

## Quantum-chromodynamic predictions for large- $p_T$ hadron production with transversely polarized beam and target

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We apply a hard-scattering model based on quantum-chromodynamic perturbation theory to the production of hadrons at large transverse momentum by transversely polarized beam and target protons. We find that, at large  $x_T = 2p_T/\sqrt{s}$ , the spin-spin asymmetry  $A_{NN}$  for the case of transversely polarized protons is significantly smaller than the corresponding asymmetry  $A_{LL}$  for longitudinally polarized protons. This situation is due to the relatively smaller size of both the quark-quark scattering asymmetry and the spin-dependent distribution functions for quarks in the transversely polarized proton.

### I. INTRODUCTION

Our ability to use quantum-chromodynamic (QCD) perturbation theory to make *quantitative* predictions for strong-interaction processes is currently being tested.<sup>1</sup> The usefulness of the theory depends on the ability to factorize (unknown) infrared structure and absorb it into process-independent distribution and decay functions.<sup>2</sup> In several types of reactions, this factorization should leave a kernel of residual dynamics which can be calculated perturbatively. For example, the production of hadrons at large transverse momentum can, we hope, be treated in a generalized hard-scattering model using QCD perturbation theory.<sup>3</sup> Preliminary experimental tests of this hypothesis have proved encouraging.<sup>4</sup>

Using the hard-scattering model and QCD perturbation theory, it is also possible to calculate a spin-spin asymmetry<sup>5</sup>

$$A_{LL} = \frac{d\sigma(p_{(\pm)}p_{(\pm)} \rightarrow \pi X) - d\sigma(p_{(\pm)}p_{(\mp)} \rightarrow \pi X)}{d\sigma(p_{(\pm)}p_{(\pm)} \rightarrow \pi X) + d\sigma(p_{(\pm)}p_{(\mp)} \rightarrow \pi X)}, \quad (1)$$

where the  $(\pm)$  refer to helicities. In this paper we investigate a related asymmetry

$$A_{NN} = \frac{d\sigma(p_{\uparrow}p_{\uparrow} \rightarrow \pi X) - d\sigma(p_{\uparrow}p_{\downarrow} \rightarrow \pi X)}{d\sigma(p_{\uparrow}p_{\uparrow} \rightarrow \pi X) + d\sigma(p_{\uparrow}p_{\downarrow} \rightarrow \pi X)}, \quad (2)$$

where the  $\uparrow\downarrow$  refer to transversities of beam and target. We find that at large values of  $x_T = 2p_T/\sqrt{s}$ ,

where a quark-quark scattering mechanism is expected to dominate the production of pions,<sup>3</sup> the transverse spin-spin asymmetry should be significantly smaller than the longitudinal asymmetry. This result reflects an analogous relationship between the two types of asymmetries appropriate to elastic quark-quark scattering when calculated to lowest order in QCD perturbation theory. It also reflects the relative difficulty for the quarks in a proton to "remember" the transverse spin of the proton.

Experimental determination of inclusive spin-spin production asymmetries at large  $p_T$  may soon be possible with the development of high-energy polarized beams.<sup>6</sup> Measurements of  $A_{NN}$  and  $A_{LL}$  can be combined with measurements of single-spin asymmetries in order to test fundamental assumptions in the application of the generalized hard-scattering model to large- $p_T$  processes.

Our investigation of the QCD perturbation theory predictions for hadron production by transversely polarized protons begins in Sec. II, in which we describe briefly the hard-scattering model and the quark-quark scattering cross sections for same- and opposite-transversity quarks. We also discuss the distribution functions for quarks in a transversely polarized proton. In Sec. III, we present an upper limit for the asymmetry  $A_{NN}$  and discuss the implications of this limit.

