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Light-plane commutators and corrections to partial conservation of axial-vector current

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Corrections to the partially conserved axial-vector current hypothesis are evaluated by taking into account the contributions of the radially excited pseudoscalar mesons, whose existence is predicted in the quark model. The coupling of these mesons to the axial-vector current is first order in chiral-symmetry breaking. Several sum rules, related to matrix elements of light-plane commutators of charges and divergences of currents, are analyzed. Predictions are made for some decay modes of the radially excited pseudoscalar and vector mesons.

I. INTRODUCTION

The successes of the partially conserved axial-vector current hypothesis[1] (PCAC) are generally understood at this time as a consequence of the approximate SU(3) × SU(3) invariance property of the strong interactions.[2,3] This idea is suggested by the fact that the smallness of the low-lying pseudoscalar-meson masses allows their interpretation as almost-Goldstone bosons and suggests a description of the real world by use of the simpler chirally "invariant" world as a reference.

An explicit representation of the PCAC mechanism has been obtained[4] in the σ model where the divergence of the axial-vector current emerges proportional to the corresponding pseudoscalar-meson field, the chiral symmetry being spontaneously broken in the limit of the vanishing of the pseudoscalar-meson mass.

On the other hand, the successes of the quark model in explaining the hadronic spectrum[5] as well as the lepton-hadron deep-inelastic scattering data[6] suggest that the hadronic world should be described by means of a field theory built up from interacting quark field variables.[7] The question which then arises is whether the original idea of PCAC can still be maintained in such theories. Fortunately it has appeared that, when the quark interaction is mediated via SU(3)-singlet vector gluons, chiral symmetry is only broken by the quark mass term, which is not expected to drastically modify the picture of the symmetric world.[8,9] Furthermore, one finds in that case simple relations between the low-lying pseudoscalar-meson masses and that of the quarks; these relations reflect in some sense the almost-Goldstone nature of the mesons.[10]

However, in contrast to the σ model, one feature seems to be definitely lost: The pseudoscalar mesons are no longer described by elementary fields of the theory, and one is led to wonder whether the stronger formulation of PCAC, as stated in the σ model, can still make sense in a quark field theory.[11-15] A full answer to this question cannot be given unless the whole dynamics of the theory is solved, and more particularly when all the hadronic states are explicitly expressed in terms of the (current) quark field variables. Nevertheless, lacking a complete solution of this problem, one may make an educated guess, based upon some observations of the general structure of the hadronic spectrum in the quark model.

The fundamental observation is that in a theory of confined quarks the hadronic spectrum is composed of an infinite number of particles; in particular, to every ground state with definite quantum numbers corresponds an infinite sequence of radial excitations.[16] Thus the low-lying pseudoscalar mesons have their own radial excitations which would also couple (although more weakly, via first-order chiral-symmetry-breaking effects) to the axial-vector currents. In other words, the lowest-lying pseudoscalar mesons, although they are the only almost-Goldstone bosons of the theory, should not be expected to completely dominate axial-vector current divergence operators.

This then suggests that, in view of PCAC, the axial-vector divergence, instead of being expressed in terms of the single ground-state pseudoscalar-meson (effective) field, should be expressed as a sum of the infinite sequence of all pseudoscalar-meson (effective) fields appearing in the theory. Such an expansion, which seems to be the most natural transcription of the σ-model PCAC into the quark model, would provide, we think, a more complete and faithful interpretation of PCAC equations.

The aim of the present paper is to investigate in some simple cases the possible corrective effects to PCAC, coming from these high-lying *pseudoscalar mesons, whose existence is predicted in the quark model.* This is achieved by a study of certain matrix elements of light-plane commutators of charges and axial-vector divergences which are saturated (on the mass shell) by the infinite sequences of pseudoscalar mesons. Similarly, matrix elements of axial-vector light-plane charges are systematically expressed, via unsubtracted dispersion relations, as a series of pole terms arising from the contributions from these mesons. In the past, soft-meson theorems have been used to test both the algebra of charges as well as the hypotheses concerning the nature of chiral-symmetry breaking.[17] The difficulty in making these tests originates in the fact that soft-meson theorems fix the values of the matrix elements in question at points off the meson mass shells. The problem of confronting theories of the commutators with experiment therefore becomes one of obtaining from the on-mass-shell data a reliable "experimental" estimate of the off-mass-shell matrix element at the soft-meson points. In the present approach, however, we may attempt to learn something of value about the PCAC hypothesis, without having to contend with complications arising from extrapolation procedures, which would otherwise make comparisons of theoretical and experimental quantities problematic.

