# Equilibrium temperature in particle production

T. F. Hoang, C. K. Chew, and K. K. Phua
*Department of Physics, Nanyang University, Singapore 22*
(Received 1 February 1978)

Recent $P_\perp$ distributions of $\pi^-$, $K^0$, $\rho^0$, $f$, etc. from $\pi^+p$ and $\pi^+d$ reactions at $P_{lab} = 6$ to 100 GeV/$c$ are analyzed using the Bose-Einstein distribution modified for the Feynman-Yang scaling. It is found that the temperature $T$, characteristic of the distribution, is independent of the nature of secondary mesons, that $T$ increases as the one-fourth power of the available energy in the c.m. system, and that $T$ may be regarded as the equilibrium temperature for meson production. The $K^+/\pi^+$ and $\varphi/\pi^-$ ratios at 400 GeV/$c$ are analyzed under the assumption of equilibrium temperature.

## I. INTRODUCTION

In a previous investigation of particle production in terms of the Bose-Einstein distribution modified to account for the Feynman-Yang scaling,[1] it has been found that the temperature for $J$-particle production in the original MIT-BNL experiment[2] is $T_J \simeq 102$ MeV compared to $T_\pi = 130 \pm 5$ MeV,[3(a)] whereas for the Columbia-Hawaii-Cornell-Illinois-Fermilab experiment at 205 GeV/$c$,[4] we find $T_J = 142 \pm 25$ MeV to compare with $T_\pi = 147 \pm 2$ MeV; see Ref. 3(a) and (d), respectively. As the mass difference between $\pi$ and $J(3.1)$ is very large, we therefore have the strong feeling that the temperature $T$ describing the meson production in terms of the modified Bose-Einstein distribution is actually independent of the nature of produced mesons. In other words, $T$ is the equilibrium temperature in the context of the thermodynamical and hydrodynamical model of particle production.

As regards the temperature estimate using the modified Bose-Einstein distribution, we note that $T$ depends only on the average transverse momentum $P_\perp$ and is independent of the parameter $\lambda$ we have introduced to describe the Feynman-Yang scaling. A discussion on this important point has been presented elsewhere; see Ref. 3(a).

Recently, several high-statistics data of $P_\perp$ distributions for $\rho^0$ production by $\pi^+p$ or $\pi^+d$ reactions at various energies became available. The purpose of this paper is to present results of our analyses using the Bose-Einstein distribution instead of the Gaussian distribution, the applicability of which is rather limited in a narrow range of $P_\perp$. The results of our analysis indicate that, for a given reaction, the temperature $T$ is found to be the same for $\pi^-$ and other mesons such as $\rho^0$, $f$, $K^0$, and so on. This further confirms the basic importance of this property, namely $T$ represents the equilibrium temperature as is assumed in the thermodynamical and the hydrodynamical model.

An attempt is made to interpret the $K^+/\pi^+$ ratio of the large-$P_\perp$ experiment by the Chicago-Princeton group,[5] and the $\phi/\pi^-$ ratio of a recent experiment by the Michigan-Fermilab group.[6]

## II. THE $P_\perp$ DISTRIBUTION

Consider the Bose-Einstein distribution modified for the scaling as follows:

$$\frac{d\sigma}{dP_\perp^2 dP_\parallel} \propto \frac{1}{e^{\epsilon(\lambda)/T} - 1} , \tag{1}$$

where $T$ is the temperature and

$$\epsilon(\lambda) = (P_\perp^2 + \lambda^2 P_\parallel^2 + m^2)^{1/2} , \tag{2}$$

$\lambda$ being the scaling parameter and $m$ the mass of the secondary meson. The validity of the distribution (1) to describe the single-particle distribution has been investigated extensively in the case of $pp$ collisions together with the scaling properties of $\lambda$; see Ref. 3.

We recall that the $P_\perp$ distribution derived from (1) is the same as for that obtained from the original Bose-Einstein distribution, namely

$$\frac{d\sigma}{dP_\perp^2} \propto m_\perp K_2\left(\frac{m_\perp}{T}\right) , \tag{3}$$

where, for simplicity, we have denoted by $m_\perp$ the transverse mass

$$m_\perp = (P_\perp^2 + m^2)^{1/2} , \tag{4}$$

and $K_2$ is the modified Bessel function of the second order.

