Comment on the magnetic moments of baryons in the broken-SU(6)-symmetry model

Yukio Tomozawa

Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan 48109

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It is pointed out that the consideration of the physical mass in the computation of the baryon magnetic moments in broken SU(6) symmetry allows us to choose a value of the quark mass ratio different from that chosen by De Rújula, Georgi, and Glashow and leads to a prediction significantly different from their results. Precision measurements by hyperon beams, which are planned at the Fermilab and at CERN in the near future, could differentiate between the two sets of predictions.

The concept of the combined interval SU(3) and spin symmetry—SU(6) symmetry[1]—is supported by many experimental data, such as hadron spectroscopy, mass formula, and various decay rate sum rules. In particular, a remarkable agreement of the SU(6) prediction[2] of the magnetic-moment ratio of the proton and neutron with the experimental data is a cornerstone of the success of the theory. The observed value of the magnetic moment of the Λ particle,[3] however, indicated a discrepancy with the SU(6) as well as the SU(3) prediction,[4] and prompted De Rújula, Georgi, and Glashow[5] (DRGG) to take into consideration the effect of symmetry breaking due to the quark masses. DRGG assume pointlike Dirac magnetic moments for quarks and choose the mass ratio of the up-down quarks to the strange quark to be

$$\xi = \frac{m_u}{m_s} = 0.622 \tag{1}$$

based on the mass formula for baryons. This method of determining ξ is valid in the first order to the interaction potential between quarks. Their prediction for the Λ magnetic moment agrees well with the recent measurement (see Table I). The agreement is spoiled, however, if one considers the effect of physical masses of the baryons, i.e., if one assumes that the broken-SU(6) prediction is to be used for a nondimensional quantity and therefore the product (magnetic moment) $\times$ (baryon mass) is to be compared with the symmetry prediction (assumption A). Besides, the mass ratio (1) is not well established since the validity of the perturbation calculation in the mass formula which led to Eq. (1) is not unquestionable. Instead, one may use the ratio of the vector-meson masses as a guide to the value of the parameter

$$\xi \cong \frac{m_\omega}{m_\phi} = 0.77 \tag{2}$$

since ϕ and ω (or ρ) are pure $\bar{s}s$ and $\bar{u}u \pm \bar{d}d$ states, respectively. Equation (2) would be a good approximation for the parameter $\xi \equiv m_u/m_s$ if the binding energies are small compared with the quark masses or are proportional to the masses of the constituent quarks.

In fact, the combination of the choice of the parameter (2) and the assumption A lead to a satisfactory prediction for the Λ magnetic moment. These assumptions, then, will give a result which is significantly different from that of DRGG. In view of the accurate measurement of the magnetic moments of hyperons in progress, these predictions can be tested by the observation in the very near future.

Following DRGG, the magnetic-moment operator transforms as

$$\vec{\mu} = \frac{Q_q}{2m_q c}\vec{\sigma} = \frac{\tilde{Q}}{2m_u c}\vec{\sigma}, \tag{3}$$

where the charge operator $\tilde{Q}$ represented in the quark space has the form

$$\tilde{Q} = \begin{pmatrix} \frac{2}{3} & & \\ & -\frac{1}{3} & \\ & & -\frac{1}{3}\xi \end{pmatrix} = a\lambda_3 + b\lambda_8 + c\lambda_0 \tag{4}$$

and

$$\lambda_3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix},$$

$$\lambda_0 = \sqrt{\tfrac{2}{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}, \tag{5}$$

with

TABLE I. The baryon magnetic moments in nuclear magneton units. (The underlined quantities are inputs.)

	DRGG ($\xi = 0.622$) or bag model	Mass correction ($\xi = 0.77$)	Experiment[a]
p	$\underline{2.793}$	$\underline{2.793}$	2.793
n	-1.86	$\underline{-1.86}$	-1.913
Λ	-0.58	-0.60	-0.606 ± 0.034
Σ^+	2.67	2.15	2.83 ± 0.25
Σ^0	0.81	0.68	
Σ^-	-1.05	-0.79	-1.48 ± 0.37
Ξ^0	-1.39	-1.12	
Ξ^-	-0.46	-0.46	-1.85 ± 0.75
$(\Lambda\Sigma^0)$	1.61	1.31	$1.82^{+0.25}_{-0.18}$[b]
Ω^-	-1.29	-0.90	

[a]Ref. 2.
[b]Ref. 9.

$$a = \tfrac{1}{2},$$
$$b = (1/6\sqrt{3})(1+2\xi), \qquad (6)$$
$$c = (1/3\sqrt{6})(1-\xi).$$

