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Semiclassical approach to the quark-string model and the hadron spectrum. II. Baryons and exotic hadrons

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Leading Regge trajectories of baryons and exotic hadrons are studied in the quark-string model. Boundary conditions at the string end point and the junction provide a simple topological rule on the configuration of hadron structure and also provide a relationship between the asymptotic Regge slope and the hadron structure. The center of mass of the string is shown to play a special role in classical solutions. The quark-diquark type of the baryon is absent in our formalism.

I. INTRODUCTION

In previous papers[1,2] we introduced a theory of the quark-string and presented a semiclassical approximation method to the theory.[2] (Reference 2 is referred to as I.) The meson mass spectrum was also investigated in detail.[2] In this article we extend the arguments over the baryons and exotic hadrons.

In our model, any hadron is made from strings and quarks which are joined together in such a way that the color-flux conservation law is fulfilled according to the lattice gauge theory.[3] Each branch of strings is oriented to indicate the direction of the color-flux flow. The quark is the source of the flux, so that the oriented string can begin from an extremity with a quark [Fig. 1(a)]. The antiquark is the sink of the flux, so that the oriented string can end up with an extremity with an antiquark [Fig. 1(b)]. Either of them will be called a string end with a quark if the quark or the antiquark need not be discriminated. Since the flux is assumed to be the SU(3) color flux, three incoming (or outgoing) strings can join together at a point to form a junction [Fig. 1(c)]. A hadron forms a system of the quark-string which is composed of the branches of strings with quarks and the junctions.[4]

As was shown in Ref. 1 a quark-string system is obtained by integrating over the color components in the lattice gauge theory for a given configuration of strings and quarks. The resulting effective Lagrangian for the given configuration, as will be presented in the next section, has no color index and is considered to be the Lagrangian in the configuration space which governs the motion of the given quark-string system. As a consequence the quark fields, Ψ's in (2.3) below, which also have no color index, can be treated as distinguishable fields. The orientation of the string is needed in determining the configuration of the quark-string as discussed above. Once the effective Lagrangian is obtained to the configuration, however, the string orientation does not play any role in the dynamical calculation. Our aim is not to explain which configurations are allowed by the quark-string model, but instead, to obtain the mass spectra for the quark-strings allowed by the lattice gauge theory. This is in contrast to the work of Giles and Tye,[5] whose model confines the quark as a consequence of dynamics of their string model.

In the following sections we study leading Regge trajectories of baryons and exotic particles by looking for classical rigid-rotator solutions to equations of motion. We give a systematic way of constructing the classical solutions and mass formulas. The quantum corrections to the classical solutions will not be considered.

II. CLASSICAL SOLUTIONS

Let us consider a quark-string system which is made from N strings and I quarks. The position of the κth string is represented by $X_\kappa^\mu(\tau,\sigma_\kappa)$ ($\theta_1\equiv 0<\sigma_\kappa<\theta_2\equiv\pi$, $1\le\kappa\le N$) and that of the ith quark (or antiquark) is represented by $X_i^\mu(\tau)$. They must satisfy joining conditions, $X_i^\mu(\tau)=X_\kappa^\mu(\tau,\sigma_\kappa=\theta_a)$ for the end of the κth string at which the ith quark is

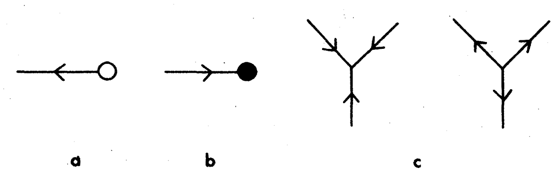

FIG. 1. The string represents the flow of color flux. The circle denotes the quark and the black dot the antiquark.

attached, and $X^\mu_\kappa(\tau, \sigma_\kappa = \theta_a) = X^\mu_\delta(\tau, \sigma_\delta = \theta_b) = X^\mu_\eta(\tau, \sigma_\eta = \theta_c)$ at the three strings junction. The Lagrangian of the quark-string is given by

$$L = \sum_{\kappa=1}^{N} L_{\kappa,\mathrm{st}} + \sum_{i=1}^{I} L_{i,q}, \qquad (2.1)$$

$$L_{\kappa,\mathrm{st}} = -\gamma \int_{\theta_1}^{\theta_2} d\sigma_\kappa [(X_{\kappa,\tau} \cdot X_{\kappa,\sigma_\kappa})^2 - (X_{\kappa,\tau})^2 (X_{\kappa,\sigma_\kappa})^2]^{1/2}, \qquad (2.2)$$

$$L_{i,q} = \tfrac{1}{2} i [X^\mu_{i,\tau}/(X_{i,\tau}{}^2)^{1/2}](\overline{\Psi}_i \gamma_\mu \Psi_{i,\tau} - \overline{\Psi}_{i,\tau}\gamma_\mu \Psi_i) - m_i (X_{i,\tau}{}^2)^{1/2} \overline{\Psi}_i \Psi_i, \qquad (2.3)$$