Noting that for particles such as $\rho^0$, $K^0$, $f$, etc. which we are dealing with here, $m \gg T$, we may approximate $K_2(x) \propto (\pi/2x)^{1/2} e^{-x}$ and write (3) as follows:

$$\frac{d\sigma}{dP_\perp^2} \propto \sqrt{m_\perp}\, e^{-m_\perp/T} . \tag{5}$$

 

It should be mentioned that in the case of $\pi$, we may simplify (5) by neglecting $m$ in the expression (4) for $m_\perp$. This leads to the well known $P_\perp$ distribution proposed by Hagedorn[7]; its validity has been tested for $\pi$'s produced by $\pi^- p$, $pp$, and $\bar{p}p$ reactions; see Ref. 3(a).

For the validity test of (5) in the case of other mesons, we may plot the log of the experimental cross sections $d\sigma/dP_\perp^2$ divided by $\sqrt{m_\perp}$ against $m_\perp$. We should expect the points to lie on a straight line with a slope equal to $-1/T$. In Fig. 1 we have presented such a plot for $\pi^+ p \to \rho^0 + \ldots$ at 15 GeV/$c$ of the Columbia experiment.[8] The dotted line represents the least-squares fit, the $\chi^2/$ point being 14.7/15. A comparison of the experiment points with the fitted line indicates that the test is very satisfactory. The temperature deduced from the slope is $118 \pm 3$ MeV.

Finally, we note that for $\rho$ as for $\pi$ the Gaussian distribution $d\sigma/dP_\perp^2 \sim \exp(-\alpha P_\perp^2)$ can only fit a small range of $P_\perp^2$; see e.g. Fig. 3 of the Columbia-Hawaii-Cornell-Illinois-Fermilab experiment.[4] Thus the coefficient $\alpha$ characteristic of the Gaussian distribution depends very much on the way to perform the fit; we refer the reader to Sec. VII for further discussion on this point.

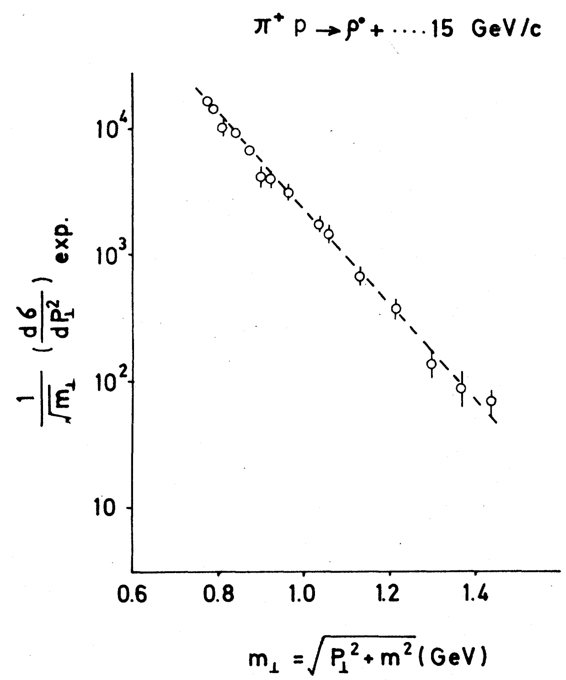


FIG. 1. Plot of

$$\ln \frac{1}{\sqrt{m_\perp}} (d\sigma/dP_\perp)_{\rm exp}$$

vs $m_\perp = (P_\perp^2 + m^2)^{1/2}$. The data are taken from the Columbia experiment, Ref. 8. The dotted straight line represents the distribution Eq. (5) derived from the Bose-Einstein distribution. A validity test requires the plot to be linear, its slope being equal to the reciprocal of the temperature.

## III. TEMPERATURE ESTIMATES

We have analyzed currently available data from $\pi^+ p$ reactions at $P_{\rm lab} = 15$ to 100 GeV/$c$ and included two $\pi^+ d$ reactions at $P_{\rm lab} = 6$ and 24 GeV/$c$.[8-16] Besides $\rho^0$ we have also considered $K^0$, $f$, and other mesons whenever there are available data. As for secondary $\pi$'s, we limit ourselves to $\pi^-$. Finally, for comparison, we have, in addition, analyzed two $\pi^- p$ reactions at $P_{\rm lab} = 16$ and 147 GeV/$c$.[17-19]

For each reaction we have estimated the temperature $T$ by least-squares fits of the experimental data to the distribution (5). The results are summarized in Table I.