### II. THE MODEL

In the hard-scattering parton model, the interaction of two protons,  $A$  and  $B$ , is assumed to proceed through the scattering of the constituents  $a$  of proton  $A$  and  $b$  of proton  $B$  with invariant cross section  $(d\hat{\sigma}/d\hat{t})(ab \rightarrow cd)$ . If we consider the spin-averaged case for the moment, the invariant cross section is given by

$$E \frac{d\sigma}{d^3p} (pp \rightarrow \pi X) \cong \sum_{ab \rightarrow cd} \int_{x_{a\min}}^1 dx_a \int_{x_{b\min}}^1 dx_b G_{a/A}(x_a) G_{b/B}(x_b) D_c^*(z_c) \frac{1}{z_c} \frac{1}{\pi} \frac{d\hat{\sigma}}{d\hat{t}} (ab \rightarrow cd), \quad (3)$$

where  $G_{a/A}(x_a)$  is the probability of finding a parton of type  $a$  in proton  $A$  with fraction  $x_a$  of  $A$ 's longitudinal momentum ( $G_{b/B}$  is defined similarly) and  $D_c^h(z_c)$  is the probability that outgoing parton  $c$  decays into a hadron  $h$  with fraction  $z_c$  of the initial longitudinal momentum of the parton. In QCD it is believed<sup>2</sup> that the form of Eq. (3) will remain unchanged, but that the distribution and decay functions will not scale in  $x$  or  $z$  and will depend weakly on the momentum transfer.

It is possible to write the parton-model equation for the difference of invariant cross sections for protons with the same vs opposite polarization in analogy to Eq. (3) as<sup>5</sup>

$$E \frac{d\sigma_{\uparrow\uparrow}}{d^3p} - E \frac{d\sigma_{\uparrow\downarrow}}{d^3p} \cong \sum_{ab=cd} \int_{x_{a\min}}^1 dx_a \int_{x_{b\min}}^1 dx_b \Delta G_{a/A}(x_a) \Delta G_{b/B}(x_b) D_c^h(z_c) \frac{1}{\pi} \frac{1}{z_c} \Delta \frac{d\hat{\sigma}}{d\hat{t}} (ab - cd), \quad (4)$$

where

$$\begin{aligned} \Delta G_{a/A} &= G_{a(\uparrow)/A(\uparrow)} - G_{a(\downarrow)/A(\uparrow)}, \\ \Delta G_{b/B} &= G_{b(\uparrow)/B(\uparrow)} - G_{b(\downarrow)/B(\uparrow)}, \\ \Delta \frac{d\hat{\sigma}}{d\hat{t}} &= \frac{d\hat{\sigma}_{\uparrow\uparrow}}{d\hat{t}} - \frac{d\hat{\sigma}_{\uparrow\downarrow}}{d\hat{t}}. \end{aligned} \quad (5)$$

In Eq. (5),  $G_{a(\uparrow)/A(\uparrow)}(x_a)$  is the probability of finding a parton of type  $a$  with fraction  $x_a$  of  $A$ 's longitudinal momentum and "up" polarization in proton  $A$ , which also has "up" polarization. The other distribution functions are defined in a similar manner. The distributions and parton cross section in the unpolarized reaction, Eq. (3), are then given by

$$\begin{aligned} G_{a/A} &= G_{a(\uparrow)/A(\uparrow)} + G_{a(\downarrow)/A(\uparrow)}, \\ G_{b/B} &= G_{b(\uparrow)/B(\uparrow)} + G_{b(\downarrow)/B(\uparrow)}, \\ \frac{d\hat{\sigma}}{d\hat{t}} &= \frac{1}{2} \left( \frac{d\hat{\sigma}_{\uparrow\uparrow}}{d\hat{t}} + \frac{d\hat{\sigma}_{\uparrow\downarrow}}{d\hat{t}} \right). \end{aligned} \quad (6)$$

The asymmetry  $A_{NN}$  that we consider here can be written in terms of the invariant cross sections of Eqs. (3) and (4) as

$$A_{NN} = \frac{\left( \frac{Ed\sigma_{\uparrow\uparrow}}{d^3p} - \frac{Ed\sigma_{\uparrow\downarrow}}{d^3p} \right)}{2 \frac{Ed\sigma}{d^3p}}. \quad (7)$$

