Implications of dynamical symmetry breaking: An addendum

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It is shown that the dynamical symmetry breakdown of a gauge symmetry can in some cases lead to simple relations among the masses of intermediate vector bosons.

This note is an addendum to a general survey[1] of the physical implications of dynamical symmetry breaking.[2] By a "dynamical" symmetry breaking is meant any spontaneous breakdown of a global or gauge symmetry for which the associated Goldstone bosons are composite rather than elementary particles. Formation of such states requires strong forces among the constituent particles, and it is not so easy to deduce all its physical consequences. In particular, the possible relations among vector-boson masses generated by the "Higgs mechanism" in a dynamical symmetry breakdown were not considered in Ref. 1.

In this note I wish to analyze the conditions under which a dynamical breakdown of a gauge symmetry can lead to simple relations among vector-boson masses. It is found that a dynamical breakdown of SU(2)× U(1) in the gauge theory[3] of weak and electromagnetic interactions can lead to the same successful relation $M_Z/M_W = \sec\theta$ that is obtained when the symmetry breakdown is due to vacuum expectation values of scalar-field doublets. For a dynamical symmetry breakdown this is not as automatic as for symmetry breaking by scalar-doublet vacuum expectation values, but depends on the assumption of a specific pattern of spontaneous symmetry breaking. The source of this relation can be traced, both for symmetry breaking by scalar doublets and in the dynamical case to be considered here, to the same simple property of the Goldstone bosons which mix with the SU(2)× U(1) gauge fields. However, there remain severe difficulties in developing realistic detailed models of elementary particles in which the spontaneous symmetry breaking is purely dynamical.

First, let us consider a gauge model that is illustrative, though its quark content is quite unrealistic. The gauge group of the weak, electromagnetic, and strong interactions is as usual taken as SU(2)× U(1)× G_S, but G_S is arbitrary. The coupling constants g, g', and g_s associated with the subgroups SU(2), U(1), and G_S are assumed to have the orders of magnitude $g \approx g' \approx e$ and $g_s \approx 1$. [Strictly speaking, g_s becomes of order unity at a renormalization scale Λ, which, as we shall see in this example, would have to be of the order of 200 GeV. Hence, G_S could not consist solely of the usual color group SU(3).] The model contains just two G_S multiplets of quarks, U and D, which form a left-handed SU(2) × U(1) doublet $(1+\gamma_5)(U,D)$ and right-handed singlets $(1-\gamma_5)U$ and $(1-\gamma_5)D$, but no scalar fields. (The "color" indices associated with G_S are dropped everywhere.) In the limit $e\to 0$, the strong interactions will automatically be invariant not only under the gauge group G_S, but also under an "accidental" global SU(2)× SU(2) symmetry,[4] consisting of independent unitary unimodular transformations on the doublets $(1+\gamma_5)(U,D)$ and $(1-\gamma_5)(U,D)$. We assume that the strong forces associated with G_S produce a dynamical breakdown of SU(2)× SU(2), which for $e=0$ leaves the global "isospin" subgroup SU(2) unbroken. [At the same time, G_S itself may also break down to some gauge subgroup, perhaps SU(3).] It is the residual global invariance of the strong interactions for $e=0$ that leads in this model to a simple relation between M_W and M_Z.

To see this, we use the general lowest-order formula[5] for the intermediate-vector-boson matrix

$$\mu^2{}_{\alpha\beta} = \tfrac{1}{64}\sum_a F_a{}^2\,\mathrm{Tr}(t_\alpha x_a)\,\mathrm{Tr}(t_\beta x_a)\,. \qquad (1)$$

Here x_a are the generators of all spontaneously broken global symmetries, in a suitably orthonormalized basis[6]; F_a are the couplings of the corresponding Goldstone bosons to the associated currents; and t_α are the generators of the weak and electromagnetic gauge groups, including all coupling-constant factors. In the present case, we have

$$\begin{aligned} &x_a = \gamma_5\tau_a, \quad a=1,2,3\\ &t_i = \tfrac{1}{4}g(1+\gamma_5)\tau_i\,, \quad i=1,2,3\\ &t_0 = -g'\left[\tfrac{1}{4}(1-\gamma_5)\tau_3 + \tfrac{1}{6}\right], \end{aligned} \qquad (2)$$

with τ_a the Pauli isospin matrices. The residual global SU(2) symmetry prevents the appearance of any positive-parity Goldstone bosons, and also imposes a relation among the F_a,

$$F_1 = F_2 = F_3 \equiv F\,. \qquad (3)$$

This is the same relation as is satisfied by the F's in the nondynamical case, where the symmetry breakdown is due to vacuum expectation values of scalar doublets, and it leads to the same relation among intermediate-vector-boson masses. From Eq. (1), we then find the nonvanishing elements of the intermediate-vector-boson mass matrix,

$$\begin{aligned}\mu^2_{11}&=\mu^2_{22}=\mu^2_{33}=\tfrac{1}{16}F^2g^2\,,\\ \mu^2_{30}&=\mu^2_{03}=\tfrac{1}{16}F^2gg'\,,\\ \mu^2_{00}&=\tfrac{1}{16}F^2g'^2\,.\end{aligned}\tag{4}$$

