Zero-mass limit and induced interactions in a two-dimensional derivative-coupling model

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(Received 11 October 1978)

We discuss a two-dimensional model of a massive spinor field interacting via a derivative coupling with a massive pseudoscalar field. This model is exactly soluble in the zero-fermion-mass limit. We show that it is possible to treat the massive case in perturbation theory in such a way that no other couplings are induced and that the exactly soluble case is recovered smoothly upon turning off the fermion mass. This means in particular that the four-point function has a better ultraviolet behavior than that of its graphs individually.

I. INTRODUCTION

This is the first of two papers dedicated to the study of a two-dimensional model described by the Lagrangian density

$$\begin{aligned}\mathcal{L} = \tfrac{1}{2} i \bar{\Psi} \overleftrightarrow{\partial} \Psi - M\bar{\Psi}\Psi + \tfrac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \tfrac{1}{2} m^2 \varphi^2 \\ + g(\bar{\Psi}\gamma^\mu\gamma^5\Psi)\partial_\mu\varphi\,. \end{aligned} \tag{1.1}$$

In an interesting paper[1] Rothe and Stamatescu have shown that the model with $M=0$ is exactly soluble. Since in this case φ is a free field, products of fields at the same point can be defined such that

$$\Psi(x) = :\exp[ig\gamma^5\varphi(x)]:\Psi^{(0)}(x)$$

is well defined and solves model (1.1) with $M=0$, if $\Psi^{(0)}(x)$ is a free massless Dirac field. The authors of Ref. 1 have also verified that, up to second order, the fermion self-energy and the three-point vertex function calculated from perturbation theory agree with the exact solution.

Although our interest in the model (1.1) is primarily connected with the possibility of making perturbations around models not exactly free (a subject that will be treated in the second paper, where we study perturbations in the fermion mass M about the exact $M=0$ solution), we think that there is still an interesting aspect of the usual perturbation in the coupling constant g which deserves mentioning. Specifically, if the Feynman graph expansion for the Green's functions of the model (1.1) is considered, one sees that graphs with four external fermion lines are logarithmically divergent.

Following the usual procedure we would subtract such divergences, but this process can in general generate a Thirring-type interaction $(\bar{\Psi}\gamma^\mu\Psi)(\bar{\Psi}\gamma_\mu\Psi)$, so that the limit would not correspond exactly to the model considered by the authors of Ref. 1. In this communication we want to show that a subtraction scheme can be constructed in such a way that the $M\to 0$ limit is smooth, and corresponds to the model of Rothe and Stamatescu. This scheme is very similar to the one considered by the authors of Ref. 2, in their study of massive quantum electrodynamics, and the infrared and ultraviolet finiteness of our scheme can be proved straightforwardly by an adaptation of their arguments to the present situation. Using normal-product methods[3] we are able to derive Ward identities and equations of motion, which in the zero-fermion-mass limit will lead us to the desired model. The absence of radiative corrections to the anomaly of the axial-vector current is a trivial consequence of our subtraction procedure.

The problem of deriving normal products in perturbation theory, which allows one to reconstruct exactly known solutions, also arises in other situations like the Federbush model.[4] Our methods can also be applied to that case, and we hope to treat it in a future publication. The paper is organized as follows. Section II introduces the Feynman rules, the subtraction prescription, and associated normalization conditions. In Sec. III properties of the Green's functions such as Ward identities and equations of motion are stated. Section IV, finally, contains a discussion of the zero-fermion-mass limit. There, we show that, in every order of perturbation, the perturbative solution agrees with the exact one. This is done explicitly for the two- and four-point Green's functions, but the result can be extended for an arbitrary N-point function.

II. FEYNMAN RULES AND SUBTRACTION SCHEME

We consider the model described by the effective Lagrangian density

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2} i \overline{\Psi} \overleftrightarrow{\partial\!\!\!/} \Psi - M\overline{\Psi}\Psi + \tfrac{1}{2}\partial_\mu \varphi \partial^\mu \varphi \\ &\quad - \tfrac{1}{2} m^2 \varphi^2 + g(\overline{\Psi}\gamma^\mu\gamma^5\Psi)\partial_\mu\varphi \\ &= \mathcal{L}_0 + \mathcal{L}_I \,, \\ \mathcal{L}_I &= g(\overline{\Psi}\gamma^\mu\gamma^5\Psi)\partial_\mu\varphi \,. \end{aligned} \tag{2.1}$$

The Green's functions of the theory are defined via the modified Gell-Mann–Low formula

$$\begin{aligned} G^{(2N,L)}&(x_1, x_2, \ldots, x_N;\, y_1, y_2, \ldots, y_N;\, z_1, z_2, \ldots, z_L) \\ &= \left\langle T \prod_{i=1}^{N} \Psi(x_i) \prod_{j=1}^{N} \overline{\Psi}(y_j) \prod_{k=1}^{L} \varphi(z_k) \right\rangle \\ &= \text{finite part of } {}^{(0)}\!\left\langle 0 \left| T \prod_{i,j}^{N} \Psi^{(0)}(x_i)\overline{\Psi}^{(0)}(y_j) \prod_{k=1}^{L} \varphi^{(0)}(z_k) \exp\left[i\int d^2x \!:\!\mathcal{L}_I(x)\!:\right] \right| 0 \right\rangle^{(0)} , \end{aligned} \tag{2.2}$$

where $\varphi^{(0)}$ and $\Psi^{(0)}$ are the free fields as specified by $\mathcal{L}_0$.