As an illustration, we have presented in Fig. 2 the fits to $\pi^-$, $\rho^0$, and $f$ data from $\pi^- p$ at 16 GeV/$c$ of the Aachen-Berlin-Bonn-CERN-Cracow-Heidelberg-Warsaw collaboration.[8] Note that for clarity only a few of the original $\pi^-$ data points have been plotted in Fig. 2. A comparison with experimental points indicates that the fits we have performed are indeed very satisfactory. In this regard, we note that because of the curvature shown by the points of the $\pi^-$ and $\rho^0$ data, it is obviously impossible to fit these data with a simple Gaussian distribution as has often been attempted. Thus it is clear that the distribution (5) here considered is more adequate to describe the properties of the $P_\perp$ distribution. Crucial tests of these two distributions have been investigated in detail in Ref. 3(a).

## IV. PROPERTIES OF $T$

In an attempt to investigate the energy dependence of $T$, we plot in Fig. 3 the values of $T$ against the incident momentum $P_{\rm lab}$ for $\pi^-$ and $\rho^0$ from $\pi^+ p$ and $\pi^+ d$ reactions. Clearly, both temperatures increase with the incident energy. If $m_\pi$ and $m_p$ are the masses of the pion and proton, the energy available for meson production is

$$W = \sqrt{s} - m_\pi - m_p \ , \tag{6}$$

$s$ being the square of c.m. energy. The behavior of $T$ can be described by the empirical power law

$$T = CW^\alpha \ , \tag{7}$$

where the parameters $C$ and $\alpha$ are determined by

TABLE I. Temperature estimates.

| Reaction | $P_{lab}$ (GeV/$c$) | $T$ (MeV) | $T/T_\pi$ | Ref. |
|---|---|---|---|---|
| $\pi^+d\to\pi^- + \cdots$ | 6 | 107 ± 1 | | 15 |
| $\to\rho^0 + \cdots$ | | 106 ± 10 | 0.99 ± 0.09 | |
| $\pi^+p\to\pi^- + \cdots$ | 15 | 118 ± 3 | | 8 |
| $\to\rho^0 + \cdots$ | | 119 ± 3 | 1.01 ± 0.04 | |
| $\to f + \cdots$ | | 97 ± 5 | 0.82 ± 0.06 | |
| $\to K^0 + \cdots$ | | 120 ± 2 | 1.02 ± 0.03 | |
| $\pi^+p\to\pi^- + \cdots$ | 16 | 121 ± 1 | | 9 |
| $\to\rho^0 + \cdots$ | | 121 ± 3 | 1.00 ± 0.02 | |
| $\to f + \cdots$ | | 95 ± 8 | 0.79 ± 0.07 | |
| $\pi^+p\to\rho^0 + \cdots$ | 16 | 118 ± 5 | 0.96 ± 0.04 | 10 |
| $\to\omega + \cdots$ | | 135 ± 3 | 1.12 ± 0.02 | |
| $\to\eta + \cdots$ | | 147 ± 11 | 1.21 ± 0.09 | |
| $\pi^+p\to K^0 + \cdots$ | 16 | 122 ± 5 | 1.09 ± 0.03 | 11 |
| $\pi^+p\to\pi^- + \cdots$ | 18.5 | 119 ± 3 | | 12 |
| $\to K^0 + \cdots$ | | 122 ± 3 | 1.03 ± 0.03 | 13 |
| $\pi^+p\to\rho^0 + \cdots$ | 22 | 127 ± 3 | | 14 |
| $\pi^+d\to\pi^- + \cdots$ | 24 | 128 ± 2 | | 15 |
| $\to\rho^0 + \cdots$ | | 129 ± 3 | 1.01 ± 0.01 | |
| $\pi^+p\to K^0 + \cdots$ | 100 | 136 ± 23 | | 16 |
| $\pi^+p\to\rho^0 + \cdots$ | 150 | 160 ± 11 | | |
| $\to J + \cdots$ | | 135 ± 17 | | |
| $\pi^-p\to\pi^- + \cdots$ | 16 | 133 ± 2 | | 17 |
| $\to\rho^0 + \cdots$ | | 131 ± 3 | 0.98 ± 0.05 | |
| $\to K^*(890) + \cdots$ | | 99 ± 8 | 0.74 ± 0.08 | 18 |
| $\pi^-p\to\pi^- + \cdots$ | 147 | 153 ± 4 | | 19 |
| $\to\rho^0 + \cdots$ | | 163 ± 5 | 1.07 ± 0.04 | |

a least-squares fit to the data listed in Table I.