where notation is explained in I, $\Psi_i(\tau)$ represents the ith quark field and is assumed to be anticommuting. The leading Regge trajectory of the quark-string is given by the stablest classical solution. This solution is a rigid-rotator solution, because all the energy of the solution is shared with rotation modes so that the highest angular momentum is attained for a given energy. We consider rigid rotators in the timelike gauge $X^0_\kappa(\tau, \sigma_\kappa) = X^0_i(\tau) = \tau$. From the Euler equations for strings we find that each branch of the strings must so rotate around the center of mass that obeys the zero mass density condition $\vec{\mathrm{P}} \cdot \vec{X}_\sigma = 0$ of the string. Because of this condition, one can easily confirm that all branches of strings, whether or not junctions are on them, must be arranged along the radial direction from the center of mass [Figs. 2(a) and 2(b)]. Take the center of mass of the rigid rotator at the origin of our coordinate frame and the direction of angular velocity along the positive z axis. Each string has the form

$$\vec{X}_\kappa(\tau, \sigma_\kappa) = \rho_\kappa(\sigma_\kappa)\hat{e}_\kappa, \qquad (2.4)$$

where

$$\hat{e}_\kappa = (\cos(\omega\tau - \zeta_\kappa), \sin(\omega\tau - \zeta_\kappa), 0) \qquad (2.5)$$

and

$$\rho_\kappa(\sigma_\kappa) = \frac{1}{\omega}\sin[c_\kappa(\sigma_\kappa - \sigma^0_\kappa)], \text{ or a constant}. \qquad (2.6)$$

The parameters c_κ and σ^0_κ are determined as follows.[2] First, we substitute (2.4)–(2.6) into the Euler equations for Ψ_i with the joining conditions, $X^\mu_i(\tau) = X^\mu_\kappa(\tau, \sigma_\kappa = \theta_a)$, and solve it for Ψ_i to represent Ψ_i as a function of c_κ and σ^0_κ. Second, c_κ and σ^0_κ are determined by the boundary conditions (Euler equations) for $X^\mu_\kappa(\tau, \sigma_\kappa = \theta_a)$ in which Ψ_i is replaced by the function of c_κ and σ^0_κ. For the sake of convenience, instead of c_κ, σ^0_κ, and σ_κ, we introduce $\alpha(\kappa, a)$ and $\alpha(\kappa)$ for the κth string; $\alpha(\kappa, a=1)$ denotes the smaller one of $c_\kappa(\theta_b - \sigma^0_\kappa)$ where b takes 1 or 2 representing either one of the end points of the κth string, and $\alpha(\kappa, a=2)$ $[> \alpha(\kappa, a=1)]$ denotes the other. The running parameter of the κth string is represented by $\alpha(\kappa) = c_\kappa(\sigma_\kappa - \sigma^0_\kappa)$ instead of σ_κ. Using these $\alpha(\kappa, a)$ we obtain the boundary condition to each joint.

(i) For the string end with a quark (or an antiquark) [Figs. 1(a) and 1(b)], the boundary condition is given by[6]

$$\omega^2\rho_i[m_i(1-\omega^2\rho_i{}^2) + \omega s_i]/(1-\omega^2\rho_i{}^2)^{3/2} = (-1)^a\gamma\, \mathrm{sgn}[\rho_{\kappa,\sigma_\kappa}(\theta_a)][1 - \omega^2\rho_\kappa^2(\theta_a)]^{1/2}, \qquad (2.7)$$

where m_i and $s_i (= \pm\frac{1}{2})$ denote the mass and the z component of spin of the ith quark (or antiquark), respectively. The joining condition $\rho_i = \rho_\kappa(\theta_a)$ must be imposed, too. There are two solutions [(i-a) and (i-b) below] to (2.7):

$$\text{(i-a)} \quad \rho_\kappa(\sigma_\kappa) = \rho_i = 0 \quad (0 \le \sigma_\kappa \le \pi). \qquad (2.8)$$

In this case, the string shrinks to the origin (the center of mass) and the quark is, therefore, at the origin, too.

$$\text{(i-b)} \begin{cases} \rho_\kappa = \dfrac{1}{\omega}\sin\alpha_\kappa \quad [\alpha(\kappa,1) \le \alpha(\kappa) \le \alpha(\kappa,2)], \\ (-1)^a \dfrac{\gamma}{\omega}\, \mathrm{sgn}[\cos\alpha(\kappa,a)] \cos^4\alpha(\kappa,a) \sin^{-1}\alpha(\kappa,a) = m_i\cos^2\alpha(\kappa,a) + \omega s_i. \end{cases} \qquad (2.9)$$

In this case the string is stretched and the possible solutions were studied in detail in I.

(ii) For the junction [Fig. 1(c)], the situation is different depending on whether the junction is at the center of mass or not.