After expanding the exponential and applying Wick's theorem, we obtain the usual sum over Feynman amplitudes of the type

$$\lim_{\epsilon\to 0} J_G(p, m, M, \epsilon) = \lim_{\epsilon \to 0} \int d^2k I_G(p, k, m, M, \epsilon), \tag{2.3}$$

where

$$d^2 k = \prod_{i=1}^{l} d^2 k_i \,,$$

$$p = \{p_1, \ldots, p_r\}$$
= basis for external momenta,

$$k = \{k_1, \ldots, k_l\}$$
= basis for internal momenta,

and the integrand $I_G(p, k, m, M, \epsilon)$ can be obtained from a graph G through the following correspondence:

fermion line: $i\dfrac{l\!\!/ + M}{l^2 - M^2 + i\epsilon(\overline{l}^2 + M^2)}$,

scalar line: $\dfrac{i}{l^2 - m^2 + i\epsilon(\overline{l}^2 + m^2)}$,

vertex: $-g l\!\!/_1 \gamma^5$,

where the propagators and vertices are shown in Fig. 1.

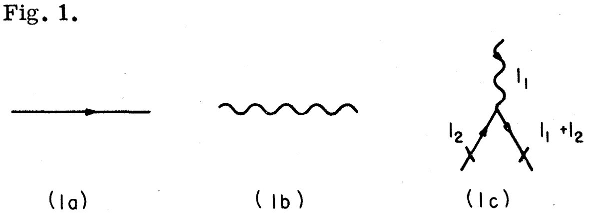

FIG. 1. Graphical representation for Feynman rules: (a) fermion propagator (b) scalar propagator, and (c) vertex.

The integral (2.3) is in general divergent, the degree of divergence for a proper subgraph γ being given by

$$d(\gamma) = 2 - \tfrac{1}{2}N_\gamma \,, \tag{2.4}$$

where N_γ is the number of external fermion lines of γ. In computing $d(\gamma)$ the momentum factor at the vertex has contributed with $+1$. However, momentum factors at vertices to which an external scalar line is attached are independent of the loop momentum variables, so that we can define an effective degree of superficial divergence as

$$\overline{d}(\gamma) = 2 - \tfrac{1}{2}N_\gamma - B_\gamma \,,$$

where B_γ is the number of external pseudoscalar lines of γ. The graph γ will be superficially divergent if

$$\overline{d}(\gamma) \geq 0 \,.$$

The process of removing these divergences to be adopted here consists in the application of Zimmermann's forest formula[3]: if G is a proper graph, I_G must be replaced by

$$R_G = S_G \sum_{U \in \mathcal{F}_G} \prod_{\gamma \in U} (-\tau_\gamma^{d(\gamma)} S_\gamma) I_G(U) \,, \tag{2.5}$$

where $\mathcal{F}_G$ is the set of all G forests, and $\tau_\gamma^{d(\gamma)}$ is the (i) Taylor operator of order $d(\gamma)$ in the external momenta p^γ and in the mass M^γ of the internal fermion lines of the graph γ, at $p^\gamma = M^\gamma = 0$, if γ is not the graph of Fig. 2, and (ii) Taylor operator of order $d(\gamma)$ in p^γ and M^γ at $p^\gamma = 0$ and $M^\gamma = \mu$, if γ is the graph of Fig. 2 (hereafter to

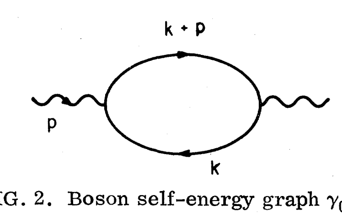

FIG. 2. Boson self-energy graph γ_0.

be called γ_0). S_γ is a substitution operator shifting from the variables of $\lambda \in U$ to those of $\gamma \in U$, if $\gamma \supset \lambda$; S_G in addition sets the mass μ equal to M.

We remark that the possibility of making subtractions at zero fermion mass, without getting infrared divergences, stems from the fact that the mass m is not modified by the action of the Taylor operators. In fact, both ultraviolet and infrared convergence can be proved by a straightforward adaptation of the reasoning given in Ref. 2. The subtraction scheme above furnishes immediately the following normalization conditions for the vertex functions:

$$\begin{aligned}
&\overline{\Gamma}^{(2,0)}\Big|_{\substack{p=0\\M=0}}=0\,,\quad \frac{\partial\overline{\Gamma}^{(2,0)}}{\partial M}\Big|_{\substack{p=0\\M=0}}=-i\,,\\
&\frac{\partial\overline{\Gamma}^{(2,0)}}{\partial p^\mu}\Big|_{\substack{p=0\\M=0}}=i\gamma_\mu\,,\\
&\frac{\partial\overline{\Gamma}^{(2,1)}(q,-q-p,p)}{\partial p^\mu}\Big|_{\substack{p=q=0\\M=0}}=-g\gamma_\mu\gamma^5\,,\\
&\overline{\Gamma}^{(4,0)}\Big|_{\substack{p_i=0\\M=0}}=0\,,
\end{aligned}\tag{2.6}$$

where $\overline{\Gamma}^{(2N,L)}$ denotes the contribution for the corresponding vertex function, resulting from graphs not containing the graph γ_0 of Fig. 2 as a subgraph. (As will become clear from the next section, the same normalization conditions also hold for $\Gamma^{(2,0)}$, $\Gamma^{(2,1)}$, and $\Gamma^{(4,0)}$ in spite of the apparent logarithmic divergence of γ_0.) As we shall see, when $M\to 0$, the above equations become the normalization conditions for the zero-fermion-mass theory. It is clear that m and M are not the physical scalar and fermion masses; however, the latter one tends to zero as $M\to 0$.

