Elementary-particle symmetries in relativistic many-body theory at finite temperature and density

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The consequences of a phase transition associated with symmetry restoration to $SU(2)\times SU(2)$ in nuclear matter are investigated. The changes in the mass spectrum due to the phase transition (a) at zero temperature and high nuclear density, and (b) at high temperature with zero nuclear chemical potential are evaluated in the σ model of particle physics. The experimentally observable effects necessitate the measurement of current correlation functions. In this paper we consider the vector–vector–axial-vector (*VVA*) and the vector-vector-pseudoscalar (*VVP*) current correlation functions. The relation between the *VVA* and *VVP* correlation functions is obtained and it is shown that the Adler-Bell-Jackiw anomaly in the divergence of the axial-vector current remains unaltered in the nuclear medium, even though the *VVA* and the *VVP* correlation functions have additional contributions from processes specific to the many-particle system. The *VVP* correlation function is related to the neutral-pion decay amplitude. The changes in the decay rate of $\pi^0\rightarrow 2\gamma$ in the nuclear medium are evaluated by including the effects of changes in the mass spectrum of particles, and by using the cutting rules of many-body field theory for the real and imaginary parts of the amplitude. The photon in the nuclear medium is dressed by polarization effects and propagates as a plasmon. This effect is taken into account by evaluating the plasmon frequency or the plasmon mass. The changes in the mass spectrum due to symmetry restoration affect the decay rate of $\pi^0\rightarrow 2\gamma$ by at least 2 orders of magnitude and these results are tabulated. It is suggested that the Primakoff effect might provide the signal for the presence of the "abnormal nuclear matter" phase.

I. INTRODUCTION

The progress in elementary-particle physics over the past two decades has been in the understanding of the symmetries exhibited by particle spectra and by particle scattering amplitudes. Given that the central reason for the thrust toward high-energy research was the hope for a better understanding of nuclear-matter theory, it is now important to employ the techniques and insights gained in high-energy physics for the study of the properties of elementary particles in the aggregate. We are aware that we should expect new properties for particles in bulk. The dispersion curves for these "excitations" differ from the free-particle situation and the effective mass differs from the free mass due to renormalization effects produced by the medium. Furthermore, changes in the collective behavior and in transport properties, as exemplified by metallic superconductivity,[1] could also occur.

The nuclear potentials do have attractive components so that superconducting states[2] may be expected in nuclear matter. There are other special features which may be possible in nuclear matter. Recently, T. D. Lee[3] considered the possibility of "abnormal nuclear states" in nuclear matter in which at very high density the nucleons revert to a massless phase called the "abnormal phase." Yet another possibility, proposed independently by Migdal[4] and by Sawyer,[5] is the existence of pion condensation in dense nuclear matter. All of these effects depend not only on the density of nuclear matter but also on the temperature of the ensemble. In other words, elementary-particle physics provides us with information at just one point on a density-versus-temperature plot for the system, and a development along the lines of many-body effects for elementary particles stemming from the application of statistical mechanics to this area is possible. This general approach has applications not only to nuclear-matter theory but also to astrophysics—since stellar cores and neutron stars are composed of nuclear matter—and to multiparticle production in high-energy collisions in the context of statistical models or the hydrodynamical model of Landau[6] for particle production.

The possibility of symmetry changes as a function of temperature in gauge theories of weak and electromagnetic interactions was proposed by Kirzhnits and Linde[7] and was investigated by Weinberg.[8] The symmetries of particle spectra are considered to be spontaneously broken symmetries since the mass spectra typically exhibit lower symmetries than are implied by the dynamics of scattering amplitudes, and it was suggested that at high temperatures the spectra change to multiplet structures corresponding to a higher symmetry. This would affect the cooling rates of stellar matter since, for example, certain decays which now take place could be energetically disallowed at higher temperatures, and vice versa.

At present, observable consequences of such phase changes and the resultant changes in the excitation spectra have not been fully explored. It is therefore desirable to investigate the response of the (many-particle) nuclear medium to external disturbances using current correlation functions. In the present paper we consider the process $\pi^0 \to 2\gamma$ in the nuclear medium.

The neutral pion decays primarily into two photons through polarization effects in the medium. In the vacuum, the process proceeds via the pion transforming into a virtual charged-particle–antiparticle pair which then annihilates into the two photons. In the nuclear medium with protons present, the conversion of π^0 into 2γ is enhanced by new physical effects related to the presence of the medium: A proton from the Fermi distribution of the nucleons in nuclear matter could absorb the π^0 and be excited above the Fermi surface, emit two photons, and then return to the Fermi sea. This may be viewed as a kinematical constraint on the available phase space for the intermediate particles (arising from the Pauli principle). There is, in addition, the dynamical effect of the changes in the mass spectrum of the intermediate particles due to phase changes in nuclear matter at high temperature and density. Thus, sharp changes in the lifetime of the pion would signal the existence of changes in the phase of nuclear matter. It is of interest to indicate the formal analogy with positron-annihilation experiments on metals[9] where the lifetime of the positron(ium) in the metal is dependent on the Fermi distribution of electrons in the metal, and such experiments provide information about the many-electron system, through correlation measurements on the photons escaping the metal.

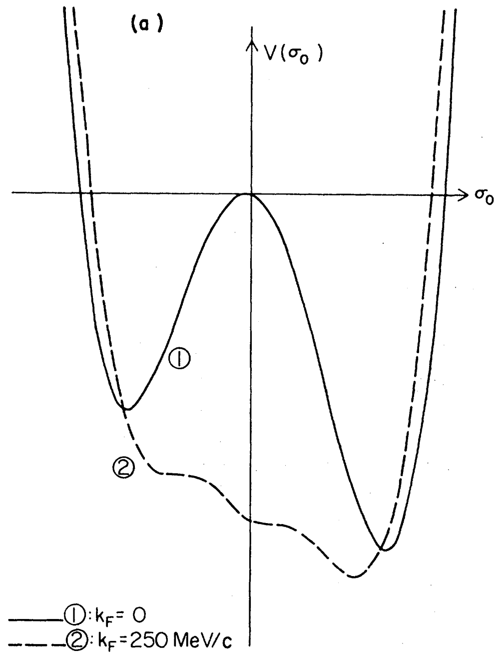
The photons emitted in pion decay will be dressed by polarization effects in the charged nuclear medium. They will therefore propagate as transversely polarized plasmons, representing the collective response of the medium to the electromagnetic disturbance. We have taken these effects into consideration while focusing attention on a single π^0 in the medium. We have reported the results for the decay widths of the π^0 as a function of nuclear density.[10] Here we include the effect of temperature on $\pi^0 \to 2\gamma$ and the details of our calculation. We find that the decay width of the pion is substantially increased above the critical temperature or critical density.

The decay width of the neutral pion is experimentally measured by the "Primakoff effect" in which π^0 production is measured at near forward directions.[11] The results of our calculation suggest that if the incident photon is capable of depositing very high energy within the nucleus, it might be feasible to detect changes of phase in nuclear matter through measurement of the lifetime of the pion, in view of the substantial change in this quantity.

In Sec. II we describe the σ model of particle physics which provides the framework for the consideration of phase changes in nuclear matter. In Sec. III the nuclear plasmon effect is calculated. This is for the sake of completeness and also to indicate some general features of interest. In Sec. IV we consider the triangle graph for $\pi^0 \to 2\gamma$. The decay amplitude as calculated in the nuclear medium differs from that in the vacuum due to the many-particle system affecting the phase space of the intermediate state and due to the collective behavior of the medium leading to a changed mass spectrum for the "elementary particles" in the medium. The numerical results for the decay width of the pion are given in this section, and concluding remarks are reserved for Sec. V.

II. THE MODEL

We first note that the idea of spontaneous symmetry breaking has its roots in many-body theory and most models in particle physics are variants of the Landau-Ginzburg theory of superconductivity. Hence it is all the more interesting to study the consequences of the spontaneously broken symmetries for the many-(elementary)-particle systems. To emphasize the parallel we note that the presence of a nonzero vacuum expectation value for fields is the abstraction of the idea from many-body theory of the existence of an order parameter describing a condensate in a many-body system. The generation of masses by the presence of a nonzero classical component to the field in particle physics is easily understood in terms of the many-body counterpart of the interaction with the condensate leading to modifications in the mass spectrum. Nambu[12] and Schwinger[13] were the first to recognize the importance of such a mechanism for the dynamic generation of masses for elementary particles. In elementary-particle physics the device of having a large number of "condensate particles" in the ground state is an inconvenience which is dispensed with and the vacuum is considered to be the ground state; and with spontaneous symmetry breaking the symmetry of the vacuum is reduced. The advantage of such a mechanism,



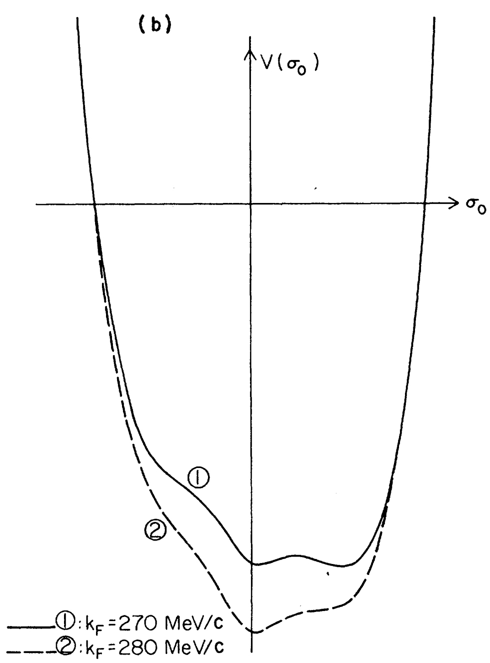


FIG. 1. The effective potential $V(\sigma_0)$ as a function of the order parameter σ_0, for Fermi momenta (a) $k_F = 0$ MeV/c and $k_F = 250$ MeV/c, (b) $k_F = 270$ MeV/c and $k_F = 280$ MeV/c, and (c) $k_F = 300$ MeV/c and $k_F = 350$ MeV/c.