(ii-a) When the junction is at the center of mass, the boundary condition is given by

$$(-1)^a\, \mathrm{sgn}[\rho_{\kappa,\sigma_\kappa}(\theta_a)]\hat{e}_\kappa + (-1)^b\, \mathrm{sgn}[\rho_{\delta,\sigma_\delta}(\theta_b)]\hat{e}_\delta + (-1)^c\, \mathrm{sgn}[\rho_{\eta,\sigma_\eta}(\theta_c)]\hat{e}_\eta = 0, \qquad (2.10)$$

where the $\hat{e}$'s are defined by (2.5). There are three types of solutions to (2.10):

$$\text{(ii-}a\text{-1)} \quad \rho_\kappa(\sigma_\kappa) = \rho_\delta(\sigma_\delta) = \rho_\eta(\sigma_\eta) = 0 \quad (0 \le \sigma_\kappa, \sigma_\delta, \sigma_\eta \le \pi). \qquad (2.11)$$

In this case, all three strings shrink to the origin.

$$\text{(ii-a-2)} \begin{cases} \rho_\kappa = \dfrac{1}{\omega}\sin\alpha(\kappa) \quad [\alpha(\kappa,1) \le \alpha(\kappa) \le \alpha(\kappa,2)], \\ \rho_\delta = \dfrac{1}{\omega}\sin\alpha(\delta) \quad [\alpha(\delta,1) \le \alpha(\delta) \le \alpha(\delta,2)], \\ \rho_\eta(\sigma_\eta) = 0 \quad (0 \le \sigma_\eta \le \pi), \\ \hat{e}_\kappa = \hat{e}_\delta, \\ (-1)^a\, \mathrm{sgn}[\cos\alpha(\kappa,a)] = -(-1)^b\, \mathrm{sgn}[\cos\alpha(\delta,b)], \\ \sin\alpha(\kappa,a) = \sin\alpha(\delta,b) = 0. \end{cases} \qquad (2.12)$$

In this case, one of the three strings shrinks to the origin and the other two extend from the origin to form a straight line.

$$\text{(ii-a-3)}\begin{cases}\rho_\kappa=\dfrac{1}{\omega}\sin\alpha(\kappa)\quad[\alpha(\kappa,1)\le\alpha(\kappa)\le\alpha(\kappa,2)]\,,\\ \rho_\delta=\dfrac{1}{\omega}\sin\alpha(\delta)\quad[\alpha(\delta,1)\le\alpha(\delta)\le\alpha(\delta,2)]\,,\\ \rho_\eta=\dfrac{1}{\omega}\sin\alpha(\eta)\quad[\alpha(\eta,1)\le\alpha(\eta)\le\alpha(\eta,2)]\,,\\ \sin\alpha(\kappa,a)=\sin\alpha(\delta,b)=\sin\alpha(\eta,c)=0\,,\\ a=b=c\,,\\ \zeta_\kappa=\zeta_\delta+\tfrac{2}{3}\pi=\zeta_\eta+\tfrac{4}{3}\pi\,,\end{cases}\qquad(2.13)$$

where the ζ's are defined by (2.5). This solution forms a symmetrical Y-shaped tree with a junction at the origin.

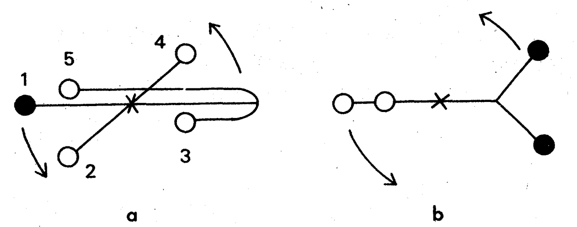

FIG. 2. Geometrical configurations of quark-strings. The diagram (a) is allowed. The diagram (b) is an example of forbidden structure. A junction having three strings not along a straight line is not allowed. The cross represents the center of mass around which the system is rotating.

(ii-b) When the junction is not the center of mass, the boundary condition is given by

$$\gamma(1-\omega^2\rho^2)^{1/2}\{(-1)^a\,\mathrm{sgn}[\rho_{\kappa,\sigma_\kappa}(\theta_a)]+(-1)^b\,\mathrm{sgn}[\rho_{\delta,\sigma_\delta}(\theta_b)]+(-1)^c\,\mathrm{sgn}[\rho_{\eta,\sigma_\eta}(\theta_c)]\}=0\,,\qquad(2.14)$$

where

$$\rho\equiv\rho_\kappa(\theta_a)=\rho_\delta(\theta_b)=\rho_\eta(\theta_c)\,.\qquad(2.15)$$

There are also three types of solutions to (2.14):

$$\text{(ii-b-1)}\quad\rho_\kappa(\sigma_\kappa)=\rho_\delta(\sigma_\delta)=\rho_\eta(\sigma_\eta)=\rho\,,\qquad(2.16)$$

a constant for $0\le\sigma_\kappa,\sigma_\delta,\sigma_\eta\le\pi$. The other end of each string cannot be a string end with a quark if the junction is not at the center of mass. All the strings shrink to the same point.