III. WARD IDENTITIES AND EQUATIONS OF MOTION

Ward identities for the vector and axial-vector currents can be derived in the standard way.[3] We will find that the anomaly of the axial-vector current is mild enough so as to permit the solubility of the $M\to 0$ limit. We have

$$X=\prod_{i,j=1}^{N}\Psi(x_i)\overline{\Psi}(y_i)\prod_{k=1}^{L}\varphi(z_k)\,,$$

$$\partial_x^\mu\langle TN_1[\overline{\Psi}\gamma_\mu\Psi](x)X\rangle=i\langle T\{N_2[\overline{\Psi}(-i\overleftarrow{\partial\!\!\!/}-M)\Psi](x)-N_2[\overline{\Psi}(i\partial\!\!\!/-M)\Psi](x)\}X\rangle=\sum_{i=1}^{N}[\delta(x-y_i)-\delta(x-x_i)]\langle TX\rangle\,,\tag{3.1}$$

$$\begin{aligned}
\partial_x^\mu\langle TN_1[\overline{\Psi}\gamma_\mu\gamma^5\Psi](x)X\rangle&=i\langle T\{N_2[\overline{\Psi}(-i\overleftarrow{\partial\!\!\!/}-M)\gamma^5\Psi](x)+N_2[\overline{\Psi}\gamma^5(i\partial\!\!\!/-M)\Psi](x)+2N_2[M\overline{\Psi}\gamma^5\Psi](x)\}X\rangle\\
&=-\sum_{i=1}^{N}[\delta(x-x_i)\gamma^5_{x_i}+\delta(x-y_i)\gamma^{5T}_{y_i}]\langle TX\rangle+2i\langle TN_2[\overline{\Psi}\gamma^5\Psi](x)X\rangle\,.
\end{aligned}\tag{3.2}$$

We now use Zimmermann's identity

$$2iN_2[M\overline{\Psi}\gamma^5\Psi](x)=2iMN_1[\overline{\Psi}\gamma^5\Psi](x)+a\,\partial^2\varphi(x),\quad a=-g/\pi\,,\tag{3.3}$$

where the last term in the right-hand side of (3.3) comes from the subtraction for the graph of Fig. 3. Using (3.3) we can rewrite (3.2) as

$$\partial^\mu\langle TN_1[\overline{\Psi}\gamma_\mu\gamma^5\Psi](x)X\rangle=2iM\langle TN_1[\overline{\Psi}\gamma^5\Psi](x)X\rangle+a\langle T\partial^2\varphi(x)X\rangle-\sum_{i=1}^{N}[\delta(x-x_i)\gamma^5_{x_i}+\delta(x-y_i)\gamma^{5T}_{y_i}]\langle TX\rangle\,.\tag{3.4}$$

Equations of motion can be derived analogously. In particular,

$$(\partial_x{}^2+m^2)\langle T\varphi(x)X\rangle=-i\sum_{k=1}^{L}\delta(x-z_k)\langle TX_{\hat{z}_k}\rangle-g\partial_x^\mu\langle TN_1[\overline{\Psi}\gamma_\mu\gamma^5\Psi](x)X\rangle\,,$$

so that, using (3.4) we get

$$\begin{aligned}
(\partial_x{}^2+m'^2)\langle T\varphi(x)X\rangle=&-\frac{2iMg}{1+ag}\langle TN_1[\overline{\Psi}\gamma^5\Psi](x)X\rangle-\frac{i}{1+ag}\sum_{k=1}^{L}\delta(x-z_k)\langle TX_{\hat{z}_k}\rangle\\
&+\frac{g}{1+ag}\sum_{i=1}^{N}[\delta(x-x_i)\gamma^5_{x_i}+\delta(x-y_i)\gamma^{5T}_{y_i}]\langle TX\rangle\,,
\end{aligned}\tag{3.5}$$

where $m'^2 = m^2/(1+ag)$. Equation (3.5) shows that in the zero-fermion-mass limit φ is a free field of mass m'. Similarly, the equation of motion for the Ψ field is given by

$$(i\partial\!\!\!/_{x_1} - M)\left\langle T\Psi(x_1)\cdots\Psi(x_N)\prod_{j=1}^{N}\bar{\Psi}(y_j)\prod_{k=1}^{L}\varphi(z_k)\right\rangle = -g\langle TN_{3/2}[\gamma^\mu\gamma^5\Psi\partial_\mu\varphi](x)X\rangle + i\sum_{i=1}^{N}(-1)^{i+N}\delta(x-y_i)\langle TX_{\hat{y}_i}\rangle\,. \tag{3.6}$$

IV. ZERO-MASS LIMIT

In order to simplify the discussion of the $M\to 0$ limit, we consider separately the contribution coming from the graph γ_0 of Fig. 2, and proceed as follows:

(1) We apply the axial-vector current Ward identity to one of the vertices of γ_0 obtaining the graphs shown in Fig. 4. We note that the first two of these graphs cancel after integration in the loop momentum variable of γ_0.

(2) We take the limit $M_{\gamma_0}\to 0$ of the resulting expressions, where M_{γ_0} denotes the mass of the internal fermion lines of γ_0. Calculating the contribution of the third graph of Fig. 4, one sees that it goes to zero in this limit. Thus, after these steps the only surviving contribution from γ_0 is the term of the anomaly (the fourth graph of Fig. 4), where $\bar{a} = g^2/\pi$ is M independent and thus finite as $M\to 0$. As shown in Appendix A we arrive at the same result even if γ_0 is a subgraph of a larger graph. In this case only the masses of the lines belonging to γ_0 are set to zero, whereas the masses of all other lines remain finite.