The calculation of the constituent scattering cross sections  $d\hat{\sigma}_{\uparrow\uparrow}/d\hat{t}$  and  $d\hat{\sigma}_{\uparrow\downarrow}/d\hat{t}$  involves a sum over all possible scatterings of quarks, antiquarks, and gluons. However, for  $x_T \equiv 2p_T/\sqrt{s} \gtrsim 0.4$ , quark-quark scattering is the dominant mechanism involved in the determination of  $A_{NN}$  and we will restrict our attention to this process. The dominance of quark-quark scattering can be seen in the following manner. If we suppress the integration and kinematic variables of Eqs. (3) and (4) (these are the same in both the spin-averaged and spin-dependent cases), we can estimate  $A_{NN}$  by

$$A_{NN} \sim \frac{d\hat{\sigma}(qq)\Delta G_{q/p}\Delta G_{q/p} + d\hat{\sigma}(q\bar{q})\Delta G_{q/p}\Delta G_{\bar{q}/p} + d\hat{\sigma}(qV)\Delta G_{q/p}\Delta G_{V/p} + \dots}{d\hat{\sigma}(qq)G_{q/p}G_{q/p} + d\hat{\sigma}(q\bar{q})G_{q/p}G_{\bar{q}/p} + d\hat{\sigma}(qV)G_{q/p}G_{V/p} + \dots},$$

where  $q(\bar{q})$  refers to the quark (antiquark) and  $V$  refers to the vector gluon. The possible magnitudes of the  $\Delta G_{q/p}$ ,  $\Delta G_{\bar{q}/p}$ , and  $\Delta G_{V/p}$  distributions are limited due to the constraints on the *sums* of the  $(\uparrow\uparrow)$  and  $(\uparrow\downarrow)$  distributions in each case. For example,  $G_{u/p} = (G_{u\uparrow/p\uparrow} + G_{u\downarrow/p\uparrow}) \gg G_{\bar{u}/p} = (G_{\bar{u}\uparrow/p\uparrow} + G_{\bar{u}\downarrow/p\uparrow})$ . In view of this sort of constraint, even if quark-antiquark,<sup>7</sup> antiquark-antiquark, etc. scattering asymmetries are very large, the overall contribution of processes involving antiquarks and gluons to  $A_{NN}$  will be small.

The angles  $\theta_{c.m.}$  and  $\phi$  which enter into the expressions for  $Ed\sigma_{\uparrow\uparrow(\uparrow\downarrow)}/d^3p$  are defined in Fig. 1. The usual Mandelstam variables are given by

$$s = (p_A + p_B)^2, \quad t = (p_A - p_\pi)^2, \quad u = (p_B - p_\pi)^2.$$

The limits of integration in Eqs. (3) and (4) are

$$\begin{aligned} x_{a\min} &= x_T \cot(\theta_{c.m.}/2) / [2 - x_T \tan(\theta_{c.m.}/2)], \\ x_{b\min} &= x_a x_T \tan(\theta_{c.m.}/2) / [2x_a - x_T \cot(\theta_{c.m.}/2)]. \end{aligned}$$

In the lowest order of QCD perturbation theory, the quark-quark scattering proceeds through the  $t$ -

channel and  $u$ -channel exchange of a single gluon. In terms of the quark mass  $m$  (taken to be equal for the  $u$  and  $d$  quarks) and the parton center-of-mass Mandelstam variables  $\hat{s}$ ,  $\hat{t}$ , and  $\hat{u}$ , given by

$$\hat{s} = (p_a + p_b)^2 \cong x_a x_b s, \quad \hat{t} = (p_a - p_c)^2 \cong x_a t / z_c, \quad \hat{u} = (p_b - p_c)^2 \cong x_b u / z_c,$$

we find

$$\begin{aligned} \frac{d\hat{\sigma}}{d\hat{t}} \left( \begin{array}{c} (\uparrow\uparrow) \\ (\uparrow\uparrow) \end{array} \right) &= \frac{g^4}{36\pi\hat{s}^2} \left\{ \frac{(\hat{s}^2 + \hat{u}^2 + 8m^2\hat{t} - 8m^4)}{\hat{t}^2} \pm \frac{4m^2}{\hat{t}} \left[ 1 - \left( 1 + \frac{\hat{t}}{\hat{s} - 4m^2} \right) \cos^2\phi \right] \right. \\ &\quad + \delta_{\alpha\beta} \frac{(\hat{s}^2 + \hat{t}^2 + 8m^2\hat{u} - 8m^4)}{\hat{u}^2} + \delta_{\alpha\beta} \frac{4m^2}{\hat{u}^2} \left[ \hat{u} - \hat{t} \left( 1 + \frac{\hat{t}}{\hat{s} - 4m^2} \right) \cos^2\phi \right] \\ &\quad \mp \frac{\delta_{\alpha\beta}}{3\hat{t}\hat{u}} \left[ \hat{s}^2 - \hat{t}^2 - \hat{u}^2 - 12\hat{s}m^2 + 24m^4 + 4(\hat{s} - 2m^2)\hat{t} \left( 1 + \frac{\hat{t}}{\hat{s} - 4m^2} \right) \cos^2\phi \right] \\ &\quad \left. + \delta_{\alpha\beta} \frac{(2\hat{s}^2 + 16m^2\hat{s} - 24m^4)}{3\hat{t}\hat{u}} \right\}, \end{aligned} \quad (8)$$

