PHYSICAL REVIEW D VOLUME 19, NUMBER 4 15 FEBRUARY 1979

Matter-antimatter accounting, thermodynamics, and black-hole radiation

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(Received 16 August 1978)

We discuss several issues bearing on the observed asymmetry between matter and antimatter in the content of the universe, in particular, the possible role in this of Hawking radiation from black holes, with allowance for weak C- and T-violating interactions. We show that the radiation, species by species, can be asymmetric between baryons and antibaryons. However, if baryon number is microscopically conserved there cannot be a net flux of baryon number in the radiation. Black-hole absorption from a medium with net baryon number zero can drive the medium to an asymmetric state. On the other hand, if baryon conservation is violated, a net asymmetry can develop. This can arise through asymmetric gravitational interactions of the radiated particles, and conceivably, by radiation of long-lived particles which decay asymmetrically. In the absence of microscopic baryon conservation, asymmetries can also arise from collision processes generally, say in the early stages of the universe as a whole. However, no asymmetries can develop (indeed any "initial" ones are erased) insofar as the baryon-violating interactions are in thermal equilibrium, as they might well be in the dense, high-temperature stages of the very early universe. Thus particle collisions can generate asymmetries only when nonequilibrium effects driven by cosmological expansion come into play. A scenario for baryon-number generation suggested by superunified theories is discussed in some detail. Black-hole radiation is another highly nonequilibrium process which is very efficient in producing asymmetry, given microscopic C, T, and baryon-number violation.

I. INTRODUCTION AND SUMMARY

An annoying feature of big-bang cosmology, as currently formulated, is the seeming necessity to specify nonzero values for baryon and perhaps lepton numbers. (The net electric charge, on the other hand, must be very nearly equal to zero.) It would be more attractive to suppose that the initial state is symmetric between matter and antimatter. The problem becomes more acute if baryon number is not microscopically conserved, and if its violation becomes large at the high temperatures characteristic of the early universe. For then when thermal equilibrium is established at early times baryons and antibaryons are equally numerous. In either case, if we start at early times with a situation symmetric between matter and anitmatter, we must understand how the matter and anitmatter later separated, on a scale which is sufficiently large to accord with the observation of local asymmetry,[1] or else we must understand how a universe which is symmetric early on can evolve in time into one which is asymmetric. We shall examine the latter alternative here, with special attention to the role of Hawking radiation from black holes.

Even if baryon number is microscopically conserved, it is not completely obvious that the net flux of baryon number in black-hole radiation[2,3] is zero, given the observed fact of CP violation in weak interactions. Neither baryon-number conservation for the CPT theorem is directly relevant—it is well known that the baryon number of a black hole is ill defined,[4] and the fact that a black hole is radiating gives us an arrow of time (so that the CPT theorem is not applicable). We shall show nevertheless that the net flux of baryon number is zero. Both a quantum-mechanical and a thermodynamic argument are offered for this. This shows that if baryon number is microscopically conserved then black holes can only tend to give matter-antimatter symmetry—no matter what goes in, equal numbers of baryons and antibaryons come out. It is worth remarking, however, that species by species the radiation need not be symmetric, e.g., one may have more Λ than $\bar{\Lambda}$ particles. We shall supply an explicit example of this. Only the net flux connected with absolutely conserved quantum numbers is forced to be zero.

To summarize, if baryon number is microscopically conserved then the "transcendence" of baryon number in a black hole can only make an initially asymmetric situation more symmetric; it cannot give a net baryon-antibaryon asymmetry starting from a symmetric situation.

On the other hand, *absorption* from a symmetric medium can lead to an asymmetric condition. For example, if the hole which preferentially absorbs Λ rather than $\bar{\Lambda}$ is surrounded by a medium containing equal amounts of Λ and $\bar{\Lambda}$, it will lead to a medium containing mostly $\bar{\Lambda}$.

The situation becomes more interesting if baryon number is not microscopically conserved. As we have mentioned, in this case a dynamical understanding of how matter-antimatter asymmetry can arise from a symmetric situation is not only esthetically desirable but also physically necessary if the baryon-violating interactions ever establish thermal equilibrium in the very early stages of the universe. Our earlier result that the flux of particles and antiparticles need not balance species by species, but only in the net flux of a conserved quantity, means that if baryon number is microscopically violated then there can be net baryon-number flux in black-hole radiation.

Our analysis leads to the conclusion that a net imbalance of baryons and antibaryons will arise in particle production by cosmological expansion if baryon number, C and T are violated microscopically. Cosmological particle production, but without symmetry violation, has been discussed by Parker[5] and others.

The above-mentioned effects result from interactions between matter and gravitational fields, more precisely from corrections to the minimal coupling of gravity to matter induced by C-, T-, and baryon-number-violating interactions. Even in the presence of such symmetry violations in thermal equilibrium the number of baryons and antibaryons must be equal—the number of particles is governed by the Boltzmann factor, and baryon and antibaryon have equal mass by the CPT theorem. One can, however, generate asymmetries if there are nonequilibrium processes. Nonequilibrium situations may arise because of cosmological expansion. Another, very powerful, method of generating nonequilibrium situations is through black-hole radiation, an explosive process.[2,3] We will discuss these possibilities in detail below.

The contents of the paper are as follows. In Sec. II, we shall discuss how asymmetry may develop kinetically in nonequilibrium processes. A simple thought experiment involving K mesons is used to illustrate this. The kinetic mechanisms involving cosmological expansion and black-hole radiation are compared and contrasted. A scenario suggested by superunified gauge theories is described. Some speculations regarding a possible black-hole-dominated phase of the universe are presented. In Sec. III, we prove our theorem that if baryon number is microscopically conserved then the net flux of baryon number in black-hole radiation is zero. We show by example that if, on the contrary, baryon number is not microscopically conserved then a net flux can arise. The generalization of this result to cosmological particle production is mentioned.