(3) The contribution from γ_0 to the scalar propagator is summed over. The effect of this is to replace $\Delta_F(x, m^2)$ by $[1/(1-\bar{a})]\Delta_F(x, m^2/(1-\bar{a}))$. After this step we obtain new graphs with γ_0 omitted, which are made finite by application of the forest formula, using Taylor operators in the external momenta and in the mass of the fermion lines around $p=0$ and $M=0$. The vertex functions defined by these new graphs clearly satisfy the normalization conditions (3.6). For these new Green's functions it is easily verified that

$$M\frac{\partial}{\partial M}G^{(2N,L)} = M\Delta_0 G^{(2N,L)}\,, \tag{4.1}$$

where Δ_0 is the soft differential vertex operation given formally by

$$\Delta_0 = -i\int N_1[\bar{\Psi}\Psi](x)d^2x\,.$$

In the limit $M\to 0$ and for nonexceptional momenta, $\Delta_0 G^{(2N,L)}$ can develop logarithmic infrared divergences, but $G^{(2N,L)}$ stays finite. [Observe that owing to our subtraction scheme, reduced vertices with two fermion lines (Fig. 5) will have a momentum factor which improves the infrared behavior of the integral in the loop momentum of these lines.] It is therefore apparent that in the zero-fermion-mass limit the Green's functions of the theory above will approach the Green's functions of the zero-fermion-mass theory, i.e., in this limit M can be put equal to zero directly in the integrands of the Feynman amplitudes contributing to the Green's functions, assuming that the $M\to 0$ and $\epsilon\to 0$ limits may be interchanged:

$$\lim_{M\to 0}\lim_{\epsilon\to 0} J_G(p, M, m', \epsilon) = \lim_{\epsilon\to 0}\mathcal{J}_G(p, m', \epsilon)\,,$$

where

$$\mathcal{J}_G(p, m', \epsilon) = \int d^2k\bar{R}_G(p, k, m', \epsilon)\,, \tag{4.2}$$

and $\bar{R}_G(p, k, m', \epsilon)$ denotes the subtracted integrand of the zero-mass theory. For a proper graph

$$\bar{R}_G(p, k, m', \epsilon) = \sum_{U\in\mathcal{F}_G}\prod_{\gamma\in U}(-t_p^{d(\gamma)})I_G(p, k, m', \epsilon)\,, \tag{4.3}$$

where $I_G(p, k, m', \epsilon)$ is constructed using the following Feynman rules:

fermion propagator:

$$\frac{i\not{l}}{l^2 + i\epsilon\vec{l}^{\,2}}\,,$$

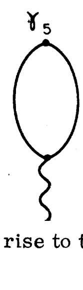

FIG. 3. Graph giving rise to the axial-vector anomaly.

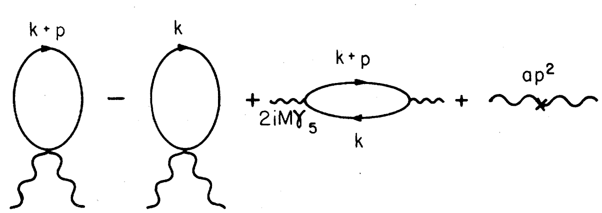

FIG. 4. Graphs resulting from the application of the axial-vector current Ward identity to γ_0. These graphs are minimally subtracted.

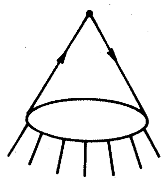

FIG. 5. Reduced vertex with two fermion lines connected to an arbitrary graph.

boson propagator:

$$\frac{i(1-\bar{g}^2/\pi)^{-1}}{l^2+m'^2+i\epsilon(\bar{l}^2+m'^2)},$$

$$m'^2=m^2(1-\bar{g}^2/\pi)^{-1},$$

vertex: $-g\not{l}_1\gamma^5$,

but omitting the graph γ_0. $t_p^{d(\gamma)}$ is the Taylor operator in the external momentum variables of γ. In the following we expand perturbatively in powers of g and at the very end set $\bar{g}=g$. This means that we will obtain the Green's functions of Rothe and Stamatescu with coupling constant $g_{\rm RS}{}^2=g^2/(1-g^2/\pi)$ and boson mass $m_{\rm RS}{}^2=m^2/(1-g^2/\pi)$. For completeness the proof of the infrared convergence of this last scheme is sketched in Appendix B.

We shall now prove that, order by order in g, the Green's functions of the zero-mass theory are equal to the ones of Rothe and Stamatescu's model. This will be done explicitly for the two- and four-point Green's functions.

A. Two-point function

The fermion two-point function satisfies the equation of motion

$$\begin{aligned} i\not{\partial}_x G^{(2,0)}(x,y) &= i\not{\partial}_x\langle T\Psi(x)\overline{\Psi}(y)\rangle \\ &= -g\langle TN_{3/2}[\partial_\mu\varphi\gamma^\mu\gamma^5\Psi](x)\overline{\Psi}(y)\rangle \\ &\quad + i\delta(x-y)\,. \end{aligned} \tag{4.4}$$

The graphs contributing to

$$\langle TN_{3/2}[\partial_\mu\varphi\gamma^\mu\gamma^5\Psi](x)\overline{\Psi}(y)\rangle$$

have in momentum space the structure shown in Fig. 6. Applying the axial-vector-current Ward identity in the way shown in Fig. 7, one therefore

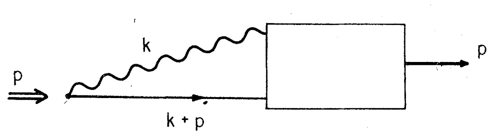

FIG. 6. Graph contributing to $\langle TN_{3/2}[\partial_\mu\varphi\gamma^\mu\gamma^5\psi]^{(x)}\overline{\psi}(y)\rangle$.