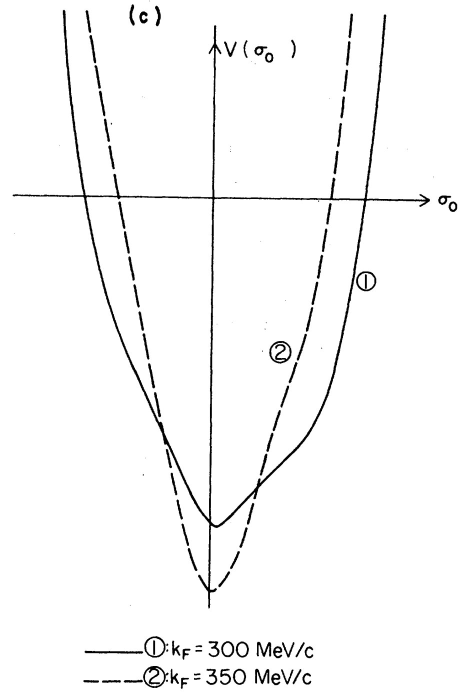


FIG. 1. (*Continued*)

when it does reflect the observed particle spectrum, is that a higher symmetry is built into the problem right from the beginning. This leads to predictions based on the higher symmetry for the particle spectra, for interaction coupling constants, relations among particle decay widths, and relations among the cross sections for various processes.

For nuclear theory a convenient model is one having nucleons and pions and SU(2) isospin symmetry as the minimal ingredients of the theory. The chirality in weak interactions suggest that the symmetry be extended to chiral SU(2)×SU(2) symmetry. A simple realization of this symmetry is given by the σ model[14] in which the nucleons are classified into the $(\frac{1}{2}, 0) + (0, \frac{1}{2})$ representation of the symmetry group and the pion is put into a $(\frac{1}{2}, \frac{1}{2})$ representation with a scalar, isoscalar σ meson. The σ model incorporates all the basic ideas of spontaneous symmetry breaking and reproduces the results of current algebra. This renormalizable model has been used as a testing ground for many new ideas in strong-interaction physics.

The model Lagrangian density, before spontaneous symmetry breaking is permitted, is given by

$$\mathcal{L} = \tfrac{1}{2}[(\partial_\mu \sigma)^2 + (\partial_\mu \vec{\pi})^2] - \frac{\mu_0^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4!}(\sigma^2 + \vec{\pi}^2)^2 + \overline{\Psi}_N (i\gamma_\mu \partial^\mu)\Psi_N - g\overline{\Psi}_N(\sigma - i\gamma_5 \vec{\tau} \cdot \vec{\pi})\Psi_N + C\sigma, \tag{1}$$

where all the terms except $C\sigma$ are invariant under chiral-SU(2) transformations. The σ field is now allowed to have a nonzero vacuum expectation value $\sigma_0 = \langle 0|\sigma|0\rangle$, which is the order parameter in the spontaneously broken symmetry phase. The spontaneous symmetry breaking generates the nucleon mass and the explicit symmetry-breaking term $C\sigma$ in (1) ensures a nonzero mass for the pions which are the Goldstone bosons in the system. The pion mass is given by (C/σ_0), where $C = F_\pi m_\pi^2$ is determined by the excitation spectrum in the absence of the nuclear many particle system. The parameters in (1) are given by

$$\lambda = (3/F_\pi^2)(m_\sigma^2 - m_\pi^2),$$
$$\mu_0^2 = -\tfrac{1}{2}(m_\sigma^2 - 3m_\pi^2). \tag{2}$$

We use $m_\sigma \simeq 850$ MeV. The nucleon mass is $M_N = g\sigma_0$. The order parameter σ_0 has the value $F_\pi = 94$ MeV at zero temperature and density, and we let $g = 10$ in the model studies to follow. This theory, which is well understood at zero temperature and density, is extended to the entire positive quadrant of the temperature-versus-density plane, for the many-particle system. The matrix elements of various operators are then defined by taking thermal averages over a grand canonical ensemble. Correspondingly, the order parameter is temperature and density dependent.[15] At high temperature the thermal fluctuations in the system reduce the order parameter to nearly zero. Similarly, at high density with zero temperature, the nucleons being fermions have high kinetic energy, and again this leads to the removal of spontaneous symmetry breaking. The excitation spectrum is then altered such that the π and σ become nearly degenerate and the nucleons become nearly massless.

We shall consider two specific circumstances: the problem of symmetry restoration with (a) increasing nucleon density at zero temperature, and (b) increasing temperature with the nucleons chemical potential μ_F being zero at all temperatures. The calculation proceeds by evaluating the minima of the effective potential $V(\sigma_0)$ as the nucleon density, or the temperature, is increased from zero.[15]

Let us consider the calculation at finite density. The presence of the explicit symmetry-breaking term $C\sigma$ in (1) leads to an asymmetric effective potential with the absolute minimum being the one chosen by the system for its ground-state energy. It has been shown by Lee[3] that there is a first-order phase transition for nuclear matter at a critical Fermi momentum $k_{F,c}$. The order parameter corresponding to the minimum of $V(\sigma_0)$ decreases discontinuously as k_F is increased beyond $k_{F,c}$. This feature is illustrated in Figs. 1(a)–1(c).

The minima of the effective potential at the one-loop level are given by the solutions of[15]

$$\sigma_0\left(\mu_0^2 + \frac{\lambda\sigma_0^2}{6} - \frac{C}{\sigma_0}\right) + \frac{\lambda\sigma_0}{2}\int \frac{d^3k}{(2\pi)^3}\left(\frac{n_B(E_\sigma)}{E_\sigma} + \frac{n_B(E_\pi)}{E_\pi}\right) + 8g^2\sigma_0 \int \frac{d^3p}{(2\pi)^3}\, \frac{n_F(E_{M^*}, \mu_F) + \bar{n}_F(E_{M^*}, \bar{\mu}_F)}{2E_{M^*}} = 0. \tag{3}$$

Here n_F, $\bar{n}_F$, and n_B are the distribution functions for fermions, antifermions, and bosons,

$$n_F = \frac{1}{e^{(E-\mu_F)/T} + 1}, \quad n_B = \frac{1}{e^{E/T} - 1}, \tag{4}$$

and T is the temperature. At zero temperature, with $\bar{n}_F = 0$ and zero chemical potentials for bosons, we have the exact result[3]

$$\left(\mu_0^2 + \frac{\lambda\sigma_0^2}{6} - \frac{C}{\sigma_0}\right) + \frac{g^2}{\pi^2}\left[\mu_F(\mu_F^2 - g^2\sigma_\rho^2)^{1/2} - g^2\sigma_0^2)^{1/2} \ln\left(\frac{\mu_F + (\mu_F^2 - g^2\sigma_0^2)^{1/2}}{g\sigma_0}\right)\right] = 0. \tag{5}$$

The order parameter σ_0 is proportional to M^* and in Fig. 2 the behavior of the effective mass as a function of k_F is shown. The variation of the σ and π masses with increasing nucleon density is shown in Fig. 3. At high temperature, we retain only the leading terms in (3) to write[15]

$$\sigma_0\left(\mu_0^2 + \frac{(\lambda + 4g^2)}{12}T^2\right) + \frac{\lambda\sigma_0^3}{6} - C = 0. \tag{6}$$

In Fig. 4 the behavior of the effective nucleon mass M^* and of the σ and π masses as functions of temperature is shown. The pion mass at finite temperature and density is given by

$$m_\pi^2 = \mu_0^2 + \frac{\lambda\sigma_0^2}{6} = C/\sigma_0(T), \tag{7}$$

while m_σ^2 is given by the second derivative of $V(\sigma_0)$

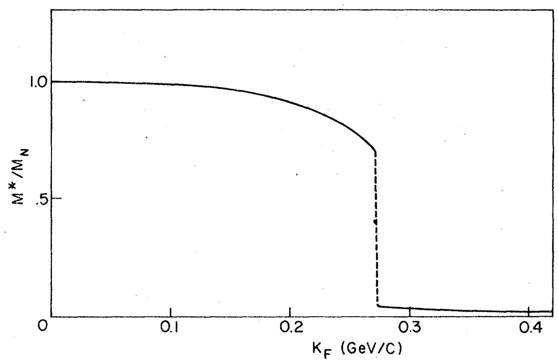


FIG. 2. The ratio of the effective nucleon mass to the free-nucleon mass as a function of Fermi momentum.

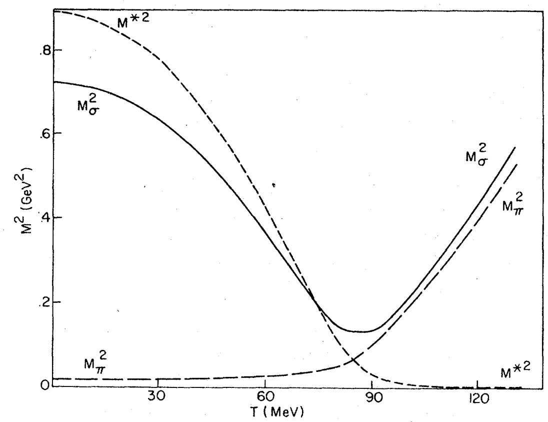


FIG. 4. Mass spectrum for the σ and π mesons and the nucleon with increasing temperature.

at its absolute minimum. The excitation spectrum thus obtained is used in the calculation of $\pi^0\to2\gamma$.

The renormalizability of the σ model has been discussed at finite temperature and density.[15] It has been shown that in relativistic many-body theory there are temperature- and density-dependent infinities in Feynman integrals at intermediate stages of the calculations in addition to the usual infinities of field theory.[15,16] However, these divergences were explicitly shown to cancel out among themselves when the dimensional regularization technique and renormalization procedure of 't Hooft[17] is used. In the present paper we are interested in the phenomenological aspects of the model, and we restrict ourselves to calculations

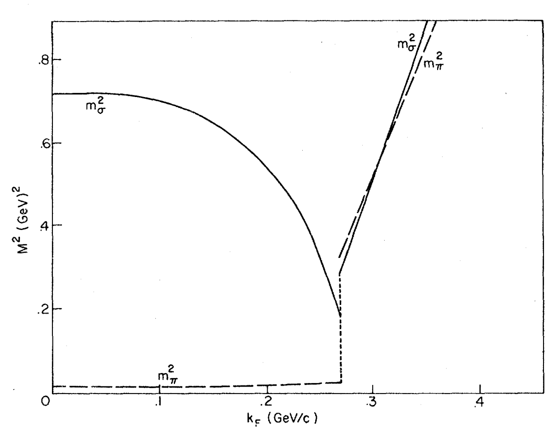


FIG. 3. Mass spectrum for the σ and π mesons with increasing Fermi momentum.

at the one-loop level. Also, the presence of the explicit symmetry-breaking term $C\sigma$ in (1) does not allow the order parameter to become identically equal to zero. This feature is taken into account in the numerical analysis.