From the above it should be clear that we cannot claim to have a theory of the baryon-antibaryon asymmetry which gives the magic number $n_B/n_\gamma \approx 10^{-8}$. Our knowledge of microscopic baryon nonconservation (if any) and T violation is much too uncertain for that. We hope, however, to have established some rules of the game so that such questions may be rationally discussed, and to have shown that the baryon-antibaryon asymmetry need not be a "given" of cosmology but could arise dynamically from regular physical processes.

II. KINETIC PROCESSES

A. A thought experiment with K mesons

Consider a blackbody with temperature $T \gtrsim m_K$ comparable to the K-meson mass radiating into empty space. It will radiate K^0 and $\bar{K}^0$ mesons equally. In free space the mesons will decay. The decay of K mesons is a classic story.[6] For our purposes it is sufficient to recall that the time development of K^0 and $\bar{K}^0$ are described by

$$|K^0(t)\rangle = \frac{(1+|\epsilon|^2)^{1/2}}{\sqrt{2}\,(1+\epsilon)}\left(e^{-\gamma_S t/2}|K_S^0\rangle + e^{-\gamma_L t/2}|K_L^0\rangle\right), \tag{2.1}$$

$$|\bar{K}^0(t)\rangle = \frac{(1+|\epsilon|^2)^{1/2}}{\sqrt{2}\,(1-\epsilon)}\left(e^{-\gamma_S t/2}|K_S^0\rangle - e^{-\gamma_L t/2}|K_L^0\rangle\right), \tag{2.2}$$

where ϵ is a parameter which measures T violation ($\epsilon = 0$ if time-reversal symmetry is good). For our illustrative purposes we ignore the $K_L - K_S$ mass difference. Using CPT and the $\Delta S = \Delta Q$ rule we find that the amplitudes for semileptonic K_S^0, K_L^0 decay are given by

$$\langle \pi^- e^+ \nu | K_S^0\rangle = f(1+\epsilon), \tag{2.3a}$$

$$\langle \pi^+ e^- \bar{\nu} | K_S^0\rangle = f^*(1-\epsilon), \tag{2.3b}$$

$$\langle \pi^- e^+ \nu | K_L^0\rangle = f(1+\epsilon), \tag{2.3c}$$

$$\langle \pi^+ e^- \bar{\nu} | K_L^0\rangle = -f^*(1-\epsilon), \tag{2.3d}$$

where f is a form factor which for our purposes we take to be constant. Combining these formulas we find that the total yields from the various possible semileptonic decays, integrated over time, are proportional to

$K^0 \to \pi^- e^+ \nu$:

$$\left(\frac{1}{\gamma_S} + \frac{1}{\gamma_L} + \frac{4}{\gamma_S + \gamma_L}\right), \tag{2.4a}$$

$K^0 \to \pi^+ e^- \bar{\nu}$:

$$\left|\frac{1-\epsilon}{1+\epsilon}\right|^2\left(\frac{1}{\gamma_S}+\frac{1}{\gamma_L}-\frac{4}{\gamma_S+\gamma_L}\right), \tag{2.4b}$$

$\bar{K}^0 \to \pi^- e^+ \nu$:

$$\left|\frac{1+\epsilon}{1-\epsilon}\right|^2\left(\frac{1}{\gamma_S}+\frac{1}{\gamma_L}-\frac{4}{\gamma_S+\gamma_L}\right), \tag{2.4c}$$

$\bar{K}^0 \to \pi^+ e^- \bar{\nu}$:

$$\left(\frac{1}{\gamma_S}+\frac{1}{\gamma_L}+\frac{4}{\gamma_S+\gamma_L}\right). \tag{2.4d}$$

Since experimentally $\epsilon = 1.5\times 10^{-3}$ and $\gamma_S^{-1} \ll \gamma_L^{-1}$ we find that in the radiation there is a preponderance of positrons over electrons by roughly a part in 10^3.

Needless to say, lepton number, which is microscopically conserved, remains zero. With the preponderance of e^+ over e^- comes a preponderance of ν over $\bar{\nu}$. In equilibrium, the back-reactions $e^+\pi^-\nu \to K$, etc., would restore the Boltzmann distribution, with equal numbers of e^+ and e^-. (This example is imperfect because the $\pi^{\pm}$ mesons will eventually decay into $e^{\pm}$, restoring the balance. So our asymmetry is short lived. This problem, however, is practical rather than conceptual. One could, for instance, imagine a world with T violation and a stable π meson.)

This thought experiment illustrates how an asymmetry between matter and antimatter may arise from the interplay of C and T violation and a nonequilibrium process. If our blackbody radiator is replaced by a radiating black hole and the K mesons by some heavy mesons whose decays violate C, T, and baryon number, we see how a baryon-antibaryon asymmetry might be induced. More explicitly, suppose there is a meson M and antimeson $\bar{M}$ with the decay channels $M \to p+\bar{p}$, $p+e$, $\bar{M} \to p+\bar{p}$, $\bar{p}+\bar{e}$. M and $\bar{M}$ mix through the $p+\bar{p}$ channel, and a proper analysis of their decays would proceed much like the case of K mesons. If C and T are violated, a net baryon number would arise from the decay in free space of an equal mixture of M and $\bar{M}$.

