Multilepton final states and the weak interactions of the fifth quark

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We summarize the evidence for the properties of a fifth quark, denoted as b, which is a constituent of $\Upsilon(9.4\ \text{GeV}/c^2)$. We show how an analysis of the lepton content of final states arising from unbound $(b\bar{b})$ production can yield the relative strengths of the $b \to u + W^-$ and $b \to c + W^-$ weak-current transitions.

I. INTRODUCTION

There is indirect but highly suggestive evidence for a fifth quark b, with mass $m_b \simeq 5\ \text{GeV}/c^2$ and charge $e_b = -\frac{1}{3}$, which is not inert with respect to weak interactions. The relative rates for the expected charged-current transitions $b \to c$ and $b \to u$ are of evident interest. Charmed particles, in contrast to particles composed only of light quarks (u, d, s), are copious sources of prompt leptons. The observation of leptons as decay products of $(b\bar{b})$ pairs above the new-flavor threshold therefore provides information on the relative $b \to c$ and $b \to u$ transition rates. In this paper we present a systematic technique for the analysis of leptonic final states, and call attention to constraints which may be useful in eliminating backgrounds. We discuss potential ambiguities and limitations of the method and comment on complementary approaches to the problem.

We review the evidence for the fifth quark and discuss its properties in Sec. II. The analysis of final states containing zero, one, two, three, and four leptons from $(b\bar{b})$ decay is presented in Sec. III. The observables defined there are related in Sec. IV to items of theoretical interest. In Sec. V we treat the effects of backgrounds, neutral-particle mixing, and CP violation. Section VI is devoted to a summary and conclusions. The evolution of correlated neutral-meson pairs is further discussed in an Appendix.

II. EVIDENCE FOR THE b QUARK

The vector meson $\Upsilon(9.4\ \text{GeV}/c^2)$ was first observed[1] as a $\mu^+\mu^-$ resonance in the reaction

$$p + N \to (\mu^+\mu^-) + \text{anything}. \tag{2.1}$$

It was seen to have at least one heavier companion state,[2] $\Upsilon'(10.0)$, just as the $\psi/J(3.095)$[3] has a radial excitation $\psi'(3.684)$.[4] The interpretation of the psion family as bound states of a charmed quark and antiquark $(c\bar{c})$ has been notably successful.[5,6] It was therefore natural that the Υ family be interpreted as bound states of a new heavy quark and antiquark $(Q\bar{Q})$.[7,8] The constituent of the upsilons would be the fifth quark, following the already established u, d, s, and c, and would have a mass of approximately $m_\Upsilon/2$.

On the basis of various models for the Υ and Υ' production cross sections, a number of authors[7] expressed a preference for $e_Q = -\frac{1}{3}$ as the charge of the new quark. This assignment is supported by recent measurements[9,10] in e^+e^- annihilations of the leptonic width of Υ, which yielded

$$\Gamma(\Upsilon \to e^+e^-) = 1.3 \pm 0.4\ \text{keV}, \tag{2.2}$$

a value much more compatible with $e_Q = -\frac{1}{3}$ than with $|e_Q| \geq \frac{2}{3}$.[7,8,11,12]

In the conventional nomenclature, which we shall adopt, a charge $-\frac{1}{3}$ quark with mass near $5\ \text{GeV}/c^2$ is called[13] b. This designation implies nothing about the weak interactions of the new quark, nor does it require the existence of another new quark of charge $+\frac{2}{3}$. We shall refer to the new additive quantum number carried by the b quark as $\mathcal{B}$. Thus, $\mathcal{B}(b) = +1$, $\mathcal{B}(\bar{b}) = -1$, $\mathcal{B}(u, d, s, c) = 0$. Although we consider it highly likely that a sixth quark does exist, our analysis will be independent of this possibility and *a fortiori* independent of the sixth quark's charge and weak couplings.

The production of $(\mathcal{B} = +1,\ \mathcal{B} = -1)$ hadron pairs in hadron collisions is expected to occur at a level no less than the cross section for Υ or Υ' production.[14] In two searches carried out at Fermilab[15,16] at a sensitivity of $\frac{1}{10}$ the Υ cross section, no charged stable particles of mass $\sim 5\ \text{GeV}/c^2$ were detected in 400-GeV/c pN collisions. If the expectations for pair production are correct, these experiments imply that the b quark is unstable, with a lifetime[17]

$$\tau_b \lesssim 5 \times 10^{-8}\ \text{sec}. \tag{2.3}$$

We shall assume that b-quark decays are mediated by the charged weak current[18]:

$$b \to u + W^- , \quad (2.4a)$$

$$b \to c + W^- . \quad (2.4b)$$

It is unlikely that any additional charge $+\frac{2}{3}$ quarks exist with masses less than m_b, because the vector $(Q\overline{Q})$ states should have been prominent in the data of Refs. 1 and 2. A question of immediate importance is the relative rate of the transitions (2.4).

Let us note that if the b quark were coupled with full strength to the u quark, the rate for $b \to u$ transitions would be[19, 20]

$$\Gamma_0(b \to u + W^-) = \Gamma_\mu \left(\frac{m_b}{m_\mu}\right)^5 [1 + 1 + f(m_\tau/m_b) + 3 + 3f(m_c/m_b)] , \quad (2.5)$$

where f is a kinematic factor given by

$$f(x) = (1 - x^4)(1 - 8x^2 + x^4) - 12x^4 \ln x^2 , \quad (2.6)$$

and

$$\Gamma_\mu = G^2 m_\mu^5/192\pi^3 = 4.55 \times 10^5 \text{ sec}^{-1}. \quad (2.7)$$

The terms in square brackets in Eq. (2.5) correspond to the decays

$$\begin{aligned} b &\to u(e\overline{\nu}_e) \\ &\to u(\mu\overline{\nu}_\mu) \\ &\to u(\tau\overline{\nu}_\tau) \\ &\left.\begin{aligned}&\to u(d\overline{u}) + u(s\overline{u}) \\ &\to u(d\overline{c}) + u(s\overline{c})\end{aligned}\right\} \text{3 colors,} \end{aligned} \quad (2.8)$$

where all masses except m_τ and m_c have been neglected. We dismiss for the moment the possibility of appreciable nonleptonic enhancement, which might increase the last two rates.[21, 22] For $m_b = 5$ GeV/c^2, $m_\tau = 1.78$ GeV/c^2,[23] and $m_c/m_b = \frac{1}{3}$ to $\frac{1}{4}$, we have

$$\tau_0 = \hbar/\Gamma_0(b \to u + W^-) \simeq 1.3 \times 10^{-15} \text{ sec.} \quad (2.9)$$