It should be noted that the model includes only σ and pion fields so that there would be a lack of saturation for the nuclear binding energy, and large three-body forces as shown by Barshay and Brown,[18] and also the phase transition is sensitive to the σ mass. Despite these aspects it was felt that the consequences of the model, which predicts a Lee-Wick phase transition, should be explored systematically for $\pi^0\to2\gamma$.

A feature of this model study worth mentioning is that the entire nucleon mass is generated by the interactions with the σ condensate. The ground-state energy given by the minimum of the effective potential at each k_F is then essentially the nuclear binding energy. Even though there is no saturation in the model the binding energy per nucleon as a function of k_F can be shown to pass through $\simeq -16$ MeV at $k_F \simeq 240$ MeV/c. Furthermore, the effective mass of the nucleons at normal nuclear densities is $M^* \simeq 0.8 M_N$ and this value is close to the usual expectations in nuclear matter theory.

III. PLASMONS AND NUCLEAR MATTER

The collective response to an electromagnetic disturbance by the nuclear medium has to be taken into account before studying physical processes in many-particle systems involving photons at high energy. The perturbation-theoretic calculations must involve the excitation spectrum which reflects these collective effects. The plasmon effect has been extensively studied for the relativistic electron gas.[19-23] Here we wish to illustrate the application of field-theory rules and cutting rules for the calculation of the real and imaginary parts of the dressed photon propagator in the nuclear medium having the charged particles of the σ model, namely protons and the charged pions. The methods of this section will be used on the triangle graph for $\pi^0 \to 2\gamma$ in the next section. The many-body effects are of interest to us here. It has been shown by Schwinger[24] that the requirement of gauge invariance for a vector field coupled to a dynamical current does not automatically imply the existence of a corresponding particle of zero mass. A direct physical realization of this idea was illustrated by Anderson[25] with the example of the plasmon. The plasmon frequency is equivalent to the mass, since below that frequency the electromagnetic disturbance in the medium cannot propagate. Despite this, gauge invariance and particle conservation are satisfied. The propagator for the photon is shown in Fig. 5 in which the polarization effects are represented by the proton loops and by charged pion loops. In second-order perturbation theory the polarization tensor $\Pi_{\mu\nu}$ is given by

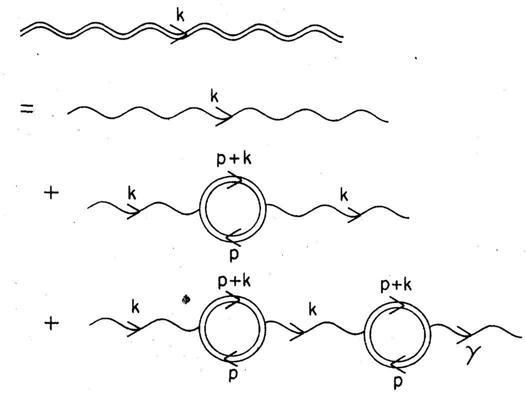


FIG. 5. The dressed photon propagator ($\approx$) in terms of a free photon propagator ($\sim$) plus polarization effects represented by proton (or charged pion) loops.

$$\Pi_{\mu\nu}(k_0,\vec{k}) = 4ie^2\int\frac{d^4p}{(2\pi)^4}\left[\frac{g_{\mu\nu}(M^{*2}-p^2-p\cdot k)+p_\mu(p+k)_\nu+p_\nu(p+k)_\mu}{\{[(p+k)^2-M^{*2}]\}\{[p^2-M^{*2}]\}}\right]$$
$$-ie^2\int\frac{d^4p}{(2\pi)^4}\left(\frac{(2p-k)_\mu(2p-k)_\nu}{[[p^2-m_\pi{}^2]][[(p-k)^2-m_\pi{}^2]]}-\frac{2g_{\mu\nu}}{[[p^2-m_\pi{}^2]]}\right), \tag{8}$$

where the first integral in (8) represents the fermion contributions from vacuum polarization, and from the density- and temperature-dependent polarization of the many-nucleon system, while the second term corresponds to the charged-pion contribution. In (8) we are using the double-bracket notation[15] for the propagators: For fermions we have

$$S_F(p,\mu_F,T)=\frac{\not p+M^*}{\{[p^2-M^{*2}]\}}=\frac{(\not p+M^*)}{2E_p}\left[\frac{1-n_F(E_p,\mu_F)}{p_0-E_p+i\epsilon}+\frac{n_F(E_p,\mu_F)}{p_0-E_p-i\epsilon}-\frac{1-\bar n_F(E_p,\bar\mu_F)}{p_0+E_p-i\epsilon}-\frac{\bar n_F(E_p,\bar\mu_F)}{p_0+E_p+i\epsilon}\right], \tag{9}$$

and for bosons the propagator is given by

$$D_F(p,T)=\frac{1}{[[p^2-m^2]]}=\frac{1}{2E_p}\left[\frac{1+n_B(E_p,T)}{p_0-E_p+i\epsilon}-\frac{n_B(E_p,T)}{p_0-E_p-i\epsilon}-\frac{1+\bar n_B(E_p,T)}{p_0+E_p-i\epsilon}+\frac{\bar n_B(E_p,T)}{p_0+E_p+i\epsilon}\right]. \tag{10}$$

The new terms dependent on temperature and density in (8) constitute the extension to many-body theory of the expressions for $\Pi_{\mu\nu}$ from the vacuum polarization first evaluated by Feynman.[26] From Eqs. (8)–(10) it is straightforward to verify that

$$k^\mu\Pi_{\mu\nu}=0,$$

which ensures gauge invariance. The dressed photon propagator is given by the Dyson equation

$$i\mathfrak{D}_{\mu\nu}(k_0,\vec{k})=iD^0_{\mu\nu}(k_0,\vec{k})+iD^0_{\mu\lambda}(k_0,\vec{k})[i\Pi^{\lambda\rho}(k_0,\vec{k})]i\mathfrak{D}_{\rho\nu}(k_0,\vec{k}), \tag{11}$$

while the lowest-order photon propagator is

$$iD^0_{\mu\nu}=i(-g_{\mu\nu}+k_\mu k_\nu/k^2)/(k^2+i\epsilon). \tag{12}$$

The solution for $\mathfrak{D}_{\mu\nu}$ is given by

$$\epsilon^{\rho}_{\mu}\mathfrak{D}_{\rho\nu}=D^0_{\mu\nu} \tag{13}$$

with

$$\epsilon^{\rho}_{\mu}=g^{\rho}_{\mu}+D^0_{\mu\lambda}\Pi^{\lambda\rho}. \tag{14}$$

The dielectric function $\epsilon_{\mu\nu}$ in (14) has an imaginary part which is not the same as the imaginary part in the usual dielectric response function. The difference arises from the fact that the dielectric response function governs the retarded response of the system, while the function we are studying is the causal polarization function. The relation between the two is given by a dispersion integral.[27] Since $\mathfrak{D}_{\mu\nu}=[\epsilon]^{-1\lambda}_{\ \mu}D^0_{\lambda\nu}$, the excitation spectrum is obtained[28] by solving for the roots of

$$\det[\epsilon]=0. \tag{15}$$

In the frame of reference with $k^\mu=(k^0,0,0,|\vec{k}|)$, Eq. (15) is expressible as

$$(1+\Pi_{11}/k_\mu{}^2)(1+\Pi_{22}/k_\mu{}^2)(1+(\Pi_{33}-\Pi_{00})/k_\mu{}^2)=0,$$

or

$$(\epsilon_T)^2(\epsilon_L)=0. \tag{16}$$

We shall be interested in the solutions of

$$\epsilon_T=(1+\Pi_T)=0, \tag{17}$$

for the transversely polarized excitations, where

$$\Pi_T=\Pi_{11}/k_\mu{}^2. \tag{18}$$

In the dimensional-regularization scheme[17] it is straightforward to show that the vacuum-polarization terms in (8) lead to

$$\Pi_T^{(0)}(k_0,\vec{k})=-\left(\frac{e^2}{4\pi^2}\right)\left[\frac{2}{3(n-4)}+\frac{1}{3}\ln\left(\frac{M^{*2}}{\mu^2}\right)-\left(\frac{5}{9}+\frac{4}{3}\frac{M^{*2}}{k^2}\right)-\frac{2}{3}\left(1+\frac{2M^{*2}}{k^2}\right)\left(1-\frac{4M^{*2}}{k^2}\right)^{1/2}\tanh^{-1}\left(\frac{1}{(1-4M^{*2}/k^2)^{1/2}}\right)\right]$$
$$-\left(\frac{e^2}{48\pi^2}\right)\left[\frac{2}{n-4}+\ln\left(\frac{m_\pi{}^2}{\mu^2}\right)-\left(\tfrac{8}{3}\right)+8(m_\pi{}^2/k^2)+(1-4m_\pi{}^2/k^2)^{3/2}\ln\left(\frac{1+(1-4m_\pi{}^2/k^2)^{1/2}}{1-(1-4m_\pi{}^2/k^2)^{1/2}}\right)\right]. \tag{19}$$

The imaginary part of $\Pi_T^{(0)}$ will be treated separately below.