The radiation of a blackbody into free space is of course closely related to the behavior of blackbody radiation subject to rapid expansion. Therefore the same process, production of an imbalance from the interplay of baryon-number-, C-, and T-violating interactions with a nonequilibrium process, could be driven by rapid cosmological expansion in the early stages of the universe.

B. General discussion of kinetics of expansion

As we have just seen, in the presence of baryon-number-, C-, and T-violating interactions, nonequilibrium processes can generate a net baryon number starting from zero baryon number. We have in mind two mechanisms which may lead to cosmologically significant disequilibrium: black-hole radiation and cosmological expansion. Black-hole radiation is to a first approximation radiation into empty space, so its kinetics is very simple. Cosmological expansion is much more complicated —one must investigate the behavior of matter at a high and rapidly changing temperature. We shall now discuss this more complicated case. The two mechanisms are compared and contrasted in Sec. II D.

We shall show in Sec. II C that insofar as particle masses are negligible (i.e., the temperature is much higher than the rest mass of the particles present) no asymmetry can arise. Furthermore, as we have mentioned, no asymmetry can arise from equilibrium processes. We therefore are concerned with estimating when a massive particle goes out of equilibrium. Its subsequent decays (or reactions) can then generate asymmetries.

For definiteness we consider a meson of mass M which couples to two-fermion (quark or lepton) channels with electromagnetic strength. In our estimates, we keep only Born terms. This is in the spirit of asymptotic freedom, which is presumably very good at the relevant high temperatures. This discussion, of course, could be generalized. Let the temperature be T. The characteristic expansion time is then $(G^{1/2}T^2)^{-1}$, where G is the gravitational constant. As T decreases below M, the reactions which create the heavy mesons cease. If the decrease of temperature is slow enough annihilation reactions leading to decrease in the number of heavy mesons will proceed, decreasing the density of mesons in line with the Boltzmann factor $e^{-M/T}$. However, if the annihilation reactions are slow compared to the characteristic time, then the distribution of mesons is no longer in equilibrium. Let us estimate the temperature at which this occurs.

(i) Pair annihilation: Two heavy mesons may annihilate into two quarks or leptons. The rate of this per meson in equilibrium is $\sim(\alpha/\pi)^2(T^{3/2}/M^{1/2})e^{-M/T}$ for $M > T$ and $\sim(\alpha/\pi)^2 T$ for $M \leq T$, and becomes comparable to the characteristic inverse expansion time when

$$\left(\frac{\alpha}{\pi}\right)^2 \frac{1}{M^{1/2}} T_d^{3/2} e^{-M/T} \sim G^{1/2} T_d^2 \quad (M \geq T), \tag{2.5a}$$

$$\left(\frac{\alpha}{\pi}\right)^2 T_d \sim G^{1/2} T_d^2 \quad (M \leq T). \tag{2.5b}$$

Numerically, this gives $T_d \approx M/8$ for $M = 10^6$ GeV, $T_d \approx M/4$ for $M = 10^{10}$ GeV. In either case the density $\sim e^{-M/T}$ of mesons is small at the decoupling time. When $M \approx 10^{12}$ GeV, T_d becomes comparable to M, and when $M \geqslant 10^{13}$ GeV the reaction is *never* in equilibrium.

(ii) Annihilation against light particles: A heavy meson and a quark or lepton may annihilate into a quark or lepton plus (say) a photon. This decouples roughly when

$$\left(\frac{\alpha}{\pi}\right)^2 \frac{1}{M^2} T_d{}^3 \sim G^{1/2} T_d{}^2 \quad (M \geqslant T), \tag{2.6a}$$

$$\left(\frac{\alpha}{\pi}\right)^2 T_d \sim G^{1/2} T_d{}^2 \quad (M \leqslant T). \tag{2.6b}$$

which gives $T_d \sim 10^{-3}$ GeV for $M = 10^5$ GeV, $T_d \sim 10^7$ GeV for $M = 10^{10}$ GeV. In either case, this process dominates the pair annihilation and shows that very few heavy mesons are present at the decoupling time. When $M \geqslant 10^{13}$ GeV, Eq. (2.6) cannot be satisfied for any value of T.

(iii) Decay: The decay of the heavy mesons will occur with a rate $(\alpha/\pi)M$. If $M \geqslant T_d$, the rate dominates the previous two until decoupling. In any case, decays will dominate once $T \leqslant (\pi/\alpha)M$.

(iv) Inverse decay: Production of heavy mesons by inverse decays occurs at a rate $\sim(\alpha/\pi)M e^{-M/T}$ $(M \gtrsim T)$ or $\sim(\alpha/\pi)T$ $(M \lesssim T)$. At early enough times particles of mass up to $(\alpha/\pi)G^{-1/2} \approx 10^{16}$ GeV will be brought into equilibrium by this process.

