PHYSICAL REVIEW D VOLUME 19, NUMBER 5 1 MARCH 1979

Quantum chromodynamics and the spin-dependent quark-antiquark forces

Howard J. Schnitzer
Department of Physics, Brandeis University, Waltham, Massachusetts 02154
(Received 16 October 1978)

Recent results from quantum chromodynamics (QCD) are shown to be consistent with the generalized Breit-type phenomenological Hamiltonian previously proposed for the spin-dependent spectroscopy of quark-antiquark states. In particular, the concept of a short-distance and long-distance quark-gluon chromomagnetic moment, with different qualitative behavior for light and heavy quarks, is compatible with QCD. An induced axial-vector exchange, previously suggested by Feinberg and Sucher, also seems to be implied by the field-theoretic calculations.

I. INTRODUCTION

Since the discovery of the ψ/J states there has been a healthy interplay between experiment, phenomenology,[1] and the field-theoretic results[2] of quantum chromodynamics (QCD). Considerable success has been achieved in the description of heavy-quark–antiquark ($Q\overline{Q}$) spectroscopy by means of a spin-independent static potential model.[1] Further, a qualitative understanding of the spin-dependent corrections to the $Q\overline{Q}$ levels is attained by means of a Breit-type Hamiltonian,[3] allowing both effective scalar and vector exchanges,[4] supplemented by a flavor-dependent, long-distance quark-gluon color anomalous moment.[5-7] In spite of the numerous successes of this straightforward approach to hadron spectroscopy, a number of problems remain to be solved. In particular, it is desirable to put this program on a firmer footing, based on known results of QCD.

Many of the same issues pursued by phenomenologists have also been examined by field theorists attempting to extract precise consequences from QCD. For example, both the static color-singlet potential[2,8] and spin-dependent Hamiltonian[9] have recently been studied in perturbation theory. Although no signal of quark confinement has been discovered in these calculations, we believe that a careful examination of results from QCD is capable of providing further insights for the phenomenologist. Conversely, phenomenological analyses and speculations should stimulate additional field-theoretic calculations directed towards the problems of spectroscopy.

It is the purpose of this paper to reexamine recent results from QCD, so as to present a synthesis suitable for application to the phenomenology of the spin dependence of the $Q\overline{Q}$ force. We make several speculations which have interesting implications, and which hopefully will provide the basis for further work directed towards clarifying the issues raised in this paper.

We shall present two major findings:

(1) The Breit-type Hamiltonian,[3] with quark-gluon anomalous moment,[5] together with an induced axial-vector exchange, is consistent with recent calculations,[9] and

(2) This induced axial-vector exchange is probably unimportant in light-quark spectroscopy. Further work will be required to clarify the role of the axial-vector exchange in heavy-quark–antiquark spectroscopy, although our preliminary analysis indicates that it could be comparable to the usual hyperfine interaction in *heavy*-quark systems.

It should be emphasized that the induced axial-vector exchange was anticipated by Feinberg and Sucher,[10] and their collaborators,[11] although they were unable to relate it to the other terms of the potential. Our interpretation of Dine's work suggests a relation of the axial-vector exchange to the other parameters of the theory.

II. SPIN-DEPENDENT FORCES

The phenomenology of the spin-dependent forces in hadrons has been vigorously pursued in the last few years.[3-7] In spite of significant progress, we are far from a fundamental understanding of these forces, so that a certain degree of intuition and plausibility, based on QCD, is required in any analysis. One particular case which exemplifies the interplay between theory and model building is the question of the quark-gluon color anomalous magnetic moment vertex. It had been shown by Yao[12] and Duncan[13] that this vertex was infrared divergent. Motivated by these analyses, we reasoned that such divergences must be summed in the infrared (long distance) region if the spin-dependent Hamiltonian was to have a phenomenological meaning. Therefore we proposed[5] that the effective spin-dependent potential required an effective quark-gluon color anomalous magnetic moment, which was small at short distances, and possibly large for *heavy* quarks at long distances. It was suggested that such an anomalous moment might assist our understanding of the charmonium states $X(2830)$ and $\chi(3450)$. In this section we attempt to clarify the theoretical basis of the Breit-type Hamiltonian, and quark-gluon color anomalous moment, within the context of QCD.

