Even-wave harmonic-oscillator theory of baryonic states. V. Polarized photoproduction of single pions

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(Received 11 April 1978)

A variant of the harmonic-oscillator (h.o.) model, termed the even-wave h.o. model (with quarks having h.o. interaction in even waves only), which was developed by Mitra and co-workers in a series of recent papers, is applied to the phenomenon of polarized photoproduction. The model, which predicts a ground state of $(\underline{70},0^+)$ and assumes an 8_d mixing angle ($\cot\theta = \sqrt{2}$) between 56 and 70 states, was recently found to give a good account of (i) the mass spectra of resonances, (ii) low-energy parameters such as G_A/G_V, $\Delta \rightarrow N\pi$ width, and $NN\pi$ coupling constant, and (iii) resonance photocouplings, with no free parameters other than a universal spring constant ($\Omega \approx 1$ GeV^2). The present investigation reveals that this model agrees better with the data on polarized-target asymmetry as well as $d\sigma_\parallel$ and $d\sigma_\perp$ than does the conventional h.o. model.

I. INTRODUCTION

During the last decade several analyses of the reaction $\gamma N \rightarrow N\pi$ (Refs. 1–3) have provided considerable information about the resonant states of the nucleon. Continued interest in the analysis of these reactions is probably because of the extra-SU(3) (Ref. 4) nature of the process $\gamma N \rightarrow N\pi$ and similar reactions, so that these can profitably serve as testing grounds for various models of hadron structure and higher-symmetry schemes. If the incoming photon is polarized, observation of the angular distribution of the emitted pion can throw valuable light on some finer features of the photoproduction amplitudes which have a direct bearing on the dynamical content of different models.

It is well known that the symmetric quark model with harmonic-oscillator (h.o.) interaction[5] between quarks has given a fairly reasonable account of resonance systematics since Berkeley,[6] including pseudoscalar decays and photocouplings of baryon resonances.[7–10] However, there are certain well established features of resonance spectroscopy which have persistently defied any natural explanation in the symmetric h.o. quark model. For example, its prediction of totally antisymmetric $\underline{20}$ states has hardly found any convincing experimental support even after a decade of "bump hunting" since the birth of the model.[11] The assignment of $P_{11}(1470)$ to a radially excited $\underline{56}$ state not only precludes the possibility of a nontrvially mixed nucleon (needed for several low-energy parameters, e.g., G_A/G_V, $\Delta \rightarrow N\pi$, etc.[12,13]), but is beset with serious problems of photoproduction.[7,10] The actual mass spectra of resonances also require an elaborate program of mass operator parametrization[14] within the conventional h.o. model.

A different point of view, advocated in a series of recent papers by Mitra and co-workers,[15–17] is based on a variant of the harmonic oscillator, wherein the h.o. force operates only in the even partial waves of a q-q system. The even-wave h.o. model thus provides a controled breaking of h.o. symmetry while maintaining SU(6) symmetry. Without going into the details, which are available in papers I and II, we summarize a few important and distinct features of the even-wave model. By the very nature of the even-wave assumption, the $\underline{20}$ states are automatically kept out while the $\underline{56}$ states remain unaffected. For the $\underline{70}$ states, there appears a ground state of $(\underline{70}, 0^+)$ whose proximity to $(\underline{56}, 0^+)$ affords the natural facility of a mixed nucleon which helps reconcile G_A/G_V with the $\Delta \rightarrow N\pi$ width and the $NN\pi$ coupling constant[13,17] in the low-energy domain, as well as accounts for the shape of R^{np} in the deep-inelastic limit.[18]

Recently the even-wave h.o. theory was applied to photocouplings[19] and pseudoscalar decays[20] of baryon resonances. In the case of photocouplings, this model has had considerable experimental success including traditionally difficult cases such as $P_{11}(1470)$, $D_{15}(p\gamma)$, and $F_{35}(1890)$, in contrast to the performance of the usual h.o. theory. An extensive analysis of the pseudoscalar decays[20] not only supports the new classification of $(\underline{70}, 1)^-$ states in two distinct groups (l and u) but also explains certain anomalous cases [e.g., the $N\bar{K}$ versus $\Sigma\pi$ modes of $D_{05}(1835)$] none of which is amenable to the usual h.o. model.