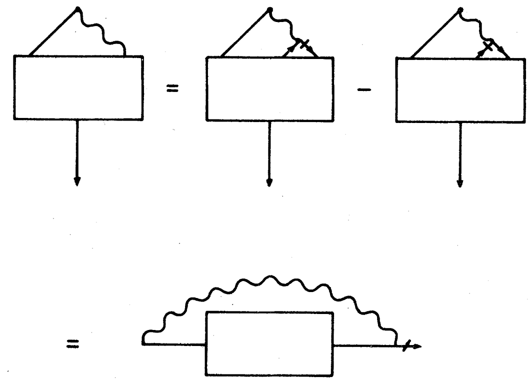

FIG. 7. Applying the axial-vector current Ward identity to the graph of Fig. 6.

obtains

$$\begin{aligned} &\langle TN_{3/2}[\partial_\mu\varphi\gamma^\mu\gamma^5\Psi](x)\overline{\Psi}(y)\rangle \\ &\qquad = -igN[\not{\partial}\Delta_F(x-y)\langle T\Psi(x)\overline{\Psi}(y)\rangle]\,, \end{aligned} \tag{4.5}$$

where the symbol N is to indicate that the expression in parentheses is to be subtracted according to the scheme (4.3). Here and in the following $\Delta_F(x)$ stands for the modified boson propagator, whose Fourier transform is given by

$$\frac{(1-\bar{g}^2/\pi)^{-1}}{l^2+m'^2+i\epsilon(\bar{l}^2+m'^2)}\,.$$

The importance of having used the scheme (4.3) becomes clear at this point for it avoids inducing Thirring-type contributions to the right-hand side of Eq. (4.5). Had we used for example the subtraction scheme of Ref. 5, which involves a subtraction point, a Thirring-type interaction would have resulted.[6] It arises from subtractions for the graph shown in Fig. 8.

Substituting (4.5) into (4.4) we get

$$\begin{aligned} \not{\partial}G^{(2,0)}(x,y) &= g^2N[\not{\partial}\Delta_F(x-y)\langle T\Psi(x)\overline{\Psi}(y)\rangle] \\ &\quad + \delta(x-y), \end{aligned} \tag{4.6}$$

which can be solved iteratively. The solution of

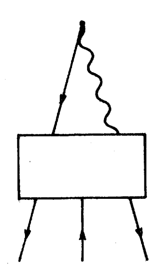

FIG. 8. Graphs whose subtraction could induce Thirring-type interactions.

this equation to order g^{2n} is given by

$$G^{(2,0)}(x,y)=\frac{g^{2n}}{n!}\,\Delta_F^n(x-y)S(x-y)\,, \tag{4.7}$$

where $S(x)$ is the free fermion propagator.

Proof: (i) zero order: $\not\partial_x G^{(0)}(x-y)=\delta(x-y)$ implies that $G^{(0)}(x,y)=S(x-y)$. (ii) Suppose that the contribution of order g^{2n} is given as above. Then, the term of order g^{2n+2} satisfies

$$\begin{aligned}
\not\partial_x G^{2n+2}(x,y)&=g^2N[\not\partial\Delta_F(x-y)G^{2n}(x,y)]\\
&=g^{2n+2}N\left[\frac{1}{n!}\not\partial\Delta_F(x-y)\Delta_F^n(x-y)S(x-y)\right]\\
&=g^{2n+2}N\left[\left(\frac{\not\partial\Delta_F^{n+1}(x-y)}{(n+1)!}\right)S(x-y)\right]\\
&=g^{2n+2}N\left[\not\partial\left(\frac{\Delta_F^{n+1}(x-y)}{(n+1)!}S(x-y)\right)\right]-g^{2n+2}N\left[\frac{\Delta_F^{n+1}(0)}{(n+1)!}\delta(x-y)\right]\\
&=g^{2n+2}\not\partial\left[\frac{1}{(n+1)!}\Delta_F^{n+1}(x-y)S(x-y)\right],
\end{aligned}$$

since $N[\Delta_F^{n+1}(0)/(n+1)!]=0$ owing to the subtractions and $N\not\partial F(x-y)=\not\partial F(x-y)$ because of the property $t_p^\alpha[\not p f(p)]=\not p\, t_p^{\alpha-1}[f(p)]$ of the Taylor operator. Thus

$$G^{2n+2}(x,y)=g^{2n+2}\,\frac{\Delta_F^{n+1}(x-y)}{(n+1)!}S(x-y)\,,$$

or equivalently

$$G(x,y)=e^{g^2\Delta_F(x-y)}S(x-y)\,, \tag{4.8}$$

which is the solution of Rothe and Stamatescu.