(All Goldstone bosons are eliminated by the Higgs mechanism here.) It is easy to see that the nonvanishing mass eigenvalues are

$$M_W{}^2=\tfrac{1}{16}F^2g^2\,,\quad M_Z{}^2=\tfrac{1}{16}F^2(g^2+g'^2)\,.\tag{5}$$

With $g'/g\equiv\tan\theta$, these have the usual ratio $M_Z/M_W=\sec\theta$.

The U and D of this model cannot, of course, be identified with any known particles. Their mass must be of order F, because for $e=0$ there are no free parameters in the theory except for the G_S renormalization scale Λ, so that M_U, M_D, and F must all be of order Λ. But if we identify $\sqrt{2}\,g^2/8M_W{}^2$ with the usual Fermi coupling constant G_F, then F takes the value $2^{-3/4}G_F{}^{-1/2}$, or 175 GeV. Also, U and D are nearly degenerate, since their masses are split by the weak and electromagnetic interactions only in order α.

In order to construct a slightly more realistic model, we consider the same gauge group $SU(2)\times U(1)\times G_S$, but now we suppose that there are *four* quark flavors U_1, D_1, U_2, D_2, which form two left-handed $SU(2)\times U(1)$ doublets $(1+\gamma_5)(U_1,D_1)$ and $(1+\gamma_5)(U_2,D_2)$, plus four right-handed singlets. In this model, the strong interactions for $e\to 0$ will automatically have an accidental global $SU(4)\times SU(4)$ symmetry.[4] We assume that this symmetry suffers a spontaneous dynamical breakdown in such a way that two quarks, U and D, receive equal masses, while the other two, u and d, remain massless. The subgroup of $SU(4)\times SU(4)$ which remains unbroken is assumed to be the largest subgroup consistent with such masses, with the generators (in a U,D,u,d basis) given by

$$\begin{pmatrix}\vec{\tau}&0\\0&0\end{pmatrix},\quad\begin{pmatrix}0&0\\0&\vec{\tau}\end{pmatrix},\quad\gamma_5\begin{pmatrix}0&0\\0&\vec{\tau}\end{pmatrix},\quad\begin{pmatrix}1&0\\0&-1\end{pmatrix}.\tag{6}$$

(Also, parity is assumed to be not spontaneously broken.) The orthonormalized generators of the broken part of $SU(4)\times SU(4)$ can then be taken as

$$\begin{aligned}&\vec{x}_A=\gamma_5\begin{pmatrix}\vec{\tau}&0\\0&0\end{pmatrix},\quad x_B=\frac{\gamma_5}{\sqrt{2}}\begin{pmatrix}1&0\\0&-1\end{pmatrix},\\ &\vec{x}_C=\frac{1}{\sqrt{2}}\begin{pmatrix}0&-i\vec{\tau}\\i\vec{\tau}&0\end{pmatrix},\quad\vec{x}_C'=\gamma_5\vec{x}_C\,,\\ &\vec{x}_D=\frac{1}{\sqrt{2}}\begin{pmatrix}0&\vec{\tau}\\\vec{\tau}&0\end{pmatrix},\quad\vec{x}_D'=\gamma_5\vec{x}_D\,,\\ &x_E=\frac{1}{\sqrt{2}}\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad x_E'=\gamma_5x_E\,,\\ &x_F=\frac{1}{\sqrt{2}}\begin{pmatrix}0&-i\\i&0\end{pmatrix},\quad x_F'=\gamma_5x_F\,.\end{aligned}\tag{7}$$