The idea of extending the PCAC hypothesis to include the contributions of heavy pseudoscalar mesons to the axial-vector current divergence has been developed by several authors, in particular by Drell, Bars, and Halpern, and by Dominguez.[16] In contrast to the former authors, we follow Dominguez in insisting upon the conservation of the axial-vector current when the pion (kaon) mass vanishes. We differ from Dominguez in that our investigation uses the framework of the quark model, thus enabling us to make some definite predictions for the couplings of the radially excited pseudoscalar mesons, whereas Dominguez must make additional assumptions regarding their couplings.

There have been attempts to estimate the corrections to PCAC arising from the continuum.[18] Pagels and Zepeda conclude that such contributions cannot account for the observed deviations from PCAC, while Jones and Scadron find that they may account for roughly half the deviation. We follow Dominguez in assuming that the most important part of PCAC corrections comes from the one-particle sector, represented by contributions from the heavy mesons.

Furthermore, in the spirit of the proposal by Dashen and Weinstein, we base our work on the hypothesis that certain matrix elements of observables may be evaluated by means of a perturbative expansion, up to second order, about a chiral symmetric limit. However, owing to the specific framework for chiral-symmetry breaking (the quark model) and the knowledge of the symmetry-breaking parameters (the quark masses), one expects to obtain here more quantitative predictions from such a hypothesis.

After briefly reviewing in Sec. II the two most commonly accepted relations which connect pseudoscalar-meson masses to the bare-quark-mass ratio m_s/m_u, we develop in Sec. III the idea of "generalized" PCAC as sketched above. Sections IV and V are devoted to some of its applications through the use of light-plane commutators of charges and axial-vector divergences taken between vacuum and one-particle states. In Sec. IV we exhibit the correction to a well-known sum rule related to the K_{l3} form factor f_+. In Sec. V we analyze some new sum rules which provide us with predictions about decay modes of radially excited pseudoscalar and vector mesons. We summarize our results in Sec. VI and we conclude with a few remarks about low-energy theorems.

II. THE QUARK MASSES

Since the bare quark masses are the basic theoretical parameters of chiral-symmetry breaking, it is essential to get at least a crude estimate of their values. A convenient way to evaluate the ratio of the strange- and nonstrange-quark masses (m_s/m_u) is to consider the matrix element of the axial-vector divergence between vacuum and a pseudoscalar-meson state $\langle 0|D_5^a|P^b\rangle$, which is proportional to the matrix element of the corresponding pseudoscalar density $v^a = i\bar{q}\gamma_5 \frac{1}{2}\lambda^a q$. (In order to avoid complications arising from the η-η' mixing problem, and from the existence of an anomaly term in the singlet axial-vector divergence, we limit ourselves throughout this paper to the pion and kaon axial divergences.)

One generally assumes that the chiral-symmetry-breaking Hamiltonian $H_1 = m_0 u^0 + m_1 u^8$, where the u's are the scalar densities $\bar{q}\frac{1}{2}\lambda^a q$ and the m's are combinations of m_s and m_u, can be treated as a perturbation,[2,8] and the various matrix elements of operators can be formally computed by the standard techniques of perturbation theory in terms of H_1. Consequently the matrix elements $\langle 0|v^a|P^b\rangle$ can be expanded in terms of the quark masses (which are assumed to be small on the hadronic mass scale), and thus give to first order

$$\langle 0|v^a|P^b\rangle = A\delta_{ab} + B\,\mathrm{Tr}([\tfrac{1}{2}\lambda^a, [m, \tfrac{1}{2}\lambda^b]_+]_+) + O(m^2), \qquad (2.1)$$

where m is the quark mass matrix, and the constants A and B are independent of m. We have assumed for this expansion that the scalar densities u^0 and u^8 (present in H_1) have the same reduced matrix elements between given SU(3) octet and/or singlet states.

