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$K_1 \to \gamma\gamma$ decay rate and vector-meson-exchange contributions

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Using the most recent data on radiative decays $V \to \pi\gamma$ of vector mesons, the overall effect of ω, ρ, ϕ, and J exchange contributions to $K_1 \to \gamma\gamma$ is determined more stringently. For cutoff masses of $\Lambda = 25\mu$ and $\Lambda = 100\mu$, where μ is the pion mass, the corrections to the rate estimated, using the unitarity relation with 2π dominant state, are 1.9% and 16% respectively.

To date, there is a paucity of experimental information on the decay of the short-lived neutral kaon into two photons. The most recent experiment, done in 1973, on the decay was carried out by Barmin *et al.*[1] It establishes the branching ratio

$$\Gamma(K_S \to 2\gamma)/\Gamma(K_S \to \text{all}) < 4.0 \times 10^{-4}\ , \tag{1}$$

with 90% confidence level. The importance of the decay lies not only on the possibility of observing CP violation,[2] but also on the ground that it manifests the mechanism of higher-order weak and electromagnetic interactions.

The status of theoretical considerations is as follows:

(a) Unitarity-relation calculations[3] assuming a dominant two-pion state yield $\Gamma(K_S \to \gamma\gamma) = 2.6 \times 10^4$/sec. The Particle Data Group[4] has averaged the results of several high-precision experiments, which are compatible with each other, on the K_S mean life. The average value is $(0.8923 \pm 0.002) \times 10^{-10}$ sec. Using this in the unitarity relation calculations, the predicted branching ratio is

$$\frac{\Gamma(K_S \to \gamma\gamma)}{\Gamma(K_S \to \text{all})} = 2.53 \times 10^{-6}\ , \tag{2}$$

which is well below the experimental upper bound.

(b) Gaillard and Lee[5] found that in a free-quark model in gauge theories, $K_S \to \gamma\gamma$ is suppressed.

(c) In the exact-SU(3) limit, $K_S \to \gamma\gamma$ is forbidden by U-spin conservation.[6]

(d) Vector-meson-exchange contributions due to ω, ρ, and ϕ to the $K_S \to \gamma\gamma$ rate had been estimated by the author[7] using the then available data in the $\omega^0 \to \pi^0\gamma$, $\rho^\pm \to \pi^\pm\gamma$, and $\phi \to \pi^0\gamma$. Only the upper bounds on the last two decays were available at that time and the calculations were done assuming they were the actual rates. It was found that the contributions did not alter in any significant way the result of the unitarity relation calculations with dominant 2π intermediate state. However, since then, the rate for $\omega \to \pi^0\gamma$ has been updated,[4] the exact rates for $\rho^- \to \pi^-\gamma$,[8] $\phi \to \pi^0\gamma$,[8] $\phi \to \pi^0\gamma$,[4,9] and $J \to \pi^0\gamma$[10] have been measured.

The purpose of this short paper is to make use of the new data on the vector-meson radiative decays to determine more stringently their contributions to the $K_1 \to \gamma\gamma$ rate. We shall write K_1 instead of K_S indicating that we are assuming CP conservation in the decay.

For clarity we would like to summarize the results of Ref. 7: The Lorentz invariant matrix element M that satisfies gauge invariance and Bose statistics can be written as

$$M = H(s)[(\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2) - (\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1)]\ , \tag{3}$$

where ϵ_i and k_i refer to the polarization and momentum four-vectors, respectively, of the ith photon. Assuming the dominance of the 2π intermediate state and the interaction Lagrangians

$$L_{K\pi\pi} = \lambda\psi_K \phi_\pi^\dagger \phi_\pi\ , \tag{4}$$

$$L_{\pi\pi-\gamma\gamma} = -ieA[\phi_\pi \overleftrightarrow{\partial}_\mu \phi_\pi^\dagger] + e^2 A_\mu A^\mu \phi_\pi \phi_\pi^\dagger\ , \tag{5}$$

for the vertices in the diagram in Fig. 1, it is straightforward to write down the corresponding matrix element M_π. Using Cutkosky's rule and dispersion relation, we get

