Suppression factor in the leptonic and hadronic decays of the Υ family

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By applying our suppression factor $Q^m(q^2)$ to the decay modes of the Υ family, the leptonic decay widths, direct hadronic decay widths, and two-body decay widths of the Υ are predicted. The values of the predicted decay widths vary greatly ranging from the order of keV to the order of 10^{-6} keV and the examination of such a great variation through experimental observation could be very interesting.

I. INTRODUCTION

The recent discovery of the dimuon enhancement in the 10-GeV region at Fermilab[1] has aroused great theoretical interest and many speculations. This phenomenon was first analyzed as a single broad resonance with the dimuon mass $M = 9.54 \pm 0.04$ GeV or two narrow resonances with the masses $M = 9.44 \pm 0.03$ and 10.17 ± 0.05 GeV. More recently, a third peak has been included in analyzing this improved data,[2] with the masses of three peaks corresponding to $M_\Upsilon = 9.39 \pm 0.013$, $M_{\Upsilon'} = 9.98 \pm 0.04$, and $M_{\Upsilon''} = 10.40 \pm 0.10$ GeV. In analogy with the ψ/J family, we shall assume that these peaks are three narrow resonances which can be considered as the bound states of a heavy quark with another flavor b and its antiquark $\bar{b}$, in the quantum states 1^3S_1, 2^3S_1, and 3^3S_1, respectively. Of course, the most important thing to do is to determine the charge of the new quark, which presumably can be obtained from the leptonic and hadronic decays of the Υ family. Some predictions have been made along these lines.[3]

In our previous paper,[4] we have introduced a suppression factor $Q^m(q^2)$, which, for $m=1$, is given to be the cube root of the proton form factor and we have used it to explain the leptonic decays of all ψ's and the known two-body hadronic decays of the ψ family. The idea of introducing the suppression factor into various decay widths is as follows: Zweig's rule is generally used as a means of explaining the narrowness of the resonance width of J/ψ but it does not give a quantitative prescription of how much the decay width of a Zweig-forbidden diagram is suppressed. For example, the decay width of $\psi' \to \psi\eta$ is about 30 times larger than that of $\psi' \to \rho\pi$. There are at present three general schemes to tackle the problem of ψ decay: (i) mixing various quark wave functions,[5] (ii) group-theoretical method,[6] (iii) dual model.[7] We have attempted a fourth possibility,[4] that the suppression of the ψ decay is due to some dynamic origin by adopting a phenomenological approach, and presented a semiquantitative prescription that can summarize the present experimental situation in an approximate way. The prescription is given below. Whenever a hadron changes its four-momentum it will be suppressed by a factor

$$Q^m(q^2)\,,$$

where m is the number of quarks in the hadron that has been bent or twisted. According to this prescription, the proton form factor from $ep \to ep$ scattering is given by

$$F_p(q^2) = Q^3(q^2)\,,$$

where $m=3$. From the above equation, the exact suppression factor is obtained as follows:

$$Q(q^2) = F_p^{\,1/3}(q^2) = \left(\frac{1}{1+q^2/0.71}\right)^{2/3}. \tag{1.1}$$

Although the momentum transfer q^2 is spacelike for elastic scatterings and form factors, we have argued in our previous paper[4] that for the timelike region, as in the case of inelastic scatterings, it is reasonable to take the symmetric case for the suppression factor as

$$Q(s) = \left(\frac{1}{1+s/0.71}\right)^{2/3}, \tag{1.2}$$

where s is the energy squared in the c.m. system.

The present work is an extension of our previous work to include the suppression factor in the leptonic and hadronic decay width formulas of the Υ family. Since the suppression factor which we have introduced plays an important role in the decays of the ψ family, we believe that it will also give good predictions for the decays of the Υ family.