where  $\alpha$  and  $\beta$  represent the initial quark flavors.

The quark-quark scattering asymmetry  $\hat{A}_{NN}$  defined by

$$\hat{A}_{NN} = \frac{d\hat{\sigma}(\uparrow\uparrow)/d\hat{t} - d\hat{\sigma}(\uparrow\downarrow)/d\hat{t}}{d\hat{\sigma}(\uparrow\uparrow)/d\hat{t} + d\hat{\sigma}(\uparrow\downarrow)/d\hat{t}} \quad (9)$$

is found to be

$$\hat{A}_{NN} = \delta_{\alpha\beta} \left( -\frac{1}{11} + \frac{2}{11} \cos^2\phi \right) \begin{cases} \leq 0.09, & \alpha = \beta \\ = 0.0, & \alpha \neq \beta \end{cases} \quad (10a)$$

for transversely polarized quarks in the limit  $m \approx 0$  (Ref. 8) and at  $\hat{\theta}_{c.m.} = 90^\circ$ . This can be compared with the asymmetry  $\hat{A}_{LL}$  for longitudinally polarized quarks,<sup>5</sup>

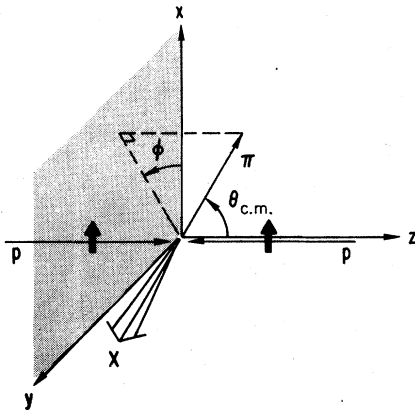


FIG. 1. Definition of the angles  $\theta_{c.m.}$  and  $\phi$  which give the direction of the produced pion momentum in the center-of-mass frame of the  $pp$  system. The spins of the initial protons are in the  $x$  direction.  $\phi$  is the azimuthal angle around the  $z$  axis between the produced-pion momentum and the spin direction.

$$\hat{A}_{LL} = \frac{3 + \frac{1}{3} \delta_{\alpha\beta}}{5 + \frac{7}{3} \delta_{\alpha\beta}} = \begin{cases} 0.45, & \alpha = \beta \\ 0.60, & \alpha \neq \beta \end{cases} \quad (10b)$$

in the same limit. We see that the transversely polarized quark asymmetry  $\hat{A}_{NN}$  is a factor of 5 to 7 smaller than the corresponding asymmetry  $\hat{A}_{LL}$  for longitudinally polarized quarks.

From Eqs. (3), (4), and (7), we can estimate

$$\begin{aligned} A_{NN} &\approx \frac{\Delta G}{G} \times \frac{\Delta G}{G} \times \hat{A}_{NN} \\ &\sim \frac{1}{3} \times \frac{1}{3} \times (\pm 0.09) \\ &\approx \pm 0.01, \end{aligned} \quad (11)$$

where we have perhaps overestimated the magnitude of  $\Delta G/G$ , as we will discuss below, by relying on the order-of-magnitude SU(6) expectation<sup>9</sup> for the polarized quark distributions in a proton. So we expect  $A_{NN}$  to be on the order of a few percent at most, much smaller than  $A_{LL}$  which can be as large as 35%–45%.