The purpose of the present paper is to extend the application of the even-wave h.o. model to a more direct comparison with the data on pion photoproduction with polarized photons[21,22] and to compare its prediction with those of the conventional (full-wave) h.o. model.[23] The calculations have been carried out at two distinct levels of phenomenology, viz. (α), theory-oriented approach which makes use of the full machinery of $SU(6)_W \times O(3)$ and partial symmetry[24] to govern both pion and vector-meson couplings[25] (with the supermultiplet from factors generated by the even-wave h.o. model), and (β), an analysis-oriented approach wherein the pion couplings are essentially taken from experiment,[23] while $SU(6)_W \times O(3)$ couplings continue to govern electromagnetic interactions. The first scheme (α), being more rigid in its framework, does not admit of an *ad hoc* introduction of background, t-channel, etc., effects[1,26] which are known to be important for $\pi^{\pm}$ production; therefore we have used the α scheme only for π^0 production from protons. On the other hand, the second scheme (β) is less sensitive to the above effects insofar as the pion vertex is effectively taken from experiment, so that its predictions need not be so rigidly restricted to π^0 production only.

In Sec. II we sketch the essential details of the calculational tools used. Section III contains comparison of our results with the data and similar calculations in the conventional h.o. model. In Sec. IV we give a summary of our conclusions in relation to the contemporary approaches.

II. NECESSARY FORMALISM

In this section we summarize the main ingredients of the formalism, the details of which have been given in recent literature,[27-29] in order to keep the paper reasonably self-contained as well as to emphasize the essentially parameter-free nature of our calculations, a fact of particular relevance for a comparison of our results with the data in the context of contemporary phenomenological approaches.[23] The model consists of three distinct ingredients, viz., (i) a unified $SU(6)\times O(3)$ framework for P and V interactions of baryons resonances, (ii) orbital form factors f_L for supermultiplet transitions ($l=L$ to $l=0$) and (iii) Clebsch-Gordan coefficients of SU(6).[29] A general framework for relativistic $\bar{B}_L(P,V)B$ couplings in a tensorial language under $SU(6)_W\times O(3)$ and partial symmetry[24] has been described in some recent articles[28,29] and the results summarized in Ref. 27. A new classification of baryon resonances under the even-wave model is described in paper I, while the construction of orbital form factors f_L in this model is given in paper II. Unless otherwise specified the notations used are those of Ref. 29, for the various quantities to be described in this section. For easy reference, complete $N^*N\gamma$ couplings in a compact form are exhibited in Table I.

The partial-symmetry principle[24] in a somewhat more generalized form than originally envisaged, is contained in the following expression for the basic single-quark transition operator[29] (in emission convention for the radiation quantum):

$$M=\sum_{\alpha}\sum_{a=0}^{8}[-i(k_i-\rho P_i^{\alpha})(-\pi^a\sigma_i^{\alpha}\lambda_a^{\alpha}+\epsilon_{ijl}\sigma_j V_l^a\lambda_a^{\alpha}) - 2\vec{P}^{\alpha}\cdot\vec{V}^a\lambda_a^{\alpha}]. \quad (1)$$

TABLE I. Complete γN couplings for important N^* and Δ states. Notations and conventions are that of Ref. 17. $N=e/2m_p\cos\theta$, $h_1=\frac{4}{3}f_1(2\tau/\sqrt{3}\epsilon)^3$, $\rho'=\sqrt{\frac{2}{3}}(M-m)\rho$; A,C,D,E are defined in Eqs. (2)–(5) of the text; for other symbols $(f_0, f_0', f_1, g_0, \tau, \gamma_p, \epsilon)$, see Table II of Ref. 17.