II. DECAY WIDTH INTO A LEPTON PAIR

If we consider the vector meson V as a bound state of $q\bar{q}$, it decays into a lepton pair through a photon intermediate state in the lowest-order Feynman diagram and the decay width[8] in the nonrelativistic limit is given by

$$\Gamma(V\to l\bar{l}) = 16\pi \frac{\alpha^2 e_q^{\,2}}{M_V^{\,2}} \,|\psi(0)|^2, \qquad (2.1)$$

where M_V is the mass of V, e_q is the charge of q in units of e, and $\psi(0)$ is the quark-antiquark wave function at the origin where the annihilation takes place.

It was shown by Jackson[8] that $|\psi(0)|^2$ has a remarkable relation to the vector mesons ρ, ω, ϕ, and J/ψ given by

$$|\psi(0)|^2 \propto M_V^{\,1.89\pm0.15}. \qquad (2.2)$$

If this relation is applied to $\psi'(3.684)$, the decay width, according to Eqs. (2.1) and (2.2), is given by

$$\Gamma(\psi'\to e^+e^-) = \Gamma(\psi\to e^+e^-)\left(\frac{M_\psi}{M_{\psi'}}\right)^2 \frac{|\psi'(0)|^2}{|\psi(0)|^2}$$

$$\approx 4.7 \text{ keV}.$$

This is more than twice the experimental value. However, if the square of the suppression factor $Q^m(s)$ of the form

$$Q^{2m}(s) = \left(\frac{1}{1+s/0.71}\right)^{4m/3} \qquad (2.3)$$

with $s = M_\psi^{\,2}$ and $m=1$ (the Feynman diagram contains only one quark line which undergoes four-momentum change) is inserted into the decay formula as

$$\Gamma(V\to l\bar{l}) = 16\pi \frac{\alpha^2 e_q^{\,2}}{M_V^{\,2}} \,|\psi(0)|^2 Q^2(M_V^{\,2}), \qquad (2.4)$$

the Jackson relation becomes

$$|\psi(0)|^2 Q^2(M_V^{\,2}) \propto M_V^{\,1.89\pm0.15} \qquad (2.5)$$

for all 1^3S_1 bound states of the vector mesons. For the radical excited state $\psi'(3.684)$, the $|\psi'(0)|^2$ can be assumed to be identical to $|\psi(0)|^2$ of $\psi(3.095)$. The reason is as follows: From the charmonium model,[9] we know that $|\psi_n(0)|^2$ is independent of n for a purely linear potential. It also shows that the Coulomb potential is not effective throughout most of the size of the charmonium. In our case, we adopt a purely linear potential and the Coulombic as well as all other effects are included in the suppression factor.

The decay rate calculated in this way, as given by our previous paper, is 2.2 keV, which agrees excellently with the experimental value 2.1 ± 0.3 keV.

If we assume the new resonances $\Upsilon(9.39)$, $\Upsilon'(9.98)$, and $\Upsilon''(10.40)$ are, respectively, the 1^3S_1, 2^3S_1, and 3^3S_1 bound states of the quarkonium $b\bar{b}$ and that the annihilation wave function squared times the suppression factor squared has the same dependence as the other vector mesons, we obtain

$$|\psi(0)|^2 Q^2(M_\Upsilon^{\,2}) = 0.317 \text{ GeV}^3. \qquad (2.6)$$

Following the same treatment as for ψ and ψ' we take $|\psi'(0)|^2$ for Υ' and $|\psi''(0)|^2$ for Υ'' to be identical to $|\psi(0)|^2$ for Υ, we predict the leptonic decay widths of Υ' and Υ'' together with that of ψ's and Υ and list them all in Table I.

III. HADRONIC DECAY WIDTH

According to quantum chromodynamics (QCD), the direct hadronic decay (the Zweig-forbidden decay) of the vector meson V may be considered as the annihilation of q and $\bar{q}$ into three gluons, which combine together to become hadrons, with the width given by[10]

$$\Gamma_{\text{direct}}(V\to\text{hadrons}) = \frac{160(\pi^2-9)}{81}\frac{\alpha_s^{\,3}}{M_V^{\,2}}\,|\psi(0)|^2. \qquad (3.1)$$

TABLE I. Leptonic decays of the ψ and the Υ families.