Using the unbroken part of $SU(4)\times SU(4)$, we easily see that there are only three independent F_a parameters; they are $F_{Ai}\equiv F_A, F_B$ and $F_{Ci}=F'_{Ci}=F_{Di}=F'_{Di}=F_E=F'_E=F_F=F'_F\equiv F_C$. (Here $i=1,2,3$.) The quark fields U,D,u,d of definite mass are not necessarily the same as those in the original weak doublets $(1+\gamma_5)(U_1,D_1)$ and $(1+\gamma_5)(U_2,D_2)$. However, we can always put the weak doublet into the form

$$\begin{aligned}&(1+\gamma_5)\begin{pmatrix}u\\d\cos\phi+D\sin\phi\end{pmatrix},\\ &(1+\gamma_5)\begin{pmatrix}U\\-d\sin\phi+D\cos\phi\end{pmatrix}.\end{aligned}\tag{8}$$

The angle ϕ must be determined by minimizing a "potential" $V(\phi)$, given to lowest order in e by the sum of graphs in which W or Z is emitted and absorbed by a strong-interaction vacuum fluctuation.[7] By using the unbroken $SU(2)\times SU(2)\times SU(2)\times U(1)$ subgroup of $SU(4)\times SU(4)$, we easily see that the Z contribution is ϕ independent, while the W contribution is a sum of terms proportional to $\cos^2\phi$ or $\sin^2\phi$. The whole potential therefore has the ϕ dependence $V(\phi)=A+B\cos^2\phi$. Thus, depending on the sign of B, the angle ϕ at which $V(\phi)$ is a minimum must take the values $\phi=\pi/2$ or $\phi=0$. Let us consider these two cases in turn:

(a) $\phi=\pi/2$: The gauge generators (in a U,D,u,d, basis) here take the form

$$\begin{aligned}t_1&=\tfrac{1}{4}g(1+\gamma_5)\begin{pmatrix}0&-i\tau_2\\i\tau_2&0\end{pmatrix},\\ t_2&=\tfrac{1}{4}g(1+\gamma_5)\begin{pmatrix}0&i\tau_1\\-i\tau_1&0\end{pmatrix},\\ t_3&=\tfrac{1}{4}g(1+\gamma_5)\begin{pmatrix}\tau_3&0\\0&\tau_3\end{pmatrix},\\ t_0&=-g'\left\{\tfrac{1}{4}(1-\gamma_5)\begin{pmatrix}\tau_3&0\\0&\tau_3\end{pmatrix}+\tfrac{1}{6}\right\}.\end{aligned}\tag{9}$$

From Eq. (1), we find that the nonvanishing elements of the intermediate-vector-boson mass matrix are

$$\mu^2_{11}=\mu^2_{22}=\tfrac{1}{4}F_C{}^2g^2\,,$$
$$\mu^2_{33}=\tfrac{1}{16}F_A{}^2g^2\,,$$
$$\mu^2_{30}=\mu^2_{03}=\tfrac{1}{16}F_A{}^2gg'\,,\qquad(10)$$
$$\mu^2_{00}=\tfrac{1}{16}F_A{}^2g'^2\,.$$

The nonvanishing intermediate-vector-boson masses are then

$$M_W{}^2=\tfrac{1}{4}F_C{}^2g^2,\quad M_Z{}^2=\tfrac{1}{16}F_A{}^2(g^2+g'^2)\,.\qquad(11)$$

Hence no simple mass relation arises in this case.

(b) $\phi=0$: The gauge generators (in a $U,D,\ u,d$ basis) here take the form

$$t_i=\tfrac{1}{4}g(1+\gamma_5)\begin{pmatrix}\tau_i & 0\\ 0 & \tau_i\end{pmatrix},$$
$$t_0=-g'\left\{\tfrac{1}{4}(1-\gamma_5)\begin{pmatrix}\tau_3 & 0\\ 0 & \tau_3\end{pmatrix}+\tfrac{1}{6}\right\}.\qquad(12)$$

From Eq. (1), we find the nonvanishing elements of the intermediate-vector-boson mass matrix are

$$\mu^2_{11}=\mu^2_{22}=\mu^2_{33}=\tfrac{1}{16}F_A{}^2g^2\,,$$
$$\mu^2_{30}=\mu^2_{03}=\tfrac{1}{16}F_A{}^2gg',\quad \mu^2_{00}=\tfrac{1}{16}F_A{}^2g'^2,\qquad(13)$$

so the masses are

$$M_W{}^2=\tfrac{1}{16}F_A{}^2g^2,\quad M_Z{}^2=\tfrac{1}{16}F_A{}^2(g^2+g'^2)\qquad(14)$$

and have the same ratio $M_Z/M_W=\sec\theta$ as in the simpler two-flavor model. The reason for this is just that the only Goldstone bosons which mix with W and Z are those associated with $\vec{x}_A$, and the unbroken subgroup of SU(4)×SU(4) requires these to have equal F_a values.