Resonance	Classification (even-wave model)	Magnetic	Convective	Recoil
$S_{11}(1535)$	$[70,1^-]_1(8_d)^l$	$-\frac{1}{3}N(2\tau_3+1)h_1\gamma_p k^2 C$	$\frac{2}{3}N\cos^2\theta\,\tau_3 h_1 C$	$\sqrt{\frac{2}{3}}N(2\tau_3+1)h_1P'C$
$P_{11}(1470)$	$-(56)_0\sin\theta+(70)_0\cos\theta$	$-\frac{1}{3}N\sin\theta[(5\tau_3+1)f_0-(\tau_3+1)f_0']A$		
$D_{13}(1520)$	$[70,1^-]_1(8_d)^l$	$\frac{\sqrt{2}}{3}N(2\tau_3+1)h_1\gamma_pA$	$\frac{2}{\sqrt{3}}N\cos^2\theta\,\tau_3h_1E$	$\frac{\sqrt{2}}{6}N(2\tau_3+1)h_1P'E$
$D_{15}(1670)$	$[70,1^-]_1(8_q)^u$	$-\frac{1}{6}N\tan\theta(2\sqrt{3}\tau_3-1)(\tau^4g_0+f_0'/\epsilon)\gamma_pD$		
$F_{15}(1688)$	$[56,2^+]_2(8_d)^l$	$-\frac{1}{3}N(5\tau_3+1)f_0\gamma_p^2A$	$-\frac{2\sqrt{2}}{\sqrt{3}}N\cos^2\theta(\tau_3+1)f_0\gamma_p^2E$	$-\frac{1}{3}N(5\tau_3+1)f_0\gamma_pP'$
$P_{33}(1232)$	$[56,0^+]_0(10_q)$	$\frac{4}{\sqrt{3}}Nf_0D$		
$D_{33}(1670)$	$[70,1^-]_1(10_d)^l$	$\frac{1}{\sqrt{3}}Nh_1\gamma_pA$	$-\sqrt{2}N\cos^2\theta\,h_1E$	$\frac{1}{2\sqrt{3}}Nh_1P'E$
$F_{37}(1950)$	$[56,2^+]_2(10_q)$	$-\frac{4}{\sqrt{3}}Nf_0\gamma_p^2D$		

The first term represents the pseudoscalar-meson operator as generally employed in the quark model calculations. The second and third terms represent vector-meson interactions of the "magnetic" and "convective" types, respectively. [Note that the "charge" interaction ($\sim m_V V_0$) vanishes for transitions involving $l \geqslant 0$.[7,9]] It is clear that the operator (1) governs the necessary connections between $\bar{B}_L PB$ and $\bar{B}_L VB$ couplings as matrix elements of (1) between qqq states, so that they are all expressible in terms of a common form factor f_L. The relative strength ρ of the recoil coupling whose proportionality to the mass difference (M-m) follows from the Gell-Mann–Oakes–Renner effect[30] has been fixed at $0.33(M-m)$, in GeV units, to agree with $D_{13}(1520) \to \Delta\pi$ decay. The tensorial structures of the relevant $\bar{B}_L VB$ and $\bar{B}_L PB$ couplings after relativistic boosting are listed below.[27]

$\bar{B}_L VB$ couplings are as follows:

$$A \equiv \bar{\psi}^{L+1/2}_{\mu_1\cdots\mu_L}(P) i\sigma_{\mu\nu}k_\nu k_{\mu_1}\cdots k_{\mu_L} V_\mu \psi(p)\,, \tag{2}$$

$$C \equiv \bar{\psi}^{L-1/2}_{\mu_2\cdots\mu_L}(P) i\gamma_5\gamma_\mu k_{\mu_2}\cdots k_{\mu_L} V_\mu \psi(p)\,, \tag{3}$$

$$D \equiv \bar{\psi}^{L+3/2}_{\lambda\mu_1\cdots\mu_L}(P)\epsilon_{\lambda\nu\mu\sigma}k_\nu V_\mu \gamma_\sigma k_{\mu_1}\cdots k_{\mu_L}\psi(p)\,, \tag{4}$$

$$E \equiv \bar{\psi}^{L+1/2}_{\mu\mu_2\cdots\mu_L}(P) V_\mu k_{\mu_2}\cdots k_{\mu_L}\psi(p)\,. \tag{5}$$

$\bar{B}_L PB$ couplings are as follows:

$$\text{I.}\quad \bar{\psi}^{L+1/2}_{\mu_1\cdots\mu_L}(P)\gamma_5\gamma_\mu k_\mu k_{\mu_1}\cdots k_{\mu_L}\psi(p)\,, \tag{6}$$

$$\text{II.}\quad \begin{aligned}&\bar{\psi}^{L-1/2}_{\mu_2\cdots\mu_L}(P) i k_{\mu_2}\cdots k_{\mu_L}\psi(p)\,,\\ &\bar{\psi}^{L+3/2}_{\mu\mu_1\cdots\mu_L}(P) i k_\mu k_{\mu_1}\cdots k_{\mu_L}\psi(p)\,.\end{aligned} \tag{7}$$