Suppose the effective Hamiltonian between heavy *color-singlet* $Q\bar{Q}$ states is written in momentum space as a sum of *effective dressed* propagator exchanges, together with *dressed* vertices. This proposition in general permits all five Dirac invariants (S, V, T, P, and A) to be included as exchanges in the effective Hamiltonian.[10,14] Nevertheless only S and V can contribute to the *spin-independent* potential in the static limit $M \to \infty$, where M is the mass of the quark. Moreover, the explicit fourth-order QCD calculation of Dine,[9] including spin dependence to $O(1/M^2)$, only requires the presence of effective V and A exchanges. Let us consider this circumstance in more detail.

Assume the effective vector-gluon propagator at zero energy transfer to be of the form[5]

$$D_{\mu\nu}(q_0=0,\vec{q}) = V(\vec{q})g_{\mu\nu}, \tag{2.1}$$

and the fermion–effective-vector-gluon vertex to be

$$\Gamma^a_\mu(p,q) = \left[\gamma_\mu - i\frac{\kappa(\vec{q})}{2M}\sigma_{\mu\nu}q^\nu\right]T^a, \tag{2.2}$$

where p is the (on-shell) quark momentum and T^a is the color representation matrix. Equation (2.2) does not allow for infrared-divergent contributions to the vertex which diverge as $p^2 \to M^2$, but as Dine shows,[9] these cancel through one loop. Similarly, assume that the effective axial-vector propagator is of the form[10,14]

$$D^A(q_0=0,\vec{q}) = (A(\vec{q})/M^2)g_{\mu\nu}, \tag{2.3}$$

and the effective fermion–axial-vector vertex is

$$\Gamma^a_{\mu 5}(p,q) = (\gamma_\mu\gamma^5)T^a. \tag{2.4}$$

It is straightforward to derive the nonrelativistic reduction of the effective interaction, and obtain the effective Hamiltonian in momentum space for V and A exchange using (2.1) to (2.4). The result is[3-5,10,14]

$$\begin{aligned}H_{\text{int}} = -C_F V(\vec{q})\Big\{&1 + (\text{spin-independent corrections}) + \frac{i}{4M^2}[1+2\kappa(\vec{q})][\vec{q}\times\vec{p}_2\cdot\vec{\sigma}_Q - \vec{q}\times\vec{p}_1\cdot\vec{\sigma}_{\bar{Q}}]\\ &+\frac{i}{2M^2}[1+\kappa(\vec{q})][\vec{q}\times\vec{p}_2\cdot\vec{\sigma}_{\bar{Q}} - \vec{q}\times\vec{p}_1\cdot\vec{\sigma}_Q] - \frac{[1+\kappa(\vec{q})]^2}{4M^2}[\tfrac{1}{3}\vec{q}^{\,2}\vec{\sigma}_Q\cdot\vec{\sigma}_{\bar{Q}} - \vec{q}\cdot\vec{\sigma}_Q\vec{q}\cdot\vec{\sigma}_{\bar{Q}}]\\ &-\frac{1}{6M^2}[1+\kappa(\vec{q})]^2[\vec{q}^{\,2}\vec{\sigma}_Q\cdot\vec{\sigma}_{\bar{Q}}]\Big\} + \frac{A(\vec{q})}{M^2}\vec{\sigma}_Q\cdot\vec{\sigma}_{\bar{Q}} + O\left(\frac{1}{M^4}\right),\end{aligned} \tag{2.5}$$

(in the above C_F is the usual Casimir operator). The *spin-dependent* corrections in (2.5) are, respectively, the Thomas term, the spin-orbit coupling, the tensor force, hyperfine interaction, and axial-vector interaction. Equation (2.5) is therefore the momentum-space version of a Breit-type interaction with V and A exchanges.