To do better than the crude estimate of Eq. (11), we now must determine what form the distribution functions  $\Delta G(x)$  take for a polarized proton. We will consider only the up and down quarks, with distributions denoted by  $\Delta u(x)$  and  $\Delta d(x)$ , respectively, and will ignore the  $Q^2$  dependence of the distributions since this effect is similar for both spin-dependent and spin-independent distributions<sup>5</sup> and will cancel out in the ratio  $A_{NN}$ . We have used the fragmentation functions  $D_c^\pi(z_c)$  of Field and Feynman<sup>10</sup> in our calculations.

We receive some guidance as to the form the quark distributions will take from the operator-product expansion for the spin-dependent part of the electroproduction cross section. Wandzura and Wilczek<sup>11</sup> have performed this analysis and

have arrived at an approximate sum rule that relates the distributions for quarks in a transversely polarized proton to those for quarks in a longitudinally polarized proton. They find

$$k_+(x) - k_-(x) = g_1(x) + g_2(x) = \int_x^1 \frac{dy}{y} g_1(y), \quad (12)$$

where  $g_1(x)$  and  $g_2(x)$  are the usual parton-model structure functions for polarized-lepton-nucleon scattering and  $[k_+(x) - k_-(x)]$  is the sum over the distributions  $\Delta G(x)$ , each weighted by the square of the parton charge. In the limit  $x \gg 0$ ,  $g_1(x) + g_2(x) \approx 0$  (Ref. 12), and so the transverse polarization distributions approach zero. This is, intuitively, a reflection of the tendency for the quarks' spins to line up with their direction of motion for large  $x$ . Hence, the quarks do not "remember" the transverse polarization of the proton well for large  $x$  in contrast to the case of longitudinally polarized quarks for which the quarks carry the helicity information of the proton almost exclusively at large  $x$ . In an intuitive picture of the proton, it is most likely that the proton's transverse polarization is due to the large orbital angular momentum of the quarks about the proton's spin direction.

In terms of longitudinal ( $L$ ) and transverse ( $T$ ) quark distributions, Eq. (12) becomes

$$4\Delta u^T(x) + \Delta d^T(x) = \int_x^1 \frac{dy}{y} [4\Delta u^L(y) + \Delta d^L(y)], \quad (13)$$

where we have ignored the possible antiquark and strange-quark contributions since these effects are small. In Eq. (13) we have also ignored the

approximate nature of the Wandzura-Wilczek sum rule which derives from their assumption that the matrix elements of quark operators of the form

$$A = (\bar{\Psi} \gamma_5 \gamma^{\lambda} \bar{D}^{\sigma} \bar{D}^{\mu_1} \dots \bar{D}^{\mu_{J-1}} \Psi)$$

should be negligible relative to the matrix elements of operators of the form

$$S = (\bar{\Psi} \gamma_5 \gamma^{\sigma} \bar{D}^{\mu_1} \dots \bar{D}^{\mu_{J-1}} \Psi).$$

Wandzura and Wilczek tested this assumption in the framework of the MIT bag model and found the ratio  $A/S \approx 0.20$ . This error factor in the sum rule of Eq. (13) may be  $x$ -dependent; we label it  $\epsilon(x)$ . Then, also noting that Eq. (12) is a charge-independent relationship so that the up and down quarks each satisfy the sum rule separately, we can write for the distributions of quarks in a transversely polarized proton

$$\Delta q^T(x) = \epsilon(x) \Delta q^L(x) + \int_x^1 \frac{dy}{y} \Delta q^L(y), \quad q = u, d. \quad (14)$$

The helicity distributions  $\Delta u^L(x)$  and  $\Delta d^L(x)$  have been studied both theoretically<sup>5,13,14</sup> and experimentally.<sup>15,16</sup> For our work, we have chosen the distributions of Carlitz and Kaur,<sup>13</sup> which are in very good agreement with the available data<sup>15,16</sup> on longitudinally polarized  $ep$  scattering as can be seen in Fig. 2(a), where we compare the prediction for

