the velocity of an electron that can (cyclotron) absorb the frequency ω according to the Doppler law. The denominator of this expression contains the lowest-order relativistic correction to $d\omega/dv$ at $\theta \cong \pi/2$, cf. Daugherty and Ventura.[8]

A word is due here on the structure and meaning of the polarization modes and of the refractive index near the resonance. From Eqs. (9), (10), and (23) the resonant term in n^2 (including damping) is

$$(n_j^2)_{\rm res} = \frac{(w/\gamma)|\hat{e}^j_-|^2}{x + i}$$

$$= -\frac{w}{\gamma}|\hat{e}^j_-|^2\left(\frac{x}{x^2+1} - \frac{i}{x^2+1}\right), \tag{28}$$

$$x \equiv (1 - u^{1/2})/\gamma.$$

As a function of u, Eq. (28) displays the typical resonance behavior whereby the real part changes from positive to negative values at $u = 1$ while the imaginary part has a maximum value given by $(w/\gamma)|\hat{e}_-|^2|_{u=1}$. The first important feature to note is that quite generally $n^2=1+O(w/\gamma)$, where $(w/\gamma)\cong 5\times 10^{-8}N_{20}\omega_{20}^{-3}$ represents a negligible effect in the accretion column of a magnetic neutron star, but can be significant in the case of a white dwarf. In our numerical evaluation (Fig. 2) this effect was completely neglected.

Another noteworthy feature is that only the extraordinary mode is resonant since the appropriate polarization component for the ordinary mode vanishes, i.e., $\hat{e}_-^1(u=1)=0$, as is easily checked from Eq. (10). [This happens because of the property $\alpha(\theta)=-\cos\theta$ at $u=1$.] The longitudinal component is also easily evaluated from Eq. (6b) giving at $u=1$

$$\lambda_1(\theta)=0,$$

$$\lambda_2(\theta)=\frac{\sin\theta(\cos^2\theta+1)}{\sin^2\theta-2i\gamma/w}\,.$$

In the limit of low density this is also of order w/γ and is again neglected in our numerical evaluation.

VII. DISCUSSION

Comment on the single-scatterer approach: We have tacitly assumed throughout this paper that the scattered intensity from a unit volume of the plasma is found as the incoherent sum of the amplitudes squared resulting from the radiation of individual particles within this volume. This treatment is justified at very high frequencies whose wavelength is much less than the mean interparticle separation.[18] In the opposite limit a statistical approach becomes necessary, since the wave no longer scatters from individual particles but rather from density (or field) fluctuations. Equation (1) may still be used in this case if the right-hand side is multiplied by the "spectral density" of the plasma fluctuations.[19] For a fully uncorrelated gas (e.g., an ideal gas) the spectral function is identically equal to 1, and Eq. (1) is once again applicable. In the opposite extreme, however, of highly correlated media it is possible to have a drastically reduced scattering in directions other than the forward.[20]

Comment on the relativistic and quantum mechanical effects: The effect of relativity on the total cross section (in the absence of an external field) is described by the Klein-Nishina formula which at low frequencies ($h\omega \ll mc^2$) gives[12]

$$\sigma_{KN}\cong\sigma_{TH}\left(1-\frac{2h\omega}{mc^2}+\cdots\right),$$

resulting in a reduction of the radiated energy equal to the mean recoil energy absorbed by the electron. A corresponding reduction in the differential cross section is mostly concentrated at large-angle scattering as is easy to understand on the ground that the biggest recoil occurs at high photon momentum transfer. A similar effect is to be expected in the presence of an external magnetic field, when proper account is taken of the electron's bound transverse motion. This effect was verified (in the weak-field limit $\omega_B \ll \omega$) in the relativistic calculation of Milton *et al.*[21] Recent results by Herold[22] confirm this expectation for $\omega\approx\omega_H$ and also show the anticipated (angle dependent) Doppler shift of the cyclotron line.

The quantum-mechanical effect turns out to give corrections on the classical treatment, which are of the same order as the relativistic corrections so that the two effects ought to be studied simultaneously. This happens because the quantum-mechanical oscillator matrix elements involve an expansion in terms of the parameter $t=(\hbar\omega/2mc^2)\sin^2\theta$, cf. Canuto and Ventura.[2] The appearance of higher harmonics in the case of very strong fields also belongs to this combined relativistic-quantum effect.

From the formalism given in Sec. II it is further clear that the scattering mechanism and the plasma effect ought (in principle) to be treated in a self-consistent manner, i.e., in the same order of approximation. Thus, for example, in order to include the plasma effect in the relativistic calculation of Herold[22] it may prove insufficient to simply insert the classical cold-plasma modes into their expressions.

