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Quark confinement in unusual environments

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We consider the properties of one-dimensional gauge field theories at finite temperatures and densities. The massive Schwinger model in the presence of a uniform charge background is shown to form a Wigner crystal which Debye screens charged impurities. The two-species Schwinger model with oppositely charged fermions is studied at finite baryon density. This system does not undergo a phase transition as the density is increased, but becomes progressively more polarizable until at infinite density Debye screening occurs. Finally we consider the massive Schwinger model at nonzero temperature and show that Debye screening occurs at infinite temperature. We speculate that in three dimensions this last transition occurs at finite temperature.

I. INTRODUCTION

Quantum field theory is usually studied at zero temperature and fermion number density. However, a good deal of interesting physics occurs in environments of extreme temperature and density. Physics at temperatures and densities corresponding to the masses of electron and positron pairs can be understood in the framework of perturbative quantum electrodynamics. However, it is believed that when the temperatures and densities increase to the point where the average energy density becomes comparable to that of hadronic matter new phenomena occur. In particular, a number of phase transitions have been conjectured as the density of matter in a neutron star is increased. These phenomena include pion condensation,[1] abnormal nuclear matter,[2] and the transition to a state of free quarks[3] at high density and/or high temperatures.

At these extreme conditions it is probably important to describe matter in terms of quark constituents which interact through forces which at zero temperature and densities can account for confinement. For these reasons we will study the high-density and high-temperature limits of one-dimensional theories which display confinement. In particular we will consider the Schwinger model with one or two massive fermion species. We would rather, of course, study quantum chromodynamics in three dimensions, but we do not have sufficient tools to do that at this time. We hope that the questions we pose and some of the phenomena we find in the one-dimensional models will have their analogs in higher dimensions. This will be discussed further in the text.

This article consists of six sections. In Sec. II we review the properties of the massless and mas-massive Schwinger models. The massless model behaves as a plasma which can Debye screen an arbitrary charge while massive model behaves as an insulator. In the third section we consider the massive Schwinger model in the presence of a uniform background charge density. We find that an opposing, nonuniform charge density is induced. In fact, the induced charged fermions form a Wigner crystal. In Sec. IV we consider the two-species Schwinger model and introduce a chemical potential to control the particle density. At a finite chemical potential a phase transition occurs and the ground-state density of particles becomes nonzero. However, the theory continues to confine its fermions. Only as the density goes to infinity does the dielectric polarizability tend to infinity. Therefore, formally speaking, a transition to an unconfined phase occurs at infinite density. Equivalently, the long-range confining force between static sources vanishes smoothly as the density goes to infinity. Finally, in Sec. V we consider the massive Schwinger model at high temperatures. The system again undergoes a transition to a plasma phase at $T=\infty$. We speculate that in the real three-dimensional world, this transition would occur at finite temperatures. Some concluding remarks and discussion appear in Sec. VI.

II. USEFUL PROPERTIES AND FORMULAS OF THE SCHWINGER MODEL

The massive Schwinger model[4] is defined by the (1+1)-dimensional Lagrangian density

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}{}^2 + \bar{\psi}\gamma^\mu(\partial_\mu - igA_\mu)\psi - m\bar{\psi}\psi, \qquad (2.1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and m is the bare mass of the fermion. The properties of this model include the following:

1. The spectrum consists of neutral, massive bosons.[5]

2. If sources of charge $\pm\epsilon g$ are embedded in the vacuum at separation distance L, a confining linear potential occurs,[6]

$$V(L,\epsilon) = \epsilon^2 g^2 f(\epsilon, m/g)L. \qquad (2.2)$$

For ϵ = integer the sources are neutralized by the creation of pairs whose members then bind to the sources. This phenomenon causes f to be a periodic function of ϵ which vanishes whenever $\epsilon = 0, \pm1, \pm2, \ldots$. This behavior can be summarized for small electric fields E by saying that the vacuum is a dielectric with a field-dependent dielectric constant $1/f(E/g, m/g)$.

3. For the special case $m=0$ the long-range force is Debye screened. This means that $\lim_{m\to 0} f(\epsilon, m/g) = 0$ so that long-range confining forces are now absent. In other words, long-range forces do not occur whatever the charges $\pm\epsilon g$ of the sources. In this case the vacuum is referred to as a "plasma" or "conductor".

The behaviors summarized in properties 2 and 3 give us a practical method of labeling a general theory as either "confining" or "plasma".

These properties of the Schwinger model are most easily obtained by studying the Bose form of the theory. Writing the equivalent Bose form of the theory in the Coulomb gauge consists of the following correspondences[7,8]:

$$:\bar{\psi}\psi: \to -cmN_m\cos(2\sqrt{\pi}\phi), \qquad (2.3a)$$

$$:\bar{\psi}\gamma_5\psi: \to -cmN_m\sin(2\sqrt{\pi}\phi), \qquad (2.3b)$$

$$j_\mu = :\bar{\psi}\gamma_\mu\psi: \to \frac{1}{\sqrt{\pi}}\epsilon_{\mu\nu}\partial^\nu\phi, \qquad (2.3c)$$

$$:\bar{\psi}i\partial\!\!\!/\psi: \to \tfrac{1}{2}N_m(\partial_\mu\phi)^2, \qquad (2.3d)$$

where N_m denotes normal ordering with respect to the fermion mass m and c is a numerical constant. Now the Hamiltonian in the Coulomb gauge

$$\mathcal{H} = \int \bar{\psi}(i\gamma_1\partial_1 + m)\psi dx - \frac{g^2}{4}\int\int j_0(x)\,|x-y|\,j_0(y)dx\,dy \qquad (2.4a)$$

becomes

$$\mathcal{H} = N_m\left\{\int [\tfrac{1}{2}\pi^2(x) + (\partial_1\phi)^2 - cm^2\cos(2\sqrt{\pi}\phi)]dx - \frac{g^2}{4\pi}\int\int \partial_1\phi(x)\,|x-y|\,\partial_1\phi(y)dx\,dy\right\}. \qquad (2.4b)$$