If $M \geqslant 10^{16}$ GeV we have a very peculiar situation. The distribution of heavy mesons is never brought into equilibrium. Furthermore, such mesons decay before they interact once $T \leqslant (\pi/\alpha)M$. For these reasons it is problematical to estimate how many such mesons would be created in the history of the universe. If we assume, however, that in the earliest stages the density of mesons is not as singular as T^3, we can estimate the contribution from their decays to the net baryon number. The mesons are produced [from reaction (iv) above] at roughly the rates

$$r \sim \left(\frac{\alpha}{\pi}\right) \frac{T^6}{M^2 + T^2} e^{-M/T} \tag{2.7}$$

per unit time per unit volume at temperature T. Suppose the meson decays into baryons more than antibaryons by a fraction ϵ. A baryon produced at temperature T contributes proportional to $(T_0/T)^3$ to the present density, where T_0 is the present temperature. Putting it all together, the contribution of heavy-meson decay to the present baryon density is approximately $(dt = dT/G^{1/2}T^3)$

$$n_B \approx \int_M^{(\pi/\alpha)M} \epsilon\left(\frac{\alpha}{\pi}\right) \frac{T^6}{M^2+T^2}\left(\frac{T_0}{T}\right)^3 \frac{dT}{G^{1/2}T^3}$$
$$\approx \frac{\epsilon(\alpha/\pi)\,T_0{}^3}{G^{1/2}M}. \tag{2.8}$$

The limits on the integral are determined by the exponential cutoff in (2.7), and the temperature at which annihilation dominates decay. The contribution to n_B/n_γ is then about $\epsilon(\alpha/\pi)/G^{1/2}M$. Such a number could be close to the desired 10^{-9} for $\epsilon = 10^{-6}$, $M = 10^{18}$ GeV as might be suggested by ideas of superunification.[7,8] Notice that in this picture the baryons are produced at such an early time that the standard big-bang scenario for the later stages is unaffected.

In this discussion (for $M \gtrsim 10^{16}$ GeV) we have assumed that initially there were no heavy mesons. An alternative scenario [developed by one of the authors (F. W.) in conversations with S. Weinberg[9]] assumes that by some (quantum gravitational?) mechanism the heavy mesons do follow a Boltzmann distribution at the highest temperatures $T \gg M$. As we have seen, the fastest processes involving the heavy mesons are decays; they become important at times $t_D \approx [(\alpha/\pi)M]^{-1}$ or temperatures $T_D = G^{-1/4} t_D{}^{-1/2} \approx [G^{-1/2}(\alpha/\pi)M]^{1/2}$. Until this time the heavy mesons initially present simply red-shift freely and therefore are overabundant compared to the Boltzmann distribution for a massive particle. In fact the number of heavy mesons is just the same as the number of photons. The heavy mesons then decay away (if $M \gtrsim T_D$ back reactions creating the heavy mesons are unimportant); the net asymmetry per photon produced is then remarkably simple:

$$\frac{n_B}{n_\gamma} \approx \epsilon \frac{n_{\text{heavy}}}{n_\gamma} \approx \epsilon\,. \tag{2.9}$$

Finally, we will briefly discuss the regime $M \lesssim 10^{16}$ GeV, which may be the most interesting case.[8] A very crude, preliminary analysis of this case is as follows. Because massless particles generate no asymmetry, we concentrate again on the heavy mesons. Now these are *forced* into equilibrium at early times. Asymmetry can be generated when the temperatures reach $T \approx M$, where there are significant numbers of heavy mesons (roughly equal to the number of photons) and cosmological expansion drives their distribution out of equilibrium (see part C below). The asymmetry produced will be the asymmetry per decay multiplied by a parameter characterizing the "nonequilibrium" character of expansion, i.e.,

$$\frac{n_B}{n_\gamma} \approx \epsilon \frac{t_c(T=M)}{t(T=M)} \approx \epsilon \frac{\alpha}{\pi} G^{1/2} M\,, \tag{2.10}$$

where $t_c \approx [(\alpha/\pi)T]^{-1}$ is a characteristic interaction time and $t = \dot{R}/R \approx (G^{1/2}T^2)^{-1}$ is a characteristic expansion time. Any baryon number produced will be partially thermalized (driven to zero) by later interactions.[10] We believe that all these effects may be accurately taken into account using the fluctuation-dissipation theorems of statistical mechanics. Calculations of this kind, together with a calculation of ϵ, will be presented in a subsequent paper.

C. Absence of asymmetry for massless particles

One can prove a little theorem that reactions among massless particles do not give particle-antiparticle asymmetries from cosmological expansion. This means that such asymmetries will be characterized by parameters $\sim m/T$ to a power, in addition to other parameters of smallness, when the relevant masses m are much less than the temperature.

It is important to distinguish between kinetic and chemical equilibrium. Kinetic equilibrium, distribution of energies and momenta according to the Boltzmann factor, is enforced by the presence of any collisions at all. Chemical equilibrium may be established only by much slower interactions and in fact never reached in an expanding universe. In the case at hand, baryon-number-violating processes may be rare and although in true equilibrium the baryon number would be zero such equilibrium may never be established. On the other hand, kinetic equilibrium should be a good approximation in the early universe.

Taking into account collisions and cosmological expansion, we have for the densities

$$\frac{dn_i(p_i)}{dt} = \sum_{\substack{jkl \\ p_j p_k p_l}} \left[-\phi_{p_i p_j p_k p_l} \left| \langle k p_k l p_l | T | i p_i j p_j \rangle \right|^2 n_i(p_i) n_j(p_j) + \phi_{p_k p_l p_i p_j} \left| \langle i p_i j p_j | T | k p_k l p_l \rangle \right|^2 n_k(p_k) n_l(p_l) \right] - K p_i \frac{\partial n_i(p_i)}{\partial p_i} . \tag{2.11}$$

Here the Latin indices denote particle type, and $K = \dot{R}/R$ is the expansion rate. The last term indicates the effect of the cosmological expansion, which for massless particles is a simple red-shift. ϕ is the phase space (and statistics) factor, and T is the usual scattering matrix. We claim (2.11) is solved by

$$n_i(p_i) = e^{-p_i/T(t)}, \qquad \frac{dT}{dt} = -KT. \tag{2.12}$$

Indeed, with the particle number independent of species the first two terms on the right-hand side of (2.11) cancel by the completeness relation $T^\dagger T = TT^\dagger$, and the equality is a trivial calculation. This shows that in the approximation of massless particles cosmological expansion does not induce particle-antiparticle asymmetry. We disagree in this with the calculation of Yoshimura.[11] For massive particles the proof does not work because the expansion term (red-shift) is not simply $-Kp\partial n/\partial p$.