Since the coupling of b to $(u + c)$ is unlikely to exceed full strength (and is considerably smaller in specific models), it is reasonable to regard (2.9) as a rough lower bound on the b-quark lifetime, so that

$$10^{-15} \text{ sec} \lesssim \tau_b \lesssim 5 \times 10^{-8} \text{ sec.} \quad (2.10)$$

In the six-quark generalization of the Weinberg-Salam model due to Kobayashi and Maskawa,[24] which has been extensively analyzed in the present context,[19, 25] the couplings of b to u and c are highly suppressed. Universality of the β-decay coupling constant requires

$$g_{V-A}^2(b \to u + W^-) \lesssim 3 \times 10^{-3} g_{V-A}^2(d \to u + W^-) . \quad (2.11)$$

Universality is not stringently tested in charmed quark decays. A far weaker limit is imposed by the requirement that $s \to c + W^-$ transitions be strong enough to suppress the K_L-K_S mass difference:

$$g_{V-A}^2(b \to c + W^-) \lesssim \tfrac{2}{5} g_{V-A}^2(d \to u + W^-) . \quad (2.12)$$

In this model, the b-quark decay rate is therefore restricted to

$$\Gamma_b \lesssim \tfrac{2}{5} \Gamma_\mu \left(\frac{m_b}{m_\mu}\right)^5 [f(m_c/m_b) + f(m_c/m_b) + \phi(m_c, m_\tau; m_b) + 3f(m_c/m_b) + 3\phi(m_c, m_c; m_b)] , \quad (2.13)$$

where $\phi(m_1, m_2; M)$ is a kinematic factor for the decay $M \to m_1 + m_2 +$ zero-mass particle. For the equal-mass case, $\phi(m, m; M) = g(4m^2/M^2)$, where

$$g(y) = (1 - 7y/2 - y^2/8 - 3y^3/16)(1 - y)^{1/2} + 3y^2(1 - y^2/16) \ln\left(\frac{1 + \sqrt{1 - y}}{\sqrt{y}}\right) . \quad (2.14)$$

This suggests that $\tau_b \gtrsim 10^{-14}$ sec. The upper limits given by (2.11) and (2.12) are not to be taken as an indication of the relative strengths of the $b \to u$ and $b \to c$ couplings, but they obviously admit the possibility that

$$\Gamma(b \to c + W^-) \gg \Gamma(b \to u + W^-) , \quad (2.15)$$

which previous authors[19, 25] have emphasized. We shall present a means for testing this suggestion through observations of the leptons emitted in b-quark decays. Measurement of the relative rates for $b \to u$ and $b \to c$ transitions in exclusive nonleptonic channels, while attractive in principle, is complicated by small branching ratios for charm decays and by the combinatorics of multiparticle final states.

III. ELECTRON SIGNALS IN $(b\overline{b})$ PRODUCTION

In this section we shall organize the observables which pertain to leptonic final states which occur in the decays of unbound $(b\overline{b})$ pairs. To be specific, we discuss observations of prompt electrons only. Completely parallel discussions apply to the cases of muon detection and of electron plus muon detection.

The decay process

$$b^{-1/3} \to \text{quark}^{+2/3} + W^- \quad (3.1)$$

can give rise to no electrons, to a single e^-, to a single e^+, or to an e^+e^- pair. The possible sources of prompt electrons are enumerated in Table I. A positron, if present, results from the semileptonic decay of a charmed quark, which occurs with a branching ratio of about 10%.[22] Electrons can arise from the decay of the virtual W^-. We may therefore write, symbolically,

$$b=(1-\alpha-\beta-\gamma)\times(\text{no } e^{\pm}) + \alpha e^+ + \beta e^- + \gamma(e^+e^-)\,, \qquad (3.2)$$

where the probabilities α,β,γ satisfy $0\leq\alpha,\beta,\gamma\leq 1$ and $\alpha+\beta+\gamma\leq 1$.

We focus upon unbound $(b\bar{b})$ production for two reasons. First, we expect the experimental isolation of a $(b\bar{b})$ signal to be considerably easier than the identification of a single hadron with $\mathcal{B}\neq 0$. Second, important correlations occur in the leptonic final states from $(b\bar{b})$ pairs.

One promising source of a $(b\bar{b})$ signal would be a ${}^3S_1(b\bar{b})$ level just above the threshold for $(\mathcal{B}=+1, \mathcal{B}=-1)$ hadron production. We have previously estimated[26] that such a state would be the $4^3S_1(10.6\text{ GeV}/c^2)$ or $5^3S_1(10.8)$ radial excitation of $\Upsilon(9.4)$. The 4S level should lie a few tens of MeV above or below flavor threshold. If the 5S level is the first unbound state, it will be no more than a couple of hundred MeV above threshold. There may also be a ${}^3D_1$ analog of the $\psi''(3.772)$,[27] a copious source of charmed particle pairs.[28] The $\psi(4.04)$, $\psi(4.16)$, and $\psi(4.41)$ states all have hadronic widths of several tens of MeV,[29] though the highest is nearly 700 MeV above charm threshold. The above discussion suggests that a small total width for the first ${}^3S_1(b\bar{b})$ state above flavor threshold is not at all unlikely. Moreover, the node effects discussed by Eichten *et al.*[5] are quite capable of suppressing the total widths of at least one of the many ${}^3S_1$ or ${}^3D_1(b\bar{b})$ states expected to lie in the range between flavor threshold and a few hundred MeV above it. In any event, for accumulating the number of events ($\sim 10^3$) we shall find required for the analysis a prominent resonance is more a convenience than a necessity.