The real density-dependent polarization is obtained by cutting one of the internal lines in the one-loop graph in Fig. 5. Physically, the process corresponds to a particle (antiparticle) absorbing the initial photon and being excited, and then returning to the original Fermi distribution by reemitting the photon. This real contribution to Π_T is

$$\overline{\Pi}_T=\frac{-e^2}{k_\mu{}^2\pi^2}\int_0^\infty\frac{dp\,p^2}{E_p}[n_F(E_p)+\bar{n}_F(E_p)]\int_{-1}^1dx\,\frac{[-2(p_\mu\cdot k^\mu)^2+k_\mu{}^2p^2(1-x^2)]}{[(k_\mu{}^2)^2-4(p_\mu\cdot k^\mu)^2]}$$
$$-\frac{e^2}{k_\mu{}^2 2\pi^2}\int_0^\infty\frac{dp\,p^2}{E_p}n_B(E_p)\int_{-1}^1dx\left[1+p^2(1-x^2)\left(\frac{2k_\mu{}^2}{(k_\mu{}^2)^2-4(p_\mu\cdot k^\mu)^2}\right)\right]. \tag{20}$$

It has been shown by Chin[28] that the neglect of $(k_\mu{}^2)^2$ in the denominator in (20) provides a closed form expression for $\overline{\Pi}_T$, at zero temperature, and $\bar{n}_F=0$ and $n_B=0$ at $T=0$, so that we have

$$\epsilon_T=1+\Pi_T^{(0)}+\frac{e^2}{2\pi^2}\frac{k_F{}^3}{\mu_F k_\mu{}^2}\left\{-1+\left(1-\frac{C_0^2}{\beta_F{}^2}\right)\left[1-\frac{C_0}{2\beta_F}\ln\left(\frac{C_0+\beta_F}{C_0-\beta_F}\right)\right]\right\}, \tag{21}$$

where k_F and μ_F are the Fermi momentum and energy, $\beta_F=k_F/\mu_F$, and $C_0=k_0/|\vec{k}|$. The dispersion relation is then given by $\epsilon_T=0$, and after renormalizing $\Pi_T^{(0)}$ we obtain

$$k_0{}^2\cong\Lambda^2+|\vec{k}|^2+\tfrac{1}{5}\beta_F{}^2|\vec{k}|^2+\cdots, \tag{22}$$

for small $|\vec{k}|$. Here

$$\Lambda^2=e^2k_F{}^3/3\pi^2\mu_F. \tag{23}$$

In the nonrelativistic limit at low densities $\mu_F\approx M^*$ and we have the familiar result

$$\Lambda^2\approx N_F e^2/M^*, \tag{24}$$

for the square of the plasmon frequency. In (24),

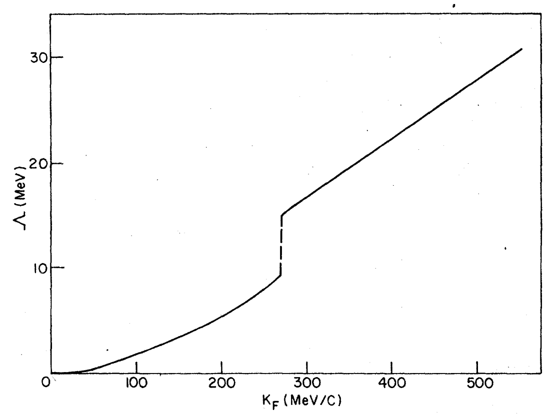


FIG. 6. The plasmon mass Λ with increasing Fermi momentum.

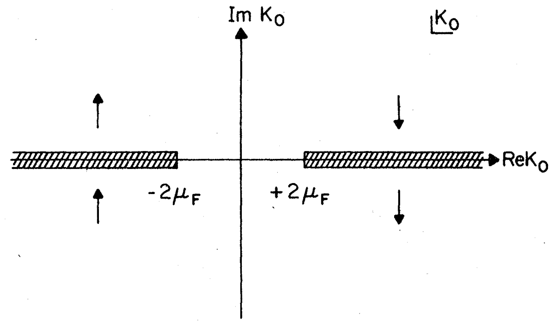


FIG. 7. The singularity structure in the complex k_0 plane for Π_T. Note that the threshold occurs at $|k_0| = 2\mu_F$ and not at $|k_0| = 2M$, as for the usual vacuum process. The arrows indicate the way in which the discontinuity across the branch cuts for Π_T was evaluated.

N_F is the number density of fermions. In the following we shall retain only the leading terms and write

$$k_0^2 \cong \Lambda^2 + |\vec{k}|^2, \tag{25}$$

as an acceptable input in the calculations for $\pi^0 \to 2\gamma$. The behavior of Λ^2 as the nucleon density is increased is shown in Fig. 6, which includes the variation in nucleon mass as the density increases.

At high temperature T, the dielectric function ϵ_T is calculated within the same approximation as at zero temperature. The leading term corresponding to the neglect of masses has the form

$$\epsilon_T \cong 1 + \Pi_T^{(0)} + \frac{e^2T^2}{3k_\mu^2}\left\{-1 + (1 - C_0^2)\left[1 - \frac{C_0}{2}\ln\left(\frac{C_0+1}{C_0-1}\right)\right]\right\}, \tag{26}$$

which includes the contributions from the protons, antiprotons, and $\pi^\pm$ particle distributions at finite temperature. The excitation spectrum for the plasmon effect[23] is given by

$$k_0^2 \approx \Lambda^2 + |\vec{k}|^2,$$

where

$$\Lambda^2 = \frac{2e^2T^2}{9}. \tag{27}$$

So far we have included only the effect of virtual intermediate states in the photon propagator. Let us now evaluate the imaginary part of Π_T which arises when the photon propagation proceeds via intermediate states which correspond to energetically accessible physical states. The damping in the photon propagator then arises due to the opening up of new channels leading to a dissipation of the original photon propagation. In order to evaluate the imaginary part of Π_T we find it convenient to return to the original integrals in (8) and consider the p_0 integration. On the complex p_0 plane the integrand has two poles below the real axis and two above for both $p_0 \gtrless 0$, corresponding to the four poles in each propagator, for each of the integrals in (8). The $i\epsilon$ prescription provides all the necessary conditions for evaluating the integrals. It is easy to show that when two poles are on the same side of the real axis the contour may be closed (on either side) to give zero. Thus the real part is obtained by enclosing one pole at a time by contour closing. Upon performing the p_0 integration, the resulting expression is to be viewed as an integral expression for $\Pi_T(k_0, \vec{k})$. On the k_0 plane this integral develops a "pinch singularity" when the two poles in the p_0 plane appear on either side of the p_0 contour at the same location.[29] The imaginary part of Π_T is obtained by taking the discontinuity across the cut in the k_0 plane as shown in Fig. 7. On performing the p_0 integration and then evaluating the discontinuity in k_0 across the branch cuts in the k_0 plane we obtain, for the nucleon contribution,

$$
\begin{aligned}
\mathrm{Im}\Pi_T^{(N)}(k_0,\vec{k}) = {} & \frac{e^2}{4\pi^2 k_\mu{}^2}\int \frac{d^3p}{E_pE_{p+k}}\left(p^2\sin^2\theta\cos^2\varphi+\frac{p_\mu\cdot k^\mu}{2}\right) \\
& \times\big(\delta(E_p-E_{p+k}+k_0)\{[1-n_F(E_p)]n_F(E_{p+k})+\bar{n}_F(E_p)[1-\bar{n}_F(E_{p+k})]\} \\
& +\delta(E_p+E_{p+k}+k_0)\{\bar{n}_F(E_p)n_F(E_{p+k})-[1-n_F(E_p)][1-\bar{n}_F(E_{p+k})]\}\big) \\
& +\frac{e^2}{4\pi^2 k_\mu{}^2}\int \frac{d^3p}{E_pE_{p-k}}\left(p^2\sin^2\theta\cos^2\varphi-\frac{p_\mu\cdot k^\mu}{2}\right) \\
& \times\big(\delta(E_p+E_{p-k}-k_0)\{\bar{n}_F(E_p)n_F(E_{p-k})-[1-n_F(E_p)][1-\bar{n}_F(E_{p-k})]\} \\
& +\delta(E_p-E_{p-k}-k_0)\{\bar{n}_F(E_p)[1-\bar{n}_F(E_{p-k})]+n_F(E_{p-k})[1-n_F(E_p)]\}\big)\,.
\end{aligned} \tag{28}
$$

For $\vec{k}=0$, $k_0>0$ the imaginary part corresponds to fermion pair creation and we have

$$
\mathrm{Im}\Pi_T^{(N)}(\text{pair creation})=\left(\frac{e^2}{4\pi}\right)\left(\frac{1}{3}\right)\left(1-\frac{4M^{*2}}{k_0{}^2}\right)^{1/2}\left(\frac{2M^{*2}}{k_0{}^2}+1\right)\{[1-n_F(k_0/2)][1-\bar{n}_F(k_0/2)]-\bar{n}_F(k_0/2)n_F(k_0/2)\}\,. \tag{29}
$$

The Fermi distribution functions in (29) correspond to a positive imaginary part in Π_T arising from pair creation, with the proviso that the particle and the antiparticle be created above their respective Fermi seas. The probability amplitude for this is proportional to $(1-n_F)(1-\bar{n}_F)$. There is also a negative term in (29) corresponding to the probability amplitude for the recombination of a fermion and an antifermion, from the many-particle background, so as to revitalize the photon propagation. This amplitude, proportional to $n_F\bar{n}_F$, subtracts from the usual damping represented by $(1-n_F)(1-\bar{n}_F)$. These features are new to relativistic many-particle systems. The effect of the Pauli exclusion principle thus leads to a threshold dependence of $(1-n_F(k_0/2,\mu_F)-\bar{n}_F(k_0/2,\bar{\mu}_F))$. In the absence of an antinucleon distribution we have the threshold for pair creation occurring not at $k_0=2M^*$, but rather at $k_0=2\mu_F$. The imaginary part of $\Pi_T^{(N)}$ in (28) in the region $\beta(=p/E)\geq C_0$ had contributions from terms proportional to $n_F(1-n_F)$ and $\bar{n}_F(1-\bar{n}_F)$. These terms arise from the excitation of a fermion or an antifermion, in the corresponding Fermi distribution, to an energy above the Fermi energy. These terms are known as the Cherenkov (or the Landau) damping terms. We shall be concerned with the pair-creation damping only. The imaginary part of the polarization function $\Pi_T^{(\pi^\pm)}$ is given in the cases of $\pi^\pm$ pair production and of Cherenkov damping as

$$
\begin{aligned}
\mathrm{Im}\Pi_T^{(\pi^\pm)}(k_0>0,\vec{k};\text{ pair creation}) = {} & \frac{e^2}{k_\mu{}^2 8\pi^2}\int\frac{d^3p}{E_pE_{p-k}}(p^2\sin^2\theta\cos^2\varphi)\delta(E_p+E_{p-k}-k_0) \\
& \times\{[1+n_B(E_p)][1+\bar{n}_B(E_{p-k})-\bar{n}_B(E_p)n_B(E_{p-k})]\}
\end{aligned} \tag{30}
$$

and

$$
\begin{aligned}
\mathrm{Im}\Pi_T^{(\pi^\pm)}(k_0>0,\vec{k};\text{ Cherenkov}) = {} & \frac{e^2}{k_\mu{}^2 8\pi^2}\int d^3p\,(p^2\sin^2\theta\cos^2\varphi)\left\{\delta(E_p-k_0-E_{p-k})\frac{[1+n_B(E_p)]n_B(E_{p-k})}{E_pE_{p-k}}\right. \\
& \left.+\delta(E_p+k_0-E_{p+k})\frac{[1+\bar{n}_B(E_{p+k})]\bar{n}_B(E_p)}{E_pE_{p+k}}\right\}.
\end{aligned} \tag{31}
$$