B. Four-point function

In this case the equation of motion reads

$$\begin{aligned}
i\not\partial_{x_1}G^{(4,0)}(x_1,x_2;y_1,y_2)=&-g\langle TN_{3/2}[\gamma^\mu\gamma^5\Psi\partial_\mu\varphi](x_1)\Psi(x_2)\overline{\Psi}(y_1)\overline{\Psi}(y_2)\rangle\\
&+i\delta(x_1-y_1)G^{(2,0)}(x_2,y_2)+i\delta(x_1-y_2)G^{(2,0)}(x_2,y_1)\,.
\end{aligned} \tag{4.9}$$

Similarly to the calculation for the two-point function we have

$$\begin{aligned}
\langle TN_{3/2}[\partial_\mu\varphi\gamma^\mu\gamma^5\Psi](x_1)\Psi(x_2)\overline{\Psi}(y_1)\overline{\Psi}(y_2)\rangle=&-ig\{N[(\not\partial\Delta_F(x_1-y_2)\gamma^5_{x_2}+\not\partial\Delta_F(x_1-y_1)\gamma^5_{y_1}\\
&+\not\partial\Delta_F(x_1-y_2)\gamma^5_{y_2})G^{(4,0)}(x_1,x_2;y_1,y_2)\}\,,
\end{aligned} \tag{4.10}$$

so that using (4.10), Eq. (4.9) becomes

$$\begin{aligned}
\not\partial_{x_1}G^{(4,0)}(x_1,x_2;y_1,y_2)=&g^2N[(\not\partial\Delta_F(x_1-x_2)\gamma^5_{x_2}+\not\partial\Delta_F(x_1-y_1)\gamma^5_{y_1}+\not\partial\Delta_F(x_1-y_2)\gamma^5_{y_2})G^{(4,0)}(x_1,x_2;y_1,y_2)]\\
&-\delta(x_1-y_1)G^{(2,0)}(x_2,y_2)+\delta(x_1-y_2)G^{(2,0)}(x_2,y_2)\,,
\end{aligned} \tag{4.11}$$

which can be solved iteratively, since the two-point functions are already known. We claim that the solution of (4.11) is given by

$$\begin{aligned}
G^{(4,0)}(x_1,x_2;y_1,y_2)=\sum_{n=0}^{\infty}\frac{g^{2n}}{n!}&[\Delta_F(x_1-x_2)\gamma^5_{x_1}\gamma^5_{x_2}+\Delta_F(x_1-y_1)\gamma^5_{x_1}\gamma^5_{y_1}+\Delta_F(x_1-y_2)\gamma^5_{x_1}\gamma^5_{y_2}+\Delta_F(x_2-y_1)\gamma^5_{x_2}\gamma^5_{y_1}\\
&+\Delta_F(x_2-y_2)\gamma^5_{x_2}\gamma^5_{y_2}+\Delta_F(y_1-y_2)\gamma^5_{y_1}\gamma^5_{y_2}]^n G^{(0)}(x_1,x_2;y_1,y_2)\,,
\end{aligned} \tag{4.12}$$

where $G^{(0)}(x_1,x_2;y_1,y_2)$ is the free four-point function.

This can be easily verified applying $\not\partial_{x_1}$ to Eq. (4.12) and using

$$\begin{aligned}
\not\partial_{x_1}G^{(0)}(x_1,x_2;y_1,y_2)=&-\delta(x_1-y_1)S(x_2-y_2)\\
&+\delta(x_1-y_2)S(x_2-y_1)
\end{aligned}$$

and the γ^5 invariance of the zero-mass theory.

One obtains Eq. (4.11) so that the iterative solution is given by (4.12).

The same procedure can be applied to all higher-point Green's functions, showing that the perturbative solution agrees order by order with the exact one.

ACKNOWLEDGMENTS

This research of M. El Afioni was supported by the Fundacão de Amparo à Pesquisa do Estado de São Paulo, (FAPESP) Brasil. M. Gomes' and R. Köberle's were partially supported by Conselho Nacional de Pesquisas (CNPq), Brasil.

APPENDIX A

In this appendix we want to show that as $M\to 0$ the only surviving contribution from the graph γ_0 of Fig. 2 is the term of the anomaly (the fourth graph in Fig. 4).

Proof (sketch): Let G be a proper Feynman graph. The G forests can be classified as follows:

$$\mathfrak{F}_1=\{U \mid U\in\mathfrak{F}_1\Rightarrow \text{there is a graph } \gamma\in U \text{ so that } \gamma \text{ and } \gamma_0 \text{ are overlapping}\},$$

$$\mathfrak{F}_2=\{U \mid \gamma\in U\Rightarrow \gamma\cap\gamma_0=\emptyset\},$$

$$\mathfrak{F}_3=\{U \mid \gamma\in U\Rightarrow \gamma \text{ and } \gamma_0 \text{ are nonoverlapping and there is at least one graph } \gamma'\in U \text{ with } \gamma'\supseteq\gamma_0\}.$$

We consider the contribution from G forests of $\mathfrak{F}_3$. We consider two possibilities:

(i) $\gamma_0\in U$, but there is no other graph $\gamma\in U$ with the property $\gamma\supset\gamma_0$. For each forest U_1 of this type there is a forest U_2 of $\mathfrak{F}_2$, where U_2 differs from U_1 only by the fact that $\gamma_0\notin U_2$. The mechanism of cancellation (up to the anomaly) for this pair of forests is exactly as in the text.

(ii) $\gamma_0\in U$ and there is at least one graph $\gamma\in U$ with the property $\gamma\supset\gamma_0$. Let τ be the smallest graph with the property $\tau\supset\gamma_0$. There are two subcases to be considered:

(a) τ is not a fermion self-energy part. For each forest of this type there is another forest $\overline{U}\in\mathfrak{F}_3$, which differs from U only by the fact that $\gamma_0\notin\overline{U}$. The combination of these two forests will give in the $M\to 0$ limit only the anomaly term.