$$\text{Im}H_\pi(s) = 2\alpha(\lambda)\frac{\mu^2}{s^2}\ln\left(\frac{1+\beta}{1-\beta}\right)\ , \tag{6}$$

$$\text{Re}H_\pi(s) = 2\alpha(\lambda)\frac{\mu^2}{2\pi s}\left\{-\frac{1}{\mu^2} + \frac{1}{s}\left[\pi^2 - \ln^2\left(\frac{1+\beta}{1-\beta}\right)\right]\right\}, \tag{7}$$

where

μ = pion mass, $M_K = K_S$ mass, $S = M_K^2$,

$$\alpha = \tfrac{1}{137}\ , \quad \beta = \left[1 - \frac{4\mu^2}{S}\right]^{1/2}$$

Numerical evaluations, using updated data on $K_S \to \pi^+\pi^-$ gives

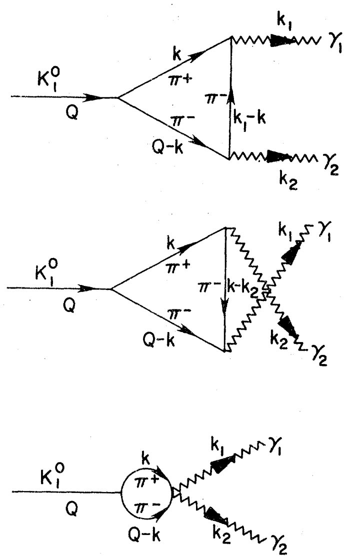

FIG. 1. The three Feynman diagrams contributing to perturbation calculations of $K_1 \to \gamma\gamma$ with π interchange. The seagull term is needed for gauge invariance.

$$\frac{\mathrm{Im} H_\pi}{(\hbar c)^2} = 2.16 \times 10^{-1} (\mathrm{cm\,MeV}^2)^{-1} \,, \tag{8}$$

$$\frac{\mathrm{Re} H_\pi}{(\hbar c)^2} = -1.23 \times 10^{-1} (\mathrm{cm\,MeV}^2)^{-1} \,, \tag{9}$$

which yields the rate

$$\Gamma_\pi(K_1 \to \gamma\gamma) = 2.26 \times 10^4/\mathrm{sec} \tag{10}$$

that agrees with previous calculations.[3]

In the calculations for the contributions of the vector mesons ω, ρ, ϕ, and J, we consider the Lagrangians

$$L_{V \to \pi\gamma} = \frac{i f_V}{m_V} \left(v^\mu \frac{\partial \phi^+}{\partial \chi^\mu} F^{\alpha\beta} \right) \epsilon_{\mu\nu\alpha\beta} + \mathrm{H.c.} \,, \tag{11}$$

and Eq. (4) in the vertices of the Feynman diagram in Fig. 2. The explicit expressions for $\mathrm{Im} H_V$ and $\mathrm{Re} H_V$ are

$$\mathrm{Im} H_V = \frac{1}{16\pi} \lambda \left(\frac{f_V}{m_V} \right)^2 \left(\frac{m_V^2}{s} \hat{V} - \beta \right), \tag{12}$$

$$\mathrm{Re} H_V = \frac{1}{16\pi} \lambda \left(\frac{f_V}{m_V} \right)^2 \frac{1}{\pi} \left[m_V^2 P(\Omega') - P(\Omega'') \right], \tag{13}$$

where[11]

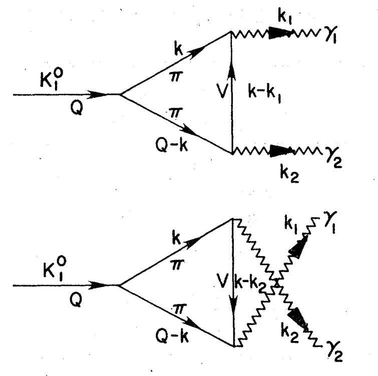

FIG. 2. The Feynman diagrams for $K_1 \to \gamma\gamma$ with vector-meson exchange in $\pi\pi \to \gamma\gamma$ scattering part.