We define the following observables:

$$\sigma_0\equiv\sigma(b\bar{b}\to\text{no } e^{\pm})/\sigma(b\bar{b})\,, \qquad (3.3)$$

$$\sigma_1\equiv[\sigma(b\bar{b}\to e^+)+\sigma(b\bar{b}\to e^-)]/\sigma(b\bar{b})\,, \qquad (3.4)$$

$$\sigma_{+-}\equiv\sigma(b\bar{b}\to e^+e^-)/\sigma(b\bar{b})\,, \qquad (3.5)$$

$$\sigma_{ss}\equiv[\sigma(b\bar{b}\to e^+e^+)+\sigma(b\bar{b}\to e^-e^-)]/\sigma(b\bar{b})\,, \qquad (3.6)$$

$$\sigma_3\equiv[\sigma(b\bar{b}\to e^+e^+e^-)+\sigma(b\bar{b}\to e^+e^-e^-)]/\sigma(b\bar{b})\,, \qquad (3.7)$$

$$\sigma_4\equiv\sigma(b\bar{b}\to e^+e^+e^-e^-)/\sigma(b\bar{b})\,. \qquad (3.8)$$

If the decays of b and $\bar{b}$ are uncorrelated, these can be expressed in terms of the parameters α,β,γ defined in Eq. (3.2) as

$$\sigma_0=(1-\alpha-\beta-\gamma)^2\,, \qquad (3.9)$$

$$\sigma_1=2(\alpha+\beta)(1-\alpha-\beta-\gamma)\,, \qquad (3.10)$$

$$\sigma_{+-}=\alpha^2+\beta^2+2\gamma(1-\alpha-\beta-\gamma)\,, \qquad (3.11)$$

$$\sigma_{ss}=2\alpha\beta\,, \qquad (3.12)$$

$$\sigma_3=2(\alpha+\beta)\gamma\,, \qquad (3.13)$$

$$\sigma_4=\gamma^2\,. \qquad (3.14)$$

There are three constraints among the fractional cross sections. The first is the trivial requirement that

$$\sigma_0+\sigma_1+\sigma_{+-}+\sigma_{ss}+\sigma_3+\sigma_4=1\,. \qquad (3.15)$$

Two others can be expressed as

$$\sigma_3{}^2/\sigma_4=\sigma_1{}^2/\sigma_0 \qquad (3.16)$$

and

$$\frac{\sigma_1}{2\sqrt{\sigma_0}}+\sqrt{\sigma_0}=1-\sqrt{\sigma_4}\,. \qquad (3.17)$$

The parameters α,β,γ may be obtained from the observed cross sections. First, the parameter γ is computed from Eq. (3.14), or with the aid of (3.16) as

$$\gamma=\sigma_3\sqrt{\sigma_0}/\sigma_1\,, \qquad (3.18)$$

which will be statistically more powerful. Then, since

$$\alpha+\beta=\sigma_1/2\sqrt{\sigma_0}\,, \qquad (3.19)$$

we can use Eq. (3.12) to set up a quadratic equation for α and β, which has the solutions

$$\alpha=\sigma_1/4\sqrt{\sigma_0}\pm\left(\frac{\sigma_1{}^2}{16\sigma_0}-\frac{\sigma_{ss}}{2}\right)^{1/2}, \qquad (3.20)$$

$$\beta=\sigma_1/4\sqrt{\sigma_0}\mp\left(\frac{\sigma_1{}^2}{16\sigma_0}-\frac{\sigma_{ss}}{2}\right)^{1/2}. \qquad (3.21)$$

TABLE I. Sources of prompt electrons from b-quark decay: $b\to\text{quark}+W^-$.

Quark \ $W^-\to$	$e^-\bar{\nu}_e$	$\mu^-\bar{\nu}_\mu$	$\tau^-\bar{\nu}_\tau$ no e^-	$\tau^-\bar{\nu}_\tau$ e^-	$\bar{u}(d,s)$	$\bar{c}(d,s)$ no e^-	$\bar{c}(d,s)$ e^-
u	e^-	0	0	e^-	0	0	e^-
$c\nearrow$ no e^+	e^-	0	0	e^-	0	0	e^-
$c\searrow e^+$	e^+e^-	e^+	e^+	e^+e^-	e^+	e^+	e^+e^-

The quadratic ambiguity of Eqs. (3.20) and (3.21) is not always present in practice, as we shall note in Sec. IV. Under some circumstances it is possible to determine independently whether α or β is the larger. It goes without saying that in the absence of magnetic analysis of the electron charge, α and β cannot be determined separately.

The requirement that α and β be real numbers, together with Eqs. (3.20) and (3.21), implies that

$$\sigma_{ss} \leq \sigma_1^2/8\sigma_0 . \tag{3.22}$$

Moreover, from Eq. (3.17),

$$1 \geq \frac{\sigma_1}{2\sqrt{\sigma_0}} + \sqrt{\sigma_0} \geq \sqrt{2\sigma_1} \tag{3.23}$$

or

$$\sigma_1 \leq \tfrac{1}{2} . \tag{3.24}$$

Combining Eqs. (3.22) and (3.23), we find

$$\sigma_{ss}\sigma_0 < \tfrac{1}{32} . \tag{3.25}$$

The constraints (3.15)–(3.17) and (3.22)–(3.25) provide important checks on signal purity, on experimental biases, and on the assumption of uncorrelated decays of b and $\bar{b}$. We shall discuss the elimination of backgrounds and the effects of neutral-particle mixing at some length in Sec. V. Now, however, we turn to the problem of relating the parameters α, β, and γ to quantities of theoretical interest, in idealized circumstances.

IV. DETERMINATION OF BRANCHING RATIOS

The relative rates for inclusive b-quark decay which appear in Eq. (3.2) are determined by the rates for the processes listed in Table I. Our task now is to relate the parameters α, β, γ defined in (3.2) to the rates for specific decay processes. We begin by making an important simplifying assumption, for which consistency checks will be noted. We assume the decays of the b quark ($b \to u + W^-$, $b \to c + W^-$) to be independent of the subsequent decays of the weak current ($W^- \to l^- \bar{\nu}_l, d\bar{u}, s\bar{c} \cdots$). This ansatz is unjustified if the nonleptonic decays of the b quark are substantially enhanced. Such circumstances are not anticipated in the Kobayashi-Maskawa model.[19] We nevertheless conclude this section with weaker results that are independent of this assumption.

A. No nonleptonic enhancement

We denote the branching ratios for b-quark decays as

$$\Gamma(b \to u + W^-)/\Gamma(b \to \text{all}) \equiv \xi , \tag{4.1}$$

$$\Gamma(b \to c + W^-)/\Gamma(b \to \text{all}) \equiv (1-\xi) , \tag{4.2}$$

with $0 \leq \xi \leq 1$. The charmed quark decays semielectronically with a probability of approximately 10%[22];

$$\Gamma(c \to e^+ + \text{anything})/\Gamma(c \to \text{all}) \equiv z \approx 10\% . \tag{4.3}$$

Consequently, the probability for the inclusive decay of b into e^+ is

$$\Gamma(b \to e^+ + \text{anything})/\Gamma(b \to \text{all}) \equiv \alpha + \gamma = z(1-\xi) . \tag{4.4}$$

If the parameters α and γ are extracted by the method of the previous section, knowledge of z permits us to obtain the relative rates for $b \to u$ and $b \to c$ transitions.