Dine[9] has evaluated *all* fourth-order contributions to the $Q\bar{Q}$ interaction correct to order $g_\mu^{\,4}\ln(|\vec{q}^{\,2}|/M^2)$, with $|\vec{q}| \ll M$, where

$$g_\mu^{\,2}(\vec{q}) = g_\mu^{\,2}\left[1 - \frac{11}{3}\frac{C_A g_\mu^{\,2}}{16\pi^2}\ln(\vec{q}^{\,2}/\mu^2)\right] \tag{2.6}$$

is the invariant charge to $O(g_\mu^{\,4})$, subtracted at a mass scale μ of the order of typical bound-state momenta, i.e., of order $|\vec{q}|$, $|\vec{p}|$. After correcting a typographical error in Dine's reports,[9,15] Dine's result can be written exactly as in (2.5), with the identifications, correct to the accuracy of his calculation,

$$V(\vec{q}) = g_\mu^{\,2}(\vec{q}), \tag{2.7}$$

$$\kappa(\vec{q}) = -\frac{C_A g_\mu^{\,2}}{16\pi^2}\ln(\vec{q}^{\,2}/M^2), \tag{2.8}$$

and

$$A(\vec{q}) = -\tfrac{1}{4}\kappa(\vec{q})\vec{q}^{\,2}V(\vec{q}). \tag{2.9}$$

Note that $\kappa(\vec{q})$ in the effective Hamiltonian arises from the same trigluon self-interaction corrections evaluated by Yao[12] and Duncan[13] in their studies of the quark-gluon vertex. It is important to observe that the anomalous moment $\kappa(\vec{q})$ involves *two separate* mass scales, μ and M, characteristic of the infrared and ultraviolet domains of the quark-gluon vertex, respectively. Most importantly, the $\ln(\vec{q}^2/M^2)$ in (2.8) cannot be absorbed into a redefinition of the invariant charge, or of the spin-independent potential. Let us consider these issues further.

A. Color magnetic moment

Aside from the induced axial-vector exchange, which we discuss shortly, Eqs. (2.5)–(2.8) are completely consistent with the spin-dependent Hamiltonians assumed in phenomenological analyses.[3-7] The presence of a $(\vec{q}^2)$ dependent, i.e., distance dependent color anomalous moment $\kappa(\vec{q})$ also confirms our earlier hypothesis.[5] Since the factor $\ln(\vec{q}^2/M^2)$ cannot be absorbed into a redefinition of the running charge $g_\mu^2(\vec{q})$, one should consider the behavior of the sum of all powers of the $\ln(\vec{q}^2)$ terms contributing to $\kappa(\vec{q})$ before application of phenomenology should be attempted. There are no specific techniques available for performing such a sum, other than the general considerations of Poggio.[16] The methods of Poggio[16] are important in that they allow the separation of the infrared from the ultraviolet domain in summing contributions to $\kappa(\vec{q})$ in the spin-dependent Hamiltonian. We conjecture, consistent with the analysis of Poggio[16] and (2.8), that the *total* chromomagnetic moment $\mathfrak{M}(\vec{q})$ is

$$\begin{aligned}\mathfrak{M}(\vec{q}) &\equiv [1+\kappa(\vec{q})]\\ &= \left(\frac{\vec{q}^2}{M^2}\right)^{-d(g_\mu(\vec{q}^2))},\end{aligned} \tag{2.10}$$

where in perturbation theory

$$d(g_\mu(\vec{q}^2)) = \frac{C_A g_\mu^2(\vec{q}^2)}{16\pi^2} + O(g_\mu^4(\vec{q}^2)), \tag{2.11}$$

with $g_\mu^2(\vec{q}^2)$ given to lowest order by (2.6). The conjectured summation (2.10) is plausible in that (a) it removes all explicit logarithms from $\kappa(\vec{q})$, and (b) it accounts for cross terms of powers of $\ln\vec{q}^2$ of the form $[\ln\vec{q}^2/\mu^2]^r[\ln\vec{q}^2/M^2]^s$, which inevitably appear in perturbation theory for the Yang-Mills vertex. Although (2.10) is deceptively simple, the interplay of the ultraviolet and infrared regions can be quite complicated in perturbation theory. Only when $|\vec{q}|^2 \gg \mu^2$ should the perturbative evaluation of (2.11) be valid, since only then will $g_\mu^2(q^2)$ be small.