The electromagnetic (em) interactions are simulated by the vector-meson dominance[31] assumption which amounts to the following em substitutions:

$$\rho^0_\mu \to \frac{e}{g_\rho}A_\mu\,,\quad \omega_\mu \to \frac{e}{g_\rho}\,\frac{1}{3}\,\frac{m_\rho}{m_\omega}A_\mu\,. \tag{8}$$

The fuller details on the coupling structures are given in Ref. 27. However, we would like to stress that the form factors f_L used in Ref. 27 were entirely phenomenological while they now have their origin in the dynamics of the even-wave h.o. model whose predictions on the orbital overlap integrals are summarized in Table II of Ref. 17. For ease of reference and explicit display of normalizations we have incorporated in Table I the results of paper II on the orbital overlap integrals in the form of complete relativistic Lagrangians for $N^*N\gamma$ interactions including SU(6) coefficients and scalar form factors f_L.

III. DISCUSSION OF THE RESULTS

A. Polarized-target asymmetry

In Fig. 1 we have displayed our results on polarized-target asymmetry as functions of energy

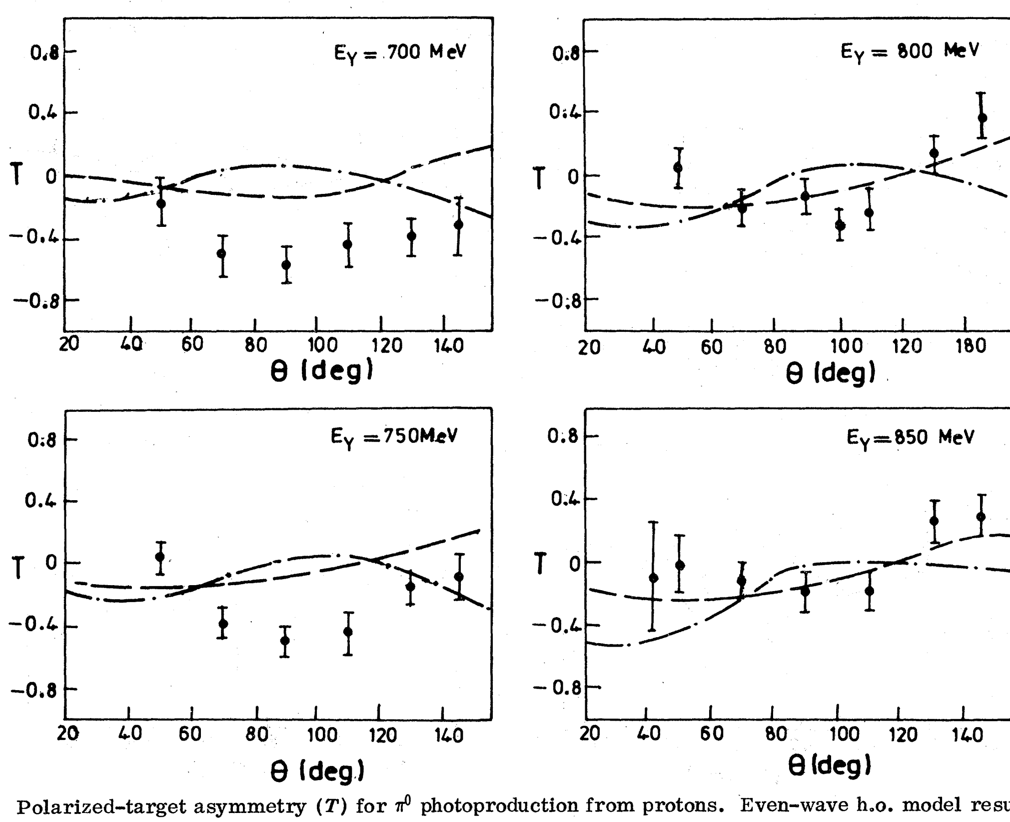

FIG. 1. Polarized-target asymmetry (T) for π^0 photoproduction from protons. Even-wave h.o. model results: dashed line, conventional h.o. model results: dot-dashed line. Data are from Ref. 21.