Meson	M (GeV)	e_q	$\lvert\psi(0)\rvert^2Q^2(M_V^2)$ (GeV^3)	Γ (keV)	Γ_{exp} (keV)
ψ	3.095	$\frac{2}{3}$	3.90×10^{-2}	4.8 (input)	4.8 ± 0.06
ψ'	3.684	$\frac{2}{3}$	2.52×10^{-2}	2.2	2.1 ± 0.3
ψ''	4.028	$\frac{2}{3}$	2.00×10^{-2}	1.46	
Υ	9.39	$-\frac{1}{3}$	3.17×10^{-1}	1.06	
		$\frac{2}{3}$		4.24	
Υ'	9.98	$-\frac{1}{3}$	2.69×10^{-1}	0.80	
		$\frac{2}{3}$		3.2	
Υ''	10.40	$-\frac{1}{3}$	2.41×10^{-1}	0.66	
		$\frac{2}{3}$		2.6	

Here α_s is the so-called running coupling constant in the asymptotically free QCD given by the expression

$$\alpha_s(s)=\frac{\alpha_s(m_0^2)}{1+(11-\frac{2}{3}N)[\alpha_s(m_0^2)/4\pi]\ln(s/m_0^2)}\,. \tag{3.2}$$

where m_0 is a free scale parameter, and N is the number of quark flavors. In our case we choose $m_0=3.095$, the mass of ψ, $N=4$ for the ψ family and $N=5$ for the Υ family.

With the suppression factor $Q(M_V^2)$, Eq. (3.1) may be written as

$$\Gamma_{\text{direct}}(V\to\text{hadrons})=\frac{160(\pi^2-9)}{81}\,\frac{\alpha_s^{2}}{M_V^{2}}\,|\psi(0)|^2Q^2(M_V^2)\,. \tag{3.3}$$

The $\alpha_s(m_0^2)$ may be determined by the experimental value of the width ratio $\Gamma_{\text{direct}}(\psi\to\text{hadrons})/\Gamma(\psi\to e^+e^-)\approx 10\pm 1.5$, which yields $\alpha_s=0.190$. We can now calculate the direct hadronic decay width without using any additional experimental value, as follows:

$$\alpha_s(s=3.684^2)=\alpha_s(m_0^2)\left(1+\frac{25}{12\pi}\,\alpha_s(m_0^2)\ln\frac{s}{m_0^2}\right)^{-1},$$
$$=0.182\,, \tag{3.4}$$

$$\Gamma_{\text{direct}}(\psi'\to\text{hadrons})=\frac{160(\pi^2-9)}{81}\,\frac{\alpha_s^{3}}{M_{\psi'}^{2}}\,|\psi'(0)|^2Q^2(M_{\psi'}^2)\,,$$
$$=19.5\text{ keV}\,. \tag{3.5}$$

The ratio $\Gamma_{\text{direct}}(\psi'\to\text{hadrons})/\Gamma_{\text{total}}=19.5/228=8.6\%$ again agrees very well with the experimental value ~9%. We wish to point out that if the suppression factor is not included, one then finds that the direct hadronic decay of ψ' would be too large by more than a factor of 2 unless one uses one more piece of experimental value, i.e., $\Gamma(\psi'\to e^+e^-)$ as Jackson[11] did.

We can now calculate the running coupling constants $\alpha_s(s)$ and the direct hadronic decay widths of the Υ family using Eqs. (3.2) and (3.3), and these are shown in Table II.

We note that since ϕ is a pure $s\bar{s}$ state, its decay into 3π must go through annihilation into gluons. So if Eq. (3.2) is applied to calculate the running coupling constant with $N=3$, we get

$$\alpha(s=(1.02\text{ GeV})^2)\approx 0.272$$

and

$$\Gamma_{\text{direct}}(\phi\to 3\pi)\approx 0.157\text{ MeV}\,.$$

This value is about four times smaller than the experimental value 0.67 MeV. In order to obtain the experimental value, the running coupling constant $\alpha_s(s)$ must be set equal to 0.44. This may be too large for Eq. (3.2) to be applicable, as was pointed out by Appelquist and Politzer.[10] Hence higher-order terms may have to be taken into account.