With $n_B=\bar{n}_B$, Eq. (30) reduces to

$$
\mathrm{Im}\Pi_T^{(\pi^\pm)}(k_0>0,\vec{k}=0;\text{ pair creation})=\left(\frac{e^2}{4\pi}\right)\left(\tfrac{1}{12}\right)(1-4m_\pi{}^2/k_0{}^2)^{3/2}[1+2n_B(k_0/2)]\,\theta(k_0{}^2-4m_\pi{}^2). \tag{32}
$$

In concluding this section we note the presence of the new feature in the imaginary part of the propagator, associated with the recombination of particles from the many-particle background, in relativistic many-particle systems. The analytic properties of the Green's functions are not the same as in usual field theory, and the shifting of branch points and other singularities has to be taken into account in many-body theory. Some aspects of the analytic properties of the Green's functions in many-body theory were studied earlier.[30] An interesting application of S-matrix methods to superconductivity theory is given in Ref. 31. We also note that the Rayleigh-Langmuir[32] plasma frequency is changed from $\Lambda=(N_F e^2/M^*)^{1/2}$ to its relativistic form $\Lambda=(e^2k_F^3/3\pi^2\mu_F)^{1/2}$, while at finite temperature it is given by $\Lambda=(Qe^2T^2/9)^{1/2}$, where Q is the number of particle-antiparticle pairs in the theory.

IV. THE TRIANGLE GRAPH FOR $\pi^0 \to 2\gamma$

The σ model provides a convenient basis for the evaluation of $\pi^0 \to 2\gamma$ in nuclear matter. We are interested in extending the Steinberger model,[33] in which the decay proceeds via virtual proton-antiproton states, to include further contributions arising from the nuclear medium. This is in contrast to pion decay proceeding via virtual quark-antiquark intermediate states. The numerical estimates in either model for $\pi^0 \to 2\gamma$ in vacuum are in agreement with each other provided color symmetry is invoked and quarks coupling to π^0 have three colors. Here we are interested in the density dependence of the lifetime for the pion in nuclear matter and the σ model seems to be the more tractable candidate.

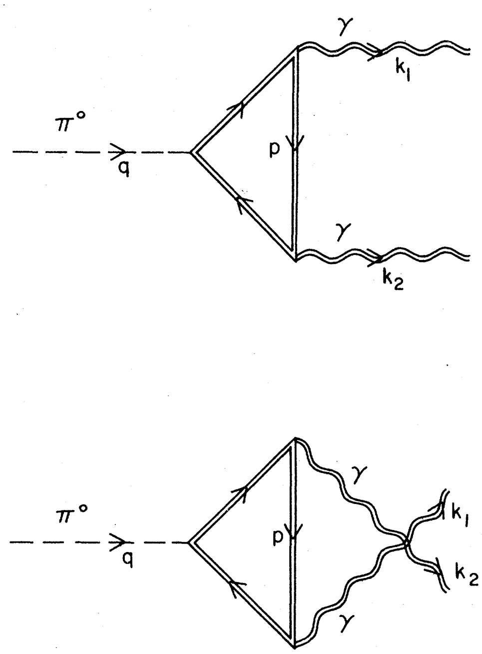


FIG. 8. The triangle graph for pion decay. The internal nucleon lines ($\approx$) and the external pion (-----) and photon ($\sim$) lines corresponding to the dressed propagators.

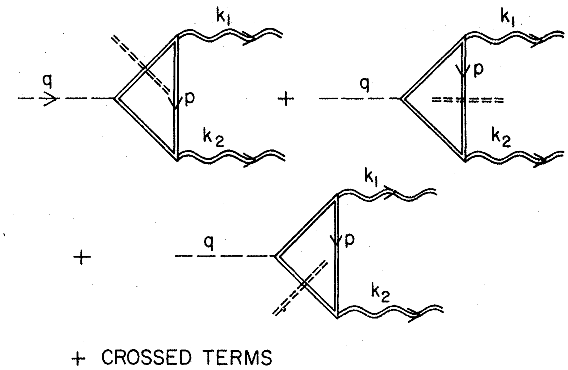


FIG. 9. Feynman diagrams with the nucleon lines cut (dashed lines) one at a time, representing the contributions to the real density-dependent part of the neutral-pion decay amplitude.

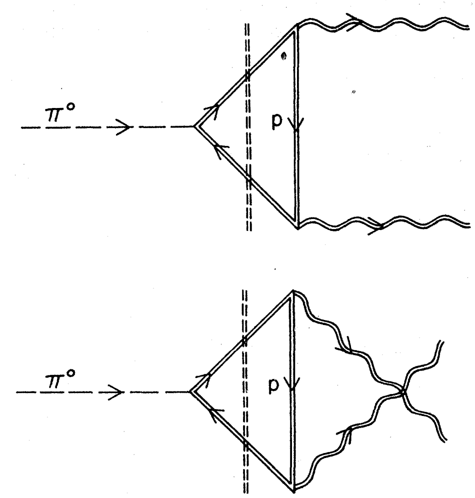


FIG. 10. Feynman diagrams with the fermions put on the mass shell by cutting (dashed lines) the fermion lines adjacent to the external pion leg, for the evaluation of the imaginary part of the neutral pion decay amplitude in nuclear matter.

A. The pion decay amplitude

Let us include electromagnetic effects in the σ model Lagrangian so as to allow π^0 to decay into two photons. The photons couple to the neutral pion through polarization effects represented by the triangle graphs of Fig. 8. The amplitude for $\pi^0 \to 2\gamma$ is evaluated using the interaction terms

$$\mathcal{L}_{\pi^0 p} = g\overline{\Psi}_p i\gamma_5 \tau_3 \Psi_p \pi^0 \tag{33}$$

and

$$\mathcal{L}_{\gamma p} = e\overline{\Psi}_p \gamma_\mu A^\mu \Psi_p . \tag{34}$$

This amplitude is related to the vector-vector-pseudoscalar (VVP) current correlation function. The axial-vector coupling of the pion to the nucleons

$$\mathcal{L}_{\mathfrak{a}\Psi} = G_A \overline{\Psi}\left(\frac{\tau_3}{2}\right)\gamma_\mu\gamma_5\Psi\,\partial^\mu \pi^0, \tag{35}$$

together with (34) leads to the consideration of the vector-vector-axial-vector (VVA) current correlation function. The relation between the VVP and VVA correlation functions and the gauge invariance of these functions have been fully investigated for the amplitudes in the absence of a nuclear medium.[33-36] Here we wish to include the effects of the many-particle system. We first evaluate the amplitude for $\pi^0 \to 2\gamma$ before comparing the VVP and VVA amplitudes.

The calculation requires the effect of the collective behavior of the many-particle system to be taken into account and this is implemented by using the effective mass of the nucleons and the dressed pion mass as given by the order parameter, and the plasmon mass, at each temperature and density.

The pion decay amplitude is given by[36]

$$T_{\mu\nu}(\pi(\vec{q}=0) \to \gamma(\vec{k}_1, \vec{\epsilon}_1), \gamma(\vec{k}_2, \vec{\epsilon}_2)) = -ge^2 \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}[(\gamma_5 S_F(p-k_1)\gamma_\mu S_F(p)\gamma_\nu S_F(p+k_2)) + (k_1 \leftrightarrow k_2,\ \mu \leftrightarrow \nu)], \tag{36}$$

where $S_F(p)$ are the nucleon propagators (9) for the many-particle system. We have not included in (36) the covariant normalization factors and factors of $[n_B(k)+1]^{1/2}$ connected with the thermal background of photons of finite temperature. These factors are deferred to the phase-space calculation. On performing the trace in (36) we obtain

$$T_{\mu\nu} = i4ge^2 M^* \epsilon_{\mu\nu\lambda\rho} k_1^\lambda k_2^\rho \int \frac{d^4p}{(2\pi)^4}\left[\left(\frac{1}{\{[(p-k_1)^2 - M^{*2}]\}\{[(p+k_2)^2 - M^{*2}]\}\{[p^2 - M^{*2}]\}}\right) + (k_1 \leftrightarrow k_2)\right]. \tag{37}$$

The terms in (9) are separable into the usual vacuum Feynman propagator and terms which are density dependent; hence by applying the cutting rules of many-body field theory,[15] the amplitude (37) is expressible as (a) the vacuum amplitude, (b) a real density-dependent amplitude obtained by cutting one nucleon line at a time as in Fig. 9, and (c) an imaginary density-dependent amplitude obtained by cutting both nucleon lines adjacent to the external pion leg as in Fig. 10.