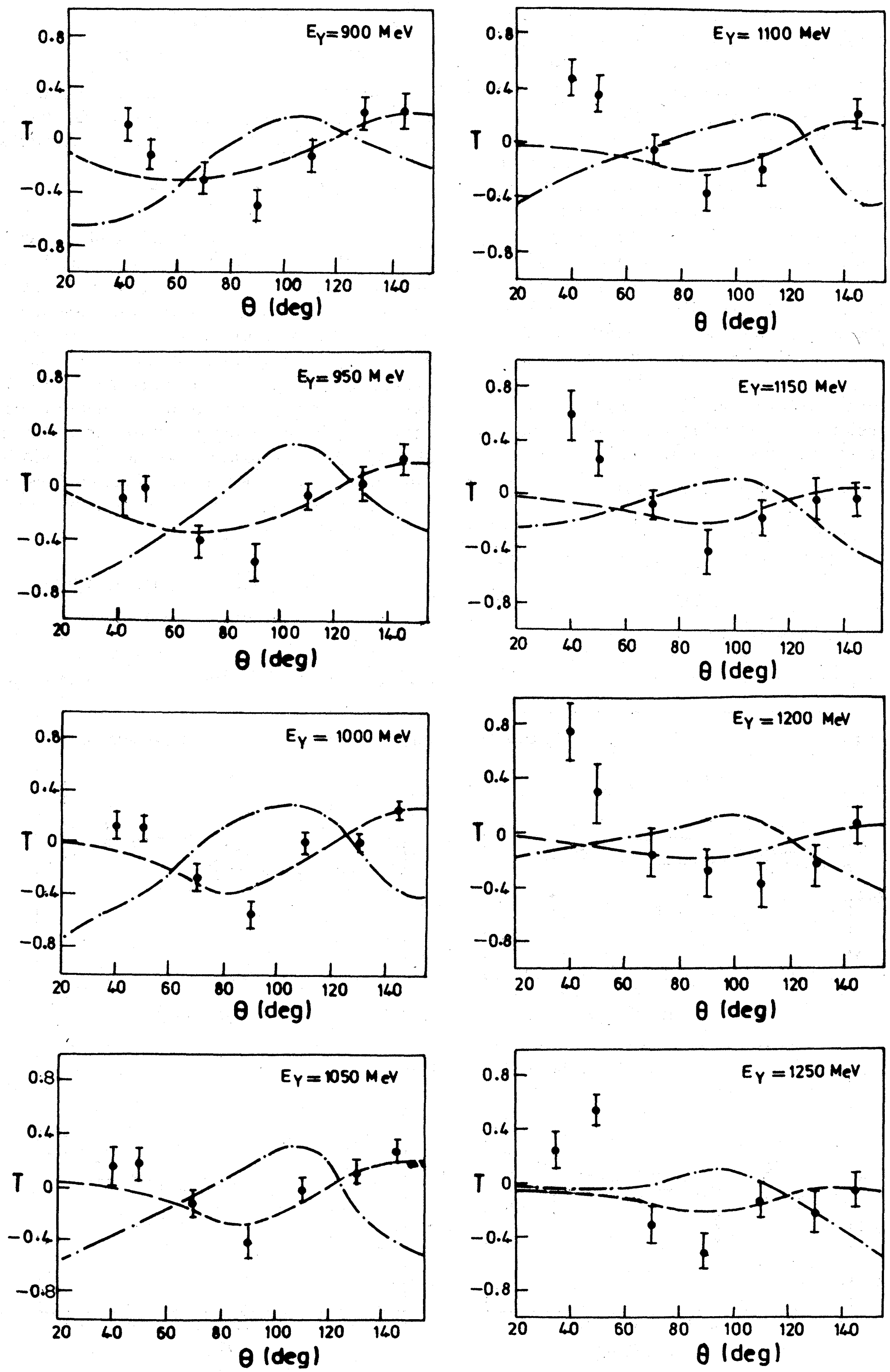

FIG. 1. (*Continued*)