It may happen in practice that one of the two solutions for α in Eq. (3.20) yields an unphysical value for ξ in Eq. (4.4). This circumstance would decisively resolve the quadratic ambiguity in (3.20) and (3.21).

By considering in detail the possible sources of electrons listed in Table I, we shall find it probable that the quadratic ambiguity can be eliminated even if both solutions for α lead to physically acceptable values of ξ. In the absence of nonleptonic enhancement, the inclusive semielectronic decay rate of the b quark is[30]

$$\begin{aligned} \Gamma(b \to q + e^- + \text{anything}) &= \Gamma(b \to q + e^- + \bar{\nu}_e) \\ &+ \frac{\Gamma(b \to q + \tau^- + \bar{\nu}_\tau)\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau)}{\Gamma(\tau^- \to \text{all})} \\ &+ \frac{\Gamma(b \to q + s_\theta + \bar{c})\Gamma(\bar{c} \to e^- + \text{anything})}{\Gamma(\bar{c} \to \text{all})} , \end{aligned} \tag{4.5}$$

where the generic q represents u or c, and s_θ denotes the Cabibbo-mixed s quark. We define the electronic branching ratio of the $\tau (\approx 18\%)$[31] to be

$$\nu = \Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau)/\Gamma(\tau^- \to \text{all}) . \tag{4.6}$$

Then, making use of Eqs. (2.5)–(2.7) and (2.13)–(2.14), we have

$$\frac{\Gamma(b \to u + e^- + \text{anything})}{\Gamma(b \to u + \text{anything})} \simeq \frac{1 + \nu f(m_\tau/m_b) + 3zf(m_c/m_b)}{5 + f(m_\tau/m_b) + 3f(m_c/m_b)} \tag{4.7}$$

and

$$\frac{\Gamma(b \to c + e^- + \text{anything})}{\Gamma(b \to c + \text{anything})} \simeq \frac{f(m_c/m_b) + \nu\phi(m_c, m_\tau; m_b) + 3z\phi(m_c, m_c; m_b)}{5f(m_c/m_b) + \phi(m_c, m_\tau; m_b) + 3\phi(m_c, m_c; m_b)} . \tag{4.8}$$

Inserting the experimental values $\nu = 0.18$ and $z = 0.10$, choosing $m_c = 1.5$ GeV/c^2 and $m_b = 5$ GeV/c^2, and approximating $\phi(m_c, m_\tau; m_b) \approx g((m_c + m_\tau)^2/m_b^2)$, we estimate

$$\Gamma(b \to u + e^- + \text{anything})/\Gamma(b \to u + \text{anything}) \approx 0.18\,, \tag{4.9a}$$

$$\Gamma(b \to c + e^- + \text{anything})/\Gamma(b \to c + \text{anything}) \approx 0.18\,. \tag{4.9b}$$

To the extent that the branching ratios (4.9) are equal, we may conclude from Eq. (3.2) that

$$\beta + \gamma = \Gamma(b \to e^- + \text{anything})/\Gamma(b \to \text{anything}) \approx 0.18\,. \tag{4.10}$$

Since, according to (4.4)

$$0 \leqslant \alpha + \gamma \leqslant z \approx 0.10\,, \tag{4.11}$$

we expect on the model of this subsection that $\beta > \alpha$. By imposing this requirement we eliminate the quadratic ambiguity of (3.20) and (3.21).

The value of the parameter γ is expected to be quite small:

$$\gamma = \frac{\Gamma(b \to c + e^- + \text{anything})}{\Gamma(b \to \text{all})} \frac{\Gamma(c \to e^+ + \text{anything})}{\Gamma(c \to \text{all})} \approx 0.18(1-\xi)z \lesssim 0.02. \tag{4.12}$$

Consequently, four-lepton events will be exceedingly rare, with $\sigma_4 = \gamma^2 \lesssim 4 \times 10^{-4}$. The multilepton cross sections are plotted in Fig. 1 as functions of ξ. We notice that $\sigma_{ss} \approx O(1\%)$ is by no means negligible. Consequently, prompt same-sign dileptons should be a useful signature for unbound $(b\bar{b})$ production in hadronic interactions.

Finally, let us make the essentially kinematical connection between the parameter ξ of Eqs. (4.1) and (4.2) with the weak current couplings:

$$\frac{g^2(b \to u + W^-)}{g^2(b \to c + W^-)} = \frac{\Gamma(b \to u + W^-)}{\Gamma(b \to c + W^-)} \times \frac{5f(m_c/m_b) + \phi(m_c, m_\tau; m_b) + 3\phi(m_c, m_c; m_b)}{5 + f(m_\tau/m_b) + 3f(m_c/m_b)} \approx 0.48 \frac{\xi}{(1-\xi)}\,, \tag{4.13}$$

where the coefficient 0.48 carries an uncertainty characterized by our ignorance of quark masses.

B. Arbitrary nonleptonic enhancement

If some of the nonleptonic decays of the b quark are enhanced, the analysis given in Sec. IV A holds through Eq. (4.4), which is to say that the parameter ξ can be determined. However, the quadratic ambiguity of (3.20) and (3.21) may persist, and the connection of ξ with the weak couplings will not be as direct as given in Eq. (4.13).

The definitions of β and γ provide a possibly useful constraint. Combining

$$\beta + \gamma = \frac{\Gamma(b \to u + e^- + \text{anything}) + \Gamma(b \to c + e^- + \text{anything})}{\Gamma(b \to \text{all})} \tag{4.14}$$

with

$$\gamma = z \frac{\Gamma(b \to c + e^- + \text{anything})}{\Gamma(b \to \text{all})}\,, \tag{4.15}$$

we have

$$\frac{\Gamma(b \to u + e^- + \text{anything})}{\Gamma(b \to c + e^- + \text{anything})} = z\left[\frac{\beta+\gamma}{\gamma}\right] - 1 \geqslant 0\,. \tag{4.16}$$

The positivity requirement implies that

$$\beta + \gamma \geqslant \gamma/z\,, \tag{4.17}$$

which replaces Eq. (4.10). If $\gamma/z \gtrsim z$, Eqs. (4.11) and (4.17) again indicate that $\beta > \alpha$.