If the constant A is different from zero, then one expects that the second term in the right-hand side of Eq. (2.1) can be neglected, and so one gets in leading order in chiral-symmetry breaking the well-known solution[8,9]

$$m_s/m_u \simeq 2M_K{}^2/M_\pi{}^2 - 1 = 24 . \tag{2.2}$$

However, it may happen that A is zero; this can be the consequence of some additional group-theoretical constraints or of some dynamical mechanism, related for instance to the way the pseudoscalar-meson masses are generated. In that case the second term in the right-hand side of Eq. (2.1) is dominant, and one gets another solution[11-14,19] for m_s/m_u:

$$m_s/m_u \simeq 2M_K/M_\pi - 1 = 6 . \tag{2.3}$$

It is evident that for the first type of solution, hereafter called the *quadratic solution*, the pseudoscalar-meson masses squared are of first order in chiral-symmetry breaking,

$$M_\pi{}^2 = a_\pi m_u, \quad M_K{}^2 = a_K(m_s + m_u)/2, \quad a_\pi, a_K = O(1), \tag{2.4}$$

while for the second type of solution, hereafter called the *linear solution*, it is the masses themselves which are of first order,

$$M_\pi = b_\pi m_u, \quad M_K = b_K(m_s + m_u)/2, \quad b_\pi, b_K = O(1) . \tag{2.5}$$

The different relationships between pseudoscalar-meson and quark masses in the two cases above will clearly have different consequences for chiral-symmetry-breaking effects; some of these will be analyzed below.

To proceed further by evaluating the magnitude of the quark masses involves more dynamical assumptions, which will not be reviewed here. In the quadratic-solution case typical values are[10]

$$m_u \simeq 5 \text{ MeV}, \quad m_s \simeq 125 \text{ MeV}, \tag{2.6}$$

implying that SU(2) × SU(2) is almost a perfect symmetry of nature, and that SU(3) × SU(3) breaking can be treated as a perturbation. In the linear solution case, typical values are[12,20]

$$m_u \simeq 30\text{–}40 \text{ MeV}, \quad m_s \simeq 180\text{–}240 \text{ MeV}, \tag{2.7}$$

which also lead one to expect a perturbative approach to chiral-symmetry breaking to be useful. Another set of values is[19]

$$m_u \simeq 140 \text{ MeV}, \quad m_s \simeq 680 \text{ MeV}, \tag{2.8}$$

which is not obviously compatible with a perturbative treatment of chiral-SU(3) × SU(3) breaking.

Actually, aside from a few numerical applications involving the first two sets of values for m_u and m_s, we shall not be much concerned here with presumed values for the quark masses. On the other hand, we shall make extensive use of the (assumed) property that the quark masses are relatively small with respect to the hadronic mass scale. This will allow us to expand various quantities in perturbation series in terms of them, at least up to second order, in the quadratic as well as in the linear solution cases.

III. GENERALIZED PCAC

As we mentioned in the Introduction, we will be guided by confined-quark potential models to the extent that we make the assumption of the existence of a possibly infinite sequence of pseudoscalar mesons for each SU(3)-octet state quantum number. The ensuing SU(6) × O(3) pattern can be interpreted as arising from radial excitations of the corresponding lowest-lying particles. The K' particle (1.4 GeV) recently discovered[21] at SLAC is a candidate for the first radial excitation of the K; presumably, its nonstrange partner π' lies nearby in mass.

Because of their large masses, these radially excited pseudoscalar mesons cannot be considered to be almost-Goldstone bosons; their masses are set by the hadronic mass scale, and so should not vanish in the chiral-symmetry limit. In this limit, the corresponding weak decay constants should vanish; these constants are then first-order quantities in chiral-symmetry breaking. (There does not appear to be any reason to expect the decay constants to be second-order quantities.)