A straightforward calculation with $k_1^2 = k_2^2 = \Lambda^2$ leads to the vacuum amplitude

$$T_{\mu\nu}^{(a)} = \frac{ge^2 M^*}{2\pi^2}\epsilon_{\mu\nu\lambda\rho} k_1^\lambda k_2^\rho \int_0^1 dx \int_0^{1-x} dy [M^{*2} - q^2 xy - \Lambda^2(x+y)(1-x-y)]^{-1}. \tag{38}$$

For the densities and temperatures we have considered, $\Lambda^2 \ll 4M^{*2}$. The "plasmon mass" Λ is typically small and Λ^2 is neglected only in the real part of the amplitude. (The imaginary part of $T_{\mu\nu}$ is treated separately below.) This leads to a closed-form evaluation of the real part, and we have

$$\mathrm{Re}T_{\mu\nu}^{(a)} = 4ge^2 M^* \epsilon_{\mu\nu\lambda\rho} k_1^\lambda k_2^\rho \begin{cases} \dfrac{1}{4\pi^2 q^2}[\sin^{-1}(\sqrt{q^2}/2M^*)]^2 & q^2 \le 4M^{*2}, \\ \dfrac{1}{16\pi^2 q^2}\left[\pi^2 - \ln^2\left(\dfrac{x_+}{x_-}\right)\right], & q^2 \ge 4M^{*2}, \end{cases} \tag{39}$$

where

$$x_\pm = 1 \pm B, \quad B = (1 - 4M^{*2}/q_0^2)^{1/2}. \tag{40}$$

The real density-dependent amplitude of type (b) discussed above corresponds to diagrams with a single cut in the fermion loop. These diagrams represent the physical processes in which a proton from the nuclear medium absorbs the pion, emits two photons, and then returns to the Fermi sea of protons. The various terms of this type refer to the temporal ordering of the absorption of the pion and the emission of photons in various ways. This amplitude is

$$\mathrm{Re}T_{\mu\nu}^{(b)} = 4ge^2M^*\epsilon_{\mu\nu\lambda\rho}k_1^\lambda k_2^\rho\left\{-\int\frac{d^3p}{(2\pi)^3}\left(\frac{n_F(E_p)+\bar{n}_F(E_p)}{2E_p}\right)\right.$$
$$\times\left[\frac{2(\Lambda^2)^2-8(p\cdot k_1)(p\cdot k_2)}{[(\Lambda^2)^2-4(p\cdot k_1)^2][(\Lambda^2)^2-4(p\cdot k_2)^2]}+\frac{2q^2\Lambda^2+8(p\cdot q)(p\cdot k_1)}{[(q^2)^2-4(p\cdot q)^2][(\Lambda^2)^2-4(p\cdot k_1)^2]}\right.$$
$$\left.\left.+\frac{2q^2\Lambda^2+8(p\cdot q)(p\cdot k_2)}{[(q^2)^2-4(p\cdot q)^2][(\Lambda^2)^2-4(p\cdot k_2)^2]}\right]\right\}. \tag{41}$$

The approximation of $\Lambda^2 \ll (p\cdot k)$ allows further simplification. Specifically, for $\vec{q}=0$ we have

$$\mathrm{Re}T_{\mu\nu}^{(b)} = 4ge^2M^*\epsilon_{\mu\nu\lambda\rho}k_1^\lambda k_2^\rho\left[\frac{1}{4\pi^2}\int\frac{\beta\,d\beta}{[q_0^2(1-\beta^2)-4M^{*2}]}\ln\left(\frac{C_0+\beta}{C_0-\beta}\right)[n_F(E_p,\mu_F,T)+\bar{n}_F(E_p,\bar{\mu}_F,T)]\right], \tag{42}$$

where

$$\beta = p/E$$

and

$$C_0 = k_0/|\vec{k}|. \tag{43}$$

At zero temperature we set $\bar{n}_F=0$ and note that $n_F(E_p,\mu_F)$ provides a cutoff for the integral in (42) at the Fermi velocity $\beta=\beta_F=k_F/\mu_F$. For the specific mass spectrum in the model the branch point at which $T_{\mu\nu}$ develops an imaginary part associated with $q_0\geq 2\mu_F$ is such that the integrand in (42) does not develop a pole in the range $0\leq\beta\leq\beta_F$. The integral in (38) is then evaluated in terms of dilogarithmic functions,[37] and we have

$$\mathrm{Re}T_{\mu\nu}^{(b)}(T=0) = [4ge^2M^*\epsilon_{\mu\nu\lambda\rho}k_1^\lambda k_2^\rho/(8\pi^2q_0^2)]$$
$$\times\left\{\ln\left(\frac{C_0+B}{C_0-B}\right)\ln\left(\frac{B+\beta_F}{B-\beta_F}\right)+\mathrm{Li}_2\left(\frac{B-\beta_F}{C_0+B}\right)-\mathrm{Li}_2\left(\frac{B+\beta_F}{C_0+B}\right)+\mathrm{Li}_2\left(\frac{B-\beta_F}{C_0-B}\right)+\mathrm{Li}_2\left(\frac{C_0-B}{B+\beta_F}\right)-\tfrac{1}{2}\mathrm{Li}_2\left[\left(\frac{\beta_F-B}{C_0-B}\right)^2\right]\right.$$
$$\left.-\tfrac{1}{2}\mathrm{Li}_2\left[\left(\frac{C_0-B}{B+\beta_F}\right)^2\right]-(\pi^2/6)-\tfrac{1}{2}\ln^2\left(\frac{\beta_F+B}{C_0-B}\right)\right\}. \tag{44}$$

At finite temperature with $n_F=\bar{n}_F$ and with zero chemical potential for the fermions we have

$$\mathrm{Re}T_{\mu\nu}^{(b)}(q_0,k_1,k_2;T) = 4ge^2M^*\epsilon_{\mu\nu\lambda\rho}k_1^\lambda k_2^\rho\left\{\frac{1}{2\pi^2}\int_0^1\frac{\beta\,d\beta}{[q_0^2(1-\beta^2)-4M^{*2}]}\ln\left(\frac{C_0+\beta}{C_0-\beta}\right)\left[\exp\left(\frac{M^*}{T(1-\beta^2)^{1/2}}\right)+1\right]^{-1}\right\}. \tag{45}$$

While the integral in (45) is expressible as an infinite series in dilogarithms, we find it convenient to evaluate it by numerical methods. At finite temperature, the range of integration includes the threshold $q_0 \geq 2M^*$ for the nucleon-antinucleon absorptive part and the Cauchy principal part of (45) is evaluated numerically.

We now evaluate the imaginary part of $T_{\mu\nu}$ by returning to (37). On the complex p_0 plane the nucleon propagators provide three poles in each quadrant. The p_0 integration is performed using the $i\epsilon$ prescription as usual. As in the discussion of the photon propagator in Sec. III, we calculate the imaginary part of $T_{\mu\nu}$ which arises from the presence of the pinch singularity associated with the propagators on either side of the pion vertex providing a pole on either side of the p_0 contour at the same value of $\mathrm{Re}(p_0)$. This corresponds to the cutting of these fermion lines as depicted in Fig. 10. For $q_0>0$, $\vec{q}=0$, we have

$$\mathrm{Im}T_{\mu\nu} = 4ge^2M^*\epsilon_{\mu\nu\lambda\rho}k_1^\lambda k_2^\rho\left(\frac{1}{8\pi q_0^2A}\left\{\left[1-n_F\left(\frac{q_0}{2}\right)\right]\left[1-\bar{n}_F\left(\frac{q_0}{2}\right)\right]-n_F\left(\frac{q_0}{2}\right)\bar{n}_F\left(\frac{q_0}{2}\right)\right\}\ln\left(\frac{2\Lambda^2-q_0^2-q_0^2AB}{2\Lambda^2-q_0^2+q_0^2AB}\right)\right), \tag{46}$$

where B is given by (40) and

$$A = (1-4\Lambda^2/q_0^2)^{1/2}. \tag{47}$$

The physical interpretation of the terms in (46) is analogous to the discussion given in Sec. III. The term proportional to $(1-n_F)(1-\bar{n}_F)$ represents the probability of the pion decaying into a fermion-antifermion pair, which has to be created while taking the restriction of the Pauli exclusion principle into account. The negative term proportional to $n_F\bar{n}_F$ represents the probability for the replenishment of the original channel through the recombination of a fermion and an antifermion from the many-particle background.

The decay width for $\pi^0 \to 2\gamma$ is given by

$$\Gamma(\pi^0 \to 2\gamma) = \int \frac{d^3k_1}{(2\pi)^3 2k_{10}} \int \frac{d^3k_2}{(2\pi)^3 2k_{20}} \frac{(2\pi)^4 \delta^4(p_f - p_i)}{2q_0} [1 + n_B(k_{10})][1 + n_B(k_{20})] \sum_{\lambda_1, \lambda_2} |T_{fi}|^2, \tag{48}$$

where

$$T_{fi} = \epsilon^{\mu}(k_1, \lambda_1)\epsilon^{\nu}(k_2, \lambda_2) T_{\mu\nu} = \epsilon_{\mu\nu\lambda\rho}\epsilon_1^{\mu}\epsilon_2^{\nu} k_1^{\lambda} k_2^{\rho} \overline{T}. \tag{49}$$

The factor $[1 + n_B(k_{10})][1 + n_B(k_{20})]$ in (48) represents the "thermal broadening" of the decay width. It corresponds to the probability for the photon detectors registering not only the direct photons from π^0 decay, but also the photons of the same energy originating in the thermal background of photons. We note that the effect of thermal photons recombining to reverse the reaction $\pi^0 \to 2\gamma$ would lead to the replacement of the factor $(1 + n_{B1})(1 + n_{B2})$ in (48) by

$$(1 + n_{B1})(1 + n_{B2}) - n_{B1} n_{B2}.$$

Similarly, if we have a thermal distribution of pions, or a pion condensate, an integration over the initial phase space of pions will become necessary. These aspects of the problem will not be considered in this paper. Substituting (49) into (48) we obtain

$$\Gamma(\pi^0 \to 2\gamma) = \frac{|\overline{T}|^2 m_\pi^3}{64\pi} \left(1 - \frac{2\Lambda^2}{m_\pi^2}\right) \left(1 - \frac{4\Lambda^2}{m_\pi^2}\right)^{1/2} \times \{n_B(E_\Lambda = m_\pi/2, T) + 1\}^2. \tag{50}$$

The numerical results for the decay width of the pion at various nucleon densities, at zero temperature,[10] are reproduced in Table I. With values of parameters in the model as in Sec. II the abnormal nuclear matter phase, in which the nucleons are nearly massless, sets in at $k_F \cong 270$ MeV/c. The decay width of the pion is seen to increase dramatically as the density of nuclear matter exceeds the critical density.