From a deeper point of view the essential ingredients of the preceding argument are the existence of thermal equilibrium and the fact that free expansion of a gas of massless particles is adiabactic. Therefore, free expansion of a gas of massless particles will always reproduce the equilibrium distribution.

We note parenthetically that the use of the completeness relation $TT^\dagger = T^\dagger T$ as a substitute for detailed balance in the proof of the existence of thermal equilibrium is completely general, not tied to massless particles. This follows immediately from the analysis of Ref. (12), although it is not noted there.

D. Comparison of black-hole radiation and cosmological expansion kinetics

Let us compare the two kinetic mechanisms, black-hole radiation and cosmological expansion.

(1) Black-hole radiation of particles is governed simply by the mass of the particles. Thus even very weakly interacting particles (such as we might need to give small baryon-number violation) can be copiously produced if they are light. Then their decay could give large asymmetries, as in our K-meson example. We never run into the problem of particles whose production cannot be estimated, as in Sec. II B. On the other hand since the characteristic temperature $T = 1/8\pi GM$ of a black hole of mass M is inversely proportional to the mass, very heavy particles do not get produced until the black hole has lost most of its mass. For instance, $T = 10^5$ GeV requires a black-hole mass $M = 10^7$ g. Only with large asymmetries in the decay and very many small black holes could one generate a sufficient baryon number from such particles.

(2) Particles which annihilated rapidly in pairs but whose decays violated baryon number would generate an asymmetry by the black-hole mechanism but would not survive long enough to produce asymmetry in cosmological expansion.

(3) There are deep questions of cosmology connected with each mechanism: How hot does it get in the early universe? Is there a limiting temperature for hadronic matter? What was the spectrum of black holes in the early universe? An interesting speculation concerning the second question is that the present stage of the universe results from a cosmological "bounce" from a pre-

vious collapsing stage. It is then an interesting question which could be investigated mathematically whether the collapse results in a large population of black holes whose subsequent explosion triggers the big bang. This idea, if it can be realized, would have some other advantages:

(a) The violation of the weak energy condition associated with black holes allows one to circumvent the singularity theorem and could conceivably lead to a "bounce."

(b) In this picture the universe might never get very hot (except locally, at the surface of black holes). Thus problems associated with restoration of symmetries at high temperatures and subsequent formation of domain walls need not arise.

(c) Most attractive conceptually, this type of cosmology completely eliminates the necessity of specifying initial conditions in addition to ordinary physical laws.

Finally, an important remark of a general nature: It has often been objected against cyclic cosmologies that each cycle increases the entropy, so that the present finite value of the entropy per baryon would militate against an infinite number of cycles. In theories of the type discussed in this paper, this objection is baseless. The entropy per baryon is determined dynamically by physical processes, at extreme temperatures, which are the same for each cycle. The value of the entropy per baryon is therefore independent of the number of cycles.

III. INTERACTIONS WITH FIELDS

A. Theorem on matter-antimatter balance

We now show that in a world with microscopic baryon-number conservation the net flux of baryon number in Hawking radiation is zero. We shall set up slightly more machinery than is strictly necessary to prove this, so that later we can easily analyze the effect of dropping the assumption of microscopic baryon conservation.

Only interactions odd under C and T can possibly generate asymmetric radiation. (Terms odd under parity are of no interest. They might generate a preponderance of particles over antiparticles in one hemisphere around a rotating black hole, but there would be a reversed imbalance in the other hemisphere and no net asymmetry.) To allow for such effects we take into account matter interactions, in particular the weak C- and T-violating interactions: For example, loop diagrams which describe quark-graviton scattering, corrected by exchange of virtual W bosons and involving C- and T-violating quark-W-boson vertices.

When the gravitational field is weak we can summarize the matter field interactions by means of an effective local Lagrangian. The corrections to the "free field" Lagrangian will of course be very complicated, including strong-interaction effects as well as the weak C- and T-violating effects of interest here. These corrections will alter the details of the Hawking radiation, but cannot introduce any asymmetry between matter and antimatter. We focus therefore only on the C- and T-violating interactions between matter and gravity. These introduce an asymmetry between particles and antiparticles in their propagation through the background gravitational field in the vicinity of the black hole. For illustrative purposes only, let us consider a set of (three or more) complex scalar fields Φ_i, $1 \le i \le N$. Restricting ourselves for simplicity to terms bilinear in these fields, we may take the following Lagrangian as representative of the effects under discussion:

$$\mathcal{L} = \sqrt{g}\,(g_{\mu\nu}\partial_\mu \Phi_i^* \partial_\nu \Phi_i - m_i^2 \Phi_i^* \Phi_i - \Phi_i^* V_{ij} \Phi_j R_{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}). \quad (3.1)$$

Here $R_{\alpha\beta\gamma\delta}$ is the curvature tensor, and V_{ij} is a matrix whose details depend on the details of the weak interactions [recall that (3.1) is not to be taken as a fundamental Lagrangian, but as an effective Lagrangian summarizing the weak corrections to gravity]. The matrix V_{ij} is of course Hermitian but not necessarily real and can lead to C and T violation. The Lagrangian (3.1) may serve to illustrate both the cases where there is a conserved "baryon number" and where there is not: The total number of Φ quanta of all types is conserved, since $\mathcal{L}$ is invariant under the multiplication of all the Φ_i by a common phase, but the number of quanta of species $1, 2, \ldots, N$ are not separately conserved. For the remainder of this section we will analyze the case of a microscopically conserved quantum number and call the quanta of Φ_i baryons, the quanta of Φ_i^* antibaryons.