Consider first the case of $\rho^0$. We find for $P_{lab} \geqslant 15$ GeV/$c$

$$C_\rho = 85.4 \pm 7.3, \quad \alpha_\rho = 0.23 \pm 0.04 \; .$$

The fit is shown by the solid line in Fig. 3, the dotted line being its extrapolation. Likewise we find for $\pi^-$

$$C_\pi = 78.7 \pm 2.6 \,, \quad \alpha_\pi = 0.27 \pm 0.02 \; .$$

It is interesting to note that these two sets of parameters, one for $\rho^0$ and the other for $\pi^-$, are found to be the same within statistical errors, and that the power $\alpha$ is consistent with $\frac{1}{4}$ as predicted by Fermi's statistical model[20] and Landau's hydrodynamical model[21]; see Ref. 3(b) and (d).

Next consider the ratio of temperatures $T/T_\pi$ with respect to $\pi^-$ of the same reactions. The values thus obtained are listed in Table I. A comparison of these ratios indicates that within statistical fluctuations, they are all consistent with unity, the average being

$$\langle T/T_\pi \rangle = 1.00 \pm 0.12 \quad .$$

This result is shown in Fig. 4.

Referring to other ratios $T/T_\pi$ listed in Table I for $\pi$ and $\rho^0$ from the $\pi^-p$ reactions, we note that the property that the average is about 1 also holds, although at a given energy $T$ for the $\pi^-p$ reaction is higher than that for $\pi^+p$.

We are thus led to the conclusion that the temperature we have estimated using the Bose-Einstein distribution (5) actually represents the equilibrium temperature of secondary mesons.

## V. $K^+/\pi^+$ ratio

As a further investigation of this important property that $T$ is the equilibrium temperature, we consider the $K^+/\pi^+$ ratio measured at 90° c.m. angle for $pp$ collisions at 400 GeV/$c$ of a large-$P_\perp$ experiment by the Chicago-Princeton group.[5] Their data, Table II of Ref. 5, are reproduced in Fig. 5. This case is of special interest, firstly because $K$ and $\pi$ have different quarks, the strange quark being heavier than the ordinary quark according to the quark model, and also because, here, we are dealing with large $P_\perp$ reactions.

From a previous study of large $P_\perp$ distributions of an earlier experiment by the same group,[22] we know that in this case the appropriate distribution is

$$E\frac{d\sigma}{d^3P} = \frac{C}{P_\perp} e^{-(E-aP_\perp)/T} \,, \qquad (8)$$

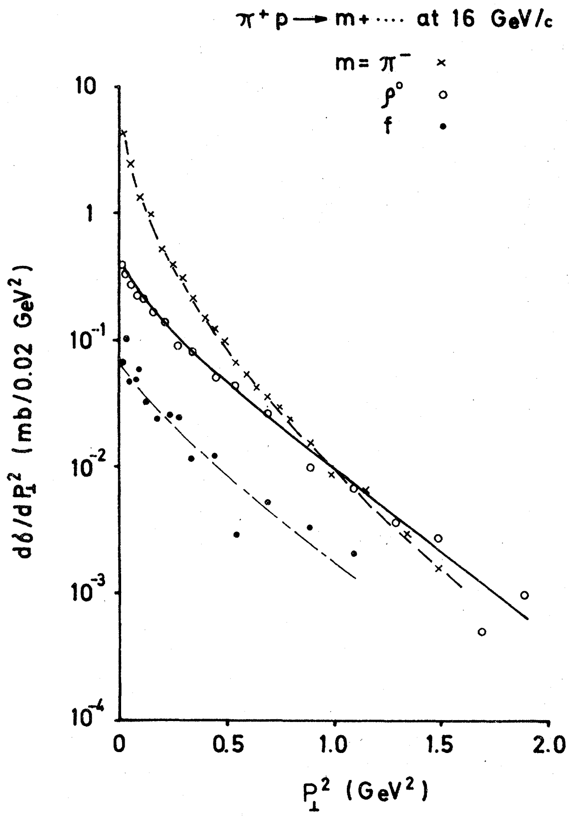


FIG. 2. $P_\perp$ distributions of $\pi^-$, $\rho^0$, and $f$ from $\pi^+p$ at 16 GeV/$c$, Ref. 9. The curves represent fits with the distribution Eq. (5). The temperatures estimated from the fits are equal within statistical errors; see Table I.