Since a leading-logarithm calculation, such as performed by Dine,[9] cannot determine the ultraviolet mass scale with any precision, at the very least a higher-order calculation keeping track of subdominant logarithms will be required. We have plausibly chosen this mass scale so that the *short-distance* chromomagnetic moment of *heavy* quarks is

$$[1+\kappa(\vec{q})]_{\vec{q}^2\simeq M^2} \cong 1, \tag{2.12}$$

i.e.,

$$\kappa(\vec{q}^2\simeq M^2)\simeq 0. \tag{2.13}$$

Given (2.10), since $(M^2/\mu^2)\gg 1$ in *heavy*-quark spectroscopy, the *long-distance* chromomagnetic moment of *heavy* quarks is

$$[1+\kappa(\vec{q})]_{\vec{q}^2\simeq\mu^2} = \left(\frac{M^2}{\mu^2}\right)^{d(g_\mu)} \tag{2.14a}$$

$$>1, \tag{2.14b}$$

where (2.14b) is based on the *assumption* that $d(g_\mu)>0$. This is true in the lowest-order estimate (2.11). Note that Eqs. (2.10)–(2.14) are consistent with the behavior postulated for the *short-distance* and *long-distance* anomalous moment of heavy quarks in earlier work.[5]

An interesting limit of (2.10) is obtained if the mass scale M^2 vanishes, i.e., if $M^2\to 0$, with $\vec{q}^2$, μ^2 fixed. Then, again assuming $d(g_\mu^2)>0$,

$$\lim_{\substack{M^2\to 0\\ \mu^2,\vec{q}^2 \text{ fixed}}} [1+\kappa(\vec{q})] \sim 0 \tag{2.15a}$$

or

$$\lim_{\substack{M^2\to 0\\ \mu^2,\vec{q}^2 \text{ fixed}}} \kappa(\vec{q}) \sim -1. \tag{2.15b}$$

This means that the hyperfine-interaction energy, at fixed $\vec{q}^2$ and μ^2, satisfies

$$\lim_{\substack{M^2\to 0\\ \mu^2,\vec{q}^2 \text{ fixed}}} \frac{[\mathfrak{M}(q)]^2}{M^2} \sim [M^2]^{2d-1} \tag{2.16a}$$

$$\sim \textit{finite}, \quad \text{if } d(g_\mu^2)\geq\tfrac{1}{2}. \tag{2.16b}$$

Equations (2.15b) and (2.16b) have previously been noted in a phenomenological context.[6] Thus, if (2.10) and (2.16b) are correct, then (2.14b) also follows.

Equations (2.15) and (2.16) are *not* limits appropriate to *light-quark* spectroscopy. In the spectroscopy of ordinary quarks, there is no clear separation of the ultraviolet and infrared mass scales, since one must consider the quark-gluon vertex in the kinematical domain,

$$\mu^2\simeq M^2. \tag{2.17}$$

In this case one does not have sufficient information to apply Eq. (2.10) to light-quark spectroscopy. Since

$$[\ln(\vec{q}^2/\mu^2)][\ln(\vec{q}^2/M^2)]\simeq[\ln(\vec{q}^2/\mu^2)]^2 \tag{2.18}$$

in region (2.17), Eq. (2.10) is probably not the most convenient form for practical applications. Another summation making the q^2 dependence more explicit is desirable. Since $\mathfrak{M} = (1+\kappa) = 0$ is equivalent to scalar exchange,[6,7] which apparently is the Lorentz character of the confining potential,[4] a careful study of $\mathfrak{M}(\vec{q}^2)$ in the domain (2.17) is clearly warrented.

B. Induced axial-vector exchange

The reader will observe from (2.8) and (2.9) that we have assumed that the $\ln(\vec{q}^2/M^2)$ term in the induced axial-vector exchange is actually $\kappa(q)$. In fourth order this identification is not obvious, but consideration of a gauge-invariant subset of graphs at the two-loop level makes this more plausible. At the one-loop level, $\kappa(\vec{q})$ appears in the Hamiltonian as a result of the Yang-Mills correction to the quark-gluon vertex (Fig. 1). In order $g_\mu{}^4$, the induced axial-vector interaction comes from the uncrossed and crossed two-gluon exchanges (Fig. 2). At the two-level, i.e., in order $g_\mu{}^6$, gauge invariance interrelates the two-gluon exchange graphs with the vertex corrections (Fig. 3), thus intertwining the induced axial-vector current with the Pauli moment corrections.