$$\text{(ii-b-2)}\begin{cases}\rho_\kappa=\dfrac{1}{\omega}\sin\alpha(\kappa)\quad[\alpha(\kappa,1)\le\alpha(\kappa)\le\alpha(\kappa,2)]\,,\\ \rho_\delta=\dfrac{1}{\omega}\sin\alpha(\delta)\quad[\alpha(\delta,1)\le\alpha(\delta)\le\alpha(\delta,2)]\,,\\ \rho_\eta(\sigma_\eta)=\rho\,,\quad\text{a constant for }0\le\sigma_\eta\le\pi\\ \dfrac{1}{\omega}\sin\alpha(\kappa,a)=\dfrac{1}{\omega}\sin\alpha(\delta,b)=\rho\,,\\ (-1)^a\,\mathrm{sgn}[\cos\alpha(\kappa,a)]=-(-1)^b\,\mathrm{sgn}[\cos\alpha(\delta,b)]\,.\end{cases}\qquad(2.17)$$

In this case, the other end of the ηth string, which is shrunk, cannot be a string end with a quark. The other two strings extend from the point ρ to form a straight line.

$$\text{(ii-b-3)}\begin{cases}\rho_\kappa=\dfrac{1}{\omega}\sin\alpha(\kappa)\quad[\alpha(\kappa,1)\le\alpha(\kappa)\le\alpha(\kappa,2)]\,,\\ \rho_\delta=\dfrac{1}{\omega}\sin\alpha(\delta)\quad[\alpha(\delta,1)\le\alpha(\delta)\le\alpha(\delta,2)]\,,\\ \rho_\eta=\dfrac{1}{\omega}\sin\alpha(\eta)\quad[\alpha(\eta,1)\le\alpha(\eta)\le\alpha(\eta,2)]\,,\\ \sin\alpha(\kappa,a)=\sin\alpha(\delta,b)=\sin\alpha(\eta,c)=1\,.\end{cases}\qquad(2.18)$$

This means that the velocity of the junction equals the speed of light. Three strings extend toward the origin from the junction to form a triple bunch of strings [the right most part of Fig. 2(a)].

III. PROPERTIES OF QUARKS

Before going into the calculations of energies and angular momenta of quark-strings, we wish to review some properties of quarks[2] when they are attached to the string according to the boundary conditions stated in the preceding section.

When the quark is sitting on the center of mass of the quark-string, the contributions from the quark to the energy and angular momentum are the rest quark mass m_i and the spin $s_i = \pm\frac{1}{2}$, respectively.

When the quark is not at the center of mass, the behavior of the quark is quite different depending on the spin state.

If the quark spin is parallel to the classical angular velocity, the solution to the boundary condition is unique. The centrifugal force of the quark works outward and is balanced by the tension of the string [see, for example Fig. 3(a)].

If the quark spin is antiparallel to the classical angular velocity, as was discussed in I, there are three solutions ($\alpha_-^{(1)}$, $\alpha_-^{(2)}$, and $\alpha_-^{(3)}$) if $m_i^2\alpha'_\infty > 1/\pi$, and one ($\alpha_-^{(1)}$) if $m_i^2\alpha'_\infty < 1/\pi$ (the definition of $\alpha_-^{(1)}$ is given in I). One of the solutions, which is called $\alpha_-^{(1)}$, shows a strange behavior. The canonical momentum of the quark $\vec{p}_i$ is antiparallel to the velocity $\vec{X}_{i,\tau}$. As a consequence, the centrifugal force of the quark works *inward* and is balanced by the outward tension of the string as shown in Fig. 3(b). This solution might be unstable against quantum corrections. Since we have not succeeded in confirming the instability, we temporarily adopt this as a possible solution in the following discussion. It may be worthwhile to note that, if $\alpha_-^{(1)}$ is unacceptable because of the instability, the pion trajectory turns out to be unstable. This implies that the pion cannot be obtained as a shrunken limit of the relativistic string, at least, within the framework of the semiclassical approximation.

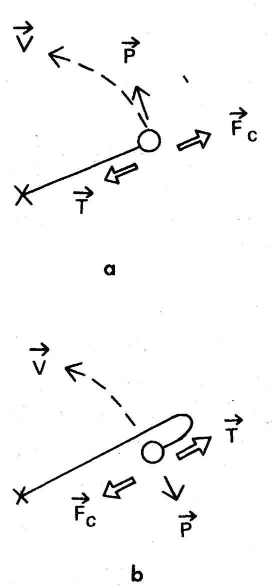

FIG. 3. Balance of forces. $\vec{F}_c$ and $\vec{T}$ denote the centrifugal force and the tension, respectively. The solution $\alpha_-^{(1)}$, shown in (b), has the canonical momentum $\vec{p}$ antiparallel to the velocity $\vec{v}$.

The other two spin-down solutions, $\alpha_-^{(2)}$ and $\alpha_-^{(3)}$, behave as normal as the spin-up quark α_+, i.e., the canonical momentum is parallel to the velocity and the configuration of the quark and string appears as shown in Fig. 3(a).