Finally, doing two integrations by parts, the Bose form of the Hamiltonian density becomes

$$\mathcal{H} = N_m\left[\tfrac{1}{2}\pi^2 + (\partial_1\phi)^2 + \frac{g^2}{2\pi}\phi^2 - cm^2\cos(2\sqrt{\pi}\phi)\right], \qquad (2.4c)$$

where we have ignored surface effects. Accounting for nonvanishing boundary conditions for ϕ is equivalent to placing charges of fractions $\pm\epsilon$ at spatial $\pm\infty$, respectively.[5,7] Then Eq. (2.4c) becomes

$$\mathcal{H} = N_m\left[\tfrac{1}{2}\pi^2 + \tfrac{1}{2}(\partial_1\phi)^2 + \frac{g^2}{2\pi}(\phi + \sqrt{\pi}\epsilon)^2 - cm^2\cos(2\sqrt{\pi}\phi)\right] \qquad (2.5a)$$

or, shifting the field $\phi \to \phi - \sqrt{\pi}\epsilon$,

$$\mathcal{H} = N_n\left[\tfrac{1}{2}\pi^2 + \tfrac{1}{2}(\partial_1\phi)^2 + \frac{g^2}{2\pi}\phi^2 - cm^2\cos(2\sqrt{\pi}\phi - 2\pi\epsilon)\right]. \qquad (2.5b)$$

Normal ordering with respect to the fermion mass m is often inconvenient. Aside from the appearance of additive constants, different normal-ordering prescriptions of $\mathcal{H}$ can be absorbed into redefinitions of the coefficient of the cosine term.[8] The prescription is

$$N_m\cos(\beta\phi) = \left(\frac{M}{m}\right)^{\beta^2/4\pi} N_M\cos(\beta\phi). \qquad (2.6)$$

Choosing $M = g/\sqrt{\pi}$, Eq. (2.5b) becomes

$$\mathcal{H} = N_M[\tfrac{1}{2}\pi^2 + \tfrac{1}{2}(\partial_1\phi)^2 + \tfrac{1}{2}M^2\phi^2 - cmM\cos(2\sqrt{\pi}\phi - 2\pi\epsilon)]. \qquad (2.7)$$

Much of the physics of the massive Schwinger model is very accessible when $\mathcal{H}$ is written in this form. For example, one can easily confirm property (2) listed above Eq. (2.7) and its generalization to the two-species model will play a central role in the following sections of this article.

III. THE MASSIVE SCHWINGER MODEL IN A BACKGROUND CHARGE DENSITY

As our first example we consider the Schwinger model in equilibrium with a uniform charged background. An external charge distribution $\rho_c(z)$ can be incorporated into the Bose Coulomb gauge formalism of Sec. II by replacing $j_0(z)$ in Eq. (2.4a) by $j_0(z) + \rho_c(z)$,

$$\mathcal{H}=\int \psi(i\gamma^1\partial_1+m)\psi dx - \tfrac{1}{4}g^2\int [j_0(x)+\rho_c(x)]\,|x-y|\,[j_0(y)+\rho_c(y)]dx\,dy. \tag{3.1}$$

If we define ϕ_c as

$$\rho_c=\frac{1}{\sqrt{\pi}}\,\partial_1\phi_c, \tag{3.2}$$

then the Bose form of the Hamiltonian density becomes

$$\mathcal{H}=N_M[\tfrac{1}{2}\pi^2+\tfrac{1}{2}(\partial_1\phi)^2+M^2(\phi+\phi_c)^2 - cmM\cos(2\sqrt{\pi}\phi)] \tag{3.3}$$

for the $\epsilon=0$ case which we study first.

A uniform charge background should be thought of as the limit of a finite line of charge extending from $-L$ to $+L$. If the region is chosen symmetrically relative to the origin, then

$$\phi_c=az \tag{3.4}$$

for $|z|<L$ and ϕ_c is zero elsewhere. The Hamiltonian density then becomes

$$\mathcal{H}=N_M[\tfrac{1}{2}\pi^2+\tfrac{1}{2}(\partial_1\phi)^2+\tfrac{1}{2}M^2(\phi+az)^2 - cmM\cos(2\sqrt{\pi}\phi)] \tag{3.5}$$

for $|z|<L$. It is convenient to define a new field $\tilde{\phi}=\phi+az$. Then Eq. (3.5) becomes

$$\mathcal{H}=N_M[\tfrac{1}{2}\pi^2+\tfrac{1}{2}(\partial_1\tilde{\phi})^2+\tfrac{1}{2}M^2\tilde{\phi}^2 - cmM\cos 2\sqrt{\pi}(\tilde{\phi}-az)], \tag{3.6}$$

where we have dropped constant terms coming from the kinetic energy. In this form the Hamiltonian density does not have translational invariance. The source of this asymmetry is the positioning of the center of the charge distribution. Note, however, that $\mathcal{H}$ is invariant under the discrete translation $z\to z+\sqrt{\pi}/a$. The equation of motion following from Eq. (3.6),

$$\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)\tilde{\phi}+M^2\tilde{\phi}+2\sqrt{\pi}cmM\sin 2\sqrt{\pi}(\tilde{\phi}-az)=0, \tag{3.7}$$

has nontrivial static solutions which also only have the discrete symmetry $z\to z+\sqrt{\pi}/a$. In particular, the state of minimum energy is such a nontrivial periodic solution of this equation. As will be discussed further below, these solutions consist of periodic waves of charge density. Their appearance is not surprising since they have a classical physics analog. In 1937 Wigner considered the possibility that the lowest energy state of a neutral system of charges in a uniform background charge density would be an ordered, crystalline state.[9] He argued, in fact, that at sufficiently low temperatures such "Wigner crystals" would indeed form. Before discussing the stability of such crystals in our (1+1)-dimensional world we still study the classical equations in more detail. It is convenient to rescale the space-time variables (z,t) and define

$$y=az,\quad t=at. \tag{3.8}$$

Then Eq. (3.7) becomes

$$\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial y^2}\right)\tilde{\phi}+\frac{M^2}{a^2}\tilde{\phi}+2\sqrt{\pi}c\frac{mM}{a^2}\sin 2\sqrt{\pi}(\tilde{\phi}-y)=0. \tag{3.9}$$

Consider the low-density case in which "a" is small and consider just the potential energy pieces of Eq. (3.9)

$$\frac{M^2}{a^2}\tilde{\phi}+2\sqrt{\pi}c\frac{mM}{a^2}\sin 2\sqrt{\pi}(\tilde{\phi}-y)\approx 0. \tag{3.10}$$