$$P(\Omega') = \frac{1}{s} \left\{ \tfrac{1}{3}\pi^2 - \tfrac{1}{2}\left[\ln\left(\frac{1+\beta}{1-\beta} \right) \right]^2 - \tfrac{1}{2}[\ln(u_1)]^2 - \tfrac{1}{2}[\ln(u_2)]^2 + \tfrac{1}{2}\ln(u_3)]^2 - \mathrm{Li}\left(\frac{1}{u_1}\right) - \mathrm{Li}\left(\frac{1}{u_2}\right) - \mathrm{Li}\left(\frac{1}{u_3}\right) + \mathrm{Li}(u_4) + \tfrac{1}{2}\mathrm{Li}\left(\frac{1}{u_3^2}\right) - \tfrac{1}{2}\mathrm{Li}(u_4^2) \right\}, \tag{14}$$

$$P(\Omega'', \Lambda) = \ln\left(\frac{1+\beta''}{1-\beta''} \right) - \beta \ln\left(\frac{\beta''+\beta}{\beta''-\beta} \right) \,, \tag{15}$$

$$\beta = \left(1 - \frac{4\mu^2}{s} \right)^{1/2}, \quad \delta_V = 1 + \frac{2\mu^2}{(m_V^2 - \mu^2)}$$

$$\beta''(\Lambda^2) = \left(1 - \frac{4\mu^2}{\Lambda^2} \right)^{1/2}, \quad \hat{V} = \ln\left[\left(\frac{1+\beta}{1-\beta} \right) \left(\frac{\delta_V - \beta}{\delta_V + \beta} \right) \right]$$

$$u_1 = \frac{\delta_V - \beta}{1-\beta}, \quad u_2 = \frac{\delta_V + \beta}{1+\beta}$$

$$u_1 = \frac{\delta_V + \beta}{1-\beta}, \quad u_2 = \frac{\delta_V - \beta}{1+\beta} \,. \tag{16}$$

The Li functions are the dilogarithm functions; Λ is the cutoff mass. Since we have included the J particle ($m_J \approx 2.3\mu$), the cutoff values $\Lambda = 25\mu$ and $\Lambda = 100\mu$ were used in the evaluation of $\mathrm{Re} H_V(1)$ and $\mathrm{Re} H_V(2)$. The results are tabulated in Table I, where we have made use of the following data:

$$\Gamma(\omega \to \pi^0\gamma) = 8.8\%\,\Gamma(\omega \to \mathrm{all}) = 8.8\%\ (10.1\ \mathrm{MeV}) = 0.89\ \mathrm{MeV},$$

$$\Gamma(\rho^- \to \pi^-\gamma) = 35\ \mathrm{keV},$$

$$\Gamma(\phi \to \pi^0\gamma) = 0.14\%\,\Gamma(\phi \to \mathrm{all}) = 0.14\% (4.1\ \mathrm{MeV}) = 5.74\ \mathrm{keV},$$

TABLE I. Values of $\mathrm{Im}H_V/(\hbar c)^2$, $\mathrm{Re}H_V(1)/(\hbar c)^2$, and $\mathrm{Re}H_V(2)/(\hbar c)^2$ for ω, ρ, θ, J exchanges and their algebraic sums.

	$\mathrm{Im}H_V/(\hbar c)^2$ [(cm MeV^2)$^{-1}$]	$\mathrm{Re}H_V(1)/(\hbar c)^2$ [(cm MeV^2)$^{-1}$]	$\mathrm{Re}H_V(2)/(\hbar c)^2$ [(cm MeV^2)$^{-1}$]
ω	-4.81×10^{-3}	-4.03×10^{-3}	-3.96×10^{-2}
$\rho^{\pm}$	-2.89×10^{-4}	-2.17×10^{-4}	-2.38×10^{-3}
θ	-8.60×10^{-6}	$+1.96\times10^{-4}$	$+9.56\times10^{-5}$
J	-3.16×10^{-11}	$+1.29\times10^{-7}$	$+1.26\times10^{-7}$
	$\sum_V \frac{I_m H_V}{(\hbar c)^2} = -5.10\times10^{-3}$	$\sum_V \frac{\mathrm{Re}H_V(1)}{(\hbar c)^2} = -4.05\times10^{-3}$	$\sum_V \frac{\mathrm{Re}H_V(2)}{(\hbar c)^2} = -4.19\times10^{-2}$

$$\Gamma(J\to\pi^0\gamma)=7.3\times10^{-5}\,\Gamma(J\to\mathrm{all})$$
$$=7.3\times10^{-5}(0.069\ \mathrm{MeV})$$
$$=5.04\ \mathrm{eV}.$$

We note that ω contributes the most and J the least.