IV. REGGE TRAJECTORIES

The mass and the spin of a hadron are given by the total energy and the total angular momentum of the corresponding quark-string in the center-of-mass system. The total energy and angular momentum of the classical solution for a given configuration can be represented by a sum of the following subenergies E's and subangular momenta J's which are associated with each sub-quark-string shown in Fig. 4. Contributions from the sub-quark-string to the energy and the angular momentum are described in the following. The shrunken string can be disregarded because it has no energy and angular momentum.

(A) For a quark at the center of mass [Fig. 4(A)], the case of (i-a) in the above classification, the subenergy and subangular momentum turn out to be

$$E(A) = m_i\,, \tag{4.1}$$

$$J(A) = s_i\,. \tag{4.2}$$

(B) The substring with no fold satisfying the following two conditions [Fig. 4(B)]. One of the string ends rests at the center of mass. The other end is rotating with the light velocity. The energy and angular momentum turn out to be,[7] respectively,

$$E(B) = \frac{\gamma}{\omega}\frac{\pi}{2}\,, \tag{4.3}$$

$$J(B) = \frac{\gamma}{2\omega^2}\frac{\pi}{2}\,. \tag{4.4}$$

(C) The substring with a quark which has no fold [Fig. 4(C)]. One of the string ends rests at the center of mass. The other end is the string end with a quark. The solution α_+, $\alpha_-^{(2)}$, or $\alpha_-^{(3)}$, de-

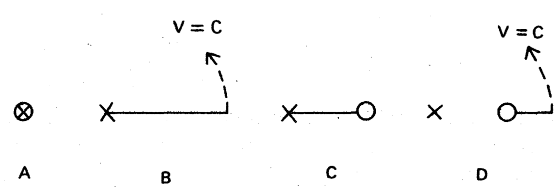

FIG. 4. Sub-quark-strings. The cross denotes the rotation center. The open ends in (B) and (D) show those which are moving with the light velocity.

fined in I, is the case. Using these α's the subenergy and angular momentum are represented as

$$E(C)=\frac{\gamma}{\omega}\alpha_{s_i}^{(r)}+\frac{1}{\cos\alpha_{s_i}^{(r)}}(m_i+s_i\omega\tan^2\alpha_{s_i}^{(r)}), \quad (4.5)$$

$$J(C)=\frac{\gamma}{2\omega^2}(\alpha_{s_i}^{(r)}-\tfrac{1}{2}\sin 2\alpha_{s_i}^{(r)}) +\frac{1}{\cos\alpha_{s_i}^{(r)}}\left(\frac{m_i}{\omega}\sin^2\alpha_{s_i}^{(r)}+\frac{s_i}{\cos^2\alpha_{s_i}^{(r)}}\right), \quad (4.6)$$

where $\alpha_{s_i}^{(r)}$ satisfies (2.15) in I and $0\le\alpha_{s_i}^{(r)}\le\pi/2$. In the above two equations the first terms are contributions from the string part and the second terms from the quark.

(D) The substring with a quark such as shown in Fig. 4(D). One of the string ends is moving with the light velocity and the other is the string end with a quark. The solution $\alpha_-^{(1)}$ is the case (the momentum of the quark is antiparallel to the velocity as discussed in Sec. III). The subenergy and angular momentum turn out to be[7]

$$E(D)=\frac{\gamma}{\omega}\left(\alpha_-^{(1)}-\frac{\pi}{2}\right) +\frac{1}{|\cos\alpha_-^{(1)}|}(m_i+s_i\omega\tan^2\alpha_-^{(1)}), \quad (4.7)$$

$$J(D)=\frac{\gamma}{2\omega^2}\left(\alpha_-^{(1)}-\frac{\pi}{2}-\tfrac{1}{2}\sin 2\alpha_-^{(1)}\right) +\frac{1}{|\cos\alpha_-^{(1)}|}\left(\frac{m_i}{\omega}\sin^2\alpha_-^{(1)}+\frac{s_i}{\cos^2\alpha_-^{(1)}}\right), \quad (4.8)$$

where $\alpha_-^{(1)}$ satisfies (2.15) in I and $\pi/2\le\alpha_-^{(1)}\le\pi$. In the above equations the first terms are contributions from the string and the second terms from the quark.

By the use of these subenergies and angular momenta, the total energy (mass) and angular momentum (spin) of a quark-string are given by

$$E_{\text{tot}}=\sum_A E(A)+\sum_B E(B)+\sum_C E(C)+\sum_D E(D), \quad (4.9)$$

$$J_{\text{tot}}=\sum_A J(A)+\sum_B J(B)+\sum_C J(C)+\sum_D J(D), \quad (4.10)$$

where each summation is taken over constituent sub-quark-strings of a given configuration. For example, the quark-string shown in Fig. 2(a) consists of two B's, four C's, and one D. Note that the string with the 5th quark in Fig. 2(a) is decomposed into two parts as shown in Fig. 5, and

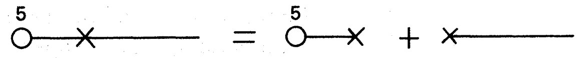

FIG. 5. Decomposition of a string into two sub-quark-strings.

counted as $C+B$.