IV. TWO-BODY DECAY WIDTHS OF THE Υ FAMILY

All two-body decay modes of the Υ family can be classified into five different groups: (1) $\Upsilon_i\to\Upsilon_j+M$, (2) $\Upsilon_i\to M_1+M_2$, (3) $\Upsilon_i\to\chi_j+M$, (4) $\chi_i\to\chi_j+M$, and (5) $\chi_i=M_1+M_2$, where $\Upsilon_i=\Upsilon(9.39)$, $\Upsilon'(9.98)$, $\Upsilon''(10.40)$ are, respectively, the ground state and the radially excited states of quarkonium $b\bar{b}$, χ_i represents the other states of quarkonium; M, M_1, and M_2 are other hadrons. Since the pseudoscalar partners and the other quarkonium states of the Υ family have not yet been found experimentally, let us for the moment consider the decay modes of the first two groups. The other groups can easily be calculated when experimental data are available. Although the number of peaks in the 9.3–10.5 region is not clear at present, for our calculation, we take the above Υ_i states with the quantum numbers $n^3S_1(1^{--})$. When a future better measurement is made, an adjustment can easily be made.

For convenience of calculation, we write down the available decay formulas as follows:

TABLE II. Direct hadronic decay widths of the ψ and the Υ families.

Meson	$\alpha_s(s)$	$\Gamma_{\text{direct}}(V\to\text{hadrons})$ (keV)	$\Gamma^{\text{exp}}_{\text{direct}}(V\to\text{hadrons})$ (keV)
ψ	0.190	48 (input)	48 ± 7.2
ψ'	0.182	19.5	~20
ψ''	0.178	11.9	
Υ	0.151	21.2	
Υ'	0.149	15.3	
Υ''	0.148	12.4	

TABLE III. Two-body decays of the type $\Upsilon_i \to \Upsilon_j + M$.

Decays	s (GeV^2)	m	$Q^{2m}(s)$	$\frac{\Gamma}{(g^2/4\pi)\,Q^{2m}(s)}$	$\bar{\Gamma}$ (MeV)
$\Upsilon' \to \Upsilon + \eta$	0.301	3	2.43×10^{-1}	3.08×10^{-2}	7.48×10^{-3}
$\Upsilon'' \to \Upsilon + \eta$	0.301	3	2.43×10^{-1}	1.61	3.91×10^{-1}
$\Upsilon'' \to \Upsilon + \eta'$	0.917	3	3.63×10^{-2}	8.75×10^{-2}	3.17×10^{-3}
$\Upsilon'' \to \Upsilon + S^*$	0.986	3	3.07×10^{-2}	5.84×10^{1}	1.79

$$\Gamma = \frac{1}{3}\left(\frac{g^2}{4\pi}\right)\frac{|\vec{p}_2|^3}{m_1^2}\,Q^{2m}(s) \tag{4.1}$$

for $1^- \to 1^- + 0^-$ decay,

$$\Gamma = \frac{1}{3}\left(\frac{g^2}{4\pi}\right)|\vec{p}_2|\left(1 + \frac{p_1 p_2}{2m_1^2 m_2^2}\right)Q^{2m}(s) \tag{4.2}$$

for $1^- \to 1^- + 0^+$ decay,

$$\Gamma = \frac{1}{3}\left(\frac{g^2}{4\pi}\right)\frac{(m_1^2 - 4m_2^2)^{3/2}}{m_1^2}\,Q^{2m}(s) \tag{4.3}$$

for $1^- \to \frac{1}{2}^- + \frac{1}{2}^+$ decay,

$$\Gamma = \frac{8}{5}\left(\frac{g^2}{4\pi}\right)\frac{|\vec{p}_2|^5}{m_1^4}\,Q^{2m}(s) \tag{4.4}$$

for $2^+ \to 0^- + 0^-$ decay, and

$$\Gamma = \frac{3}{4}\left(\frac{g^2}{4\pi}\right)|\vec{p}_2|\,Q^{2m}(s) \tag{4.5}$$

for $0^+ \to 0^- + 0^-$ decay.