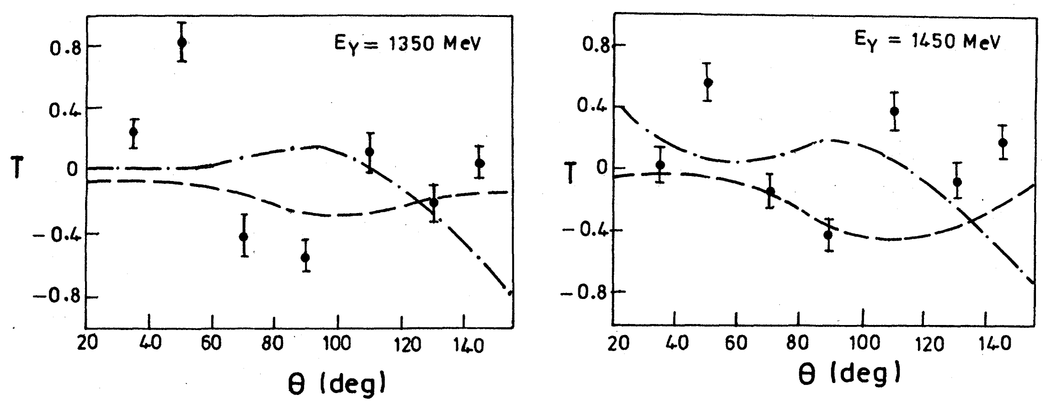

FIG. 1. (*Continued*)

and angle, along with the latest data on π° production.[21] For comparison with our calculations, we have also presented the conventional-h.o.-model predictions[8,9] carried out within our formalism. Before proceeding with the discussion of the results we may remind the reader once again that our calculations are free from any adjustable parameters, having been made under the α scheme of the Introduction. A cursory look at the diagrams indicates that we have been able to obtain a fairly good overlap with the experiment over the entire range of data. Indeed, the agreement in the energy range 800–1050 MeV is quite striking, particularly in view of the fact that there is virtually no input other than the premises of the even-wave model. The disagreement around ~700 MeV may be partly ascribed to our neglect of (smoothly varying) background effects—as distinct from the t-channel exchange contributions—which are known[1] to be quite important with lower partial waves, viz. S_{11}, P_{11}, etc.

Comparing our results with the usual h.o. results, we find that the even-wave model generally fares distinctly better than the latter, almost over the entire range of the data. The distinction is particularly conspicuous in the range 800–1050 MeV, where the conventional h.o. model is hardly able to account for the data. This is not entirely unexpected if one notes that this energy range corresponds to the set

$$P_{11}(1470),\ S_{11}(1535),\ D_{13}(1520),\ D_{33}(1670),$$
$$S_{31}(1650),\ D_{15}(1670),\ F_{15}(1688).$$

The description of $P_{11}(1470)$ in the usual h.o. model has always been a problem[2,7] and a need for mixing with the corresponding quartet states has generally been felt for the resonances $S_{11}(1535)$ (Refs. 8, 9) and $D_{13}(1520)$.[8,32] On the other hand, the success of the present model seems to reiterate its excellent description of these resonances in the case of photocouplings,[19] including the "difficult" cases of $P_{11}(1470)$ and $D_{15}(p\gamma)$ which do not admit of any simple explanation in the usual SU(6) description. At other energies the distinction is not so marked.

B. Polarized differential cross sections: $d\sigma_\parallel$, $d\sigma_\perp$

In Figs. 2 and 3 we have plotted the energy dependence of $d\sigma_\parallel$ and $d\sigma_\perp$, along with the data of