The solution to this equation can be determined graphically and is shown in Fig. 1 for the case $m/g\ll 1$. To good approximation,

$$\tilde{\phi}\approx 2\sqrt{\pi}c\frac{m}{M}\sin(2\sqrt{\pi}y), \tag{3.11a}$$

so

$$\phi\approx 2\sqrt{\pi}c\frac{m}{M}\sin(2\sqrt{\pi}az)-az, \tag{3.11b}$$

which generates an induced charge density

$$\frac{1}{\sqrt{\pi}}\partial_1\phi=4\sqrt{\pi}ac\frac{m}{M}\cos(2\sqrt{\pi}az)-\frac{a}{\sqrt{\pi}}. \tag{3.12}$$

The ripples in Eq. (3.12) indicate the crystal structure alluded to above. Next consider the case $m/g\gg 1$. Then the induced charge density consists of narrow spikes, periodically spaced. This is a reasonable result, since in this limit one should recover the classical Wigner crystal.

Do quantum fluctuations destroy the classical Wigner crystal? One might expect an affirmative

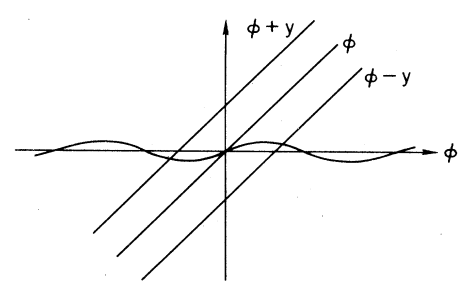

FIG. 1. A graphical solution to Eq. (3.10). The abscissa is $\phi=\tilde{\phi}-y$ and the curves show the two terms of Eq. (3.10) for different values of y.

answer in view of the general theorems which state that continuous symmetries (such as translation invariance) cannot be spontaneously broken in one dimension.[10] However, these theorems do not apply to the Wigner crystal because it is held together by long-range Coulomb forces. The stability of the crystal can be tested explicitly in perturbation theory. For small m/g one can treat the cosine term in Eq. (3.6) as a perturbation on a *massive* free field. An explicit evaluation of the lowest-order quantum correction to the vacuum expectation of $\tilde{\phi}$ gives

$$\langle\tilde{\phi}(z,t)\rangle = 2\sqrt{\pi}cmM\int \sin(2\sqrt{\pi}az') \times \Delta_F(z-z',t-t';M)dz'dt', \tag{3.13}$$

corresponding to the graph in Fig. 2. Equation (3.13) is a well-defined, finite periodic function. The lack of infrared problems in the perturbation series indicates that long-wavelength fluctuations do not disorder the system. For large m/g the classical analysis is reliable and it also indicates the existence of the ordered crystal.

Next we wish to understand the confinement properties of the theory with a given background charge density. Is there a long-range confining potential between two widely separated static charges $\pm\epsilon g$? The external charges may be described by an extra contribution to the background charge density, $\epsilon g\,\delta(x-L)-\epsilon g\,\delta(x+L)$. Corresponding to this $j_0(x,t)$, one must add to ϕ a field which has the constant value $\sqrt{\pi}\epsilon$ between the two sources and is zero elsewhere. The coefficient of the long-range potential then equals the change in the energy per unit volume in the region between the two sources. Equation (3.6) now becomes

$$\mathcal{H}=N_M[\tfrac{1}{2}\pi^2+\tfrac{1}{2}(\partial_1\phi)^2+\tfrac{1}{2}M^2(\phi+az+\sqrt{\pi}\epsilon)^2 - cmM\cos(2\sqrt{\pi}\phi)]. \tag{3.14}$$

Shifting the field $\phi\rightarrow\phi+az+\sqrt{\pi}\epsilon$, we have

$$\mathcal{H}=N_M[\tfrac{1}{2}\pi^2+\tfrac{1}{2}(\partial_1\phi)^2+\tfrac{1}{2}M^2\phi^2 - cmM\cos 2\sqrt{\pi}(\phi-az-\sqrt{\pi}\epsilon)].$$

Observe that *any* nonzero ϵ can be absorbed into a shift in the position of the Wigner crystal and therefore adds no volume-dependent energy to the system. So, the Wigner crystal causes screening—the system behaves like a plasma, not a dielectric.

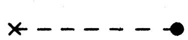

FIG. 2. Lowest-order graph contributing to the vacuum expectation value of $\tilde{\phi}_+$. The dashed line is a boson propagator.

IV. THE TWO-SPECIES SCHWINGER MODEL AT FINITE DENSITY

As our next example we consider the two-species Schwinger model in which the two species have equal mass and opposite charge. This model allows us to study a system at nonzero baryon number density but vanishing background charge density. Therefore, we have a one-dimensional analog of the environment of colorless nuclear matter as might exist in real neutron stars, say.

Again it is convenient to recast this theory into an equivalent theory of Bose fields.[11] We associate a Bose field ϕ_i with each fermion ψ_i $(i=1,2)$ and take over the results reviewed in Sec. II for each species. It is useful to introduce fields ϕ_+ and ϕ_- defined by

$$\phi_\pm=\frac{1}{\sqrt{2}}(\phi_1\pm\phi_2). \tag{4.1}$$

Then the total charge density is

$$\rho=\frac{1}{\sqrt{\pi}}\partial_1(\phi_1-\phi_2)=\frac{2}{\sqrt{\pi}}\partial_1\phi_- \tag{4.2}$$

and the baryon density is

$$B=\frac{1}{\sqrt{\pi}}\partial_1(\phi_1+\phi_2)=\frac{2}{\sqrt{\pi}}\partial_1\phi_+. \tag{4.3}$$

The Bose form of the Hamiltonian density reads

$$\mathcal{H}=N_m\left[\tfrac{1}{2}\pi_1^2+\tfrac{1}{2}\pi_2^2+\tfrac{1}{2}(\partial_1\phi_1)^2+\tfrac{1}{2}(\partial_1\phi_2)^2+\frac{g^2}{2\pi}(\phi_1-\phi_2)^2 - cm^2\cos(2\sqrt{\pi}\phi_1)-cm^2\cos(2\sqrt{\pi}\phi_2)\right]. \tag{4.4}$$

In terms of ϕ_+ and ϕ_- this becomes

$$\mathcal{H}=N_m[\tfrac{1}{2}\pi_-^2+\tfrac{1}{2}(\partial_1\phi_-)^2+M^2\phi_-^2+\tfrac{1}{2}\pi_+^2+\tfrac{1}{2}(\partial_1\phi_+)^2 - 2cm^2\cos(\sqrt{\pi}\phi_-)\cos(\sqrt{\pi}\phi_+)] \tag{4.5}$$

in the case $\epsilon=0$ which we study first.