The indices 1,2,3 refer to the particles in the decay $1 \to 2+3$, and the momenta are evaluated in the rest frame of particle 1. As in our previous paper, we use a reduced decay width

$$\bar{\Gamma} = \Gamma \Big/ \frac{g^2}{4\pi},$$

which contains all the kinematic factors and the square of the suppression factor but leaves the coupling constant $g^2/4\pi$ free. The Clebsch-Gordan coefficient that may arise from any symmetry scheme is embedded into the $g^2/4\pi$.

Table III shows that for the decay of the type $\Upsilon_i \to \Upsilon_j + M$ the suppression factors are not large. Hence the reduced decay widths are not small. These are the interesting ones which may be observed experimentally. The reduced decay widths for the type $\Upsilon_i \to m_1 + m_2$ are calculated for many cases. They are very small, ranging from 10^{-3} to 10^{-6} keV because the suppression factors are very large. For reference we list a few of them in Table IV.

We cannot compare the hadronic reduced widths of the ψ and Υ families directly with experiments because the coupling constant $g^2/4\pi$ is not known precisely and the precise group-theoretical classification of the ψ and Υ families has not yet been found. However, the suppression factor for the ψ family[4] is generally large and one can invert the procedure and calculate the coupling constant from the suppression factor and the experimentally measured decay width

$$g^2/4\pi = \Gamma_{\text{exp}}/\bar{\Gamma}.$$

From Table VI of our previous paper, we can see that the coupling constants obtained in this way range from 0.03–0.15 between a factor-of-5 difference for the mesonic decays and about ~0.5 for the baryonic decays. This is reasonable because the Clebsch-Gordan coefficients could produce a factor-of-5 difference for coupling among members in the same group representation and the baryonic coupling is also larger by about a factor of 5 than the mesonic coupling in the ordinary hadronic coupling.

In order to make more specific predictions for the two-body hadronic decays of the Υ family, we estimate them by taking the same type of coupling constants for the ψ family in our previous paper

TABLE IV. Two-body decays of the type $\Upsilon_i \to M_1 + M_2$.

Decays	s (GeV^2)	m	$Q^{2m}(s)$	$\frac{\Gamma}{(g^2/4\pi)\,Q^{2m}(s)}$	$\bar{\Gamma}$ (MeV)
$\Upsilon \to \rho + \pi$	88.17	3	4.072×10^{-9}	3.83×10^{2}	1.56×10^{-6}
$\Upsilon \to \phi + \eta$	88.17	3	4.072×10^{-9}	3.74×10^{2}	1.52×10^{-6}
$\Upsilon \to \phi + \eta'$	88.17	3	4.072×10^{-9}	3.65×10^{2}	1.49×10^{-6}
$\Upsilon \to p + \bar{p}$	88.17	4	6.502×10^{-12}	2.944×10^{3}	1.91×10^{-8}
$\Upsilon \to \Lambda + \bar{\Lambda}$	88.17	4	6.502×10^{-12}	2.869×10^{3}	1.87×10^{-8}
$\Upsilon \to K^+ + K^{-*}$	88.17	3	4.072×10^{-9}	3.775×10^{2}	1.54×10^{-6}

and multiply them by the reduced widths in Tables III and IV to obtain the actual decay widths. Unfortunately, most of the predicted two-body decay widths of Υ family are too small to be experimentally observed except one case that is $\Upsilon' \to \Upsilon + \eta$ for which the predicted decay width is 1.2 keV. This predicted width can be compared with the experimental value when it becomes available. It is interesting to note that for $\Upsilon' \to \Upsilon\pi\pi$ the predicted decay width is ~5 keV. (We will discuss three-body and many-body hadronic decays of ψ and Υ families in detail in another paper.)

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