To connect the resulting value of ξ with the weak current couplings, we generalize the numerators of Eqs. (4.7) and (4.8) to include nonleptonic enhancements, and write

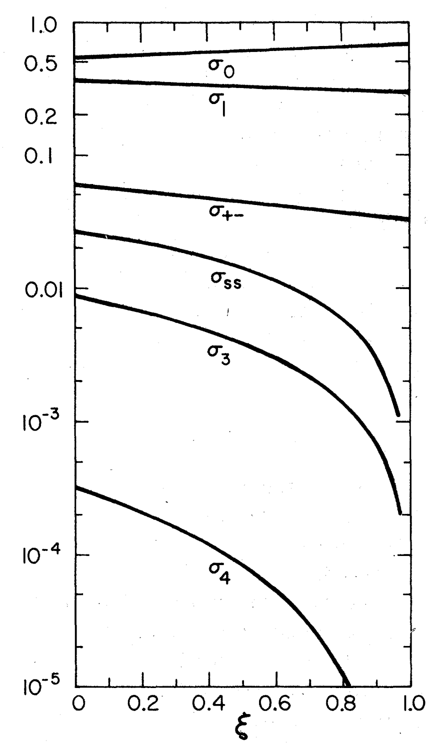

FIG. 1. Fractional multilepton cross sections from the reaction $e^+e^- \to (b\bar{b})$ in the absence of nonleptonic enhancement. The parameter ξ, defined in Eqs. (4.1) and (4.2), specifies the relative importance of the decays $b \to u + W^-$ and $b \to c + W^-$.

$$\frac{g^2(b\to u+W^-)}{g^2(b\to c+W^-)}=\frac{\Gamma(b\to u+e^-+\text{anything})}{\Gamma(b\to c+e^-+\text{anything})}\,\frac{f(m_c/m_b)+\nu\phi(m_c,m_\tau;m_b)+3zE(b\to c\bar{c}s_\theta)\phi(m_c,m_c;m_b)}{1+\nu f(m_\tau/m_b)+3zE(b\to u\bar{c}s_\theta)f(m_c/m_b)}$$

$$=\left(z\left[\frac{\beta+\gamma}{\gamma}\right]-1\right)R\big(E(b\to c\bar{c}s_\theta),E(b\to u\bar{c}s_\theta)\big), \tag{4.18}$$

where $E(b\to\text{quarks})$ is the appropriate nonleptonic enhancement factor. In general, we can say nothing precise about the numerical factor R. However, an interesting special case occurs if the nonleptonic enhancement factor is the same in numerator and denominator. The variation of R with E is then rather limited:

$$R(E=1)\approx 0.49\,,\qquad R(E=10)\approx 0.43\,, \tag{4.19}$$

and a reliable estimate of $g^2(b\to u+W^-)/g^2(b\to c+W^-)$ should follow.[32] The uncertainty reflected in (4.19) is not enormous compared to that derived from quark-mass uncertainties.

Let us summarize the results of this section. We have shown how observations of multilepton final states lead directly to a measure of the relative decay rates $\Gamma(b\to u+W^-)/\Gamma(b\to c+W^-)$. In easily forseeable circumstances, the procedure described is free of a quadratic ambiguity which could occur in principle. If nonleptonic enhancement is unimportant for b-quark decay, the inferred relative rates may be converted to a rather precise measure of the relative sizes of the weak current couplings. In the presence of nonleptonic enhancement, the final step cannot be made with great confidence.

In any event, the parameters α and γ, which pertain to wrong-sign leptons ($b\to e^+$), play a decisive role in investigating the relative strengths of $b\to u$ and $b\to c$ transitions. Without magnetic analysis of the lepton charge, the observables σ_{+-} and σ_{ss} are merged into

$$\sigma_2=\sigma_{+-}+\sigma_{ss}=(\alpha+\beta)^2+2\gamma(1-\alpha-\beta-\gamma). \tag{4.20}$$

The parameters α and β then occur in the combination $(\alpha+\beta)$ in all observables [cf. (3.9), (3.10), (3.13), (3.14)] and the analysis we have proposed cannot be executed.

The method we have described is based upon the observation of the secondary leptons from semileptonic charm decays, as well as the primary leptons from the decay $b\to q+e^-+\nu_e$. A complementary approach has been advocated by Ali,[25] who has made extensive simulations of the primary and secondary lepton spectra. In his analysis scheme, secondary leptons are eliminated by a kinematical cut.

Obviously, it would be ideal if one could tag the primary decay unambiguously. This would be possible if, instead of an undetectable neutrino, the final state of semileptonic b-quark decay contained an unstable neutral lepton. For example, a neutral heavy lepton N^0 coupled right-handedly to the electron could be reconstructed in the chain

$$b\to q+e^-+N^0\searrow_{e^+\pi^-}. \tag{4.21}$$

Branching ratios for the decay $N^0\to e^+\pi^-$ have been estimated, for example, in Ref. 33.

V. BACKGROUNDS AND $b,\bar{b}$ MIXING

The isolation of a pure $(b\bar{b})$ state is an unlikely idealization. Even in e^+e^- annihilations at the peak of a $(b\bar{b})$ resonance, there will be backgrounds due to continuum production of other hadrons and to lepton pair production. We shall use the example of $\tau^+\tau^-$ production to illustrate the effect of backgrounds upon the consistency relations recorded in Sec. III. It is to be expected that such backgrounds can be eliminated by the usual subtraction techniques using control bands on either side of the resonance. A second effect which can modify the analysis described in Sec. III is $b,\bar{b}$ mixing. This can occur if the b quark is incorporated into a neutral meson $(b\bar{d})^0$ or $(b\bar{s})^0$, which can mix by second-order weak interactions with $(\bar{b}d)^0$ and $(\bar{b}s)^0$, respectively. If the mixing is appreciable within the meson lifetimes, $b\to\bar{b}$ effective transitions take place.