In some of our detailed analyses we will consider this theory only in the limiting case $M\gg m$. Then one would expect that the low-energy behavior of the theory would be described by Eq. (4.5) by setting ϕ_- to zero and adjusting the coupling constant appropriately. The necessary rescaling of the coupling constant (the coefficient of the cosine term) has been computed in Ref. 11 with the result that

$$\mathcal{H}\approx N_m\left[\tfrac{1}{2}\pi_+^2+\tfrac{1}{2}(\partial_1\phi_+)^2-2cm^2\left(\frac{\sqrt{2}M}{m}\right)^{1/2}\cos(\sqrt{2\pi}\phi_+)\right] \tag{4.6}$$

if $M\gg m$.

We wish to study the two-species model of Eq. (4.5) at finite baryon density. To do this we introduce a chemical potential μ and add to the Hamiltonian the term

$$\mu \int B(z)dz = \mu\left(\frac{2}{\pi}\right)^{1/2} \int \partial_1 \phi_+ dz. \tag{4.7}$$

To avoid ambiguities we let the chemical potential be nonzero between $-L$ and $+L$ and set it to zero elsewhere. Now the Hamiltonian becomes

$$\mathcal{H} = + \mu\left(\frac{2}{\pi}\right)^{1/2} [\phi_+(+L) - \phi_+(-L)]. \tag{4.8}$$

Next define $\phi_+ = \tilde{\phi}_+ + bz$. This separation evidently produces a constant background Baryon density $(2/\pi)^{1/2}b$. Substituting into Eq. (4.5) we have

$$\mathcal{H} = N_m \int [\tfrac{1}{2}\pi_-^2 + \tfrac{1}{2}(\partial_1\phi_-)^2 + M^2\phi_-^2 + \tfrac{1}{2}\pi_+^2 + \tfrac{1}{2}(\partial_1\tilde{\phi}_+ + b)^2 - 2cm^2\cos(\sqrt{2\pi}\phi_-)\cos\sqrt{2\pi}(\tilde{\phi}_+ + bz)]dz + \mu\left(\frac{2}{\pi}\right)^{1/2}[\phi_+(L) - \phi_+(-L)]. \tag{4.9}$$

So,

$$\mathcal{H} = N_m \int [\tfrac{1}{2}\pi_-^2 + \tfrac{1}{2}(\partial_1\phi_-)^2 + M^2\phi_-^2 + \tfrac{1}{2}\pi_+^2 + \tfrac{1}{2}(\partial_1\tilde{\phi}_+)^2 - 2cm^2\cos(\sqrt{2\pi}\phi_-)\cos\sqrt{2\pi}(\tilde{\phi}_+ + bz)]dz + \tfrac{1}{2}b[\tilde{\phi}_+(L) - \tilde{\phi}_+(-L)] + \mu\frac{2}{\sqrt{\pi}}[\tilde{\phi}_+(L) - \tilde{\phi}_+(-L)], \tag{4.10}$$

dropping several irrelevant constant terms. We now define $b = -2\mu 2/\sqrt{\pi}$ to eliminate the last two terms.

Next we wish to minimize the classical energy and find the system's ground state. Consider two possible configurations for $\tilde{\phi}_+$ and ϕ_-:

1. $\tilde{\phi}_+ \approx -bz$ and $\phi_- = 0$. Then the average baryon density is zero.

2. $\tilde{\phi}_+ \approx 0$ and $\phi_- = 0$. Then the average baryon density is approximately $(2/\sqrt{\pi})b$.

Which of these two configurations is energetically favored? We cannot solve the classical equations analytically, so consider only the case $M \gg m$. Then the $M^2\phi_-^2$ term in Eq. (4.10) forces ϕ_- to be small and we set it to zero and adjust the coefficient of the cosine term as explained above $[c' \equiv c(\sqrt{2}M/m)^{1/2}]$,

$$\mathcal{H} = N_m \int [\tfrac{1}{2}\pi_+^2 + \tfrac{1}{2}(\partial_1\tilde{\phi}_+)^2 - 2c'm^2\cos\sqrt{2\pi}(\tilde{\phi}_+ + bz)]dz. \tag{4.11}$$

Now we consider only static fields and compare the energies of cases 1 and 2 listed above. For the first case $\tilde{\phi}_+ \approx -bz$,

$$\mathcal{H}(\text{static}) \approx + b^2L - 4cm^2L. \tag{4.12a}$$

For the second case $\tilde{\phi}_+ \approx 0$ and

$$\mathcal{H}(\text{static}) \approx 0. \tag{4.12b}$$

Evidently, for $4cm^2 > b^2$, case 1 is favored and the ground state has zero average baryon density. However, if $4cm^2 < b^2$, the ground state will have nonvanishing baryon density. Since we have not made an exhaustive search for the optimal $\tilde{\phi}_+$ it is not clear whether this transition is abrupt or continuous. Luckily it is not difficult to understand the physical origin of this transition and argue that it is in fact a continuous function of b.