Let F_{π^n} and F_{K^n} be the weak decay constants of the nth radially excited pseudoscalar mesons π^n and K^n. From the matrix elements

$$\begin{aligned} \langle 0 | D_5^\pi | \pi^n \rangle &= -iM_{\pi^n}{}^2 F_{\pi^n} , \\ \langle 0 | D_5^K | K^n \rangle &= -iM_{K^n}{}^2 F_{K^n} , \end{aligned} \tag{3.1}$$

and the fact that $M_{\pi^n}{}^2, M_{K^n}{}^2$ are zeroth-order-quantities for $n \geq 1$, one finds in lowest order in the quark masses, with $n \geq 1$,

$$\begin{aligned} F_{\pi^n} &\propto m_u , \\ F_{K^n} &\propto (m_u + m_s)/2 . \end{aligned} \tag{3.2}$$

What could be the magnitude of these decay constants? An estimate has been made in the linear-solution case, yielding[20]

$$F_{\pi^n}/F_\pi \simeq M_\pi/M_{\pi^n}\,,\qquad F_{K^n}/F_K \simeq M_K/M_{K^n}\,. \tag{3.3}$$

Now, for the case $n=1$, $M_{K'}$ and $M_{\pi'}$ are approximately 1.4 GeV, which implies

$$F_{\pi'}/F_\pi \simeq 0.1\,, \quad F_{K'}/F_K \simeq 0.35\,. \tag{3.4}$$

In the quadratic solution case one gets the upper bounds[22]

$$F_{\pi^n}/F_\pi \lesssim (\tfrac{2}{3})^{1/2} M_\pi^2/M_{\pi^n} M_\rho\,,\qquad F_{K^n}/F_K \lesssim (\tfrac{2}{3})^{1/2} M_K{}^2/M_{K^n} M_{K^*}\,, \tag{3.5}$$

$$F_{\pi'}/F_\pi \lesssim 0.015\,,\qquad F_{K'}/F_K \lesssim 0.18\,. \tag{3.6}$$

Consequently, we are encouraged to proceed with a perturbative approach to chiral-symmetry breaking, without prejudice as to the linear or quadratic solution; furthermore, we feel justified in using the $n=1$ meson contribution when a rough estimate of correction terms is desired.

In order to have an explicit framework to explore the consequences of this approach to chiral-symmetry breaking, we will adopt the hypothesis that the infinite family of pseudoscalar mesons, with $n=0, 1, \ldots$, effectively provides a complete set of states in the relevant sector of quantum numbers. In the narrow-width approximation, this implies that the axial-vector divergence can be expressed as an infinite sum of effective fields[16]:

$$i D_5^\pi = M_\pi{}^2 F_\pi \varphi_\pi + \sum_{n=1}^{\infty} M_{\pi^n}{}^2 F_{\pi^n} \varphi_{\pi^n}\,,\qquad i D_5^K = M_K{}^2 F_K \varphi_K + \sum_{n=1}^{\infty} M_{K^n}{}^2 F_{K^n} \varphi_{K^n}\,. \tag{3.7}$$

The above equations are to be understood in the sense of the saturation of absorptive parts of dispersion integrals and sum rules by sequences of particles, since the existence of appropriate interpolating fields φ_{π^n}, φ_{K^n}, constructed in terms of elementary quark fields, is not guaranteed. For instance, the ansatz Eq. (3.7) could be used to express, through the use of unsubtracted dispersion relations, the one-particle matrix elements of light-plane charges in terms of the contributions of the infinite sequence of pseudoscalar mesons. Denoting by g_A the pion axial-vector form factor at zero momentum transfer, say between two nucleons, one gets

$$(g_A/g_V) M_N = F_\pi g_{NN\pi} + \sum_{n=1}^{\infty} F_{\pi^n} g_{NN\pi^n}\,, \tag{3.8}$$

where $g_{NN\pi^n}$ is the coupling constant of π^n to two nucleons. Similar expressions could also be derived from the matrix elements of the kaon axial-vector charge.[23] Since the decay constants F_{π^n} (F_{K^n}) are first-order quantities in chiral-symmetry breaking, then the contribution of the pion (kaon) term, which is of zeroth order, is dominant and the usual Goldberger-Treiman relations can be understood as exact in the chiral-symmetry limit.[2] However, the additional contribution of the new terms of (3.7) can also be used to evaluate the corrective effects (at first order) to these relations.[16]

It is important to emphasize that the series Eq. (3.7) will effectively reduce in most cases to a small number of terms, as a consequence of the empirical fact that one-particle states seem to couple preferentially to nearby states.