Let us focus on the hyperfine interaction obtained from the vector exchange. It is proportional to[5]

$$\vec{q}^2 V(\vec{q}) \frac{(1+\kappa)^2}{M^2} \vec{\sigma}_Q \cdot \vec{\sigma}_{\bar{Q}} = \frac{(1+2\kappa+\kappa^2)}{M^2} \vec{\sigma}_Q \cdot \vec{\sigma}_{\bar{Q}} \vec{q}^2 V(\vec{q}) , \tag{2.19}$$

since this represents the magnetic-magnetic interaction

$$\frac{\mathfrak{M}_Q(\vec{q})\mathfrak{M}_{\bar{Q}}(\vec{q})}{M^2} [\vec{q}^2 V(\vec{q})] . \tag{2.20}$$

The term (2κ) in (2.19) receives $O(g_\mu{}^4)$ contributions from the two graphs of Fig. 1, and $O(g_\mu{}^6)$ contributions from the vertex graphs of Fig. 3. The term (κ^2) receives its *leading* correction, of $O(g_\mu{}^6)$, from the double vertex correction of Fig. 3(a). As we have noted, the graphs of Fig. 3 are interrelated by gauge invariance, so that anomalous magnetic moment corrections are closely related to the induced axial-vector exchange. Therefore, we also anticipate a κ^2 contribution to the axial-vector exchange.

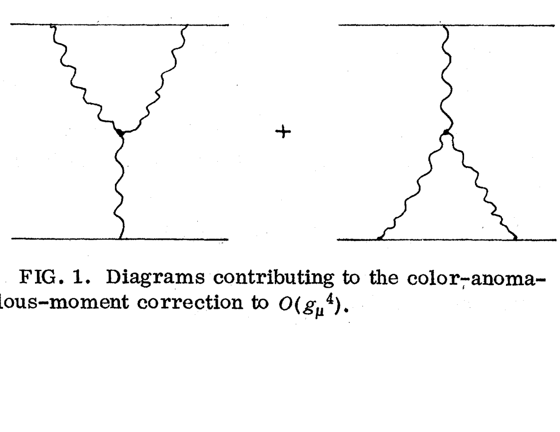

FIG. 1. Diagrams contributing to the color-anomalous-moment correction to $O(g_\mu{}^4)$.

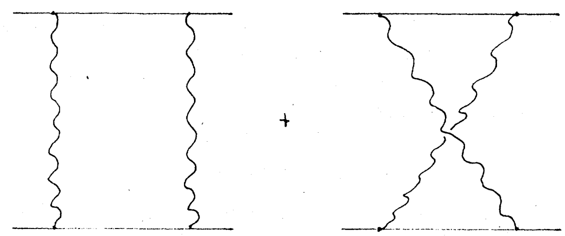

FIG. 2. Diagrams contributing to induced axial-vector exchange to $O(g_\mu{}^4)$.

Notice that to $O(g_\mu{}^4)$, the exchange of two *scalar* gluons, or one scalar gluon and one vector gluon cannot induce an axial-vector exchange in the static limit; double vector exchange seems to be required. Since $(1+\kappa) = 0$ is equivalent to scalar exchange,[6,7] we conjecture that $A(\vec{q})$ must vanish as $[1+\kappa(\vec{q})]^2$, with the power reflecting the above line of reasoning. Assembling the various elements of this discussion, we conjecture that

$$A(\vec{q}) = -\tfrac{1}{4}\kappa(\vec{q})[1+\kappa(\vec{q})]^2 \vec{q}\, V(\vec{q}) , \tag{2.21}$$

to *two-loop accuracy*. Field-theoretic calculations of the $Q\bar{Q}$ interactions must be extended to $O(g_\mu{}^6)$, at the very least, to test the validity of (2.21). In the next section we argue that (2.21) is not quite the correct axial-vector interaction to be used in phenomenology. We will attempt to take into consideration the possibility that only a fraction f of the confining potential is due to effective vector exchange, and $(1-f)$ due to scalar exchange.