The basic idea of the computation is to assume that the operator in Eqs. (3)–(6) is a member of the $\underline{35}$ representation of the SU(6) group and the baryon octet is a member of $\underline{56}$. Using the SU(6) isoscalar Clebsch-Gordan (CG) factor,[6]

$$\left(\begin{array}{cc|c} 35 & 56 & 56 \\ 8^3 & 8^2 & 8^2 \end{array}\right)_s = -\frac{\sqrt{2}}{3}, \quad \left(\begin{array}{cc|c} 35 & 56 & 56 \\ 8^3 & 8^2 & 8^2 \end{array}\right)_a = \frac{2\sqrt{2}}{3\sqrt{5}},$$
$$\left(\begin{array}{cc|c} 35 & 56 & 56 \\ 1^3 & 8^2 & 8^2 \end{array}\right) = -\frac{1}{3\sqrt{5}}; \qquad (7)$$
$$\left(\begin{array}{cc|c} 35 & 56 & 56 \\ 8^3 & 10^4 & 10^4 \end{array}\right) = \tfrac{2}{3}, \quad \left(\begin{array}{cc|c} 35 & 56 & 56 \\ 1^3 & 10^4 & 10^4 \end{array}\right) = -\frac{2}{\sqrt{15}}$$

and the SU(3) CG coefficients,[7] all the CG factors of the matrix element $\langle \text{baryon} | \lambda_i | \text{baryon} \rangle$ ($\lambda_i = 3, 8, 0$) and $\langle \text{baryon} | \vec{\mu} | \text{baryon} \rangle$ are tabulated in Table II. The computation of DRGG with $\xi = 0.622$ and the mass-corrected value (assumption A) with $\xi = 0.77$ are given in Table I. The prediction by the bag model[8] is identical to that of DRGG. In general, the mass correction reduces the value of magnetic moments up to ~30% except that of Ξ^- which is nearly equal to the DRGG prediction. It is pertinent to point out a basic difference between the two calculations at this point. While the DRGG consider the unequal-mass quarks to be noninteracting in computing the magnetic moments, our treatment is based on the transformation properties of the magnetic moment operator and thus interaction between quarks is allowed. Therefore, in the approach of DRGG, it is natural to assume that the computed result represents the observed quantity without the observed-mass correction. In our approach, however, it is more natural to treat the physical mass of the baryon to be taken out from the symmetry consideration.

A comparison with the experimental data may indicate that the DRGG prediction seems closer to reality than that of the present article, but large errors in the present experiments do not allow definite conclusion. The recent program in the precision measurement of the hyperon magnetic moment will shed some light on the problem.

TABLE II. The matrix elements $\langle B | \lambda_i | B \rangle$.

	λ_3	λ_8	λ_0	$\langle B \vert \vec{\mu} \vert B \rangle / \langle p \vert \vec{\mu} \vert p \rangle$
p	$-\frac{5}{3}\frac{1}{\sqrt{30}}$	$-\frac{1}{3}\frac{1}{\sqrt{10}}$	$-\frac{1}{3\sqrt{5}}$	1
n	$\frac{5}{3}\frac{1}{\sqrt{30}}$	$-\frac{1}{3}\frac{1}{\sqrt{10}}$	$-\frac{1}{3\sqrt{5}}$	$-\frac{2}{3}$
Λ	0	$\frac{2}{3}\frac{1}{\sqrt{10}}$	$-\frac{1}{3\sqrt{5}}$	$-\frac{1}{3}\xi$
Σ^+	$-\frac{4}{3}\frac{1}{\sqrt{30}}$	$-\frac{2}{3}\frac{1}{\sqrt{10}}$	$-\frac{1}{3\sqrt{5}}$	$\frac{1}{9}(8+\xi)$
Σ^0	0	$-\frac{2}{3}\frac{1}{\sqrt{10}}$	$-\frac{1}{3\sqrt{5}}$	$\frac{1}{9}(2+\xi)$
Σ^-	$\frac{4}{3}\frac{1}{\sqrt{30}}$	$-\frac{2}{3}\frac{1}{\sqrt{10}}$	$-\frac{1}{3\sqrt{5}}$	$-\frac{1}{9}(4-\xi)$
Ξ^0	$\frac{1}{3}\frac{1}{\sqrt{30}}$	$\frac{1}{\sqrt{10}}$	$-\frac{1}{3\sqrt{5}}$	$-\frac{2}{9}(2\xi+1)$
Ξ^-	$-\frac{1}{3}\frac{1}{\sqrt{30}}$	$\frac{1}{\sqrt{10}}$	$-\frac{1}{3\sqrt{5}}$	$-\frac{1}{9}(4\xi-1)$
$(\Lambda\Sigma^0)$	$\frac{2}{3}\frac{1}{\sqrt{10}}$	0	0	$\frac{1}{\sqrt{3}}$
Ω^-	0	$\frac{\sqrt{2}}{3}$	$-\frac{1}{3}$	$-\frac{\sqrt{5}}{3}\xi$

TABLE III. The baryon magnetic moments in nuclear magneton units for broken SU(6) with $m_u \pm m_d$. (The underlined quantities are inputs.)