The rate of Hawking radiation is a product of two factors. One is a universal factor, the same for particles and antiparticles and irrespective of type i; roughly speaking, it corresponds to blackbody emission at the event horizon. The other factor describes transmission of the emitted objects through the gravitational field outside the event horizon. The net effect corresponds to graybody emission, with graybody coefficients that depend on the transmission. In our example the transmission phenomena for baryons involves transformations among the various types, and similarly for antibaryons. The former effects are governed by the field equations for Φ_i, the latter by the equations for Φ_i^*.

To evaluate the transmission coefficients we should now solve for the probability, given the Lagrangian (3.1), of particles and antiparticles emitted at the horizon to emerge at infinity. We shall abstract from this a model problem which contains the essence of the phenomena (it amounts to a generalization of the radial equation one would derive by separating the field equations in appropriate coordinates). The model problem is that of one-dimensional transmission through a position-dependent matrix potential, governed by the Lagrangian

$$\mathcal{L} = \dot{\Phi}_i^* \dot{\Phi}_i - \frac{\partial}{\partial x}\Phi_i^* \frac{\partial}{\partial x}\Phi_i - \Phi_i^*(x) V_{ij}(x)\Phi_j(x). \qquad (3.2)$$

All of our remarks about C, T, and baryon number apply equally as well to the Lagrangian of Eq. (3.2) as to that of Eq. (3.1). We assume (for simplicity) that $V_{ij}(x)$ vanishes as $|x| \to \infty$.

Let us introduce a $2N$-component complex vector

$$v \equiv \begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_N \\ b_1 \\ \cdot \\ \cdot \\ \cdot \\ b_N \end{bmatrix} \equiv \begin{pmatrix} a \\ b \end{pmatrix}, \qquad (3.3)$$

which describes the situation where we have an outgoing wave $a_1 e^{ik(x-t)}$ and an incoming wave $b_1 e^{ik(-x-t)}$ as $x \to -\infty$ associated with a particle of species 1, outgoing wave $a_2 e^{ik(x-t)}$ and incoming wave $b_2 e^{ik(-x-t)}$ as $x \to -\infty$ for species 2, etc. Similarly we introduce

$$w \equiv \begin{bmatrix} c_1 \\ \cdot \\ \cdot \\ \cdot \\ c_N \\ d_1 \\ \cdot \\ \cdot \\ \cdot \\ d_N \end{bmatrix} \equiv \begin{pmatrix} c \\ d \end{pmatrix}, \qquad (3.4)$$

which describes an outgoing wave $c_1 e^{ik(x-t)}$ and an incoming wave $d_1 e^{ik(-x-t)}$ for species 1 as $x \to +\infty$, and so forth. There is a linear relationship between the fields at $\pm\infty$ if the fields are governed by the Lagrangian Eq. (3.2); we write this as

$$\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \equiv \mathfrak{M} \begin{pmatrix} a \\ b \end{pmatrix}. \qquad (3.5)$$

The baryon number flux $|a|^2 - |b|^2$ at $-\infty$ must be equal to the baryon number flux $|c|^2 - |d|^2$ at $+\infty$, for any choice of the vectors a, b. This implies

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} A^\dagger & C^\dagger \\ B^\dagger & D^\dagger \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \qquad (3.6)$$

or, after a little algebra, the three matrix equations

$$A^\dagger A - C^\dagger C = 1, \qquad (3.7a)$$

$$D^\dagger D - B^\dagger B = 1, \qquad (3.7b)$$

$$A^\dagger B = C^\dagger D. \qquad (3.7c)$$

The transmission probabilities are readily expressed in terms of A. Indeed, the total baryon transmission for a particle of species j is governed by requiring that the outgoing waves at $-\infty$ be only a unit flux of species j and that there be no incoming wave at $+\infty$, i.e.,

$$\begin{pmatrix} t^{(j)} \\ 0 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \vec{e}_j \\ r^{(j)} \end{pmatrix}, \qquad (3.8)$$

where $\vec{e}_j$ is the vector with unit entry in the jth slot and zeros elsewhere. Multiplying both sides by

$$\mathfrak{M}^{-1} = \begin{pmatrix} A^\dagger & -C^\dagger \\ -B^\dagger & D^\dagger \end{pmatrix} \qquad (3.9)$$

leads to

$$A^\dagger t^{(j)} = \vec{e}_j, \quad t^{(j)} = (A^\dagger)^{-1}\vec{e}_j \,. \qquad (3.10)$$

[It follows from (3.7a) that A is invertible.] Thus the transmission probability for a unit incoming wave of particle species j is

$$|t^{(j)}|^2 = \vec{e}_j A^{-1}(A^+)^{-1}\vec{e}_j \,. \qquad (3.11)$$

And the net flux for all species j is

$$\sum_j |t^{(j)}|^2 = \mathrm{tr} A^{-1}(A^+)^{-1} \,. \qquad (3.12)$$

We must compare this to the total flux of antiparticles. If the particles are governed by the potential $V_{ij}(x)$, the antiparticles are governed by $V_{ji}(x) = V_{ij}^*(x)$, as one can see by comparing the equations of motion for Φ_i and Φ_i^* following from the Lagrangian Eq. (3.2). Let us call the corresponding transmission matrix for antiparticles

$$\bar{\mathfrak{M}} = \begin{pmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{pmatrix}. \qquad (3.13)$$

Then one can show by manipulating the relevant Schrödinger equations and boundary conditions that

$$A = \bar{D}^*, \quad D = \bar{A}^*. \qquad (3.14)$$

Before deriving this from the Schrödinger equation, we should remark that the same results follow from the deeper principle of CPT invariance. Namely, the amplitude for an outgoing particle of type i at $-\infty$ to propagate to a particle of type j

at $+\infty$ is the complex conjugate of the amplitude for an incoming antiparticle of type j at $-\infty$ to propagate to an antiparticle of type i at $+\infty$. Writing this verbal statement in equations gives simply Eq. (3.14).