To do so we consider a simpler theory—free, massive fermions in the presence of a background fermion number density. We may write this theory using the Bose correspondences as a sine-Gordon equation in the presence of a chemical potential

$$\mathcal{H} = N_m \int [\tfrac{1}{2}\pi^2 + \tfrac{1}{2}(\partial_1\phi)^2 - cm^2\cos(2\sqrt{\pi}\phi)]dz + b\int \partial_1\phi dz. \tag{4.13}$$

The chemical potential term can be integrated to give an energy proportional to the total charge. In terms of fermion degrees of freedom the Hamiltonian is

$$\mathcal{H} = \int a_k^\dagger a_k[(k^2 + m^2)^{1/2} + b]dk + \int b_k^\dagger b_k[(k^2 + m^2)^{1/2} - b]dk. \tag{4.14}$$

Now consider the possible phases of this theory as b is varied. For small b it is not energetically favorable to populate any mode of the fermion field, but when $b > m$ it becomes favorable to populate antiparticle states for which $(k^2 + m^2)^{1/2} < b$. From this observation we can calculate the particle density in the ground state. All the modes are populated up to the "Fermi surface" $k_F = (b^2 - m^2)^{1/2}$. But the connection between particle density and k_F in one dimension is simply $\rho = (1/2\pi)k_F$, so the ground-state density satisfies the equation $b^2 - 4\pi^2\rho^2 = m^2$. This equation is plotted in Fig. 3

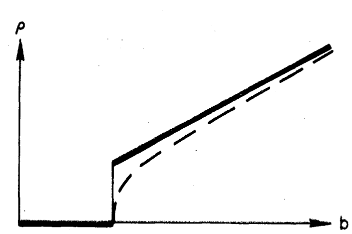

FIG. 3. Ground-state baryon density vs chemical potential. The dashed line applies to the theory of free massive fermions. The broken dark curves depict cases 1 and 2 of the two-species Schwinger model.

along with cases 1 and 2 considered above. Note the general agreement between the various curves. Therefore, we feel that the real transition in the two-species model is continuous and our crude estimates given by cases 1 and 2 are reliable for b everywhere except in the immediate neighborhood of the transition point. In that region a continuous curve resembling that in the figure is probably correct.

Now we return to the equation of motion following from Eq. (4.11),

$$\Box\tilde{\phi}_+ + 2\sqrt{2\pi}c'm^2 \sin\sqrt{2\pi}(\tilde{\phi}_+ + bz) = 0. \tag{4.15}$$

A classical solution of Eq. (4.15) would have periodic solutions indicating the possibility of a crystalline structure. However, quantum fluctuations are sure to destroy this ordered phase because the perturbation is about a *massless* field and in 1+1 dimensions there are definitely divergences here.[10] Indeed, the approximate description in which $\phi_- = 0$ is equivalent to the massive Thirring model in the presence of a chemical potential. Since this model has no long-range forces, the existence of a crystal will violate the theorems on spontaneous order in (1+1)-dimensional systems.[10]

We now consider this model in the presence of a pair of fractional charges $\pm\epsilon g$ separated by a distance $2L$. Following the logic of Sec. III we must modify the Hamiltonian to be

$$\begin{aligned}\mathcal{H} = N_m \int \{&\tfrac{1}{2}\pi_-^2 + \tfrac{1}{2}(\partial_1\phi_-)^2 + M^2\phi_-^2 + \tfrac{1}{2}(\partial_1\tilde{\phi}_+)^2 \\ &- 2cm^2 \cos\sqrt{2\pi}[\phi_- + (\pi/2)^{1/2}\epsilon] \\ &\cos\sqrt{2\pi}(\tilde{\phi}_+ + bz)\}dz.\end{aligned} \tag{4.16}$$

Again choosing $M \gg m$, the ϕ_- degree of freedom is frozen out and we can place $\phi_- = 0$ in $\mathcal{H}$,

$$\begin{aligned}\mathcal{H} = N_m \int [&\tfrac{1}{2}\pi_+^2 + \tfrac{1}{2}(\partial_1\tilde{\phi}_+)^2 \\ &- 2c'm^2 \cos(\pi\epsilon) \cos\sqrt{2\pi}(\tilde{\phi}_+ + bz)]dz.\end{aligned} \tag{4.17}$$

To study confinement we must compute the ground-state energy of Eq. (4.17). This is done in the Appendix for large densities, i.e., $M/b \ll 1$. There we determine that the vacuum energy density shifts by an amount proportional to $(m^2M/b)\cos^2(\pi\epsilon)$. The string tension is then

$$\text{tension} = \text{const} \times m^2 \frac{M}{b} \sin^2(\pi\epsilon). \tag{4.18}$$

Note that as the background density increases a larger and larger percentage of the external charge is screened away by polarization. However, only at infinite density does the screening become complete. This behavior contrasts sharply with quantum electrodynamics where a plasma occurs for arbitrarily small density.

V. THE MASSIVE SCHWINGER MODEL AT FINITE TEMPERATURE

It is interesting to ask whether theories of confinement pass to an unconfined phase as they are heated. This is known to be true of lattice quantum chromodynamics in 3+1 dimensions.[12] Here we will compute the temperature dependence of the string tension in the massive Schwinger model. General theorems concerning the absence of spontaneous symmetry breaking in theories of one spatial dimension prohibit us from finding a transition to an unconfined phase at a finite temperature.[10] Instead, we shall find that the string tension vanishes smoothly as T, the temperature, goes to infinity. We believe that it is reasonable to expect this transition to appear at a finite value of kT (k is Boltzmann's constant)—probabily on the order of a GeV—in theories of confinement in 3+1 dimensions.[12]

To begin we review the calculation of the string tension in the theory at $T=0$. We will do this using the Hamiltonian without any normal ordering because then the generalization to finite T is most elementary. From the discussion of Sec. II we anticipate that the coefficient of the cosine term in the Hamiltonian will then be renormalized order by order in perturbation theory. So, we will write

$$\mathcal{H} = \tfrac{1}{2}\pi^2 + \tfrac{1}{2}(\partial_1\phi)^2 + \tfrac{1}{2}M^2\phi^2 - mK\cos(2\sqrt{\pi}\phi - 2\pi\epsilon) \tag{5.1}$$

and adjust K to keep the energy density finite order by order in perturbation theory. We wish to calculate the shift in the ground-state energy between the charges $\pm\epsilon g$ which are located at points $\pm L$, respectively. This calculation yields the static potential acting between the charges. The calculation will be done to first order in the fermion mass m. Therefore, we need the vacuum expectation value of $\cos(2\sqrt{\pi}\phi - 2\pi\epsilon)$,

$$\begin{aligned}\langle\cos(2\sqrt{\pi}\phi - 2\pi\epsilon)\rangle &= \cos(2\pi\epsilon)\langle\cos 2\sqrt{\pi}\phi\rangle + \sin(2\pi\epsilon)\langle\sin 2\sqrt{\pi}\phi\rangle = \cos(2\pi\epsilon)\langle\cos 2\sqrt{\pi}\phi\rangle \\ &= \cos(2\pi\epsilon)\sum_{n=0,1,\ldots}\frac{(i)^{2n}}{(2n)!}(2\sqrt{\pi})^{2n}\langle\phi^{2n}\rangle.\end{aligned} \tag{5.2}$$

FIG. 4. Feynman graph depicting the vacuum expectation value of the square of a boson field.