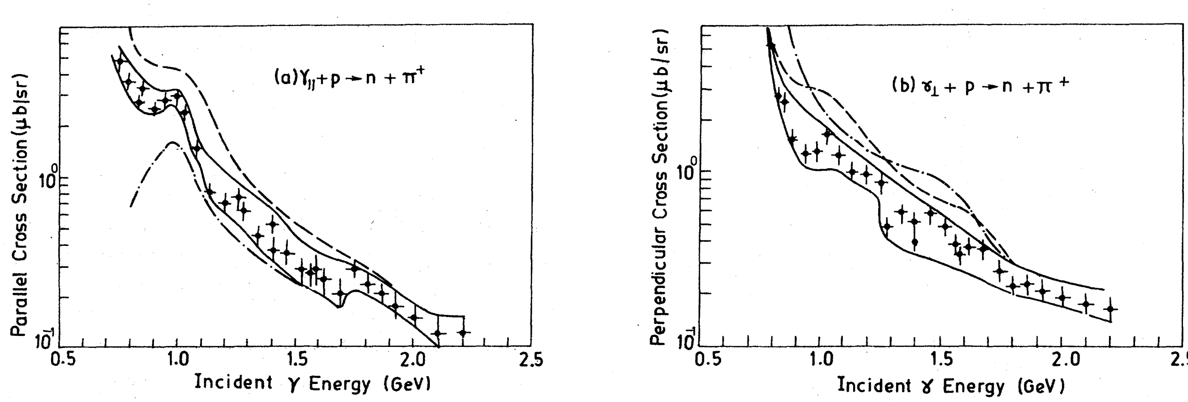

FIG. 2. $d\sigma_\parallel$ and $d\sigma_\perp$ for $\gamma_{\parallel,\perp} + p \to n + \Pi^+$ plotted in Figs. 2(a) and 2(b), respectively, as functions of lab energy of the incident photon (E_γ). Present results: dashed line; BF results: dot-dashed line. Data are from Ref. 22.

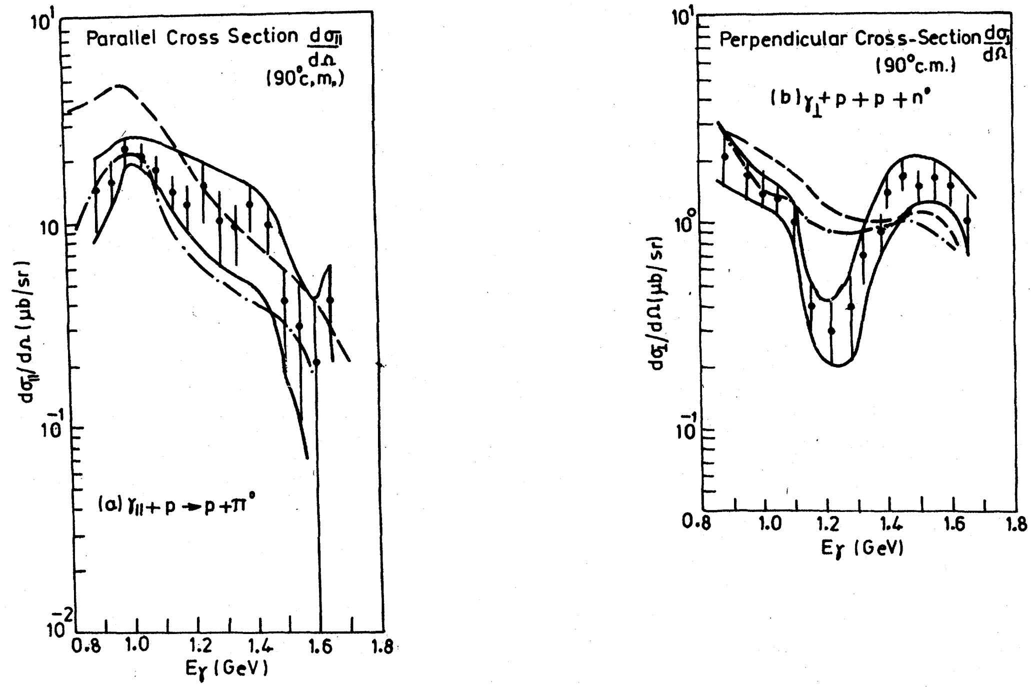

FIG. 3. $d\sigma_{\parallel}$ and $d\sigma_{\perp}$ for $\gamma_{\parallel,\perp}+p \to p+\pi^0$ plotted in Figs. 3(a) and 3(b), respectively, as functions of lab energy of the incident photon (E_γ). Present results: dashed line; BF results: dot-dashed line. Data are from Ref. 22.