	Matrix element	DRGG or bag model ($\lambda = 1.050$, $\xi = 0.628$)	Mass correction ($\lambda = 1.052$, $\xi = 0.744$)
p	$\frac{1}{9}(8+\lambda)$	$\underline{2.793}$	$\underline{2.793}$
n	$-\frac{2}{9}(1+2\lambda)$	$\underline{-1.913}$	$\underline{-1.913}$
Λ	$-\frac{1}{3}\xi$	$\underline{-0.61}$	$\underline{-0.61}$
Σ^+	$\frac{1}{9}(8+\xi)$	2.67	2.14
Σ^0	$\frac{1}{9}(4-2\lambda+\xi)$	0.79	0.65
Σ^-	$-\frac{1}{9}(4\lambda-\xi)$	−1.09	−0.83
Ξ^0	$-\frac{2}{9}(1+2\xi)$	−1.43	−1.13
Ξ^-	$-\frac{1}{9}(4\xi-\lambda)$	−0.49	−0.46
$(\Lambda\Sigma^0)$	$\frac{1}{9}\sqrt{3}\,(2+\lambda)$	1.42	1.21
Ω^-	$-\frac{1}{3}\sqrt{5}\,\xi$	−1.36	−0.91

Finally, we will consider the effect of unequal masses of the (constituent) up and down quarks. In this case, Eqs. (4) and (6) are changed into

$$\tilde{Q}=\begin{pmatrix} \frac{2}{3} & & \\ & -\frac{1}{3}\lambda & \\ & & -\frac{1}{3}\xi \end{pmatrix}=a\lambda_3+b\lambda_8+c\lambda_0, \tag{4$'$}$$

with

$$\begin{aligned} a&=\tfrac{1}{6}(2+\lambda),\\ b&=(1/6\sqrt{3})(2-\lambda+2\xi),\\ c&=(1/3\sqrt{6})(2-\lambda-\xi), \end{aligned} \tag{6$'$}$$

where

$$\lambda=\frac{m_u}{m_d}. \tag{8}$$

The relative matrix elements of Eq. (3) are given in the first column of Table III. Eliminating the parameters, one obtains the magnetic-moment sum rules

$$\begin{aligned} \Sigma^+&=p+\tfrac{1}{15}(p+4n-5\Lambda),\\ \Sigma^0&=\tfrac{1}{3}(2p+2n-\Lambda),\\ \Sigma^-&=n+\tfrac{1}{15}(4p+n-5\Lambda),\\ \Xi^0&=\Lambda-\tfrac{1}{15}(4p+n-5\Lambda)\equiv n+\Lambda-\Sigma^-,\\ \Xi^-&=\Lambda-\tfrac{1}{15}(p+4n-5\Lambda)\equiv p+\Lambda-\Sigma^+,\\ (\Lambda\Sigma^0)&=\sqrt{3}\,\tfrac{1}{15}(4p+n-5\Lambda),\\ \Omega^-&=\sqrt{5}\,n. \end{aligned} \tag{9}$$

Here the particle symbol represents the magnetic moment of the particle in the DRGG approach and the reduced magnetic moment [=(magnetic moment) × (mass)] in the approach with mass correction. The prediction by Eq. (9) is given in Table III, with the magnetic moments of p, n, and Λ as inputs. The values of the parameters λ and ξ are

$$\lambda\equiv\frac{m_u}{m_d}=1.050,\quad \xi\equiv\frac{m_u}{m_s}=0.628$$

for DRGG and

$$\lambda=1.052,\quad \xi=0.744$$

for the reduced magnetic moments. It is interesting to point out that the up-quark mass is heavier than the down-quark mass. These masses are those of the constituent quarks and different from the current-quark masses. In the latter ξ is much smaller and one of the up- or down-quark mass could be 0.

It should be stressed that the sum rules given by Eq. (8) are identical to those of the bag-model calculation.[8]

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