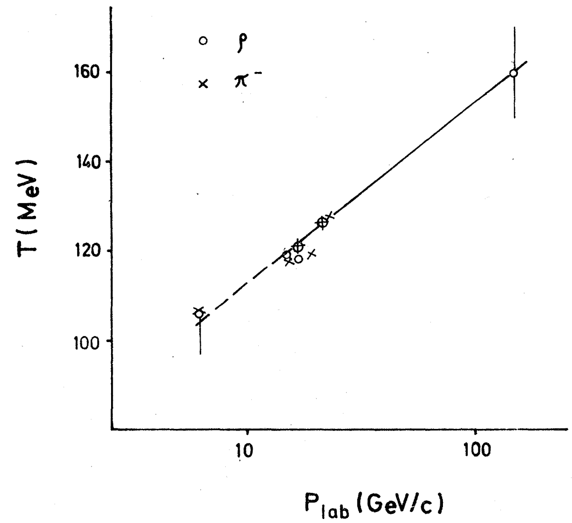


FIG. 3. Dependence of $T$ on $P_{lab}$ for $\pi^-$ and $\rho^0$ from $\pi^+p$ and $\pi^+d$ reactions. The solid line is the least-squares fit with the power-law behavior, Eq. (7), for $P_{lab} \geq 15$ GeV/$c$ and the dashed line is its extrapolation to $P_{lab} = 6$ GeV/$c$; see text of Sec. IV.

where $E = (P^2 + m^2)^{1/2}$ is the total energy of the meson under consideration, $C$ is the normalization constant, and $a$ is a parameter describing the transverse velocity of the fireball motion. For a detailed discussion on the properties of (8), we refer to Ref. 3(d).

Therefore, assuming the same temperature for $K^+$ and $\pi^+$ production, we may write

$$\frac{K^+}{\pi^+} = \left(\frac{C_K}{C_\pi}\right) e^{-(E_K - E_\pi)/T} . \quad (9)$$

By a least-squares fit we find

$T = 152 \pm 8$ MeV ,

$C_K/C_\pi = 0.55 \pm 0.03$ .

The fit is shown by the solid line in Fig. 5, the $\chi^2$/point being 16.9/10 which is mostly due to the point at $P_\perp = 5.38$ GeV/$c$. If we exclude this point, we find it reduced to 2.9/9. This indicates that the fit is actually very satisfactory.

The equilibrium temperature thus estimated is in good accord with what we deduce from $T = 152 \pm 4$ MeV for $pp \rightarrow \rho^0 + \ldots$ at c.m. angle 90° and $\sqrt{s} = 23.5$ GeV of the CERN-Columbia-Rockefeller

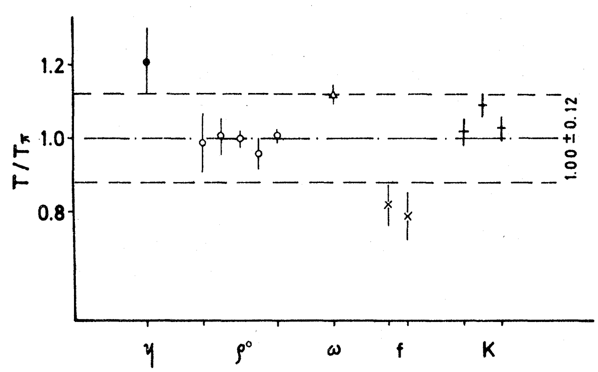


FIG. 4. Plot of the ratio of temperature of $\rho^0$, $K^0$, ... to that of $\pi^-$ of the same reaction of $\pi^+p$ and $\pi^+d$. The average of $T/T_\pi$ is 1.00 ± 0.12, shown by the straight lines.

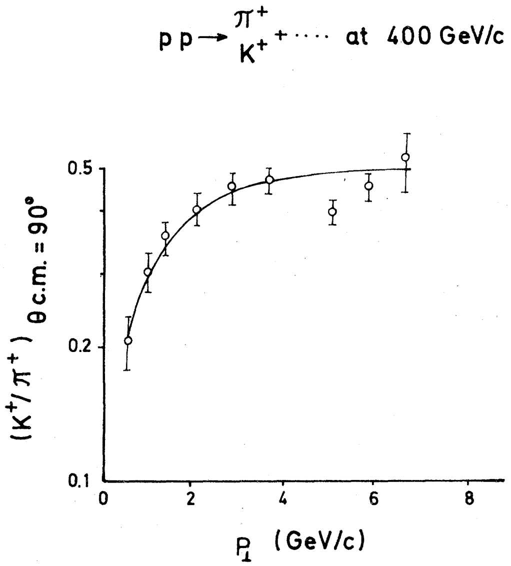


FIG. 5. Plot of $K^+/\pi^+$ ratio vs $P_\perp$ of a large-$P_\perp$ experiment at 400 GeV/$c$ by the Chicago-Princeton group, Ref. 5. The solid curve is the fit to the data using a Bose-type distribution, Eq. (8) assuming the same $T$ for $K^+$ and $\pi^+$. The least-squares fit yields $T=152\pm 8$ MeV.

experiment[23]; see Ref. 3(a). Using the empirical power law (7), we find $T$ corresponding to $P_{lab}=400$ GeV/$c$ to be about 155 MeV. This further confirms our assumption regarding the equilibrium temperature.