(b) τ is a fermion self-energy part. Because of the fact that $t_\tau^{d(\tau)}$ contains the operation $M\partial/\partial M$, the combination of forests is a little bit more complicated than in the previous case. To get cancellation (up to the anomaly), we have to combine each forest of this type with forests of the following type:

U_1, $U_1\in\mathfrak{F}_3$ and U_1 differs from U only by the fact $\gamma_0\notin U_1$;

U_2, $U_2\in\mathfrak{F}_1$, $\tau\in U_2$, and there is a maximal subgraph γ_1 of τ in U_2, so that $\gamma_1\neq\gamma_0$ and $\gamma_1\cap\gamma_0\neq\emptyset$;

U_3, $U_3\in\mathfrak{F}_1$ and U_3 differs from U_2 only by the fact $\tau\notin U_3$.

Cancellation càn be verified at the expense of straightforward, if lengthy, algebra.

APPENDIX B

We will show that the Feynman amplitudes of the zero-fermion-mass theory are, for nonexceptional momenta, infrared finite (ultraviolet convergence is obvious as can be seen by the application of the theorem of Ref. 7). We will do this by following the same method and the notation of Ref. 5, to which the reader is referred for details.

Let $u_1,\ldots,u_a$; $v_1,\ldots,v_b$ be an arbitrary basis of $\mathcal{L}(\Gamma)$, the space of linear forms in p and k of a connected graph Γ, with $\partial(u,v)/\partial k\neq 0$. Furthermore, let C be a Γ forest, which is complete with respect to S, the subspace of $\mathcal{L}(\Gamma)$ spanned by $u_1,\ldots,u_a$. Then, we have to show that

$$\underline{\deg}_u R_\Gamma(\mathrm{C})+2a>0\,, \tag{B1}$$

where $\underline{\deg}_u f(u,v)=k$, if for almost all v

$$\lim_{\lambda\to 0}\lambda^{-k}f(\lambda u,v)\neq 0,\infty$$

and

$$R_\Gamma(\mathrm{C})=(1-t_\Gamma)Y_\Gamma(\mathrm{C})\,, \tag{B2}$$

with $Y_\gamma(\mathrm{C})$ defined recursively by

$$Y_\gamma(\mathrm{C})=I_{\overline{\gamma}(\mathrm{C})}S_\gamma\prod_\alpha f_{\gamma_\alpha}Y_{\gamma_\alpha}(\mathrm{C})\,,\qquad \overline{\gamma}(\mathrm{C})=\gamma/\gamma_1\cdots\gamma_n\,. \tag{B3}$$

$\{\gamma_1,\ldots,\gamma_n\}$ = set of maximal elements of C contained in γ and

$$f_{\gamma_\alpha}=\begin{cases}1-t_{\gamma_\alpha}, & \text{if, } \overline{\gamma}_\alpha(\mathrm{C})\nparallel S,\ \ \overline{\gamma}(\mathrm{C})\parallel S\\ -t_{\gamma_\alpha}, & \text{otherwise.}\end{cases} \tag{B4}$$

In proving (B1) the following lemmas (1, 2, and 3) are important:

Lemma 1.

(a) If $t_\gamma^{d(\gamma)}Y_\gamma\neq 0$, then

$$\underline{\deg}_u t_\gamma^{d(\gamma)}Y_\gamma\geq \underline{\deg}_{up\gamma}Y_\gamma-d(\gamma)\ \text{ if } \overline{\gamma}\parallel S\,, \tag{B5}$$

$$\underline{\deg}_u t_\gamma^{d(\gamma)}Y_\gamma\geq \underline{\deg}_u Y_\gamma\ \ \text{ if } \overline{\gamma}\nparallel S\,, \tag{B6}$$

$$\underline{\deg}_{up\gamma}t_\gamma^{d(\gamma)}Y_\gamma\geq \underline{\deg}_{up\gamma}Y_\gamma\,. \tag{B7}$$

(b)

$$\underline{\deg}_{up\gamma}(1-t_\gamma^{d(\gamma)})Y_\gamma\geq \underline{\deg}_{up\gamma}Y_\gamma+\max\{d(\gamma)+1,0\}\quad \text{if } \overline{\gamma}\nparallel S\,. \tag{B8}$$

The proof of this lemma is the same as in Ref. 5.

Lemma 2.

$$\underline{\deg}_{up^\gamma} Y_\gamma \geq d(\gamma)+1-M(\gamma) \quad \text{if } \bar{\gamma}\| S \text{ and } t_\gamma^{d(\gamma)} Y_\gamma \neq 0\,, \tag{B9}$$

$$\underline{\deg}_u Y_\gamma \overset{*}{\geq} -M(\gamma) \text{ if } \bar{\gamma}\not\| S\,, \tag{B10}$$

where

$$M(\gamma)=2\sum_{\substack{\lambda\in C\\ \lambda\leq\gamma\\ \bar{\lambda}\| S}}[\text{No. of independent loops of } \bar{\lambda}(C)]$$

and

$$\overset{*}{\geq} \text{ means} \begin{cases} > \text{ if right-hand side} \neq 0,\\ = \text{ if right-hand side} = 0\,. \end{cases}$$

Lemma 3.