In Table II we present the results of the calculation at finite temperature. The nucleon mass decreases with increasing temperature. In the absence of the explicit symmetry-breaking term $C\sigma$ in (1), the symmetry restoration to SU(2)×SU(2) occurs at $T \cong 80$ MeV. In the model with $C \neq 0$ in (1), the decay width is seen from Table II to increase sharply at $T \gtrsim 80$ MeV. A discussion of these results is given in Sec. V. We now turn to the relation between the VVA and VVP correlation functions.

TABLE I. The decay width of $\pi^0 \to 2\gamma$ at various nucleon Fermi momenta in nuclear matter. The parameters used give the critical Fermi momentum at which phase transition occurs as $k_F = 270$ MeV/c.

Nuclear Fermi momentum k_F (MeV/c)	Pion mass (MeV)	Nucleon effective mass M^* (MeV)	Plasmon mass Λ (MeV)	Decay with $\Gamma(\pi^0 \to 2\gamma)$ (eV)
0.0	135.0	940.0	0.0	7.50
50.0	135.08	938.8	0.20	7.53
100.0	135.68	930.5	1.82	7.75
150.0	137.40	907.03	3.37	8.43
200.0	141.42	855.7	5.31	10.20
250.0	152.08	740.6	7.87	16.40
260.0	156.37	700.56	8.54	19.65
270.0	163.55	640.43	9.37	26.31
271.0	582.23	50.54	15.0	1.79×10^4
275.0	603.04	47.11	15.2	1.46×10^4
280.0	627.77	43.47	15.5	1.20×10^4
300.0	717.5	33.27	16.65	7.01×10^3
350.0	913.26	20.54	19.46	2.91×10^3
400.0	1091.78	14.37	22.26	1.51×10^3

TABLE II. The decay width of $\pi^0 \to 2\gamma$ at various temperatures in nuclear matter. The parameters used give the critical temperature at which phase transition occurs, in the absence of explicit symmetry breaking, at $T_c \cong 80$ MeV.

Nuclear temperature T (MeV)	Pion mass (MeV)	Nucleon effective mass M^* (MeV)	Plasmon mass Λ (MeV)	Decay width $\Gamma(\pi^0 \to 2\gamma)$ (eV)
0.0	135.0	940.0	0.0	7.50
10.0	135.50	933.05	1.43	7.71
20.0	137.06	911.92	2.86	8.90
30.0	139.87	875.67	4.28	11.76
40.0	144.32	822.53	5.71	17.07
50.0	151.19	749.41	7.14	27.13
60.0	162.22	650.98	8.57	49.19
70.0	181.89	517.82	9.99	125.82
80.0	224.89	338.71	11.42	495.17
90.0	320.29	166.99	12.85	1.12×10^4
100.0	435.12	90.49	14.28	1.61×10^4
110.0	539.99	58.77	15.70	1.12×10^4
120.0	636.16	42.33	17.13	7.80×10^3

B. The VVA current correlation function and gauge invariance

The axial-vector coupling of the pion to nucleons as given in Eq. (35) allows us to relate the VVA current correlation function to the pion decay amplitude. In the σ model, with no nucleon resonances present, the Goldberger-Treiman relation reduces to

$$g = M^* G_A, \tag{51}$$

and with

$$M^* = g\sigma_0 = gF_\pi,$$

we obtain

$$G_A = 1/F_\pi. \tag{52}$$

For the moment, if we allow $\partial_\mu A^\mu(x)$ to be the interpolating field for the π^0, the decay amplitude is expressible in terms of

$$q^\alpha T_{\alpha\mu\nu}(q, k_1, k_2) = \frac{G_A e^2}{2} q^\alpha \left[\int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}[(\gamma_5\gamma_\alpha S_F(p-k_1)\gamma_\mu S_F(p)\gamma_\nu S_F(p+k_2)) + (k_1 \leftrightarrow k_2,\quad \mu \leftrightarrow \nu)]\right]. \tag{53}$$

As before, the amplitude (53) is separable into the "vacuum" part and a density-dependent part, since the the nucleon propagators (9) themselves are so separable. We denote the vacuum amplitude by $q^\alpha T^{(a)}_{\alpha\mu\nu}$ and the density-dependent amplitude by $q^\alpha T^{(b)}_{\alpha\mu\nu}$.

The VVA amplitude $T_{\alpha\mu\nu}$ must satisfy the vector Ward identity[38]

$$k_1^\mu T_{\alpha\mu\nu} = 0 = k_2^\nu T_{\alpha\mu\nu}, \tag{54}$$

arising from vector-current conservation. The generalization to many-body theory of the propagator identity

$$\{[\not p - \not k_1 - M]\}^{-1} \not k_1 \{[\not p - M]\}^{-1} = \{[\not p - \not k_1 - M]\}^{-1} - \{[\not p - M]\}^{-1} \tag{55}$$

allows one to derive

$$k_1^\mu T_{\alpha\mu\nu} = \frac{G_A e^2}{2} \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[(\gamma_5\gamma_\alpha\{[\not p - \not k_1 - M^*]\}^{-1}\gamma_\nu\{[\not p + \not k_2 - M^*]\}^{-1}) - (\gamma_5\gamma_\alpha\{[\not p - \not k_2 - M^*]\}^{-1}\gamma_\nu\{[\not p + \not k_1 - M^*]\}^{-1})\right]. \tag{56}$$

The density- and temperature-dependent terms in (56) are finite, as can be verified by doing the p_0 integration, so that a shift in the integration variable is permissible for these terms. Hence, $k_1^\mu T_{\alpha\mu\nu}$ would vanish provided the vacuum terms also permit a shift of the integration variable. As is well known,[34-36] such a change of variables for linearly divergent integrals leads to a surface term.[39] Maintaining gauge invariance requires the surface term to be specified appropriately. This leads to the "anomalous" term in the axial-vector Ward identity to be discussed below.

With $k_1{}^2 = k_2{}^2 = \Lambda^2$, a lengthy but straightforward calculation leads to

$$T^{(a)}_{\alpha\mu\nu} = \frac{G_A e^2}{4\pi^2}\{(k_1+k_2)_\alpha\epsilon_{\mu\nu\rho\sigma}k_1^\rho k_2^\sigma F_1(q^2,\Lambda^2)+[(k_1-k_2)_\mu\epsilon_{\alpha\nu\lambda\sigma}k_1^\lambda k_2^\sigma+(k_1-k_2)_\nu\epsilon_{\alpha\mu\lambda\sigma}k_1^\lambda k_2^\sigma]F_2(q^2,\Lambda^2)+\epsilon_{\alpha\mu\nu\lambda}(k_1-k_2)^\lambda F_3(q^2,\Lambda^2)\}, \tag{57}$$

where

$$F_1(q^2,\Lambda^2) = 2\int_0^1 dx\int_0^{1-x} dy[(x+y)-(x+y)^2+4xy]/R,$$

$$F_2(q^2,\Lambda^2) = 2\int_0^1 dx\int_0^{1-x} dy[(x+y)(1-x-y)]/R,$$

and

$$F_3(q^2,\Lambda^2) = 2a + 4\int_0^1 dx\int_0^{1-x}\frac{dy}{R}\left\{[1-x\delta(x+y-1)]R\ln(R/M^{*2}) + \left(\frac{q^2}{4}\right)(x+y)(1-x-y) - M^{*2} - \frac{\Lambda^2}{2}(x+y)[1-2(x+y)]\right\},$$

with

$$R = [M^{*2} - q^2xy - \Lambda^2(x+y)(1-x-y)]\,. \tag{58}$$

In the expression for F_3 in (58) we have shifted the fermion loop momentum integration variable p of Eq. (53) by the arbitrary amount $[(k_1-k_2)^\lambda a]$. We now have

$$k_1^\mu T_{\alpha\mu\nu} = k_1^\mu T^{(a)}_{\alpha\mu\nu} = [k_1\cdot(k_1-k_2)F_2+F_3]\epsilon_{\alpha\nu\lambda\sigma}k_1^\lambda k_2^\sigma. \tag{59}$$

Gauge invariance of $T_{\alpha\mu\nu}$ requires $k_1^\mu T_{\alpha\mu\nu}=0$, so that from (59) we have

$$(2\Lambda^2 - q^2/2)F_2 + F_3 = 0 \tag{60}$$

or

$$a = 1 - \int_0^1 dx\int_0^{1-x} dy\{\Lambda^2[1-2(x+y)](y-x)\}/R\,. \tag{61}$$

For the choice of a given by (61) the amplitude (53) is given by

$$q^\alpha T_{\alpha\mu\nu} = q^\alpha T^{(a)}_{\alpha\mu\nu} + q^\alpha T^{(b)}_{\alpha\mu\nu}$$

with

$$\begin{aligned} q^\alpha T^{(a)}_{\alpha\mu\nu} &= \frac{G_A e^2}{16\pi^2}\epsilon_{\mu\nu\lambda\sigma}k_1^\lambda k_2^\sigma(q^2F_1 - 2F_3)\\ &= \frac{G_A e^2}{4\pi^2}\epsilon_{\mu\nu\lambda\sigma}k_1^\lambda k_2^\sigma\left(-1+2\int_0^1 dx\int_0^{1-x} dy\frac{M^{*2}}{R}\right)\\ &= \epsilon_{\mu\nu\lambda\sigma}k_1^\lambda k_2^\sigma\left(\frac{-e^2}{4\pi^2F_\pi}+\frac{ge^2M^*}{2\pi^2}\int_0^1 dx\int_0^{1-x}\frac{dy}{R}\right). \end{aligned} \tag{62}$$

Comparing the amplitude for the pseudoscalar coupling $T^{(a)}_{\mu\nu}$ given in (38) with the expression $q^\alpha T^{(a)}_{\alpha\mu\nu}$ in (62) we see that

$$q^\alpha T^{(a)}_{\alpha\mu\nu} = \frac{-e^2}{4\pi^2F_\pi}\epsilon_{\mu\nu\lambda\sigma}k_1^\lambda k_2^\sigma + T^{(a)}_{\mu\nu}\,. \tag{63}$$