To see Eq. (3.14) directly from the Schrödinger equation, notice that the matrix A is defined as the solution of the problem

$$\frac{\partial^2\psi_i^{(j)}}{\partial t^2}-\frac{\partial^2\psi_i^{(j)}}{\partial x^2}+V_{ik}\psi_k^{(j)}=0 \quad , \tag{3.15a}$$

subject to

$$\psi_i^{(j)}(x,t)\to e^{ik(x-t)}\delta_{ij}+\text{incoming waves}, \quad x\to-\infty \tag{3.15b}$$

$$\psi_i^{(j)}(x,t)\to e^{ik(x-t)}A_{ij}, \quad x\to+\infty. \tag{3.15c}$$

Similarly $\overline{D}$ is the solution of the problem

$$\frac{\partial^2\psi_i^{(j)}}{\partial t^2}-\frac{\partial^2\psi_i^{(j)}}{\partial x^2}+V_{ki}\psi_k^{(j)}=0, \tag{3.16a}$$

subject to

$$\psi_i^{(j)}(x,t)\to e^{ik(-x-t)}\delta_{ij}+\text{outgoing waves}, \quad x\to-\infty \tag{3.16b}$$

$$\psi_i^{(j)}(x,t)\to e^{ik(-x-t)}\overline{D}_{ij}, \quad x\to+\infty. \tag{3.16c}$$

[Notice the transposition of V in Eq. (3.16a), which arises because the antiparticles are governed by the equation of motion for Φ^* derived from the Lagrangian (3.2).] Now complex conjugating all the equations [(3.16a)–(3.16c)] and changing the sign of time (which leaves the equation of motion invariant) maps a solution of [(3.16a)–(3.16c)] onto a solution of [(3.15a)–(3.15c)], with $A_{ij}=\overline{D}_{ij}^*$. This proves (3.14).

Now the total flux of particles minus the total number of antiparticles, starting from unit amplitude for all species at $-\infty$, is, from (3.12) and(3.14),

$$\operatorname{tr}A^{-1}(A^\dagger)^{-1}-\operatorname{tr}\overline{A}^{-1}(\overline{A}^\dagger)^{-1}=\operatorname{tr}A^{-1}(A^\dagger)^{-1}-\operatorname{tr}(D^{*\dagger})^{-1}D^{*-1}. \tag{3.17}$$

Use of the unitarity equations (3.7) and cyclic invariance of the trace gives

$$\operatorname{tr}A^{-1}(A^\dagger)^{-1}=\operatorname{tr}(A^\dagger)^{-1}A^{-1}=\operatorname{tr}[1-(A^\dagger)^{-1}C^\dagger CA^{-1}],$$

$$\operatorname{tr}(D^{*\dagger})^{-1}(D^*)^{-1}=\operatorname{tr}(D^\dagger)^{-1}D^{-1}=\operatorname{tr}[1-(D^\dagger)^{-1}B^\dagger BD^{-1}]. \tag{3.18}$$

But according to Eq. (3.7c) $(A^+)^{-1}C^+=BD^{-1}$, so we see that the final traces in both equations are equal. Thus the total flux of particles is equal to the total flux of antiparticles.

This proof[13] requires several comments:

(1) We have assumed that the potential turns off at both $\pm\infty$. In the realistic black-hole case the potential does turn off at $-\infty$ (the event horizon) but not at $+\infty$ (i.e., the particles may have nonvanishing mass). The proof actually works for this situation too, with only notational modifications.

(2) Although we couched our derivation in the language of one-particle quantum mechanics, clearly the ingredients of the proof are very general: basically CPT invariance and unitarity.

(3) The proof involves use of manipulations inside the trace which are not general matrix identities— which is a particularly obscure way of saying that it does not imply equality of particle and antiparticle flux species by species.

B. Thermodynamic argument

We now show that the theorem on matter-antimatter balance just proved follows also from a thermodynamic argument. It is reassuring to see that thermodynamic principles retain their vitality even in a world with T violation.

Consider the box depicted in Fig. 1, consisting of two compartments at temperature T and chemical potential zero separated by a semipermeable membrane characterized by the potential $V_{ij}(x)$. Suppose that baryon number is conserved, and that baryons (summed over species) penetrate $V_{ij}(x)$ from left to right more readily than antibaryons (summed over species). It follows from the PCT theorem that from right to left this membrane will allow anitbaryons to penetrate more readily than baryons. We see that such a membrane would act as a Maxwell demon for baryon number, separating baryons from antibaryons. In time, the two sides of the box would become distinguishable, contrary to the zeroth law of thermodynamics (uniqueness of thermal equilibrium). Alternatively one could set up a pipe connecting the two compartments, and extract work from the diffusion gradient, violating the second

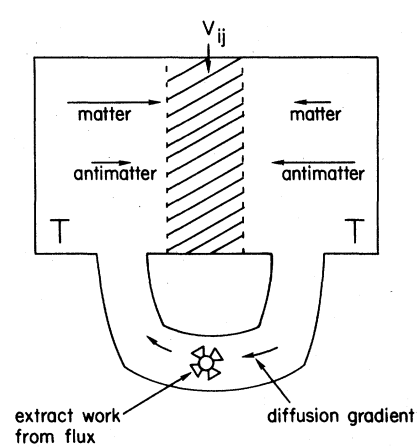

FIG. 1. Box illustrating the thermodynamic argument for zero net baryon number. See text.

law. It follows that thermodynamics does not permit a membrane with these properties, in agreement with the previous quantum-mechanical argument.