Finally we remark that we have assumed that the Lorentz character of the induced interaction is axial vector, although it could just as well be de-

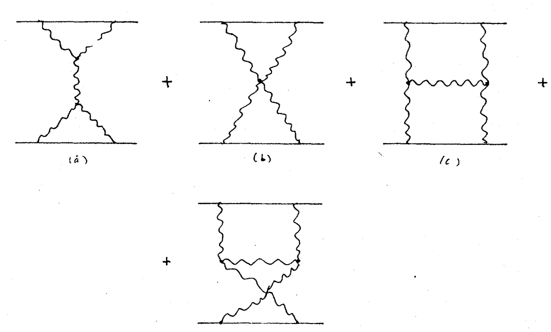

FIG. 3. Diagrams related by gauge invariance, contributing to *both* the color anomalous moment and the induced axial-vector exchange in $O(g_\mu{}^6)$. (a) κ^2 contribution to the hyperfine interaction with vector exchange; (b)–(d) diagrams contributing to the inducing axial-vector exchange.

scribed as tensor.[10,14] In order to distinguish these two possibilities, one will require a calculation of the effective Hamiltonian correct to $O(1/M^4)$.

III. IMPLICATIONS FOR PHENOMENOLOGY

The discussion of Sec. II indicates that the work of Dine[9] and Poggio[16] lend strong support to the general framework of the phenomenological efforts to date,[3-7] including the induced axial-vector exchange suggested by Feinberg and Sucher.[10] Even though the general strategy of the phenomenological program seems to be confirmed, there is as yet no evidence for quark confinement or even of induced scalar exchange emerging from the perturbative QCD calculations. Since the weight of empirical evidence favors a scalar confining potential,[4] one might hope for the appearance of an induced scalar exchange in perturbation theory.

Since one is as yet unable to derive the function $d(g_\mu(\vec{q}^2))$ appearing in (2.10), applications must treat the anomalous moment $\kappa(\vec{q})$ in an approximate manner, consistent with QCD. For the purposes of phenomenology, (2.10)–(2.14) make it reasonable to approximate $\kappa(\vec{q})$ by an *average* short-distance and long-distance value. Therefore, as in previous work,[5] one may take

$$\kappa(\vec{q}) \simeq 0 \quad \text{at short distances}\,, \tag{3.1}$$

and

$$\kappa(\vec{q}) \simeq \kappa = \text{constant} \quad \text{at long distances}\,, \tag{3.2}$$

with κ obtained from experiment. We hope that at some future date we will be able to use (2.10), or its improvement directly. The analysis of Sec. II A indicates that the long-distance anomalous moment is flavor *dependent* in that the long-distance anomalous moment of light and heavy quarks behave qualitatively differently. It is gratifying that this appears to be compatible with phenomenological analyses.[6,7]

The possible phenomenological role of the induced axial-vector exchange requires further clarification. We have equated $A(\vec{q})$ with Eq. (2.9) to $O(g_\mu^4)$, and with Eq. (2.21) to $O(g_\mu^6)$, with (2.21) the more speculative. Combining Eqs. (3.1) and (3.2) with (2.9) or (2.21) we expect

$$A(\vec{q}) \simeq 0 \text{ for either (a) short-distance vector exchange or (b) scalar exchange}\,. \tag{3.3}$$

Further, since one apparently requires the exchange of *two* effective vector gluons for a nonvanishing effect, we propose, compatible with (2.9) and (2.21), that one analyze meson spectroscopy with

$$A(\vec{q}) \simeq 0 \quad \text{at short distances} \tag{3.4a}$$

and

$$A(\vec{q}) \simeq -\frac{f^2}{4}\kappa(1+\kappa)^2[\vec{q}^2 V_c(\vec{q})] \quad \text{at long distances}\,, \tag{3.4b}$$

where $V_c(\vec{q})$ is the quark confining potential, κ is the (constant) long-distance anomalous moment, and f the *fraction* of vector exchange contributing to the confining potential. Equation (3.4b) is speculative, particularly with regard to the factors of f^2 and $(1+\kappa)^2$, although the reasoning leading to (3.4) has evolved from known results of QCD. Unfortunately, one may not be able to verify the form of $A(\vec{q})$ in perturbation theory. For many purposes it is preferable to work in coordinate space, where (3.4) becomes

$$A(\vec{r}) \simeq \frac{f^2}{4}\kappa(1+\kappa)^2\nabla^2 V_c(\vec{r})\,. \tag{3.5}$$