The Regge trajectory is obtained by eliminating the classical angular velocity ω from the mass (4.9) and the spin (4.10). There is a simple rule in calculating the asymptotic Regge slope. In the asymptotic region ($\omega\to 0$), $\alpha_s^{(r)}$ in (4.5)–(4.8) behaves as

$$\alpha_s^{(r)}\sim\frac{\pi}{2}+O(\sqrt{\omega}). \quad (4.11)$$

From (4.11) one can find the behavior of the subenergy and the sub-angular momentum as $\omega\to 0$,

$$E(\text{sub})\sim\frac{\gamma}{\omega}\frac{\pi}{2}n(\text{sub}), \quad (4.12)$$

$$J(\text{sub})\sim\frac{\gamma}{2\omega^2}\frac{\pi}{2}n(\text{sub}), \quad (4.13)$$

where $n(\text{sub})$ is defined by

$$n(A)=0,\quad n(B)=1,\quad n(C)=1,\quad n(D)=0, \quad (4.14)$$

for each class $A,\ldots,D$. Then the total energy and angular momentum turn out to be, respectively,

$$E_{\text{tot}}\sim\frac{\gamma}{\omega}\frac{\pi}{2}n_{\text{tot}}, \quad (4.15)$$

$$J_{\text{tot}}\sim\frac{\gamma}{2\omega^2}\frac{\pi}{2}n_{\text{tot}}, \quad (4.16)$$

where n_{tot} is the sum of the number of B sub-quark-strings and that of C sub-quark-strings in the given configuration. The asymptotic Regge slope is, therefore, given by

$$\alpha'_\infty=\frac{1}{\pi\gamma}\frac{1}{n_{\text{tot}}}. \quad (4.17)$$

V. BARYONS AND EXOTIC HADRONS

In the previous sections we have shown the systematic way of constructing the classical solutions to a hadron. Some examples are shown below.

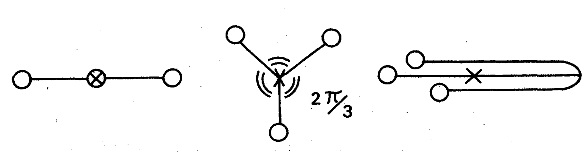

FIG. 6. Possible baryon structures.