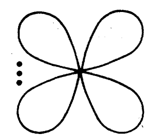

FIG. 5. Feynman graph for $\langle\phi^{2m}\rangle$.

Consider the first nontrivial contribution to the right-hand side of Eq. (5.2),

$$\langle\phi^2\rangle = \int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2+M^2}\,. \tag{5.3}$$

Define this integral to be I. The corresponding Feynman graph appears in Fig. 4. It is ultraviolet divergent, so we cut it off at momentum Λ,

$$I = \frac{1}{2\pi}\ln(\Lambda/M)\,. \tag{5.4}$$

It is easy to see that the general term in Eq. (5.2) contains a factor of

$$\langle\phi^{2n}\rangle = (2n-1)!!\,I^n, \tag{5.5}$$

where the $(2n-1)!!$ simply counts the number of ways the contradictions can be accomplished to produce a graph of the topological structure shown in Fig. 5. Now the sum in Eq. (5.2) can be done

$$\begin{aligned}\sum_n \frac{(i)^{2n}}{(2n)!}(2\sqrt{\pi})^{2n}\langle\phi^{2n}\rangle &= \sum (i)^{2n}(2\sqrt{\pi})^{2n}\frac{(2n-1)!!}{(2n)}I^n\\ &= \sum(-1)^n(4\pi)^n\frac{1}{2^n n!}I^n\\ &= \sum\frac{(-1)^n}{n}(2\pi I)^n\\ &= \exp(-2\pi I).\end{aligned} \tag{5.6}$$

So, the shift in the ground-state energy

$$E = -mK\left\langle\int\cos(2\sqrt{\pi}\phi - 2\pi\epsilon)dz\right\rangle \tag{5.7}$$

becomes

$$E = -mK\cos(2\pi\epsilon)e^{-2\pi I}2L\,. \tag{5.8}$$

To obtain the string tension we divide by $2L$ and subtract the energy density in the absence of the external charges

$$\begin{aligned}\text{tension} &= mKe^{-2\pi I}(1-\cos 2\pi\epsilon)\\ &= 2mKe^{-2\pi I}\sin^2(\pi\epsilon).\end{aligned} \tag{5.9}$$

Finally, as discussed above, we adjust K so the combination $Ke^{-2\pi I} = K_F$ is finite as the ultraviolet cutoff $\Lambda\to\infty$. Therefore, our final result is

$$\text{tension} = 2mK_F\sin^2(\pi\epsilon). \tag{5.10}$$

Now we compute the finite-temperature string tension. We will use standard finite-temperature Green's-function methods.[13] The string-tension calculation then runs parallel to the $T=0$ case with finite-temperature Green's functions replacing the $T=0$ propagators. This causes the substitution[13]

$$I \to I_\beta = \frac{1}{\beta}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dk}{(2\pi)}\frac{1}{k^2+M^2+4\pi^2n^2/\beta^2}\,,\quad \beta = \frac{1}{kT} \tag{5.11}$$

in the previous calculation. The finite-temperature analog of Eq. (5.8) becomes

$$E(T) = -mKe^{-2\pi I_\beta}\cos(2\pi\epsilon)2L\,. \tag{5.12}$$

Now we evaluate I_β. Following standard tricks of statistical mechanics,[13] the sum over n is replaced by a contour integral by considering

$$\left(\frac{1}{k^2+M^2+z^2}\right)\left[\tfrac{1}{2}\beta\cot(\tfrac{1}{2}\beta z)\right].$$

This expression useful because the quantity in the square brackets has poles of unit residue at the points $z = 2\pi n/\beta$. Consider the contour integral

$$\oint_\Gamma dz\,\frac{\frac{1}{2}\beta\cot\frac{1}{2}\beta z}{k^2+M^2+z^2}\,,$$

where the contour Γ (shown in Fig. 6) encloses all the singularities of the integrand. Letting Γ recede to infinity in all directions and observing that the integrand vanishes rapidly in all directions, we learn that

$$\oint_{\substack{\text{circle at}\\ \text{infinity}}} dz\,\frac{\frac{1}{2}\beta\cot(\frac{1}{2}\beta z)}{k^2+M^2+z^2} = 0. \tag{5.13}$$

Applying the residue theorem gives

$$\begin{aligned}0 = \sum_n\frac{1}{k^2+M^2+4\pi^2n^2/\beta^2} &- \tfrac{1}{2}\beta\frac{\cot[\frac{1}{2}i\beta(k^2+M^2)^{1/2}]}{2i(k^2+M^2)^{1/2}}\\ &+ \tfrac{1}{2}\beta\frac{\cot[-\frac{1}{2}i\beta(k^2+M^2)^{1/2}]}{2i(k^2+M^2)^{1/2}}\,.\end{aligned} \tag{5.14}$$

Therefore, the sum entering the expression for I_β is

$$\begin{aligned}\sum_n\frac{1}{k^2+M^2+4\pi^2n^2/\beta^2} = \beta\frac{\cot[\frac{1}{2}i\beta(k^2+M^2)^{1/2}]}{2i(k^2+M^2)^{1/2}} &= \frac{\beta}{2(k^2+M^2)^{1/2}}\coth[\tfrac{1}{2}\beta(k^2+M^2)^{1/2}]\\ &= \frac{\beta}{2(k^2+M^2)^{1/2}}\left[1+\frac{2}{\exp[\beta(k^2+M^2)^{1/2}]-1}\right].\end{aligned} \tag{5.15}$$

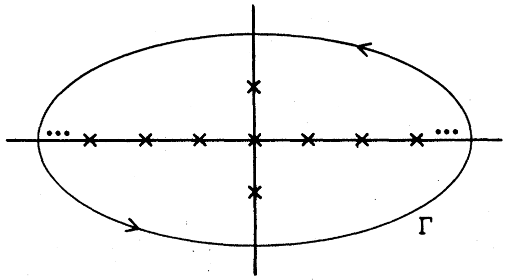

FIG. 6. The contour Γ enclosing the singularities in the finite-temperature version of Fig. 4.