$$A_{ep}^L / A_{eq}^L \equiv \sum_i e_i^2 \Delta q_i^L / \sum_i e_i^2 q_i,$$

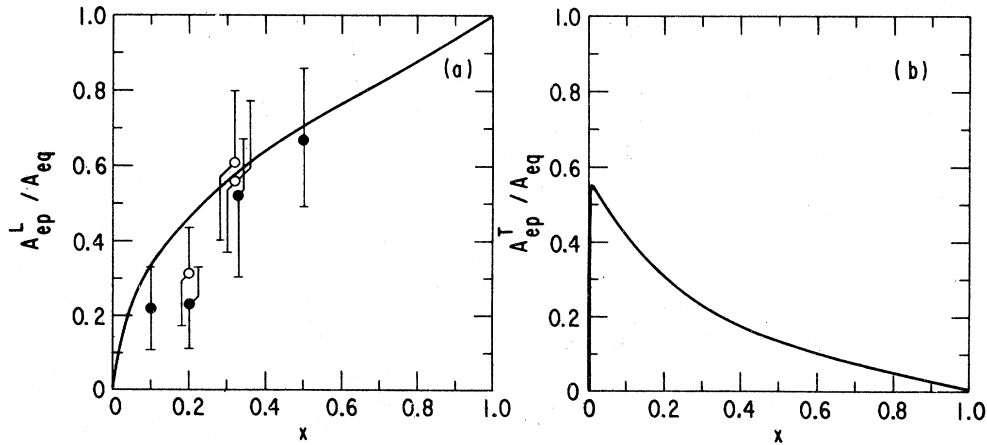


FIG. 2. Asymmetry ratios for polarized  $ep$  scattering: (a) longitudinal polarization—the curve is the prediction using Carlitz and Kaur (Ref. 13) quark distributions and the data are from Ref. 15 (open circles) and Ref. 16 (solid circles); (b) transverse polarization—the curve is the prediction using the Carlitz and Kaur (Ref. 13) longitudinal polarization quark distributions as input to the sum rule of Wandzura and Wilczek (Ref. 11), Eqs. (12) and (14) with  $\epsilon(x) = 0$ .

where  $e_i$  is the quark charge, with the data.

In the Carlitz and Kaur model, valence quarks lose their "memory" of the parent proton's spin orientation through interactions with the ocean. In particular, at small  $x$  the valence quarks lose completely their "memory" of the spin orientation of the proton. Let  $\sin^2\theta$  represent the probability that a valence quark's spin will change in interactions with the ocean. Denote the density of the ocean relative to the valence quarks by  $N(x)$  and let  $H(x)$  be the probability of a spin-flip interaction between valence and ocean. Then

$$\sin^2\theta(x) \equiv \frac{1}{2}H(x)N(x)/[H(x)N(x)+1].$$

Carlitz and Kaur then assume that ocean quarks and antiquarks are unpolarized and that gluons have a  $(1-x)^2$  falloff and arrive at the expression  $H(x)N(x) = H_0(1-x)^2x^{-1/2}$ , where  $H_0 = 0.052$ . Then

$$\Delta u^L(x) = \cos[2\theta(x)][u^{\text{val}}(x) - \frac{2}{3}d^{\text{val}}(x)],$$

$$\Delta d^L(x) = -\frac{1}{3}\cos[2\theta(x)]d^{\text{val}}(x),$$

where the valence quark distributions  $u^{\text{val}}(x) = u(x) - \bar{u}(x)$  and  $d^{\text{val}}(x) = d(x) - \bar{d}(x)$  are taken from Field and Feynman.<sup>10</sup> In Fig. (2b), we present the predictions for *transversely* polarized  $ep$  scattering using the distributions  $\Delta u^T(x)$  and  $\Delta d^T(x)$  obtained from the Carlitz and Kaur distributions according to Eq. (12).  $A_{ep}^T/A_{eq}$  is defined in a manner similar to  $A_{ep}^L/A_{eq}$ ; the only change necessary is the replacement of  $\Delta q_i^L$  by  $\Delta q_i^T$ .