Polarization of the vacuum: The vacuum in an external field can be polarized by the presence of virtual pairs and thereby exhibit birefringent properties. The underlying physics of this interesting phenomenon has been known for some time[23] while its possible importance in pulsar physics has been emphasized more recently.[9,24] The medium in the pulsar magnetosphere is actually the combined plasma + vacuum system, and the role of the vacuum component is not negligible for magnetic fields approaching the critical value $B_{cr}=m^2c^3/e\hbar=4.4\times10^{13}$ G. Its effect is represented in the polarization tensor by an additive component of the order[24]

$$\frac{1}{15}\frac{\alpha}{\pi}\left(\frac{B}{B_{cr}}\right)^2,$$

where α is the fine-structure constant. Since the plasma-polarization term in $\epsilon_{\alpha\beta}$ is of the order ω_p^2/ω^2 its relative importance decreases at high frequencies, so that the vacuum component may actually dominate the polarization properties of the x-ray radiation from accreting pulsars. We have already noted how sensitively the photon cross sections depend on the properties of the polarization modes, and may therefore anticipate significant changes in the radiative transport when the vacuum polarization effect is included. Details on this manifestation of the vacuum birefringence have been given elsewhere.[25]

APPENDIX A: DIFFERENTIAL CROSS SECTION

(a) Semitransverse limit: Consider the propagation of a monochromatic plane wave in a medium

$$\vec{E}(\vec{r}, \omega) = \hat{e} E_0 e^{i\vec{k}\cdot\vec{r}} .$$

The motion of the individual electron is directly related to the polarization currents induced in the medium by $E(\vec{r}, \omega)$, i.e.,

$$v_\alpha = (en_e)^{-1} j_\alpha = \frac{-ie\omega}{m\omega_p^2}(\epsilon_{\alpha\beta} - \delta_{\alpha\beta})E_\beta, \tag{A1}$$

where $\epsilon_{\alpha\beta}$ is the dielectric permittivity tensor, and $\omega_p = (4\pi n_e e^2/m)^{1/2}$ is the plasma frequency. According to the classical theory this motion results in a radiation field at $\vec{R}, \omega$ (for semitransverse propagation),

$$\vec{E}_{sc}(\vec{R}, \omega) = \frac{-i\omega e}{c^2}\frac{e^{ik'R}}{R}[\hat{k}' \times (\hat{k}' \times \vec{v})]$$

$$= E_0 \frac{e^{ik'R}}{R} f_{\alpha\beta}(\hat{k}')\hat{e}_\beta, \tag{A2}$$

where the last equation serves to define the scattering amplitude $f_{\alpha\beta}(\hat{k}')$, which is a matrix in polarization space. From (A1) and (A2) follows that

$$f_{\alpha\beta} = -r_0(\delta_{\alpha\gamma} - \hat{k}'_\alpha \hat{k}'_\gamma)\Pi_{\gamma\beta} , \tag{A3}$$

with the same notation as in Eqs. (1) and (2). The cross section for scattering into a specific final polarization is then

$$\left.\frac{d\sigma}{d\Omega}\right|_{\hat{e}\hat{k} \to \hat{e}'\hat{k}'} = |(\hat{e}', \vec{f}\cdot\hat{e})|^2, \tag{A4}$$

from which we can recover Eq. (1) after making use of the transversality property of the normal modes[15]

$$(\vec{1} - \hat{k}\hat{k})^2 \cdot \hat{e} = (\vec{1} - \hat{k}\hat{k}) \cdot \hat{e} = \hat{e}_t = \hat{e} .$$

If one needs to take the quantum-mechanical effects of the electron's motion into account, one needs only to substitute $v_\alpha \to \langle v_\alpha \rangle$, i.e., by the expectation value of the velocity operator, in order to have a quantum-mechanical expression for the induced current j_α.[7,2] A similar connection between cross section and polarization tensor can be made in terms of the S matrix within the framework of quantum electrodynamics along the lines discussed recently by Hamada.[26]

(b) General treatment for anisotropic media.[14,19]: The Larmor radiation formula used in Eq. (A2) is not valid if the semitransverse limit is violated. A general expression for $\vec{E}_{sc}(\vec{R}, \omega)$ is obtained by solving the wave equation

$$\left[\frac{k^2c^2}{\omega^2}(-\delta_{\alpha\beta} + \hat{k}_\alpha \hat{k}_\beta) + \epsilon_{\alpha\beta}\right]E_\beta = -\frac{4\pi ie}{\omega} v_\alpha(\vec{k}, \omega). \tag{A5}$$