Let λ be a maximal element of C properly contained in $\gamma\subset\Gamma$. Then the following inequalities hold:

(a) If $t_\lambda^{d(\lambda)} Y_\lambda \neq 0$, then

$$\underline{\deg}_u S_\gamma t_\lambda^{d(\lambda)} Y_\lambda \overset{*}{\geq} -M(\lambda)\,, \tag{B11}$$

$$\underline{\deg}_{up^\gamma} S_\gamma t_\lambda^{d(\lambda)} Y_\lambda \geq d(\lambda)+1-M(\lambda) \quad \text{if } \bar{\lambda}\| S\,. \tag{B12}$$

(b)

$$\underline{\deg}_{up^\gamma} S_\gamma (1-t_\lambda^{d(\lambda)}) Y_\lambda \geq \max\{d(\lambda)+1,0\}-M(\lambda) \quad \text{if } \bar{\lambda}\not\| S,\ \bar{\gamma}\| S\,. \tag{B13}$$

The proof of the above lemmas is essentially the same as in Ref. 5. We only have to note that

$$r(\gamma)-d(\gamma)=2n_B^\gamma \geq 0\,, \tag{B14}$$

where $r(\gamma)$ is the infrared degree of superficial divergence and n_B^γ is the number of internal scalar lines in γ. Furthermore,

$$n_B^\gamma > 0 \quad \text{if } t_\gamma^{d(\gamma)} Y_\gamma \neq 0.$$

We now apply the above inequalities to the graph Γ.

Case I: $\bar{\Gamma}\not\| S$. From (B10) and (B6) we have

$$\underline{\deg}_u (1-t_\Gamma^{d(\Gamma)}) Y_\Gamma > -M(\Gamma)\,. \tag{B15}$$

Case II: $\bar{\Gamma}\| S$. We have

$$\underline{\deg}_u S_\Gamma (1-t_\Gamma^{d(\Gamma)}) Y_\Gamma \geq \min\{\underline{\deg}_u Y_\Gamma,\ \underline{\deg}_u t_\Gamma^{d(\Gamma)} Y_\Gamma\}\,.$$

From (B5) and (B9) it follows that

$$\underline{\deg}_u t_\Gamma^{d(\Gamma)} Y_\Gamma > -M(\Gamma)\,. \tag{B16}$$

Besides that, we have

$$\begin{aligned}\underline{\deg}_u Y_\Gamma &= \underline{\deg}_u I_{\bar{\Gamma}} + \sum_{\gamma_\alpha\subset C} \underline{\deg} S_\Gamma f_{\gamma_\alpha} Y_{\gamma_\alpha}\\ &= r(\Lambda)-M(\Lambda)+\sum_{\gamma_\alpha\subset C} \underline{\deg}_u S_\Gamma f_{\gamma_\alpha} Y_{\gamma_\alpha}\,,\end{aligned} \tag{B17}$$

where Λ is the graph obtained from $\bar{\Gamma}(C)$ by reducing all constant lines of $\bar{\Gamma}$ [i.e., with momentum of the form $l_\Gamma^L = P^L(p)+U^L(u)$ with $P^L(p)\neq 0$] to a point. Let V_0 be the special vertex of Λ resulting from such a contraction (here the nonexceptionality of the external momenta of Γ is important, since this implies the existence of only one such vertex). Now, from (B10), (B11), (B12), and (B13) we get

$$\underline{\deg}_u S_\Gamma f_{\gamma_\alpha} Y_{\gamma_\alpha} \geq -M(\gamma_\alpha) \quad \text{if } V(\gamma_\alpha)\notin v(\Lambda)\,, \tag{B18}$$

$$\underline{\deg}_u S_\Gamma f_{\gamma_\alpha} Y_{\gamma_\alpha} \geq -M(\gamma_\alpha)+\max\{d(\gamma_\alpha)+1,0\} \quad \text{if } V(\gamma_\alpha)\in v(\Lambda) \text{ and } V(\gamma_\alpha)\neq V_0\,, \tag{B19}$$

where $V(\gamma_\alpha)$ is the reduced vertex obtained by contracting γ_α to a point, and $v(\Lambda)$ is the set of vertices of Λ. Using (B18) and (B19) we obtain

$$\underline{\deg}_u Y_\Gamma \geq -M(\Gamma)+r(\Lambda) + \sum_{\substack{V_\alpha\\ V_\alpha = V(\gamma_\alpha)\neq V_0\\ V(\gamma_\alpha)\in v(\Lambda)}} \max\{d(\gamma_\alpha)+1,0\}\,. \tag{B20}$$

Now by a straightforward calculation we can verify that

$$\begin{aligned} r(\Lambda) + \sum_{\substack{V_\alpha\\ V_\alpha = V(\gamma_\alpha)\neq V_0\\ V(\gamma_\alpha)\in v(\Lambda)}} \max\{d(\gamma_\alpha)+1,0\} &= \tfrac{1}{2}\nu_F(V_0)+\nu_B(V_0)\\ &\quad + \sum_{\substack{V_i\in v(\Lambda)\\ V_i\in v(\Gamma)}} [\tfrac{1}{2}\nu_F(V_i)+\nu_B(V_i)-1]\\ &\quad + \sum_{\substack{V_i = V(\gamma_i),\ V_i\neq V_0\\ d(\gamma_i)+1\leq 0}} [\tfrac{1}{2}\nu_F(V_i)+\nu_B(V_i)-2] + \sum_{\substack{V_i = V(\gamma_i),\ V_i\neq 0\\ d(\gamma_i)+1>0}} [1+\nu_B(V_i)] > 0\,, \end{aligned} \tag{B21}$$

where $\nu_F(V_i)$ = No. of fermion lines ending at V_i, $\nu_B(V_i)$ = No. of scalar lines ending at V_i. We thus obtain

$$\underline{\deg}_u Y_\Gamma > -M(\Gamma) \quad \text{if } \bar{\Gamma}\| S\,. \tag{B22}$$

Using (B15), (B16), (B22), and the result

$$2a \geq -M(\Gamma)\,,$$

we obtain (B1).

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