A. Consequences of a $\tau^+\tau^-$ background

Additional incoherent sources of hadrons and leptons necessitate a straightforward generalization of the analysis given in Sec. III. As an example, we explore the consequences of a $\tau^+\tau^-$ contribution. In analogy with Eq. (3.2), we may write

$$\tau^-=(1-\nu)(\text{no }e^\pm)+\nu e^-\,, \tag{5.1}$$

where the leptonic branching ratio ν has been defined in Eq. (4.6). If the observed cross section is made up of $(b\bar{b})$ and $\tau^+\tau^-$ in the proportions $(1-\rho)$ and ρ, the observables of Eqs. (3.9)–(3.14) become

$$\sigma_0=(1-\rho)(1-\alpha-\beta-\gamma)^2+\rho(1-\nu)^2\,, \tag{5.2}$$

$$\sigma_1=2(1-\rho)(\alpha+\beta)(1-\alpha-\beta-\gamma)+2\rho\nu(1-\nu)\,, \tag{5.3}$$

$$\sigma_{+-}=(1-\rho)[\alpha^2+\beta^2+2\gamma(1-\alpha-\beta-\gamma)]+\rho\nu^2, \quad (5.4)$$

$$\sigma_{ss}=2(1-\rho)\alpha\beta\,, \quad (5.5)$$

$$\sigma_3=2(1-\rho)(\alpha+\beta)\gamma\,, \quad (5.6)$$

$$\sigma_4=(1-\rho)\gamma^2\,. \quad (5.7)$$

Of the constraints listed in Sec. III, Eq. (3.15) continues to hold, but Eqs. (3.16) and (3.17) are no longer valid. Violations of (3.16) and (3.17) may be taken as indications that background subtractions have been inadequately made. To illustrate the expected level of these violations, we plot in Fig. 2 the ratio of $\sigma_3{}^2\sigma_0/\sigma_4\sigma_1{}^2$, which is equal to unity in the absence of any background, for various choices of the contamination parameter ρ, as a function of ξ. The values $\nu=0.18$ and $z=0.10$ are again adopted and, as before, the parameters α, β, and γ are given in terms of ξ by Eqs. (4.4), (4.10), and (4.12). Although this figure is specific to the model without nonleptonic enhancement, the acute sensitivity to background that it demonstrates is an encouraging result. In contrast, for the situation described by this example, violations of the constraint (3.17) occur at the level of $<1\%$.

B. Consequences of neutral-particle mixing

We suppose that because of neutral-particle mixing there is a probability f that a produced b quark will convert before decay to a $\bar{b}$ quark, i.e.,

$$b\rightarrow(1-f)b+f\bar{b}\,, \quad (5.8a)$$

$$\bar{b}\rightarrow(1-\bar{f})\bar{b}+\bar{f}b\,, \quad (5.8b)$$

with

$$\bar{f}=f/[f(1-r^2)+r^2]\,, \quad (5.8c)$$

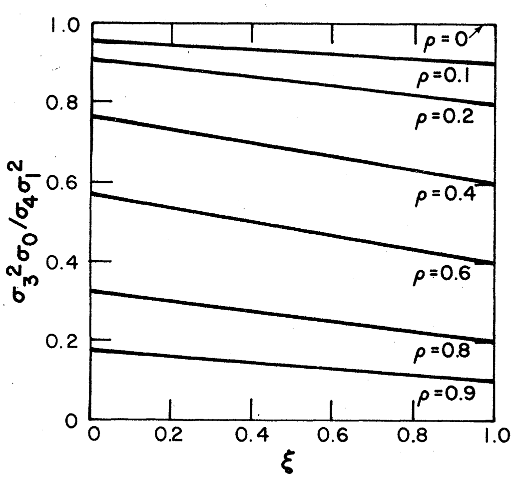

FIG. 2. Effect of an incoherent background source $e^+e^-\rightarrow\tau^+\tau^-$, upon the constraint equation $\sigma_3{}^2\sigma_0/\sigma_4\sigma_1{}^2=1$, assuming no nonleptonic enhancement. The produced population is represented as $(1-\rho)(b\bar{b})+\rho(\tau^+\tau^-)$.

$$r=\left|\frac{1-\epsilon}{1+\epsilon}\right|^2, \quad (5.8d)$$

where ϵ is the conventional CP-violation parameter.[34] For the present discussion, we assume that the produced $(b\bar{b})$ quarks, while dressed with quarks of a single flavor, are incorporated into a sufficient variety of hadrons that no correlations between the mixings of b and $\bar{b}$ are present. A treatment of the correlated case, in which one may hope to study CP-violating effects in detail, is given in the Appendix.

In the presence of mixing, Eqs. (3.9)–(3.14) are modified as follows:

$$\sigma_0=(1-\alpha-\beta-\gamma)^2\,, \quad (5.9)$$

$$\sigma_1=2(1-\alpha-\beta-\gamma)(\alpha+\beta)\,, \quad (5.10)$$

$$\sigma_{+-}=\frac{(\alpha+\beta)^2+(\alpha-\beta)^2(1-2f)(1-2\bar{f})}{2}+2\gamma(1-\alpha-\beta-\gamma)\,, \quad (5.11)$$

$$\sigma_{ss}=\frac{(\alpha+\beta)^2-(\alpha-\beta)^2(1-2f)(1-2\bar{f})}{2}, \quad (5.12)$$

$$\sigma_3=2(\alpha+\beta)\gamma\,, \quad (5.13)$$

$$\sigma_4=\gamma^2\,. \quad (5.14)$$

The quantities $(\alpha+\beta)$ and γ appear as before; but $(\alpha-\beta)^2$ is replaced $(1-2f)(1-2\bar{f})(\alpha-\beta)^2$. In addition, CP violation is manifested in the form of the charge asymmetries

$$(\sigma_+-\sigma_-)/\sigma_1=(\alpha-\beta)(\bar{f}-f)/(\alpha+\beta) \approx -8\,\mathrm{Re}\epsilon f(1-f)(\alpha-\beta)/(\alpha+\beta)\,; \quad (5.15)$$

$$(\sigma_{++}-\sigma_{--})/\sigma_{ss}=\frac{2(\bar{f}-f)(\alpha+\beta)(\alpha-\beta)}{(\alpha+\beta)^2-(\alpha-\beta)^2(1-2f)(1-2\bar{f})} \approx\frac{-16\,\mathrm{Re}\epsilon f(1-f)(\alpha+\beta)(\alpha-\beta)}{(\alpha+\beta)^2-(\alpha-\beta)^2(1-2f)^2}\,, \quad (5.16)$$

$$(\sigma_{++-}-\sigma_{+--})/\sigma_3=(\alpha-\beta)(\bar{f}-f)/(\alpha+\beta) =(\sigma_+-\sigma_-)/\sigma_1\,, \quad (5.17)$$

to leading order in ϵ.