## VI. $\phi/\pi^-$ RATIO

Consider next $\phi(1020)$ production by $p$-Be at 400 GeV/$c$ by Akerlof *et al.*[6] These authors have measured the ratio of invariant cross sections of $\phi/\pi^-$ at fixed Feynman variable $x=-0.08$, corresponding to c.m. $P_{\parallel}=-1.170$ GeV/$c$. We propose to use the Bose-Einstein distribution (1) to analyze their measurements which are reproduced in Fig. 6.

Assuming thermal equilibrium, we expect

$$\frac{\phi}{\pi}=\left(\frac{C_\phi}{C_\pi}\right)\frac{E_\phi}{E_\pi}\,e^{-(\epsilon_\phi-\epsilon_\pi)/T}\,, \tag{10}$$

where

$$\epsilon_\phi=(P_\perp{}^2+\lambda^2P_\parallel{}^2+m^2)^{1/2}$$

and a similar expression for $\epsilon_\pi$. As regards the scaling parameter $\lambda$, we may estimate it using the scaling property $\lambda\gamma_{c.m.}=2$, as discussed in Ref. 3(c).

By means of a least-squares fit we find for the two parameters

$$T=129\pm 47 \text{ MeV}\,,$$

$$C_\phi/C_\pi=0.22\pm 0.08\,.$$

The fit is shown by the solid line in Fig. 6. Note the temperature thus estimated is lower than 152 MeV obtained previously for $K^+/\pi^+$ from $pp$ collisions at the same $P_{lab}=400$ GeV/$c$ but with $x=0$ instead of 0.08, and that a comparison with the data points indicates that this fit is less good, although $\chi^2$/point = 12.9/12 is not unreasonable.

Finally, we have tried to fit the data assuming $T=152$ MeV and found

$$C_\phi/C_\pi=0.18\pm 0.05\,.$$

The fit is shown by the dotted line in Fig. 6, $\chi^2$/point = 21.6/12 being comparable to that of the previous fit.

## VII. CONCLUDING REMARKS

From the results of our analyses presented in this paper, we conclude that the Bose-type distribution (1) we have considered for the single-particle distribution, account being taken of the Feynman-Yand scaling, is adequate to describe $P_\perp$ distributions of secondary mesons $\pi^-$, $\rho^0$, $K^0$, etc. observed in various $\pi^+p$ and $\pi^+d$ reactions

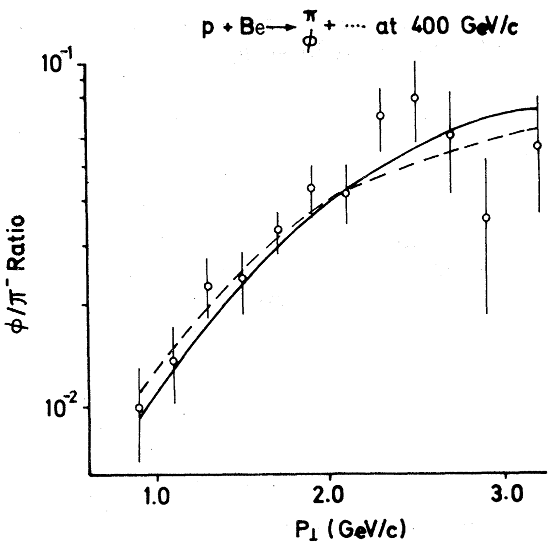


FIG. 6. Plot of $\phi/\pi^-$ ratio vs $P_\perp$ at fixed $x=0.08$ from $p$-Be at 400 GeV/$c$ of an experiment by Akerlof *et al.*, Ref. 6. The solid line represents the fit with the Bose distribution Eq. (5) assuming the same temperature: $T=129\pm 47$ MeV. The dotted line is the fit with $T=152$ MeV of Fig. 5.

from $P_{\rm lab}=6$ to 100 GeV/$c$. We find that the temperature $T$, characteristic of the distribution, is independent of the nature of secondary mesons under consideration, in spite of the difference in mass and in strangeness.

This property which also holds for $\pi^- p$ reactions (see Table I) is likely a very general one for particle production and constitutes the basic assumption of the thermodynamic and hydrodynamical model.

