## Comments and Addenda

*The section Comments and Addenda is for short communications which are not appropriate for regular articles. It includes only the following types of communications: (1) Comments on papers previously published in* **The Physical Review** *or* **Physical Review Letters**. *(2) Addenda to papers previously published in* **The Physical Review** *or* **Physical Review Letters**, *in which the additional information can be presented without the need for writing a complete article. Manuscripts intended for this section must be accompanied by a brief abstract for information-retrieval purposes. Accepted manuscripts follow the same publication schedule as articles in this journal, and page proofs are sent to authors.*

# Neutrino counting in $e^+e^-$ collisions

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(Received 26 October 1978)

The possibility of counting the number of neutrino types in $e^+e^-\to\gamma\nu\bar{\nu}$ is reexamined by taking into account effects of the $Z$ pole.

In a recent Letter,[1] Ma and Okada suggest studying the reaction $e^+e^-\to\gamma\nu\bar{\nu}$ as a way of counting the number of neutrino types. From their discussion of the various backgrounds to this reaction, it appears that $\gamma\nu\bar{\nu}$ detection has a reasonable signal-to-noise ratio.

Spurred by the prospect of a future LEP machine,[2] we reexamined this process at these high energies, taking into account the presence of the $Z$ particle, which couples directly to the $\nu$ pair and which could lead to an important enhancement. In the limit of infinitely large $Z$ mass, we obtain the four-fermion result, but disagreement with the formula of Ma and Okada emerges. Their expression contains a term which becomes very singular near the forward and backward direction for the detected photon ($\cos\theta_\gamma = y = \pm 1$). We have redone their calculation[3] and do not obtain this term.

In our calculation, we consider the Feynman diagrams of Fig. 1. In the $t$-channel diagrams [(c)–(e)], one cannot reach the $W$ pole, and for energies of the order of the $W$ mass, we expect a depression of not more than a factor of 2 for these diagrams with respect to the point coupling limit. For diagrams (a) and (b), the situation is very different: With sufficiently high energies one may actually reach the $Z$ pole. For these reasons, we take the $W$ mass to be infinite, but keep the $Z$ mass at the current Weinberg-Salam value of 90 GeV.

Using the same notation as Ma and Okada, we find for the cross section, neglecting the electron mass $m$

$$\frac{d\sigma}{dx\,dy}=\frac{G_F^2\alpha}{6\pi^2}\left\{\frac{M_Z^4\{N_\nu(g_V^2+g_A^2)+2(g_V+g_A)[1-s(1-x)/M_Z^2]\}}{[s(1-x)-M_Z^2]^2+M_Z^2\Gamma_Z^2}+2\right\}$$
$$\times\frac{s}{x(1-y^2)}[(1-x)(1-\tfrac{1}{2}x)^2+\tfrac{1}{4}x^2(1-x)y^2]\,, \qquad (1)$$

where $x=E_\gamma/E$, $s=4E^2$, and $N_\nu$ is the total number of neutrino types. In terms of the Weinberg angle we have

$$g_V=\tfrac{1}{2}+2\sin^2\theta_W\,,\quad g_A=-\tfrac{1}{2} \qquad (2)$$

and the total $Z$ width is

$$\Gamma_Z=\frac{M_Z^3G_F}{12\pi\sqrt{2}}(21+N_\nu-48\sin^2\theta_W+64\sin^4\theta_W)\,, \qquad (3)$$

where we have assumed that the $Z$ can decay into

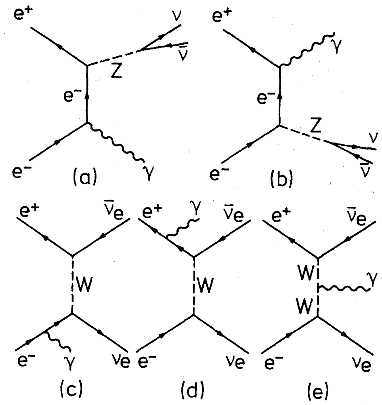


FIG. 1. The Feynman diagrams contributing to the process $e^+e^- \to \gamma\nu\bar{\nu}$.

six quarks, three charged leptons, and $N_\nu$ neutrinos. We take $\sin^2\theta_W = 0.22$.

The photon spectrum shows strong peaking near $x = 0$ and also for $|y|$ close to 1, but the validity of formula (1) is limited to $1 - |y| > m^2/E^2$.

To get an idea about the counting rate one may expect, we integrate this differential cross section over the range $-\frac{1}{2} < y < \frac{1}{2}$, $0.2 < x < 1$, which corresponds to standard photon detection setups, and which excludes the peaking regions. We present the resulting cross section as a function of the energy for $N_\nu = 3$, 10, 100, and 1000 in Fig. 2.

Increasing the number of neutrinos essentially increases the cross-section both below and above the $Z$ peak. It also broadens this peak. For PETRA and PEP energies, it seems very unlikely that the process can be measured, even for $N_\nu = 1000$. However, for LEP energies, events of this type may well be observed.

In principle, the direct measurement of $\Gamma(Z \to \text{all})$ and $\Gamma(Z \to \text{visible})$ also determines the number of neutrinos, but a very accurate determination of both quantities is required.

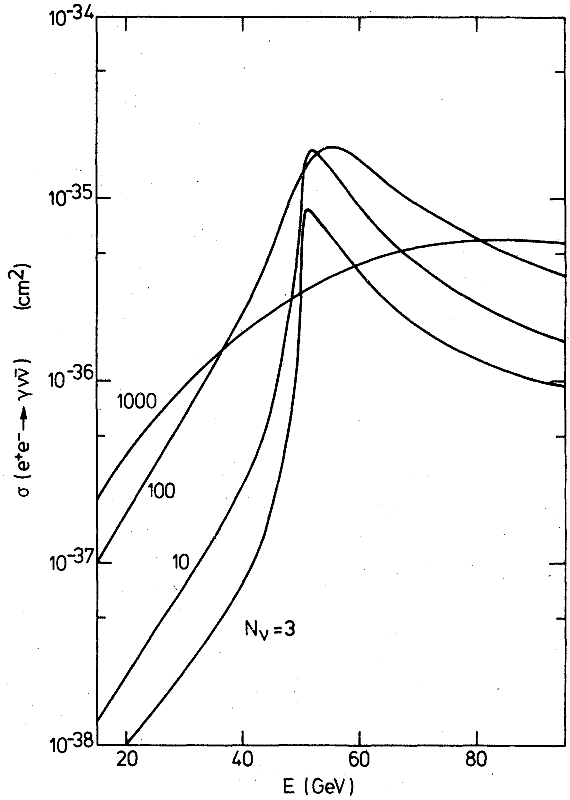


FIG. 2. The cross section for $e^+e^- \to \gamma\nu\bar{\nu}$ as a function of energy for different values of $N_\nu$, the total number of neutrino types.

Our interest in this problem was aroused during the 1978 LEP Summer Study in Les Houches by the numerous discussions with the other participants. The work of R.G. was supported in part by the Nationaal Fonds voor Wetenschappelijk Onderzoek, Belgium.

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[1]L. Ma and J. Okada, Phys. Rev. Lett. 41, 287 (1978).

[2]L. Camilleri *et al.*, CERN Report No. CERN 76–18, 1976 (unpublished).

[3]Use was made of the algebraic manipulation program REDUCE, A. C. Hearn, REDUCE-2 User's Manual, Utah (1973) (unpublished).