Thus from (2.5) we must add the additional term

$$\Delta H_{\text{spin}} = -\frac{f^2}{4M^2}\kappa(1+\kappa)^2\nabla^2 V_c(\vec{r})\vec{\sigma}_Q\cdot\vec{\sigma}_{\bar{Q}} \tag{3.6}$$

to the phenomenological Hamiltonians of earlier work.[3-7]

Since a recent survey of light-meson spectroscopy strongly suggests that[7]

$$f \simeq 0 \quad \text{for mesons with light quarks}\,, \tag{3.7}$$

ordinary meson spectroscopy is essentially unaffected by the induced axial-vector exchange. Application to charmonium is another matter. In charmonium one may obtain f_c and $(1+\kappa_c)$ from the noncontroversial ^3P states alone.[7] [We have used the subscripts to stress that f_c and $(1+\kappa_c)$ may take different values from their light-quark counterparts.] The two ^3P energy differences in charmonium give[4,7]

$$f_c \simeq 0.059 \tag{3.8}$$

and

$$\kappa_c \simeq 5.26\,. \tag{3.9}$$

The *complete* hyperfine interaction is then

$$H_{\text{spin-spin}} = -\frac{f(1+\kappa)^2}{6M^2}\left[1+\tfrac{3}{2}(f\kappa)\right]\nabla^2 V_c(\vec{r})\vec{\sigma}_Q\cdot\vec{\sigma}_{\bar{Q}}\,, \tag{3.10}$$

where the $\frac{3}{2}(f\kappa)$ factor comes from the axial-vector exchange. Since it has been estimated that[4,7]

$$\tfrac{3}{2}(f_c\kappa_c) \simeq 0.46 \tag{3.11}$$

in charmonium, the *predictions* of the $^3S_1$-$^1S_0$ energy differences are increased accordingly. Using our most recent estimates,[7] we now predict

$$n=1:\quad E(^3S_1)-E(^1S_0)=[204+94]\text{MeV} = 298\text{ MeV}, \tag{3.12}$$

and

$$n=2:\quad E(^3S_1)-E(^1S_0)=[170+78]\text{MeV} = 248\text{ MeV}, \tag{3.13}$$

where the earlier prediction with anomalous moment and the new axial-vector contributions are shown in (3.12) and (3.13) separately. These predictions are in qualitative agreement with

$$E(\psi)-E(X(2830))=265\text{ MeV} \tag{3.14}$$

and

$$E(\psi')-E(\chi(3455))=229\text{ MeV}. \tag{3.15}$$

Therefore, if our only problem in understanding $X(2830)$ and $\chi(3455)$ were their mass values, the existence of the induced axial-vector exchange would go a long way towards resolving this problem. Unfortunately this is not the case.[1]

IV. CONCLUSIONS

In this paper we have shown that recent results from QCD,[8,9,16] evaluated in perturbation theory, are compatible with the phenomenological Hamiltonians used to extract the spin-dependent forces of quark-antiquark interactions from spectroscopic data.[4-7] In particular, the concept of a short-distance and long-distance quark-gluon chromomagnetic moment,[5] with different qualitative behavior for light and heavy quarks,[6,7] is shown to be consistent with QCD. An induced axial-vector exchange, suggested by Feinberg and Sucher,[10] is also revealed by the calculations of Dine.[9]

We use these results as the basis for several speculations, which should provide a framework for further work in both field theory and phenomenology.