As regards the $P_\perp$ distribution of $\pi^-$, we note that the distribution (5) gives an excellent fit in the whole range of $P_\perp^2$ up to ~2 (GeV/$c$)$^2$, a result already known from our previous investigation on this subject [see Ref. 3(b)]. Consequently, according to our present analysis, there is no indication that the $\pi$'s can be separated into two groups, one from the direct production and another from the decay of certain resonances $\rho^0$, etc. as interpreted by Kirk *et al.*[24] using instead the Gaussian distribution to fit their data, Ref. 9.

In this respect, we note that since more $\rho^+$ are produced than $\rho^0$ in the $\pi^+ p$ reaction, $\pi^0$ is expected to outnumber $\pi^-$, and according to their interpretation, the slopes $\alpha$ of

$$\frac{d\sigma}{dP_\perp^2} \propto e^{-\alpha P_\perp^2}$$

are different for secondary $\pi^-$ and $\rho^0$ observed in the same reaction. Lacking information on $\pi^+ p \to \pi^0 + \ldots$ at 16 GeV/$c$, we have instead analyzed the 15-GeV/$c$ data of the Columbia experiment.[8] For this purpose, we have tentatively set $P_\perp^2 < 0.4$ (GeV/$c$)$^2$ and found by least-squares fits

$$\alpha_- = 9.37 \pm 0.92$$

for $\pi^-$ and

$$\alpha_0 = 10.77 \pm 0.77$$

for $\pi^0$. We note that $\alpha_- = \alpha_0$ within statistical errors contradicts their interpretation.[25]

On the other hand, we note that since the mass of $\pi$ is much lighter than that of $\rho^0$ and other resonances, and since the average $P_\perp$ is approximately proportional to $m^{1/2}$, it would be difficult to think that its "direct" production amounts to 10–20% as given by these authors[24] unless there exists some unknown mechanism which inhibits $\pi$ production to favor those resonances.

Furthermore, we note that because of the bunching effect, characteristic of Bose particles, the production of is such that the $\pi$'s tend to get close together; this favors the formation of resonances, provided that the temperature $T$ is the same. Indeed, consider two $\pi$'s of energy-momentum ($E_1$, $P_1$) and ($E_2, P_2$) such that the resultant ($E,P$) turns out to be near the mass shell of a certain resonance; then the condition imposed by the probability due to the Bose statistics, i.e., $e^{-E_1/T}e^{-E_2/T} = e^{-E/T}$ is identically satisfied in view of the energy-momentum conservation. In other cases, resonances that can be formed by $\pi$'s produced directly through the final state interaction cannot be excluded.

Of particular interest is $T_K = T_\pi$ (see Table I): this property is further supported by the result of the analysis of the $K^+/\pi^+$ ratio of the Chicago-Princeton experiment.[5] Thus, in the context of the quark model, the principle of equilibrium temperature applies to ordinary as well as strange quarks.

As for the $\phi/\pi^-$ ratio,[6] it is worth noting that the coefficient $C_\phi/C_\pi = 0.22$ turns out to be comparable to $C_K/C_\pi = 0.55$ for $K^+/\pi^+$ of the Chicago-Princeton experiment[5] at the same energy. This indicates that for large-$P_\perp$ production near $\theta_{\rm c.m.} = 90°$, $\phi$ production is less inhibited than what is expected from the Zweig rule,[26] a result different from forward production. It would also be interesting to know by future experiments if this ratio reaches a plateau as in the case of $K^+/\pi^+$ (see Fig. 5) or rather if it turns down at larger $P_\perp$ as has been observed for $K^-/\pi^-$ of the Chicago-Princeton experiment.[27]

### ACKNOWLEDGMENT

One of the authors (T. F. H.) wishes to thank Nanyang University for the kind hospitality.