As shown by Shafranov[14] this can be accomplished by expanding $\vec{E}$ in terms of the medium normal modes, giving rise to the expression

$$\vec{E}_{sc}(\vec{R}, \omega) = \lim_{\eta \to 0} \sum_{l=1} \int \frac{4\pi ie}{\omega}\frac{d^3k}{(2\pi)^3} \times \frac{\hat{e}^l(\hat{e}^l \cdot \vec{v})e^{i\vec{k}\cdot\vec{R}}}{k^2c^2/\omega^2 - n_l^2 - i\eta} . \tag{A6}$$

In the semintransverse limit $n_l^2 = 1$ and the integral over the poles in (A6) may be evaluated in a straightforward manner recovering Eq. (A2). When $n_l^2(\theta)$ has a strong dependence on the angle of propagation, however, one cannot expect spherical waves at infinity as in Eq. (A2) since these are no longer solutions of the homogeneous wave equation. Equation (A6) thus represents some appropriate wave form at infinity corresponding to the anisotropic propagation properties of the normal modes. While the integration of (A6) itself presents some ambiguity one may still obtain a simple result for the differential cross section. Following an argument presented in Sitenko's monograph[19] we thus find

$$\frac{d\sigma}{d\Omega} = \frac{dI/d\Omega}{F_i}, \tag{A7}$$

where F_i is the incident Poynting flux and the radiated intensity is found from the relation

$$I = \int d\Omega \frac{dI}{d\Omega} = -\frac{e}{2}\mathrm{Re}\int d^3r\, \vec{E}_{sc}(\vec{r}, \omega)\cdot \vec{v}^{\,\mathrm{inc}}(\vec{r}, \omega)$$

$$= \frac{e}{2}\mathrm{Re}\int \frac{d^3k}{(2\pi)^3}(\vec{E}_{sc}\cdot \vec{v}_k^*)$$

$$= \frac{e^2/\omega}{(2\pi)^3}\mathrm{Im}\int \frac{d^3k\,|\hat{e}^l \cdot \vec{v}_k^*|^2}{k^2c^2/\omega^2 - n_l^2 - i\eta}. \tag{A8}$$

The dk integration is now carried through via the prescription

$$\mathrm{Im}\frac{1}{k^2c^2/\omega^2 - n_l^2 - i\eta} = i\pi\frac{\omega^2}{c^2}\delta(k^2 - \omega^2 n_l^2/c^2),$$

and we find from (A1) and (A7) our final expression for the radiated cross section

$$\frac{d\sigma^i}{d\Omega} = r_0^2\frac{k'}{k}\sum_j |(\hat{e}^j, \vec{\Pi}\cdot\hat{e}^i)|^2. \tag{A9}$$

The expression used for the incident flux in (A7) is

$$F_i = \frac{cn_i}{8\pi} |E_0|^2 |\hat{e}^i_t|^2 = \frac{cn_i}{8\pi} |E_0|^2 .$$

APPENDIX B: TOTAL CROSS SECTION

A completely general expression for the total cross section may be obtained from the imaginary part of the refractive index given by Eq. (9) through the well-known relation

$$\mathrm{Im} n_i = \frac{c}{2\omega} n_e \sigma_i = \frac{\mathrm{Im} n_i{}^2}{2\,\mathrm{Re} n_i} . \tag{B1}$$

Including the radiation-damping term in Eq. (9) via Eq. (23), and employing once again the rotating-coordinates representation of the polarization vector we thus find

$$\sigma_i = \frac{\omega}{cn_e} \frac{\gamma w}{\mathrm{Re} n_i} \left[\frac{\hat{e}^{i*}_{tz} \hat{e}^i_z}{1+\gamma^2} + \cdots \right]$$
$$= \frac{\sigma_{\mathrm{Th}}}{n_i} \left[\hat{e}^{i*}_{tz} \hat{e}^i_z + \frac{\hat{e}^{i*}_{t+} \hat{e}^i_+}{(1+u^{1/2})^2} + \frac{\hat{e}^{i*}_{t-} \hat{e}^i_-}{(1-u^{1/2})^2 + \gamma^2} \right] , \tag{B2}$$

which is our generalized expression for the total cross section of a photon initially in mode i. In the semitransverse propagation limit this clearly confirms our previous result of Eqs. (17) and (18).

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