The density-dependent terms $q^\alpha T^{(b)}_{\alpha\mu\nu}$ and $T^{(b)}_{\mu\nu}$ can be shown to be identical using the standard manipulations on the traces of the γ matrices appearing in $T_{\alpha\mu\nu}$, together with relations of the type

$$(\not p - \not k_1 - M^*)\{[\not p - \not k_1 - M^*]\}^{-1} = 1,$$

which are trivial generalizations to many-body theory, of the usual relations with propagators in the vacuum. We thus have for the full amplitudes in nuclear matter

$$q^{\alpha}T_{\alpha\mu\nu}=\frac{-e^2}{4\pi^2F_\pi}\epsilon_{\mu\nu\lambda\sigma}k_1^\lambda k_2^\sigma+T_{\mu\nu}. \tag{64}$$

The relation (64) is the extension to many-body theory of the result of Adler[34] and of Bell and Jackiw.[35] It shows that both $T_{\alpha\mu\nu}$ and $T_{\mu\nu}$ pick up additional contributions from processes arising in the presence of nuclear matter while the anomalous term in (64) with the coefficient $(-e^2/4\pi^2F_\pi)$ continues to have the same magnitude as in the vacuum. In other words, the operator identity

$$\partial^\mu A_\mu^{(3)}=m_\pi^2F_\pi\Phi_{\pi^0}+(e^2/32\pi^2):F_{\mu\nu}F_{\lambda\rho}:\epsilon^{\mu\nu\lambda\rho}$$

remains unchanged. This result is not unexpected in view of the Adler-Bardeen theorem which states that the triangle anomaly remains unaltered up to any finite order in perturbation theory.[40] Let us now consider the question of taking the soft-pion limit in (64). Since $\Lambda^2\neq 0$, the point $q_0^2=0$ is no longer a chiral-symmetry point. This is also true for the usual vacuum amplitude when the photon is continued off its mass shell.[41] In this section we have exploited the fact that the decay rate for $\pi^0\to 2\gamma$ in the vacuum as evaluated in the σ model is in surprising agreement with the experimental value.[33,36] We have pursued the consequences of accepting the σ model as a model for the description of low-energy π-N phenomena and for nuclear matter, for the pion's decay rate in the nuclear medium.

The following comments are in order at this stage. The calculations have not included exchange terms in the self-energy of the nucleons so that, in general, the mass shifts in nuclear matter will have momentum-dependent terms.[42] Furthermore, we have neglected the modifications in the pion-nucleon coupling constant due to the presence of the medium. The renormalization of the coupling constant is expected to be small,[43] as is the renormalization of the charge due to many-body effects. We have also ignored higher-order diagrams which will have contributions in many-body theory, including two-particle–two-hole terms. All these effects are expected to be small in comparison with the terms retained.

V. CONCLUDING REMARKS

The consequences of symmetry restoration and a corresponding phase change in nuclear matter have not been fully explored in the literature beyond the computation of the changes in the excitation spectrum. The situation is akin to being given two samples of metal at low temperature, one of which is said to be superconducting, while the other is not. It is the passage of a current with zero resistance which readily establishes, by the propagation of a disturbance through the medium and the corresponding measurement of a current correlation function, which sample is indeed superconducting. It is then of interest to study the current correlation functions in relativistic many-body theory. Some aspects of this and the consequences of spectral-function sum rules in many-body theory arising from current algebra have been investigated.[44] In the context of gauge invariance we may mention that the temperature dependence of the Lamb shift in atomic hydrogen and the low-energy theorems associated with Compton scattering at finite temperatures have been studied by Walsh[45] and by Barton.[46] We have considered here the specific case of the VVP correlation function and have shown that the lifetime of the neutral pion is changed in the nuclear medium so that this could be used as a signal for phase changes. From Table I we see that below the critical Fermi momentum, at zero temperature, the decay width of the neutral pion increases from its value of 7.5 eV at zero density to nearly 26.3 eV at $k_F\cong 270$ MeV/c. The lifetime of the pion is experimentally measured by the Primakoff effect.[11] The numbers quoted in the paper by Bellettini *et al.*[47] do show a slight trend toward an increase in the decay width as the energy of the incident photon in the photonuclear production of π^0 is increased from 1.5 to 2.0 GeV. The present study suggests that a further refinement, beyond the usual corrections for coherent and incoherent productions off a heavy nucleus, is needed. This is because the γ-γ-π^0 vertex itself is dressed. It also suggests that if the incoming photon is capable of depositing very high energy within the nucleus itself, it might be feasible to detect changes of phase in nuclear matter through the measurement of the pion decay width in nuclear matter, in view of the substantial change in this quantity. We see in Table I that the decay width goes up by at least two orders of magnitude for $k_F>270$ MeV/c in our model calculations. The calculations we have performed for infinite nuclear matter can be extended (1) to other kinematic regions for obtaining theoretical estimates for experimentally achievable configurations, and (2) for finite nuclei.

We note that the production of a π^0 in the photoproduction off a heavy nucleus has contributions from (a) Coulomb production external to the nucleus, (b) Coulomb production internal to the nucleus, (c) coherent production off the nucleons in the nucleus, and (d) incoherent production off the nucleons in the nucleus. As has been emphasized by Morpurgo,[48] the first two contributions have angular and energy dependences distinct from the latter two. For a fixed energy the peaks in the differential cross section occur at different angles,

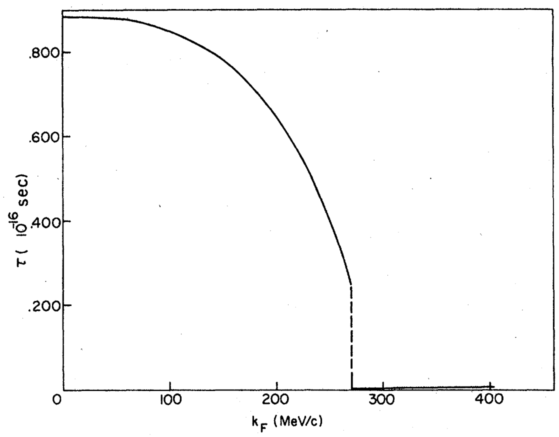


FIG. 11. Lifetime of the neutral pion in nuclear matter with increasing Fermi momentum.

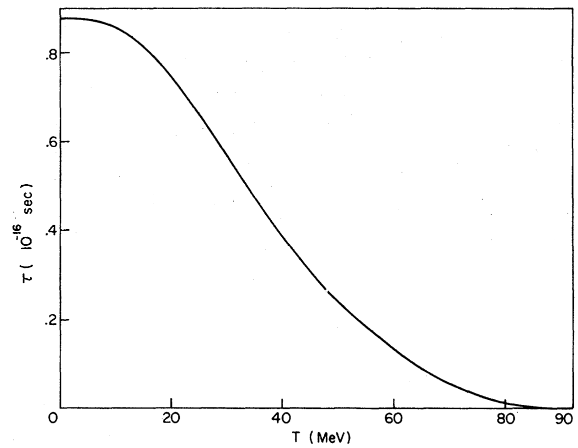


FIG. 12. Lifetime of the neutral pion in nuclear matter with increasing temperature.

and this important feature is used in the experimental detection of the Primakoff peak and for the determination of its shape. Absorption effects lower the peaks but do not appreciably affect their position. The shapes of the angular distributions for $\theta \lesssim 5°$ could permit a resolution of the internal Primakoff effect. This would entail a more detailed fit to the already available data to account for the different values of the decay width of the π^0 inside the nuclear matter and outside it. Our prediction of a very large change in the value of the decay constant inside abnormal nuclear matter might well allow an experimental detection of the existence of the phase changes of the Lee-Wick type. An estimation of the expected differential cross section at higher energies, while taking *reabsorption effects* into account, is the next step in our program. In the context of reabsorption we note that the pion dispersion relation in nuclear matter is needed.[49]

In Table II, we have presented the results for the decay width of the π^0 at finite temperature in the case where the nucleon chemical potential is neglected. The decay width increases more smoothly in this case because of the absence of a first-order phase transition in our analysis at finite temperature. In Figs. 11 and 12 we have shown the lifetime of the pion as a function of k_F and of temperature, respectively, in order to summarize the results in Tables I and II.

We have discussed the gauge invariance of the *VVA* amplitude and shown how the Adler anomaly remains inviolate in the nuclear medium, though $q^\alpha T_{\alpha\mu\nu}$ and $T_{\mu\nu}$ have additional contributions in many-body theory. We considered only the dressed transverse photons or the nuclear plasmons in the nuclear medium, since the electromagnetic radiation escaping the nuclear medium will have such a polarization in a realistic situation. The problem of treating the longitudinal plasmons is made more challenging by the fact that this mode will mix with the σ meson (or scalar density oscillation) mode. This new feature requires the treatment of mode-mixing leading to a plasmon dressed by scalar density oscillations or the "nuclear polariton." This nomenclature is in analogy with the similar situation for photons dressed by phonons in solid-state theory. The mixing arises from the fact that *at finite density* the polarization tensor develops nonzero off-diagonal elements corresponding to a photon coupling to a σ meson via polarization effects. This effective coupling at the one-loop level can easily be shown (a) to be *purely density dependent,* and (b) to vanish when there are *equal numbers of charged particles and antiparticles present in the medium.* (A discussion of a direct mode mixing in the context of the Abelian Higgs model in field theory has been given by B. W. Lee,[50] and this may be compared with the mode mixing as discussed by Chin.[28]) These new features together with the mapping out of the "polariton" dispersion relation in nuclear matter are problems for further study, as is the investigation of the feasibility of the detection of such mixed modes.

Recently, the possibility of scalar and pseudoscalar couplings in the neutral-current weak interactions has been raised, and it has been suggested that this would allow the reaction $\gamma\gamma \to \nu\bar{\nu}$ to proceed, among other possibilities via the neutral-pion intermediate state.[51] This would affect the rate of stellar cooling. The present calculation suggests that at higher temperatures, due to a larger $\gamma\gamma\pi^0$ effective coupling, the mechanism of photon conversion to neutrinos via π^0 could be even more important than in earlier estimates.

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