As we can see in Fig. 2, the asymmetry for transversely polarized quarks is larger than the corresponding asymmetry for longitudinally polarized quarks for  $x \lesssim 0.12$ , but rapidly drops off as  $x$  increases and remains much smaller than  $A_{ep}^L/A_{eq}$  even for relatively small  $x$ . In the region where we expect quark-quark scattering to dominate  $A_{NN}$ , i.e.,  $x \gtrsim 0.4$ , the transverse  $ep$  scattering asymmetry is quite small compared to the longitudinal asymmetry, reflecting a ratio between quark distributions  $\Delta q_i^T/\Delta q_i^L \approx 10^{-2} - 10^{-1}$ . This result is in agreement with our expectations from the sum rule of Wandzura and Wilczek, Eqs. (12) and (14), for  $\epsilon(x) = 0$ . Note that taking  $\epsilon(x) > 0$  introduces a component into  $\Delta q_i^T$  that has the  $x$  behavior of  $\Delta q_i^L$ , i.e., a small piece that actually rises with increasing  $x$ . So for  $\epsilon(x) > 0$ ,  $A_{ep}^T/A_{eq}$  will not fall as rapidly with increasing  $x$ . In particular, for  $\epsilon(x) = 0.20$ ,  $A_{ep}^T/A_{eq}$  falls only to a value of 0.21 for  $x = 0.90$ .

### III. CONCLUSIONS

We can combine our results for the transversely polarized quark-quark scattering asymmetry and for the distributions of quarks in a transversely

polarized proton to obtain a prediction for the asymmetry  $A_{NN}$ . We find that  $A_{NN}$  approximately scales in  $x$ . The asymmetries for the production of  $\pi^+$  and  $\pi^-$  reflect the relative magnitudes of the  $\Delta u^T(x)$  and  $\Delta d^T(x)$  distributions,  $\Delta d^T(x)/\Delta u^T(x) \approx 10^{-2} - 10^{-1}$ .  $\pi^+$  production has the largest asymmetry of any of the charge states produced, approximately 35% larger than  $\pi^0$  production, whereas  $A_{NN}$  for  $\pi^-$  production is very much smaller than  $A_{NN}$  for  $\pi^+$  production. The predicted asymmetry for jet production is essentially equal to  $A_{NN}$  for  $\pi^0$  production. We find that in all cases  $A_{NN}$  falls as  $x$  increases, as we expect from our discussion of the distributions  $\Delta q_i^T(x)$ . From our analysis, we can place a limit on  $A_{NN}$ . We find that  $A_{NN}$  reaches its positive maximum at  $\phi = 0^\circ$  and its negative maximum at  $\phi = 90^\circ$ . If we allow for the possible 20% error in the Wandzura-Wilczek sum rule for the quark distributions, Eq. (14), we find the limit

$$|A_{NN}| \lesssim 5 \times 10^{-3} \text{ for } x_1 \gtrsim 0.4. \quad (15)$$

If the sum rule of Eq. (12) were exact, our bound would be lower, i.e.,  $|A_{NN}| \lesssim 2 \times 10^{-3}$  for  $x_1 \gtrsim 0.4$ .

This is not an easily measured number. Our main point is that the use of QCD perturbation theory and the hard-scattering model predicts that the asymmetry for the scattering of transversely polarized protons must be on the order of a factor of 100 smaller than the corresponding asymmetry for protons of definite helicity. If we find that this is not the case experimentally, there are several places in which one could question this sort of analysis. One possibility in such a case is that, contrary to our current expectations, the spin-dependent semi-inclusive reactions of protons do not factorize in QCD and we cannot, therefore, apply perturbation theory as we have done here. Another possibility is that there are large effects due to confinement or other nonperturbative factors such as instantons and coherent reactions. The latter possibility makes the measurement of  $A_{NN}$  at large  $p_T$  particularly interesting since the perturbative effects are so small for this observable.

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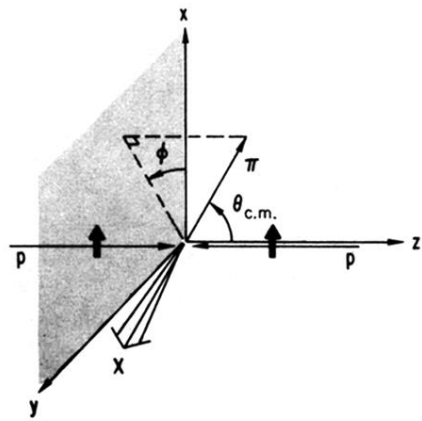


FIG. 1. Definition of the angles  $\theta_{c.m.}$  and  $\phi$  which give the direction of the produced pion momentum in the center-of-mass frame of the  $pp$  system. The spins of the initial protons are in the  $x$  direction.  $\phi$  is the azimuthal angle around the  $z$  axis between the produced-pion momentum and the spin direction.