For numerical work based on Eqs. (2.5)–(2.9) see J. M. Richard and D. Sidhu.[17]

ACKNOWLEDGMENTS

We wish to thank the Physics Department of Brookhaven National Laboratory, where this analysis was begun. We also wish to acknowledge conversations with Professor T. Appelquist, Dr. I. Muzinich, Dr. E. Poggio, Dr. J-M. Richard, and Dr. D. Sidhu.

This work was supported in part by the U. S. Department of Energy under Contract No. E(11-1)3230.

[1]For reviews see J. D. Jackson, in *Proceedings of the 1977 European Conference on Particle Physics*, Budapest, edited by L. Jenik and I. Montvay (CRIP, Budapest, 1978); K. Gottfried, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies*, Hamburg, 1977, edited by F. Gutlrod (DESY, Hamburg, 1977); T. Appelquist, M. Barnett, and K. Lane, Annu. Rev. Nucl. Sci. 28, 387 (1978); O. W. Greenberg, *ibid.* 28, 327 (1978), J. Sucher, Rep. Prog. Phys. 41, 1781 (1978); A. W. Hendry and D. W. Lichtenberg, *ibid.* 41, 1707 (1978).

[2]For an introductory review see T. Appelquist, in Proceedings of the 1977 Banff Summer Institute (unpublished).

[3]H. J. Schnitzer, Phys. Rev. D 13, 74 (1976); Phys. Rev. Lett. 35, 1540 (1975); J. Pumplin, W. Repko, and A. Sato, *ibid.* 35, 1538 (1975).

[4]A. B. Henriques, B. H. Kellett, and R. G. Moorhouse, Phys. Lett. 64B, 85 (1976); D. Gromes, Nucl. Phys. B131, 80 (1977); Lai-Him Chan, Phys. Lett. 71B, 422 (1977); C. E. Carlson and F. Gross, *ibid.* 74B, 404 (1978); N. Isgur and G. Karl, *ibid.* 72B, 109 (1977); 74B, 353 (1978); Phys. Rev. D 18, 4187 (1979); H. J. Schnitzer, Phys. Lett. 76B, 461 (1978); Phys. Rev. D 1, 3482 (1978); X. Pham and J. M. Richard, Phys. Lett. 74B, 108 (1978).

[5]H. J. Schnitzer, Phys. Lett. 65B, 239 (1976). The coefficient of $\vec{L}\cdot\vec{S}$ should read $(3+4\kappa)$ in this reference.

[6]H. J. Schnitzer, Phys. Lett. 69B, 477 (1977).

[7]H. J. Schnitzer, Phys. Rev. D 1, 3482 (1978).

[8]E. Poggio, Phys. Rev. D 16, 2586 (1977); Phys. Lett. 71B, 381 (1977); T. Appelquist, M. Dine, and I. J. Muzinich, *ibid.* 69B, 231 (1977); Phys. Rev. D 17, 2074 (1978); W. F. Fischler, Nucl. Phys. B129, 157 (1977); F. Feinberg, Phys. Rev. Lett. 39, 316 (1977).

[9]Michael Dine, Yale Report No. COO-3075-204 (unpublished); and Yale University, thesis (unpublished); E. Poggio, (unpublished).

[10]G. Feinberg and J. Sucher, Phys. Rev. Lett. 35, 1740 (1975). The *axial-vector gluon* exchange postulated in this paper remains constant as the mass of the axial-vector meson $M^2\to\infty$. The induced axial-exchange discussed in this paper vanishes as $M^2\to\infty$. This results in certain important phenomenological differences which are explored in G. Feinberg, B. Lynn, and J. Sucher, Columbia University Report No. CU-TP-130 (unpublished).

[11]J. S. Kang and J. Sucher, work in progress.

[12]Y. P. Yao, Phys. Rev. Lett. 36, 653 (1976).

[13]A. Duncan, Phys. Rev. D 13, 2866 (1976).

[14]See D. Gromes, Nucl. Phys. B131, 80 (1977) for the reduction of these exchanges to a nonrelativistic Hamiltonian.

[15]We thank Dr. Dine for confirming this correction.

[16]E. Poggio, Phys. Rev. D 16, 2586 (1977).

[17]J. M. Richard and D. Sidhu, Brookhaven report, 1978 (unpublished).