Alspector *et al.*,[22] as measured in polarized photoproduction of π^+ and π^0 at 90° in the center-of-mass system. To give a better feel for comparison between the *even*- vs full-wave h.o. models the results of BF have also been displayed along with our results. The calculations of these quantities have been made under the β scheme which, as noted in the Introduction, takes the pion couplings from data and the photon couplings from $SU(6)_W \times O(3)$. A general survey of our results indicates that our results conform to the contours of the data fairly well. However, a closer scrutiny of the results as well as the data reveals that we have been able to reproduce some of the finer features of the data as well. For example, the agreement with the hump at $E_\gamma \sim 0.9$ GeV in $d\sigma_{\parallel}$ for π^+ production is quite striking. Similarly some of the structures predicted by our model may be seen in the data for $d\sigma_{\perp}$ (π^+) and $d\sigma_{\parallel}$ (π^0) [see Figs. 2(b) and 3(a), respectively]. Unfortunately the observed dip in $d\sigma_{\perp}$ for π^0 production compares unfavorably with the plateau in our curve. This descrepancy can be traced to the role of the $A_{3/2}$ amplitude for F_{37} which, according to the Moorhouse *et al.* analysis,[2] needs to be rather large compared with the h.o. model in order to tally with experiment. Unfortunately this defect cannot be eliminated by the even-wave h.o. model either, since F_{37} is a $(\underline{56}, 2^+)$ state whose status remains unaffected in this variant of the h.o. model. On the other hand, the even-wave h.o. prediction[19] for F_{37} seems to agree with the photocoupling analysis of Barbour and Crawford.[3] We would therefore be inclined not to view this discrepancy of F_{37} on $d\sigma_{\perp}$ for π^0 photoproduction too literally, pending further experimental analysis.

We have also compared our results with the "best fits" of BF which were obtained by varying the spring constant of the usual harmonic-oscillator model. We find that the even-wave model with the standard FKR value[7] of the spring constant ($\Omega \approx 1$ GeV^2), frequently fares better than the "best fit" of BF, especially in the case of $d\sigma_{\parallel}$ for both π^+ and π^0 production. For the less favorable case of $d\sigma_{\perp}$ for π^0 production, the predictions of both the models are at least similar, the dip being replaced by a plateau in both the models.

As to the negative role of $S_{11}(1535)$ found by BF in distorting the fits, this feature seems to be a characteristic of the conventional h.o. model which predicts an exceptionally large value of the photocouplings of $S_{11}(1535)$.[7,10] On the other hand, our even-wave model predicts a more moderate value for $S_{11}(1535)$ photocouplings,[19] which is also reflected in the quality of fits to the present data on photoproduction.

IV. SUMMARY AND CONCLUSION

To summarize, our purpose in the present investigation has been not so much to give an extensive phenomenological analysis of photoproduction data in the context of the even-wave model as to examine those aspects of photoproduction data where a clean comparison could be made between its predictions and those of the conventional h.o. model. With this objective in mind, we have calculated polarized-target asymmetry for π^0 photoproduction using a unified framework for both π^0 and γ couplings while the calculations of $d\sigma_{\parallel}$ and $d\sigma_{\perp}$ for π^+ and π^0 have been based on a more phenomenological framework[23] wherein the pion couplings have been effectively taken from the data. We have found that the even-wave model without any mixing parameters other than the $\underline{56}$-$\underline{70}$ $\underline{8}_a$ mixing angle determined earlier from G_A/G_V, $\Delta \to N\pi$ width and $NN\pi$ coupling constant[13,17] not only gives a good account of the photoproduction data but also fares better than the conventional h.o. model on many counts. It may be stressed that in the calculations presented in this paper no additional assumptions about classification of resonances or the mass positions have been invoked apart from the predictions of the model itself. Further, it would not be correct to regard these results on photoproduction as an isolated item of success of the even-wave h.o. model but these should more properly be taken in the context of other successes of the model, especially parameter-free fits to the mass spectra,[16] low-energy π-N parameters,[13,17] photocouplings,[19] and pseudoscalar decays,[20] in all of which it seems to fare distinctly better than the conventional h.o. model,[8-10] including the relativistic model of FKR. Now, since the even-wave h.o. model breaks only h.o. symmetry but not $SU(6)_W \times O(3)$ symmetry, we are inclined to believe on the basis of all these results that $SU(6)_W \times O(3)$ may well be a fairly respectable framework of description of resonance physics[33] but that the (much bigger) h.o. symmetry may not be as much respected by nature. The even-wave h.o. model seems to provide one possible mechanism for implementing this point of view in concrete terms. Other applications of the even-wave model [SU(3) mass breakings, etc.] are in progress.

ACKNOWLEDGMENTS

M. G. and S. K. S. would like to thank the Department of Atomic Energy (India) and C. S. I. R. (India) for predoctoral and postdoctoral fellowships, respectively.

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