Substituting into Eq. (5.11),

$$I_\beta = \frac{1}{2\pi}\int_0^\infty \frac{dk}{(k^2+M^2)^{1/2}} + \frac{1}{\pi}\int_0^\infty \frac{dk}{(k^2+M^2)^{1/2}}\left[\frac{1}{\exp[\beta(k^2+M^2)^{1/2}]-1}\right], \tag{5.16a}$$

or

$$I_\beta = I + A_\beta\,, \tag{5.16b}$$

where we have identified the expression for I from Eq. (5.3) and have defined the second term in Eq. (5.16b) to be A_β. Now Eq. (5.12) becomes

$$E(T) = -mKe^{-2\pi I}\cos(2\pi\epsilon)e^{-2\pi A_\beta}2L \quad . \tag{5.17a}$$

We identify the quantity $K_F = Ke^{-2\pi I}$ as in the zero-temperature calculation. Then

$$E(T) = -mK_F\cos(2\pi\epsilon)e^{-2\pi A_\beta}2L. \tag{5.17b}$$

All the temperature dependence of $E(T)$ lies in the factor $\exp(-2\pi A_\beta)$. As $T\to 0, A_\beta \to 0$ [at the rate $\exp(-M/kT)$] and we retrieve the zero-temperature result. As $T\to\infty$, we can replace $\exp[\beta(k^2+M^2)^{1/2}]-1$ by $\beta(k^2+M^2)^{1/2}$ in Eq. (5.16a) to obtain the leading asymptotic behavior of A_β

$$A_\beta = \frac{1}{\pi}\int_0^\infty \frac{dk}{(k^2+M^2)^{1/2}}\left[\frac{1}{\exp[\beta(k^2+M^2)^{1/2}]-1}\right] \sim \frac{1}{\pi\beta}\int_0^\infty \frac{dk}{k^2+M^2} \sim \frac{kT}{2M}\ . \tag{5.18}$$

So, in the high-temperature limit

$$E(T) \sim -mK_F\cos(2\pi\epsilon)e^{-\pi kT/M}2L\,, \tag{5.19}$$

giving a string tension

$$\text{tension} \sim 2mK_F e^{-\pi kT/M}\sin^2(\pi\epsilon). \tag{5.20}$$

So, as the temperature increases, the string tension falls to zero continuously. Formally speaking, the system becomes a plasma at infinite temperature.

VI. CONCLUSIONS AND DISCUSSION

To conclude we would like to compare the results of our paper with various speculations concerning field theories in extreme environments in three dimensions. Consider first the possibility of pion condensation[1] in high-density nuclear matter. Pion condensation is the appearance of a macroscopic condensate of the pion field. Because the pion couples to nuclear matter through a p wave, it is thought that the condensate is nonuniform with, in fact, a periodic spatial dependence. It is interesting to compare this phenomenon to the two-species Schwinger model at finite density. The field ϕ_+ is a pseudoscalar field which is massless in the absence of fermion masses and can be taken as an analog of the pion field. In the classical approximation the pseudoscalar field was shown to have periodic spatial variation. Thus, at the classical level, pion condensation in the two-species Schwinger model occurs at an arbitrary nonzero density. However, general theorems assure us that in *one* dimension this spatial inhomogeneity is destroyed by quantum fluctuations.[10] In three dimensions, quantum fluctuations do not necessarily destroy the analogous ordered state. We have seen in the text that the development of periodic inhomogeneities in the field ϕ_+ is related to crystallization. It is interesting to speculate that in real nuclear matter the formation of a pion condensate induces a crystallization of the nuclear matter also.

Another phase transition of interest involves the possible disappearance of confinement at high density.[3] One might speculate that Debye screening occurs in high-density nuclear matter eliminating the color-confining long-range potential. It is not known if this occurs in three dimensions but in one dimension we have seen that a sharp transition of this type does not occur at finite density. To our knowledge no general theorem concerning one-dimensional physics forbids phase transitions as a function of density. Nevertheless the confining forces between colored particles become sufficiently weak at high density that they probably can be ignored in considering, for example, the system's equation of state. In other words, at large density, free fermion physics should govern the equation of state[14] even though confinement may actually persist.

As temperature increases we also did not find a sharp transition to an unconfined phase in the massive Schwinger model. But, in this case, our conclusion follows just from general theorems of one-dimensional statistical mechanics.[10] The fact that the confining forces do, however, disappear as the temperature goes to infinity suggests that in higher dimensions Debye screening should occur at *finite* temperatures. Lattice calculations in three dimensions support this conjecture.[12]

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APPENDIX

In this Appendix we shall sketch the calculation of the string tension in the two-species Schwinger model at large baryon density. We begin with the Hamiltonian of Eq. (4.17),

$$\mathcal{H}=N_m\int\left[\tfrac{1}{2}\pi_+^2+\tfrac{1}{2}(\partial_1\tilde{\phi}_+)^2+\tfrac{1}{2}k^2\tilde{\phi}_+^2-2c'm^2\cos(\pi\epsilon)\cos\sqrt{2\pi}(\tilde{\phi}_++bz)\right]dz. \tag{A1a}$$

Equation (4.17) contains a mass term $\frac{1}{2}k^2\tilde{\phi}_+^2$ which will be set to zero at the end of the calculation. Nonzerc k is introduced to avoid ambiguities in the intermediate stages of the computation. It is also best to normal order the Hamiltonian with respect to k. Then, using the normal-ordering theorems reviewed in Sec. II and recalling that $c'=c(2M/m)^{1/2}$, we have

$$\mathcal{H}=\int :\tfrac{1}{2}\pi_+^2+\tfrac{1}{2}(\partial_1\tilde{\phi}_+)^2+\tfrac{1}{2}k^2\tilde{\phi}_+^2-2\sqrt{2}cm\sqrt{Mk}\cos(\pi\epsilon)\cos\sqrt{2\pi}(\tilde{\phi}_++bz):dz. \tag{A1b}$$

Here, : : denotes normal ordering with respect to k. Since k is an auxiliary parameter it should not enter our final answers. Its disappearance from those answers constitutes a partial check of our arithmetic.