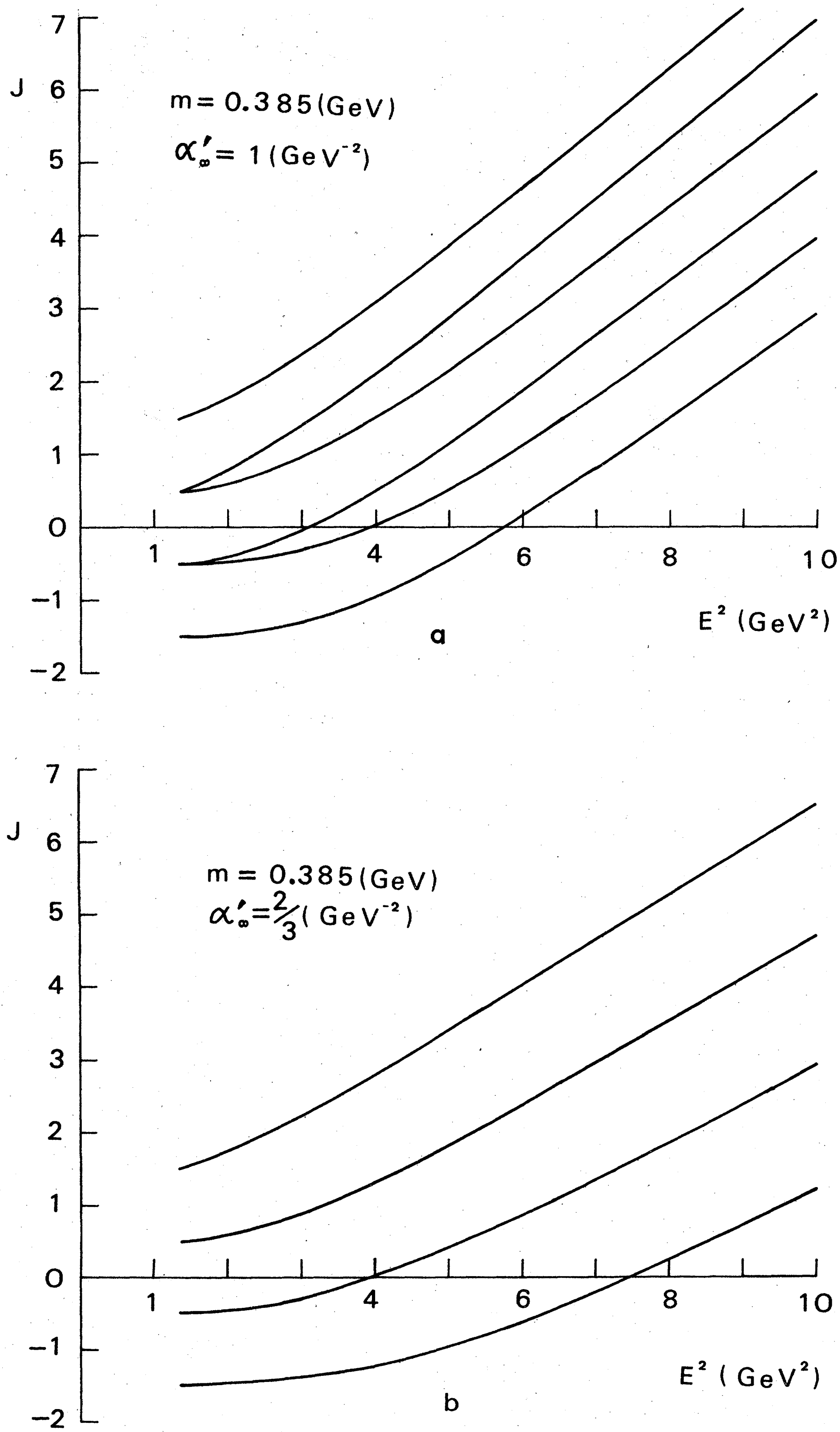

FIG. 7. (a) Regge trajectories of the linear baryons shown in Fig. 6(a). Each trajectory has a different spin configuration. (b) The Y-shaped baryons, shown in Fig. 6(b), for various spin configurations.

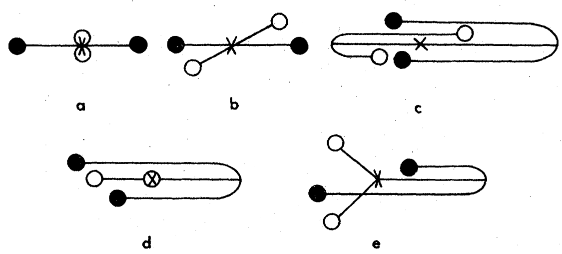

FIG. 8. Possible exotic mesons.

Among possible baryon trajectories the leading one is given by the linear baryon shown in Fig. 6(a).[8] In our semiclassical approximation there does not exist the diquark-quark configuration such as later shown in Fig. 11(c) (see the discussion in Sec. VI). There are more possible baryons like Fig. 6(b). The Regge trajectories of the linear baryon and the symmetrical Y-shaped baryon are shown in Fig. 7.