This model is still far from realistic, whether $\phi=\pi/2$ or $\phi=0$. In both cases, the u and d quarks remain massless to all orders in e. In addition for $\phi=\pi/2$, the two light quarks u,d are in SU(2) × U(1) doublets with D and U, not with each other. Furthermore, in both cases the model contains 17 physical Goldstone bosons, some of them "true" Goldstone bosons of zero mass. It is not clear how the light quarks could get reasonable masses or how the "true" Goldstone bosons could be eliminated with a dynamical symmetry breaking.

Note added in proof. After this paper was submitted for publication, I received a report by L. Susskind, which deals with similar questions. [The motivation in his paper for dynamical symmetry breaking, in terms of grand unified gauge theories, is the same as that described by H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).] The undiscovered new strong interaction of Susskind is a special case of what was called an "extra strong" interaction in Ref. 1, and a "superstrong" interaction by S. Weinberg, Phys. Today 30 (No. 4), 42 (1977). Susskind independently observes that the relation $M_Z/M_W=\sec\theta$ follows if dynamical symmetry breaking leaves an "isospin" subgroup unbroken. In addition, he points out that the origin for this relation is essentially the same as that for symmetry breaking by vacuum expectation values of scalar doublets. In the latter case, the part of the Lagrangian which is relevant to the calculation of gauge boson masses is the "kinematic" Lagrangian $\mathcal{L}_\phi=-\tfrac{1}{2}\sum_n(D_\mu\phi_n)^\dagger(D_\mu\phi_n)$, the sum running over N scalar doublets (ϕ_n^0,ϕ_n^-). By setting $\phi_n^0=\phi_{n1}+i\phi_{n2}$, $\phi_n^-=\phi_{n3}+i\phi_{n4}$, one finds that in the limit $e=0$, $\mathcal{L}_\phi$ has an $O(4)^N=[SU(2)\times SU(2)]^N$ symmetry, with an O(3) = SU(2) subgroup which is automatically left unbroken by the vacuum expectation values of the ϕ_{n1} fields, and which transforms the weak SU(2) generators as a three-vector. As shown both here and in Susskind's paper, this is the same feature that allows one to derive the Z-W mass ratio in the case of dynamical symmetry breaking.

I also wish to comment here on the problems of developing a grand unified theory of strong as well as weak and electromagnetic interactions in which the spontaneous symmetry breaking at all levels is due to vacuum expectation values of elementary scalar fields. As is well known, such theories require constraints on the parameters in the Lagrangian. However, it is *not* true that these constraints necessarily incorporate extremely small parameters, such as 10^{-19}, or that they need to involve quadratic divergences at all. It is only necessary to suppose that at a stationary point of the potential where the SU(3) × SU(2) ×U(1) subgroup is unbroken, some of the non-Goldstone eigenvalues of the scalar mass matrix vanish. Nonperturbative effects will then produce a minimum of the potential very near this stationary point, at which SU(2) × U(1) is spontaneously broken, with W and Z masses which are automatically less than the superheavy gauge boson masses by a factor $\exp(-C/e^2)$, where C is a numerical constant of order unity. (Also, any quadratic divergences are always an artifact of the cutoff procedure; they do not appear if we use dimensional regularization.) These matters are discussed by E. Gildener and S. Weinberg, Phys.

Rev. D 13, 3333 (1976), Sec. VI, and in a paper now in preparation.

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[1]S. Weinberg, Phys. Rev. D 13, 974 (1976).

[2]Dynamical mechanisms for spontaneous symmetry breaking were first discussed by Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); J. Schwinger, *ibid.* 125, 397 (1962); 128, 2425 (1962). The application to modern gauge theories is due to R. Jackiw and K. Johnson, Phys. Rev. D 8, 2386 (1973); J. M. Cornwall and R. E. Norton, *ibid.* 8, 3338 (1973).

[3]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Physics: Relativistic Groups and Analyticity*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[4]It is of course understood that the whole theory is also invariant under the global U(1) symmetry associated with fermion number conservation. However, the full global symmetry group for N quark flavors is $SU(N) \times SU(N) \times U(1)$, and not $U(N) \times U(N)$ (as supposed in Ref. 1), because invariance under the *chiral* U(1) symmetry is broken by triangle anomalies in the presence of instantons; see G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976), and references cited therein.

[5]See Ref. 1, Eq. (7.11). The factor $\frac{1}{64}$ appears here because all traces include sums over Dirac indices.

[6]See Ref. 1, Sec. IV.

[7]See Ref. 1, Sec. VI.