The string tension will be calculated in perturbation theory in the parameter m. It is easy to see that the lowest-order contributions vanish identically,

$$\begin{aligned}\Delta E^{(1)}&=-2\sqrt{2}cm\sqrt{Mk}\cos(\pi\epsilon)\left\langle\int :\cos\sqrt{2\pi}(\tilde{\phi}_++bz):dz\right\rangle,\\&=-2\sqrt{2}cm\sqrt{Mk}\cos(\pi\epsilon)\left\langle\int[:\cos\sqrt{2\pi}\tilde{\phi}_+:\cos\sqrt{2\pi}bz-:\sin\sqrt{2\pi}\tilde{\phi}_+:\sin\sqrt{2\pi}bz]dz\right\rangle,\end{aligned} \tag{A2}$$

because the factors $\cos\sqrt{2\pi}bz$ and $\sin\sqrt{2\pi}bz$ cause the integral over all of space to average to zero. The second-order contribution to the ground-state energy is

$$\begin{aligned}\Delta E^{(2)}\Delta t=8c^2m^2Mk\cos^2(\pi\epsilon)\Big\langle&\int\cos(\sqrt{2\pi}bz)\cos(\sqrt{2\pi}bz')\langle T:\cos\sqrt{2\pi}\tilde{\phi}_+(z,t):\cos\sqrt{2\pi}\tilde{\phi}_+(z',t'):\rangle dz\,dz'dt\,dt'\\&+\int\sin(\sqrt{2\pi}bz)\sin(\sqrt{2\pi}bz')\langle T:\sin\sqrt{2\pi}\tilde{\phi}_+(z,t)::\sin\sqrt{2\pi}\tilde{\phi}_+(z',t'):\rangle dz\,dz'dt\,dt'\Big\rangle,\end{aligned} \tag{A3}$$

where Δt is the time interval over which the perturbation is turned on. Counting arguments similar to those appearing in the string-tension calculation of Sec. VI can be used to compute the vacuum-expectation values in Eq. (A3)

$$\begin{aligned}&\langle T:\cos\sqrt{2\pi}\tilde{\phi}_+(z,t)::\cos\sqrt{2\pi}\tilde{\phi}_+(z',t'):\rangle=\cosh[2\pi G(z'-z,t'-t;k)],\\&\langle T:\sin\sqrt{2\pi}\tilde{\phi}_+(z,t)::\sin\sqrt{2\pi}\tilde{\phi}_+(z',t'):\rangle=\sinh[2\pi G(z'-z,t'-t;k)],\end{aligned} \tag{A4}$$

where G is the boson propagator with the mass k. We are interested in the long-distance contributions to the integrals in Eq. (A3). Then the Euclidean propagator can be approximated by

$$G(z-z',t-t';k)\sim\frac{1}{2\pi}\ln\{k[(t-t')^2+(z-z')^2]^{1/2}\}. \tag{A5}$$

Now we have

$$\Delta E^{(2)}\Delta t \approx 2c^2m^2Mk\cos^2(\pi\epsilon)$$

$$\times\int dz\,dt\,dz'dt'\Bigg([\cos\sqrt{2\pi}b(z+z')+\cos\sqrt{2\pi}b(z-z')]\left\{\frac{1}{k[(z-z')^2+(t-t')^2]^{1/2}}+k[(z-z')^2+(t-t')^2]^{1/2}\right\}$$

$$\times[\cos\sqrt{2\pi}b(z-z')-\cos\sqrt{2\pi}b(z+z')]\left\{\frac{1}{k[(z-z')^2+(t-t')^2]^{1/2}}\right.$$

$$\left.-k[(z-z')^2+(t-t')^2]^{1/2}\right\}\Bigg), \tag{A6}$$

which is identical to

$$\Delta E^{(2)}\Delta t \approx 4c^2m^2Mk\cos^2(\pi\epsilon)\int dz\,dt\,dz'dt'\left\{\frac{\cos\sqrt{2\pi}b(z-z')}{k[(z-z')^2+(t-t')^2]^{1/2}}+k[(z-z')^2+(t-t'^2)]^{1/2}\cos\sqrt{2\pi}b(z+z')\right\}. \tag{A7}$$

It is convenient to define variables

$$z_\pm = z\pm z', \quad t_\pm = t\pm t' \tag{A8}$$

so then

$$\Delta E^{(2)}\Delta t \approx c^2m^2Mk\cos^2(\pi\epsilon)\int dz_+dt_+dz_-dt_-\left[\frac{\cos\sqrt{2\pi}bz_-}{k(z_-^2+t_-^2)^{1/2}}+k(z_-^2+t_-^2)^{1/2}\cos\sqrt{2\pi}bz_+\right]. \tag{A9}$$

The second term gives zero since the average of $\cos\sqrt{2\pi}bz_+$ is zero. To evaluate the first term, evaluate the integrals over z_+ and t_+. They give the factor $4\Delta t 2L$. Next the z_- integral is evaluated:

$$\int_0^\infty \frac{\cos\sqrt{2\pi}bz_-}{(z_-^2+t_-^2)^{1/2}}\,dz_- = K_0(\sqrt{2\pi}bt_-),$$

where K_0 is a modified Bessel function. So, finally

$$\Delta E^{(2)} \approx \left[16c^2m^2M\cos^2(\pi\epsilon)\int_0^\infty K_0(\sqrt{2\pi}bt_-)dt_-\right]2L \tag{A10a}$$

and changing integration variables

$$\Delta E^{(2)} \approx \left[\frac{16}{\sqrt{2\pi}}\,cm^2\frac{M}{b}\cos^2(\pi\epsilon)\int_0^\infty K_0(s)ds\right]2L. \tag{A10b}$$

Dividing by $2L$ and subtracting the $\epsilon=0$ version of this equation we have the string tension

$$\text{tension} \approx \text{const}\times m^2\frac{M}{b}[\cos^2(\pi\epsilon)-1] \tag{A11}$$

which displays confinement (the tension vanishes whenever ϵ equals an integer) and vanishes in the infinite-density limit.

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