The constraint equations given in (3.15)–(3.17) and (3.22)–(3.25) continue to be valid. However, the quadratic equation for α and β now has the solutions

$$\alpha=\sigma_1/4\sqrt{\sigma_0}\pm\frac{1}{1-2f}\left(\frac{\sigma_1{}^2}{16\sigma_0}-\frac{\sigma_{ss}}{2}\right)^{1/2}, \quad (5.18)$$

$$\beta=\sigma_1/4\sqrt{\sigma_0}\mp\frac{1}{1-2f}\left(\frac{\sigma_1{}^2}{16\sigma_0}-\frac{\sigma_{ss}}{2}\right)^{1/2}, \quad (5.19)$$

neglecting the small difference between f and $\bar{f}$.

The required positivity of α and β may constrain the mixing parameter f. It requires

$$(1-2f)^2 \geq 1 - 8\sigma_{ss}\sigma_0/\sigma_1^{\,2}. \qquad (5.20)$$

On the assumption of no nonleptonic enhancement, it is possible to determine both the decay parameters α, β, γ and the mixing parameter f. We first compute γ from (3.14) or (3.18), and use Eq. (4.12) to evaluate ξ. Then α is given by Eq. (4.4), and β can be derived from (3.19) or (4.10). Let us denote the solutions to Eqs. (3.20) and (3.21) as α_0 and β_0. Comparing (5.18) and (5.19) with (3.20) and (3.21), we find

$$f = \frac{1}{2}\left[1 - \frac{\alpha_0 - \beta_0}{\alpha - \beta}\right], \qquad (5.21)$$

which has two solutions because of the quadratic ambiguity in $(\alpha_0 - \beta_0)$. Imposing the reasonable requirement[35] that $f \leq \frac{1}{2}$, we obtain a unique solution.

If there is an arbitrary nonleptonic enhancement, we can do nothing so specific. We may evaluate γ as before, compute $\alpha + \beta = \alpha_0 + \beta_0$, and use the bound $|\alpha - \beta| > |\alpha_0 - \beta_0|$ to derive a bound on $g^2(b \to u + W^-)/g^2(b \to c + W^-)$ along the lines of (4.18).

Ali and Aydin[36] have proposed a study of neutral-particle mixing and CP-violation based upon the observation of high-momentum leptons arising in the primary decays

$$b \to q + e^- + \bar{\nu}_e, \qquad (5.22)$$

which may be kinematically distinguishable from the leptons emitted in semileptonic charm decay.

VI. SUMMARY AND CONCLUSIONS

The relative weak-current couplings for the transitions $b \to u + W^-$ and $b \to c + W^-$ can be measured by observing the "wrong-sign" leptons arising from the secondary charmed-quark decay. We have shown that final states containing up to four charged leptons, which occur in the decay of an unbound $(b\bar{b})$ system, are particularly useful in sorting out the decays of the b quark into states containing zero, one, or two leptons. The effects of $b, \bar{b}$ mixing can be determined independently if nonleptonic decays are not enhanced, a likely possibility for very-massive-quark decays.

The most popular framework which accommodates a b quark is a six-quark generalization[24] of the Weinberg-Salam model which groups the quarks into three left-handed doublets,

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L, \qquad (6.1)$$

where L denotes $V-A$ coupling. The primes indicate that the quarks are mixed, in a manner parametrized by three (Euler) angles and a CP-violating phase. The ratio of couplings determined by Eq. (4.13) fixes one angle; another is the Cabibbo angle. To fix the third requires a study of the relative couplings of $t \to (d, s, b) + W^+$.

Another variety[37] of six-quark models involves two quarks (u, c) with charge $+\frac{2}{3}$ and four quarks (d, s, b, h) with charge $-\frac{1}{3}$. Since no evidence for a new family of $(Q\bar{Q})$ states has been found[38] up to about 15 GeV/c^2, the mass of a sixth quark is presumably greater than 7 GeV/c^2. In these models it is unclear how b and h should couple to u and c. A suggestion[39] that the new couplings be

$$\begin{pmatrix} u \\ b \end{pmatrix}_R, \quad \begin{pmatrix} c \\ h \end{pmatrix}_R \qquad (6.2)$$

seems ruled out by the absence of a high-y anomaly in $\bar{\nu}N$ scattering.[40] An alternative assignment,

$$\begin{pmatrix} u \\ h \end{pmatrix}_R, \quad \begin{pmatrix} c \\ b \end{pmatrix}_R, \qquad (6.3)$$

is ruled out by neutral-current information from neutrino scattering, which indicates that the u quark is a right-handed singlet.[41] The possibility remains[42] that b and h would be absolutely stable in the absence of mixing with d and s. In this case constraints on $b \to u + W^-$ couplings are still imposed by β-decay universality, but the restrictions are somewhat different from those within the model defined by (6.1).

Whereas it is natural in the model with three left-handed doublets that there be three charged leptons (e^-, μ^-, τ^-), the six-quark models with four charge $-\frac{1}{3}$ quarks tend to incorporate a fourth charged lepton, L^-. The correlations among multilepton final states which we have described may be of value in recognizing the unexpected background provided by a new source, such as L^+L^-.

Note added. Recent observations of $\Upsilon'(10.02)$ in e^+e^- annihilations [J. K. Bienlein *et al.*, Phys. Lett. 78B, 360 (1978) and C. W. Darden *et al.*, Phys. Lett. 78B, 364 (1978)] yield a leptonic width $\Gamma(\Upsilon' \to e^+e^-) = 0.36 \pm 0.09$ keV, which strongly supports the assignment $e_Q = -\frac{1}{3}$. For further discussion see J. D. Jackson, C. Quigg, and J. L. Rosner, LBL Report No. LBL-7977, summary of parallel session B8 at the XIX International Conference on High Energy Physics, Tokyo (unpublished).

Note added in proof. Bounds on mixing angles in the Kobayashi-Maskawa model have been reexamined by R. E. Shrock and L. L. Wang, Phys. Rev. Lett. 41, 1692 (1978).

After submission of this paper, we received a manuscript by G. C. Branco and H. P. Nilles [Bonn University Report No. BONN-HE-78-10 (unpublished)]. The importance of multilepton final states is also stressed there, and it is suggested that a comparison of events with muons and with electrons can yield information on flavor-changing neutral currents. However, we are unable to reproduce results [particularly their Eq. (12)] which should be parallel to ours.

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APPENDIX: EVOLUTION OF A $\mathcal{B}^0\bar{\mathcal{B}}^0$ SYSTEM

The evolution of a neutral-meson–anti-neutral-meson system produced by single-photon annihilation of electron and positron has been described by many authors.[43] Here we propagate the effects of neutral-particle mixing and *CP* violation through the cascade decays of b-quark-bearing mesons, paying attention to the implications for multilepton final states.