[1]T. F. Hoang, Phys. Rev. D 13, 1881 (1976).
[2]J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen, J. Leong, T. McCorrison, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, San Lan Wu, and Y. Y. Lee, Phys. Rev. Lett. 33, 1404 (1974).
[3]T. F. Hoang, Nucl. Phys. B38, 333 (1972); Phys. Rev. D 6, 1328 (1972); 8, 2315 (1973); 12, 296 (1975) referred to as (a), (b), (c), and (d), respectively.
[4]B. Knapp, W. Lee, P. Leung, S. D. Smith, A. Wijango, J. Knaur, D. Yount, D. Nease, J. Bronstein, R. Coleman, L. Cormell, G. Gladding, M. Gormley, R. Messner, T. O'Halloran, J. Sarracino, A. Wattenberg, D. Wheeler, M. Binkley, R. Orr, J. Peoples, and R. Read, Phys. Rev. Lett. 34, 1044 (1975).
[5]D. Antreasyan, J. W. Cronin, H. J. Frisch, M. J. Shochet, L. Kluberg, P. A. Piroué and R. L. Summer, Phys. Rev. Lett. 38, 115 (1977).
[6]C. W. Akerlof, P. Alley, D. Bintinger, W. R. Ditzler, D. A. Finley, O. E. Johnson, D. Koltick, F. J. Loefler, R. L. Loveless, D. I. Meyer, E. I. Shibata, K. C. Stanfield, R. Thun, and D. D. Yovanoritch, Phys. Rev. Lett. 39, 861 (1977).
[7]R. Hagedorn, Nuovo Cimento Suppl. 3, 147 (1965).

[8]C. Baltay, C. V. Cautis, D. Cohen, M. Kalelkar, D. Pisello, W. D. Smith, and N. Yeh, report, Columbia University (unpublished). We thank Dr. Cohen for communicating to us the data of their experiment.

[9]Aachen-Berlin-Bonn-CERN-Cracow-Heidelberg-Warsaw Collaboration, Nucl. Phys. B103, 426 (1976).

[10]Aachen-Berlin-Bonn-CERN-Cracow-London-Vienna-Warsaw Collaboration, Nucl. Phys. B118, 360 (1977).

[11]Aachen-Bonn-CERN-Cracow Collaboration, Nucl. Phys. B94, 21 (1975).

[12]N. N. Biswas, N. M. Cason, V. P. Kenney, J. T. Powers, W. D. Shephard, and D. W. Thomas, Phys. Rev. Lett. 26, 158 (1971).

[13]P. H. Suntebeck, N. M. Cason, J. M. Bishop, V. P. Kenny, and W. B. Shepard, Phys. Rev. D 9, 608 (1974).

[14]H. A. Gordan, Mehdi M. Habibi, Kwan-Wu Lai, and Inlin Stumer, Phys. Rev. Lett. 34, 284 (1975).

[15]M. Alston-Garnjost, J. Erwin, J. H. Klems, W. Ko, R. L. Lander, D. E. Pellett, and P. M. Yager, Phys. Rev. Lett. 35, 142 (1975).

[16]J. Anderson, A. Engler, R. Kraemer, S. Toaff, F. Weissier, J. Diaz, F. Dibianca, W. Fickinger, D. Robinson, C. Sullivan, C. Y. Chien, B. Cox, D. Feiok, V. Sreedhar, R. Zdans, C. Bromberg, T. Ferbel, and P. Slattery, Phys. Lett. 45B, 521 (1973).

[17]Aachen-Berlin-Bonn-CERN-Cracow-Heidelberg-Warsaw Collaboration, Nucl. Phys. B107, 93 (1976).

[18]Bari-Bonn-CERN-Darebury, Glasgow-Liverpool-Milano-Vienna Collaboration, Phys. Lett. 70B, 373 (1977).

[19]Brown-CERN-Fermilab-Illinois-Indiana-Johns Hopkins-MIT-Oak Ridge-Rutgers-Stevens-Tennessee-Yale Collaboration, Phys. Lett. 60B, 124 (1975).

[20]E. Fermi, Prog. Theor. Phys. 5, 570 (1950).

[21]L. D. Landau, Izv. Akad. Narek. SSSR, Ser. Fiz. 17, 51 (1953).

[22]J. W. Cronin, H. J. Frisch, M. J. Shochet, J. P. Boymond, P. A. Piroue, and R. L. Summer, Phys. Rev. Lett. D 11, 3105 (1975).

[23]CERN-Columbia-Rockfeller Collaboration, Phys. Lett. 46B, 471 (1973).

[24]Aachen-Berlin-Bonn-CERN-Cracow-London-Vienna-Warsaw Collaboration, Nucl. Phys. B128, 397 (1977).

[25]Note that the universal slope mentioned in Ref. 24, namely $\sim 3.4\ (\text{GeV}/c)^{-2}$ corresponds to a fitting range $P_\perp{}^2 \geq 1.0\ (\text{GeV}/c)^2$.

[26]G. Zweig, Report No. CERN-TH-412, 1964 (unpublished); S. Okubo, Phys. Lett. 5, 165 (1963); I. Iizuka, Prog. Theor. Phys. Suppl. 37–38, 21 (1966).

[27]See e.g. Ref. 5. Fig. 3.