Notice that a net flux of any nonconserved quantity across the membrane is permitted. Such fluxes do not lead to different equilibrium situations in the two compartments, because information concerning the nonconserved quantum numbers is completely lost in thermal equilibrium. We now establish that such fluxes do indeed occur.

C. Example of imbalance

For our example we will use notations as before with two species 1, 2 of scalar fields and the potential

$$V(x) = M_1\delta\left(x+\frac{\alpha}{2}\right)+M_2\delta\left(x-\frac{\alpha}{2}\right), \tag{3.19}$$

where M_1 and M_2 are two-by-two Hermitian matrices. We assume species 1 is distinguished from species 2 by interactions [other than the corrections to gravity summarized by Eq. (3.19)] whose details need not concern us—e.g., species 1 might be strongly interacting while species 2 is not. We shall call particles of species 1 "baryons" and particles of species 2 "leptons"—so if M_1 or M_2 has off-diagonal components baryon and lepton number are not separately conserved.

By standard quantum-mechanical methods it is not difficult to compute the transmission matrix for this problem; in our previous notations the matrix A is (up to an overall phase)

$$A = 1 - i(M_1+M_2) + (\alpha-1)M_2M_1, \tag{3.20}$$

where $\alpha = e^{-2ika}$ is the phase change for a round trip between the two δ functions, and we have absorbed a factor $1/k$ into the definition of M_1 and M_2. In the present case the transmission probability of baryons is given not by the trace of $A^{-1}(A^\dagger)^{-1}$, but by its 11 component,

$$T_B \equiv \sum_j |t_1^{(j)}|^2 = [A^{-1}(A^\dagger)^{-1}]_{11} = (A^\dagger A)^{-1}{}_{11}, \tag{3.21}$$

as one readily sees from Eq. (3.10). One could of course solve for this directly given Eq. (3.20), but much simpler expressions result if one expands T_B in a power series in M_1, M_2:

$$(A^\dagger A)^{-1} = 1 - \alpha M_2M_1 - \alpha^* M_1M_2 - M_1^2 - M_2^2 - \cdots. \tag{3.22}$$

For antiparticles we use the complex-conjugate potential to Eq. (3.20), which is equivalent to replacing M_1 and M_2 by their transposes. Then

$$(\overline{A}^\dagger\overline{A})^{-1} = (1 - \alpha M_1M_2 - \alpha^* M_2M_1 - M_1^2 - M_2^2 - \cdots)^T. \tag{3.23}$$

Letting

$$M_1 = \begin{pmatrix} a_1 & c_1 \\ c_1^* & b_1 \end{pmatrix}, \tag{3.24}$$

$$M_2 = \begin{pmatrix} a_2 & c_2 \\ c_2^* & b_2 \end{pmatrix}, \tag{3.25}$$

we find, assembling the formulas,

$$T_B - T_{\overline{B}} = (A^\dagger A)^{-1}{}_{11} - (\overline{A}^\dagger\overline{A})^{-1}{}_{11} = (\alpha-\alpha^*)(c_1^*c_2 - c_1c_2^*). \tag{3.26}$$

This expression for the net flux of baryon number does not vanish except for special values of α, c_1, c_2.

Notice that the asymmetry vanishes if $\alpha = 1$. In this case one has effectively only one δ function in the potential and one associated matrix, namely M_1+M_2. This matrix may be made real by a redefinition of the phase of Φ_2, so that there is actually no time-reversal noninvariance in this case. Similar remarks imply that the asymmetry must vanish if C_1 or C_2 vanishes, or if they have the same phase, so that the form of the asymmetry in Eq. (3.26) is almost dictated *a priori*.

We have now demonstrated that a net baryon number flux will arise in Hawking radiation if (and only if) there is microscopic baryon nonconservation, and C and T violation.

The species-by-species imbalance found here also implies that a symmetric medium (consisting of, say, an equal number of p's and $\bar{p}$'s) surrounding a black hole will evolve asymmetrically.

D. Cosmological particle production

Particles may also be produced by the time-changing gravitational fields associated with the expansion of the universe.[5] The analysis we have performed for black-hole radiation carries over essentially unchanged to this case. It is only necessary to replace the space-dependent potential $V_{ij}(x)$ by a time-dependent potential $V_{ij}(t)$. We conclude that in this case also a net baryon asymmetry arises if and only if there is a microscopic C-, T-, and baryon-number-violating interaction.[14]

ACKNOWLEDGMENT

We thank S. Bludman, S. Deser, J. Hartle, S. Hawking, D. Sciama, B. Simon, S. Weinberg, and C. N. Yang for helpful discussions.

This work was supported in part by the U. S. Department of Energy under Contract Nos. AT(E11-1)3071 and EY-76-C02-3072, and by the National Science Foundation under Contract No. PHY-78-01221. The work of F. W. and A. Z. was also supported in part by the Alfred P. Sloan Foundation.

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