If neutral-particle mixing results in effective $b \to \bar{b}$ conversions, the evolution of an initial $\mathcal{B}^0\bar{\mathcal{B}}^0$ state with charge conjugation $C=-1$ can be described by

$$(b\bar{b}) \to \frac{(1-f)(1-\bar{f})(b\bar{b})+\frac{1}{2}\bar{f}(1-f)(bb)+\frac{1}{2}f(1-\bar{f})(\bar{b}\bar{b})}{1-\frac{1}{2}(f+\bar{f})}. \tag{A1}$$

The observed cross sections, which result from decays of this semifinal state, are

$$\sigma_0=(1-\alpha-\beta-\gamma)^2, \tag{A2}$$

$$\sigma_1=2(\alpha+\beta)(1-\alpha-\beta-\gamma), \tag{A3}$$

$$\sigma_{+-}=\frac{(\alpha+\beta)^2+(\alpha-\beta)^2(1-f-\bar{f})}{2} +2\gamma(1-\alpha-\beta-\gamma)+O(\epsilon^2), \tag{A4}$$

$$\sigma_{ss}=\frac{(\alpha+\beta)^2-(\alpha-\beta)^2(1-f-\bar{f})}{2}+O(\epsilon^2), \tag{A5}$$

$$\sigma_3=2(\alpha+\beta)\gamma, \tag{A6}$$

$$\sigma_4=\gamma^2. \tag{A7}$$

For this correlated case, the *CP*-violating charge asymmetries are, to $O(\epsilon)$,

$$(\sigma_+-\sigma_-)/\sigma_1=-4\,\mathrm{Re}\epsilon f(\alpha-\beta)/(\alpha+\beta), \tag{A8}$$

$$(\sigma_{++}-\sigma_{--})/\sigma_{ss}=-\frac{8\,\mathrm{Re}\epsilon f(\alpha+\beta)(\alpha-\beta)}{(\alpha+\beta)^2-(\alpha-\beta)^2(1-2f)}, \tag{A9}$$

$$(\sigma_{++-}-\sigma_{+--})/\sigma_3=-4\,\mathrm{Re}\epsilon f(\alpha-\beta)/(\alpha+\beta) =(\sigma_+-\sigma_-)/\sigma_1. \tag{A10}$$

Comparing with the charge asymmetries in the uncorrelated case (5.15)–(5.17), we find the familiar result[44] that the asymmetries are approximately twice as large in the absence of correlations.

By a procedure analogous to that of Sec. V B, we may determine the decay and mixing parameters uniquely, in the absence of nonleptonic enhancement. In the present case, the quadratic equation for α and β yields (again neglecting the small difference between f and $\bar{f}$)

$$\alpha=\sigma_1/4\sqrt{\sigma_0}\pm\frac{1}{(1-2f)^{1/2}}\left(\frac{\sigma_1^{\,2}}{16\sigma_0}-\frac{\sigma_{ss}}{2}\right)^{1/2}, \tag{A11}$$

$$\beta=\sigma_1/4\sqrt{\sigma_0}\mp\frac{1}{(1-2f)^{1/2}}\left(\frac{\sigma_1^{\,2}}{16\sigma_0}-\frac{\sigma_{ss}}{2}\right)^{1/2}. \tag{A12}$$

The positivity of α and β now implies

$$1-2f\geq 1-\frac{8\sigma_{ss}\sigma_0}{\sigma_1^{\,2}}, \tag{A13}$$

or

$$f\leq 4\sigma_{ss}\sigma_0/\sigma_1^{\,2}. \tag{A14}$$

We now proceed according to the prescription given below Eq. (5.20), and solve for the mixing parameter as

$$f=\frac{1}{2}\left[1-\left(\frac{\alpha_0-\beta_0}{\alpha-\beta}\right)^2\right]. \tag{A15}$$

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*Permanent address; operated by Universities Research Association, Inc. under contract with the Department of Energy.

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[17]Lifetime limits are determined by the path length of the experimental arrangements. The lifetimes we ascribe to b quarks refer to those of hadrons incorporating those quarks. Unless we state otherwise, we assume the two to be equivalent and thus neglect binding effects. Although the experiments do not detect stable neutral hadrons directly, the lifetime for β decay of charged hadrons to neutral hadrons is almost certain to be far longer than the minimum observable lifetime. Consequently, it is reasonable to interpret the negative results of the search generally.

[18]Possibilities involving flavor-changing neutral transitions are treated in Sec. V.

[19]Ellis *et al.*, Ref. 7; J. Ellis, Comments Nucl. Part. Phys. 8, 21 (1978).

[20]See Cahn and Ellis, Ref. 13.

[21]Nonleptonic charm decays appear to be enhanced by a factor of approximately two in rate, as evidenced by semileptonic branching rations which are smaller (see Ref. 22) than the prediction of 20% in the absence of enhancement. In contrast, light-quark nonleptonic transitions are enhanced by a factor of approximately 20.

[22]The charmed-meson semileptonic branching ratio

$$B_{s1} \equiv \Gamma(D \to \text{hadrons} + e^+ + \nu_e)/\Gamma(D \to \text{all})$$

has been reported by R. Brandelik *et al.*, Phys. Lett. 70B, 125 (1977); 70B, 387 (1977), $B_{s1} = (8 \pm 2)\%$; by J. M. Feller *et al.*, Phys. Rev. Lett. 40, 274 (1978), $B_{s1} = (7.2 \pm 2.8)\%$; and by W. Bacino *et al.*, Phys. Rev. Lett. 40, 671 (1978), $B_{s1} = (11 \pm 2)\%$.

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[29]Summaries of charmonium spectroscopy are given by G. J. Feldman and M. L. Perl, Phys. Rep. 33C, 285 (1977), and B. Wiik and G. Wolf, DESY Report No. 78/23 (unpublished). See also Particle Data Group, Phys. Lett. 75B, 1 (1978), for references on individual states.

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[38]See Refs. 1 and 2 and A. S. Ito, in *Neutrinos—78*, edited by Earle C. Fowler (Purdue University, West Lafayette, Indiana, 1978), p. 665.

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[41]See, for example, L. F. Abbott and R. M. Barnett, Phys. Rev. Lett. 40, 1303 (1978).

[42]See, for example, Bjorken and Lane, Ref. 37.

[43]Among the papers of Ref. 34, see especially those of Kingsley and of Goldhaber and Rosner.

[44]See, for example, Kingsley, Ref. 34.