In the next two sections we shall consider applications of generalized PCAC to matrix elements which are not of zeroth order in chiral-symmetry breaking.

IV. K_{l3} FORM FACTOR

In the following we shall study matrix elements of the type

$$\langle 0 | [Q^a_{(5)},\; D^b_5] | p^c \rangle \quad (a, b = 1, \ldots, 7)\,, \tag{4.1}$$

where $Q^a_{(5)}$ is a vector (axial-vector) light-plane charge, $|p\rangle$ is a one-particle state, and the commutator is taken at equal lightlike "times." The special property of the light-plane charges in annihilating the vacuum, independently of the nonconservation of the corresponding currents, forbids the presence of disconnected diagrams in the expansion over intermediate states of the above matrix elements.[24] As far as the convergence of this expansion is concerned, one can show on the basis of light-cone analysis and the support properties of the relevant scaling functions[25] that asymptotically the contributions decrease (in the quark-parton model) faster than $(M_n{}^2)^{-1}$, where M_n is the invariant mass of the nth intermediate state (see Refs. 24 and 26, Sec. 8, and Ref. 27, Sec. 4); therefore the series is convergent.

We abstract[12-14,19,28] the transformation properties of the current divergences with respect to the $[SU(3)\times SU(3)]_L$ algebra of light-plane charges from the formal vector-gluon model (or the tree approximation of quantum chromodynamics). The divergences belong to a mixture of three representations: one $[(\bar{3},3)\mp(3,\bar{3})]$ representation with a coefficient linear in the quark mass matrix (proportional to $[m, \frac{1}{2}\lambda^a]_\pm$), and two $[(8,1)\mp(1,8)]$ representations which are of quadratic order in the quark mass matrix (proportional to $[m,[m,\frac{1}{2}\lambda^a]_\pm]_\pm$ and $[m^2, \frac{1}{2}\lambda^a]_-$, respectively) and interaction independent. When taken between vacuum and a meson state, the last two pieces will act in the sector of the Hilbert space containing only a quark-antiquark pair.

As stated earlier, we assume that physical quantities can be expanded in perturbation series (at least up to second order) in terms of the quark masses.

A. PCAC for π

We begin by considering the matrix elements

$$\langle 0|[Q^K, D_5^\pi]|K\rangle ,$$

which were first studied by Leutwyler.[24] Using the light-plane transformation properties of the axial divergence and saturating the matrix element according to the scheme Eq. (3.7), we get the relation

$$M_K{}^2F_K = \left(\frac{m_s+m_u}{2m_u}\right)\left[M_\pi{}^2F_\pi f_+(0) + \sum_{n=1}^{\infty} M_{\pi^n}{}^2 F_{\pi^n} f_+{}^{(n)}(0)\right] + \left(\frac{m_s{}^2-m_u{}^2}{4}\right) I_K F_K , \tag{4.2}$$

where $f_+^{(n)}(0)$ is the analog of the K_{l3} form factor defined by the matrix elements $\langle \pi^n|Q^K|K\rangle$, and is a first-order SU(3)-breaking quantity; furthermore,

$$I_K F_K = \int_{-1/2}^{+1/2} d\xi\, \varphi_K(\xi)/(\tfrac{1}{4}-\xi^2) , \tag{4.3}$$

φ_K being the "scaling function" of the K meson, related to the matrix element

$$\langle 0|\bar{q}(x)(\gamma_0+\gamma_3)\gamma_5 \tfrac{1}{2}\lambda^\alpha q(0)|K^\beta\rangle_{x^0=x^3} ,$$

and satisfying

$$F_K = \int_{-1/2}^{+1/2} d\xi\, \varphi_K(\xi) . \tag{4.4}$$

Spectral conditions[25] force the scaling function $\varphi_K(\xi)$ to vanish outside the region $|\xi| \leq \frac{1}{2}$. Charge-conjugation invariance implies that φ_K is an even function of ξ. Assuming a simple dependence for $\varphi_K(\xi)$ upon the scaling variable ξ, of the type $(\frac{1}{4} - \xi^2)^m$, we can get a rough estimate of the quantity I_K. For $m=1$ we get $I_K=6$ while for other values of $m>0$, the above estimate is not modified significantly. In any event, an estimate of the order of $I_K = 6\pm 3$ seems reasonable.

