Comments and Addenda

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Unitarity effect of the $\psi'(3684)$ on the shape of the $\psi''(3772)$

Gordon L. Shaw
Physics Department, University of California, Irvine, California 92717

James S. Ball
Physics Department, University of Utah, Salt Lake City, Utah 84112

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The detailed shape of the $\psi''(3772)$ has been measured and analyzed at SPEAR by two separate groups. Both analyses find that an unphysically large range parameter is needed for the P wave to fit the data. We use a two-resonance formalism to include the required unitarity effects of the $\psi'(3684)$ on the ψ'' shape. An excellent fit is obtained only if the ratio of couplings $(g_{\psi' e^+e^-}\, g_{\psi' D\bar{D}})/(g_{\psi'' e^+e^-}\, g_{\psi'' D\bar{D}})$ is negative. For this fit the range parameter can be very small. We determine $g_{\psi' DD}{}^2/g_{\psi'' DD}{}^2 \sim 0.8$.

The ψ'' (3772) resonance has been accurately measured in detail at SPEAR both by the Magnetic Detector Group[1] (MDG) and by the Direct Electron Counter Group[2] (DELCO). We will concentrate our discussion for the moment on the MDG data since the data, analyses and conclusions of the two groups are similar.[3] Since the ψ'' is so close to the $D\bar{D}$ threshold, it is important to treat the kinematics of the problem carefully. The data for R was analyzed[1] using an incoherent background

$$R_B = a + b(p_+{}^3 + p_0{}^3) \tag{1}$$

and a single Breit-Wigner resonance with amplitud

$$T = (\Gamma''_{ee}\Gamma''_{ee}/4)^{1/2}/(m'' - E - i\,\Gamma''_{DD}/2)\,,$$
$$\Gamma''_{DD} = g_{\psi'' DD}{}^2\rho\,, \tag{2}$$
$$\rho = p_0{}^3/[1 + (rp_0)^2] + p_+{}^3/[1 + (rp_+)^2]\,,$$

$$R = \sigma_T/\sigma_{\mu\mu} = R_B + \frac{9}{\alpha^2}\,|T|^2\theta(E - 2m_{D^0})\,, \tag{3}$$

where $p_{+(0)}$ is the momentum of a charged (neutral) D from D pair production. They found that the fits required the range-parameter r to be quite large: The fit in their Fig. 3 was for $r = 3$ fm. This value of the range is physically unacceptable since at the ψ'' mass m'', $(rp)_{m''}^{2} \gg 1$. [See the discussion following Eq. (5).] The data below $E = m''$ rises much faster than a "reasonable," energy-dependent P-wave width would give. In fact, they found that an energy-independent Γ fits even better.

The purpose of this paper is to report the results of a fit to the data which includes the *required* and *important* unitarity effects of the ψ'-(3684) on the shape of the ψ''. These unitarity effects, neglected in the previous analyses[1,2] enable us to obtain excellent fits with a small range r.

We fitted the data with the background term (1) and the unitarized two-resonance amplitude[4]

$$T = \frac{\rho^{1/2}}{2}\,\frac{g_{\psi'' ee}g_{\psi'' DD}\Lambda' + g_{\psi' ee}g_{\psi' DD}\Lambda'' + i(g_{\psi' ee}g_{\psi' DD} + g_{\psi'' ee}g_{\psi' DD})\lambda}{\Lambda'\Lambda'' + \lambda^2} \tag{4}$$

where

$$\Lambda = (m - E - i\Gamma_{DD}/2)\,,$$
$$\Gamma_{DD} = g_{\psi DD}{}^2\rho\,, \tag{5}$$
$$\Gamma_{ee} = g_{\psi ee}{}^2\,,$$

and

$$\lambda = \rho g_{\psi' DD}\, g_{\psi'' DD}/2\,.$$

These equations are analytically continued below the $D\bar{D}$ threshold by $p \to i|p|$. Note that the T amplitude must have a ψ' pole located about as far below the $D\bar{D}$ threshold as the ψ'' resonance is above threshold. Thus if $(rp)^2_{m''} \sim 1$, then we see from (2) that ρ (and thus Γ) developes a pole near the ψ' mass. We rule out as unacceptable, solutions with these spurious Castillejo-Dalitz-Dyson

TABLE I. Parameters for our fits to the MDG data (Ref. 1) (21 points) and the DELCO data (Ref. 2) (18 points) for δ negative. (Units are in the appropriate powers of MeV). The ψ' and ψ'' parameters are constrained to lie close to the previously published values.