If we denote by H_T the sum of the vector mesons and π-exchange contributions,

$$\frac{\mathrm{Im}H_T}{(\hbar c)^2}=2.11\times10^{-1}(\mathrm{cm\,MeV})^{-1}\ , \tag{17}$$

$$\frac{\mathrm{Re}H_T(1)}{(\hbar c)^2}=-1.27\times10^{-1}\ (\mathrm{cm\,MeV})^{-1}\ , \tag{18}$$

$$\frac{\mathrm{Re}H_T(2)}{(\hbar c)^2}=-1.65\times10^{-1}\ (\mathrm{cm\,MeV})^{-1}\ , \tag{19}$$

with the corresponding rates

$$\Gamma_T(1)=2.21\times10^4/\mathrm{sec}\ , \tag{20}$$

$$\Gamma_T(2)=2.61\times10^4/\mathrm{sec}\ , \tag{21}$$

and percentage differences

$$\left|\frac{\Gamma_T(1)-\Gamma_\pi}{\Gamma_\pi}\right|=1.9\%\ , \tag{22}$$

$$\left|\frac{\Gamma_T(2)-\Gamma_\pi}{\Gamma_\pi}\right|=16\%\ . \tag{23}$$

Therefore, the overall effect of the vector contributions is to change the rate due to π exchange alone by 1.9% and 16% for cutoff masses $\Lambda=25\mu$ and $\Lambda=100\mu$, respectively.

[1]V. V. Barmin *et al.*, Phys. Lett. 47B, 463 (1973).

[2]N. Christ and T. D. Lee, Phys. Rev. D 4, 209 (1971); B. R. Martin, E. de Rafael, J. Smith, and Z. Uy, *ibid.* 4, 913 (1971).

[3]B. R. Martin, E. de Rafael, and J. Smith, Phys. Rev. D 2, 179 (1970); F. Y. Yndurain, Prog. Theor. Phys. 46, 990 (1971); Y. Kohara, *ibid.* 48, 261 (1972); R. L. Goble, Phys. Rev. D 7, 937 (1973).

[4]Particle Data Group, Phys. Lett. 75B, 1 (1978).

[5]M. K. Gaillard and B. W. Lee, Phys. Rev. D 10, 897 (1974).

[6]M. Gourdin, *Unitary Symmetries* (North-Holland, Amsterdam, 1967); H. Lipkin, *Lie Groups for Pedestrians* (North-Holland, Amsterdam, 1965), 2nd edition.

[7]Z. E. S. Uy, Phys. Rev. D 3, 234 (1971).

[8]B. Gobbi, J. L. Rosen, H. A. Scott, S. L. Shapiro, L. Strawczynski, and C. M. Meltzer, Phys. Rev. Lett. 33, 1450 (1974).

[9]G. Cosme *et al.*, Phys. Lett. 63B, 352 (1976).

[10]DASP Collaboration, W. Braunschweig *et al.*, Phys. Lett. 67B, 243 (1977); 67B, 246 (1977); S. Yamada, in *Proceedings of the Symposium on Lepton and Photon Interactions at High Energies, Hamburg, 1977,* edited by L. Gutbrod (DESY, Hamburg, 1977).

[11]P indicates the principal value and the Ω's are integrals;

$$\Omega'=\int_{4\mu^2}^{\infty} ds'\frac{\hat{V}(s')}{s'(s'-s)},\quad \Omega''(s,\Lambda^2)=\int_{4\mu^2}^{\Lambda^2} ds'\,\frac{\beta(s')}{(s'-s)},$$

The expression in Eq. (14) is the same as Eq. (16) of Ref. 7.