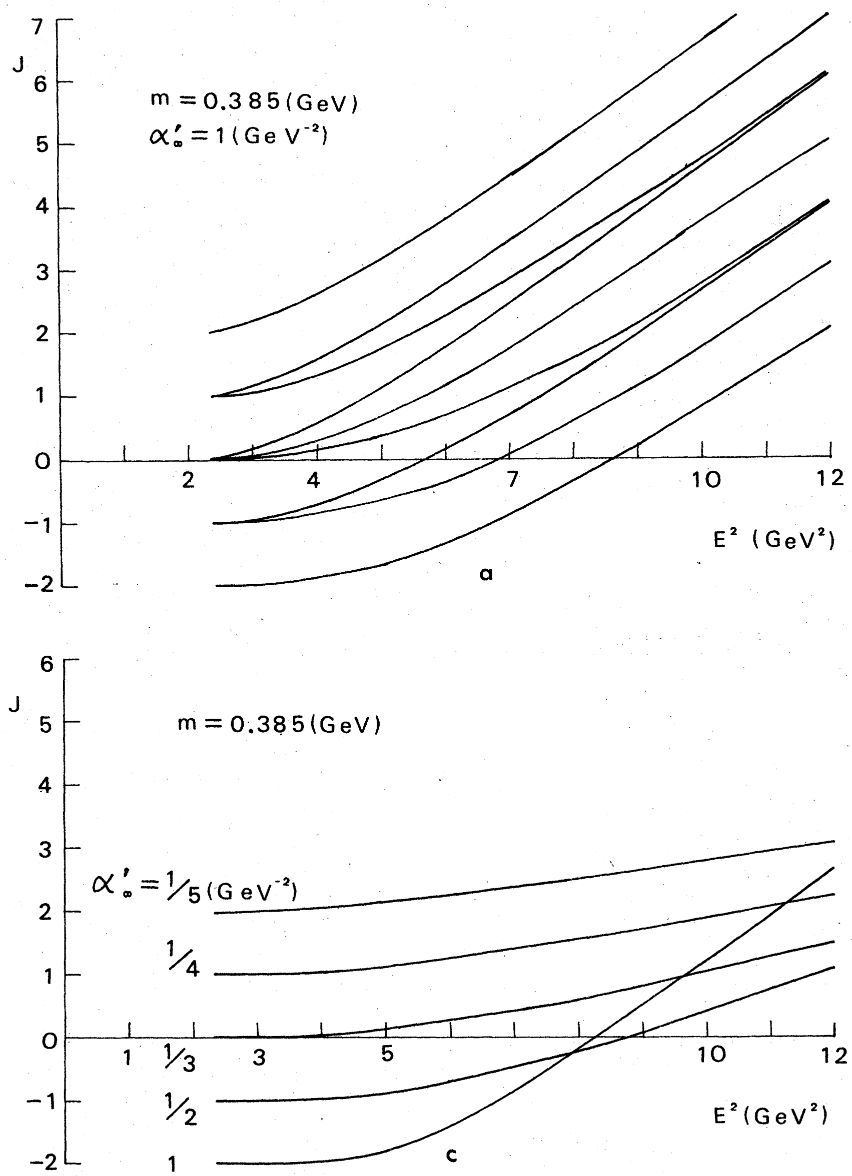

FIG. 9. (a) Trajectories of the linear exotic mesons, shown in Fig. 8(a), for various spin configuration. (c) Baryonium trajectories, shown in Fig. 8(c), for various spin configurations. The number at the left of each trajectory indicates the asymptotic slope when the ρ-meson slope is taken to be 1 GeV^{-2}.

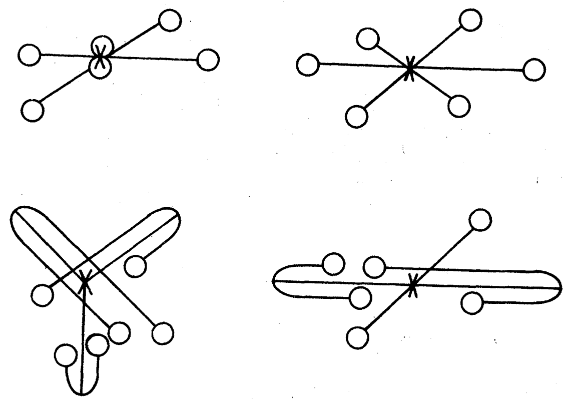

FIG. 10. Examples of complicated exotic hadrons.

The leading trajectory of the exotic mesons $(qq\bar{q}\bar{q})$ is given by the linear configuration [Fig. 8(a)] in which a pair of quarks (or antiquarks) are at the center of mass. This is a special case of the X-shaped meson [Fig. 8(b)] with two branches of strings shrunken. These types of exotic mesons may be expected to decay into two mesons rather than a baryon-antibaryon pair. More massive exotic mesons are those which include junctions moving with the light velocity [Fig. 8(c)–8(e)]. They may mostly decay into a baryon-antibaryon pair. These states can be associated with the baryoniums which are the center of many physicists' attention.[9] The Regge trajectories of two exotic mesons, Fig. 8(a) and Fig. 8(c) are shown in Fig. 9.

Some other examples of exotic hadrons are shown in Fig. 10. We note that if the number of quarks (or antiquarks) at the center of mass is more than three, some configurations will be forbidden due to the Pauli principle.

VI. CONCLUDING REMARKS

We have studied leading Regge trajectories of baryons and exotic hadrons. The mass and the spin of particles are analyzed based on the effective Lagrangian (2.1)–(2.3). In concluding this article we make some remarks on baryon trajectories.

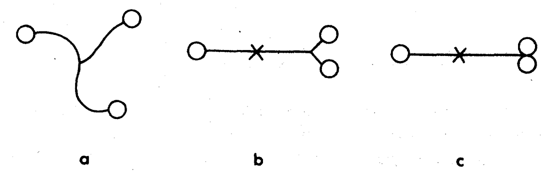

FIG. 11. (a) is an allowed topological configuration of the baryon. (b) and (c), which is considered to be a limiting case of (b), are forbidden geometrical structures although topologically allowed.

The diquark-quark baryon [Fig. 11(c)] has not been considered in our text. If one looks for the diquark-quark solution to the effective Lagrangian which has three branches of strings and a junction [Fig. 11(a)], one cannot find such a rigid-rotator solution that has two branches of shrunken strings and forms the diquark-quark configuration. The reason is that, since the tension of the string is the function of only the distance from the rotation center, a single string cannot balance with two strings at the junction if the configuration is such as the one shown in Fig. 11(b). It is, however, possible to construct a diquark-quark solution if one begins with the effective Lagrangian having a single string with quark fields at both ends. The (lattice) gauge theory, however, prohibits us from choosing such a configuration.

Another remark is on the stability of the linear baryon trajectory. One may suspect that the quark at the center of mass in the linear baryon [Fig. 6(a)] might be unstable against small disturbances because the centrifugal force works if the position is a bit off the center. It is, however, that the linear baryon has the lightest mass for a given spin, at least, among our rigid-rotator solutions. We conclude, therefore, that the linear baryon is stable.

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