	Fit to MDG data	Fit to DELCO data
a	2.80	2.52
$b/(p_+{}^3+p_0{}^3)_{m''}$	0.107	4.75×10^{-2}
m' (fixed)	3690[a]	3690[a]
m''	3782	3783
Γ'_{ee}	2.18×10^{-3}	1.72×10^{-3}
Γ''_{ee}	2.70×10^{-4}	1.00×10^{-4}
$\Gamma''_{DD}(m'')$	35.0	35.0
$g_{\psi' DD}{}^2/g_{\psi'' DD}{}^2$	0.811	0.769
r (fixed)	0.0	0.0
χ^2	16.7	11.1

[a]Note that this gives a ψ' pole position of about 3680.

poles (associated with large r) in the denominator of T since we require the proper analyticity (as well as unitarity). Note that for r small and $|\frac{1}{4}\Gamma''/(m''-m')|\ll 1$, the ψ' pole position is given by the zero of $m'-|\Gamma'|/2-E$.

The Okubo-Zweig-Iizuka-rule-forbidden decays are neglected as well as coupling to the closed $D\bar{D}^*$ and $D^*\bar{D}^*$ channels.[5] Excellent fits (given in Table I) to the data are obtained *only* if the ratio of couplings $\delta=(g_{\psi' ee}g_{\psi' DD}/g_{\psi'' ee}g_{\psi'' DD})$ is negative. This is readily understood from (4), since for δ positive there will be a zero in T between the ψ' and ψ''.[6] For δ negative, there is construction interference in this region and T rises quickly for increasing E even with the range parameter set equal to zero for the fits in Table I. We determine the one new parameter $g_{\psi' DD}{}^2/g_{\psi'' DD}{}^2$ in our fits to be ~0.8.

The most detailed understanding of the ψ spectrum below 4 GeV comes from the charmonium-model calculations.[7] The ψ'' is understood to be a $^3D_1$ $c\bar{c}$ state, with an admixture of $^3S_1$ (via a tensor force and through coupling to $D\bar{D}$) to give the appropriate decay width Γ''_{ee}. It would be of considerable interest to know if these theoretical models are consistent with the results of our analyses on the sign of δ and the magnitude of $g_{\psi' DD}{}^2/g_{\psi'' DD}{}^2$.

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[1]P. Rapidis *et al.*, Phys. Rev. Lett. 39, 526 (1977).

[2]W. Bacino *et al.*, Phys. Rev. Lett. 40, 671 (1978).

[3]DELCO finds a value of Γ''_{ee} which is a factor of 2 less than that of MDG.

[4]P. W. Coulter and G. L. Shaw, Phys. Rev. D 4, 2919 (1971); 8, 2216 (1973). Equation (4) is consistent with unitarity and analyticity. The parameters here are the usual one-state nonoverlapping or "isolated" quantities. The dynamical quantity C (in the former reference) has been taken equal to zero. In this paper we are *not* concerned with small changes in the "isolated" widths and masses due to a dynamical interaction. [For a discussion of these points see P. W. Coulter and G. L. Shaw, Phys. Rev. 188, 2443 (1969); and D. Horn and D. E. Novoseller, Phys. Rev. D 17, 1763 (1978).] We concentrate here on the large effects of the ψ' on the shape of the ψ''.

[5]These closed channels would introduce an additional energy dependence, but mainly on the high side of the ψ'' which is not a problem in the fitting. Furthermore, it would not be meaningful to introduce so many additional parameters for the limited energy region of our fit.

[6]Note that in an elastic amplitude there is no possible minus sign, so that there is necessarily a zero in T between two states. For example, in the P_{11} πN amplitude the phase shift starts off negative due to the nucleon pole before going back up through the Roper resonance [see, e.g, J. S. Ball, G. L. Shaw, and D. Y. Wong, Phys. Rev. 155, 1725 (1967)].

[7]E. Eichten *et al.*, Phys. Rev. Lett. 36, 500 (1976); K. Lane and E. Eichten, *ibid.* 37, 477 (1976).