We can now analyze relation (4.2) in both quadratic and linear cases. For simplicity we retain only the contribution of the π' meson in the series of the right-hand side of Eq. (4.2).

(i) In the quadratic-solution case (2.4), $M_\pi{}^2$ and $M_K{}^2$ are first-order quantities. To this order Eq. (4.2) reduces to

$$M_K{}^2 = \left(\frac{m_s+m_u}{2m_u}\right) M_\pi{}^2 , \tag{4.5}$$

which is nothing other than Eq. (2.2). After using (4.5), Eq. (4.2) becomes in second order

$$\left(\frac{F_K}{F_\pi f_+(0)} - 1\right) \simeq \left(\frac{m_s{}^2-m_u{}^2}{4M_K{}^2}\right) I_K \frac{F_K}{F_\pi f_+(0)} + \frac{M_{\pi'}{}^2 F_{\pi'}}{M_\pi{}^2 F_\pi}\frac{f_+^{(1)}(0)}{f_+(0)} . \tag{4.6}$$

Numerical analysis of this relation shows that it is consistent so long as $f_+^{(1)}(0)/f_+(0) \lesssim 0.1$, which is a typical first-order SU(3)-breaking effect. Thus the sum rule Eq. (4.2) appears to be reasonably satisfied in the quadratic-solution case.

(ii) In the linear-solution case (2.5), $M_\pi{}^2$ and $M_K{}^2$ are second-order quantities, and therefore the deviations of F_K/F_π and $f_+(0)$ from 1 will contribute only in third and fourth order. To second order, after using Eq. (2.5), Eq. (4.2) becomes

$$(1 - M_\pi/M_K) \simeq \left(\frac{m_s{}^2-m_u{}^2}{4M_K{}^2}\right) I_K + \frac{M_{\pi'}{}^2 F_{\pi'}}{M_K M_\pi F_\pi} f_+{}^{(1)}(0) . \tag{4.7}$$

In view of the uncertainty in $m_s{}^2 - m_u{}^2$ and in I_K, the contribution of the π' term is not determined by the above sum rule. Nevertheless, it is worth noting that a value of $f_+^{(1)} \simeq 0.1$ is sufficient for saturation if $m_s{}^2 - m_u{}^2$ and I_K fall in the range of values described earlier. For some cases, the π' term contributes nearly 40% of the right-hand side; an analysis of relation (4.2) which ignores the contribution of the heavy state π' might lead one to reject the linear solution, Eq. (2.5). However, it is important to stress that this results only from an incorrect application of PCAC, since the disregarded terms are of the same order in chiral-symmetry breaking as those retained. Also notice that both sides of Eq. (4.7) have the same quark mass dependence $(m_s - m_u)/(m_s + m_u)$, ensuring the formal consistency of the equation.

Thus the sum rule resulting from the matrix element $\langle 0|[Q^K, D_5^\pi]|K\rangle$ appears to be satisfactorily verified both by the quadratic and linear solutions Eqs. (2.4) and (2.5) provided one uses PCAC in its generalized form Eq. (3.7).

B. PCAC for K

Our prescription for generalized PCAC should be valid for both pion and kaon axial-vector divergences. If correctly applied, it should not lead to inconsistencies, at least on the formal level. The only difference arises on numerical grounds, where one observes from Eq. (3.2) that corrections to ordinary PCAC will be more important for the kaon axial-vector divergence.

As a first check for the consistency of the application of (generalized) kaon PCAC, we consider the matrix element

$$\langle 0|[Q^K, D_5^K]|\pi\rangle, \quad (4.8)$$

which involves form factors similar to those of Eq. (4.2). Proceeding as above, we obtain the relation

$$M_\pi^2 F_\pi = \left(\frac{2m_u}{m_s+m_u}\right)\left[M_K^2 F_K f_+(0) + \sum_{n=1}^{\infty} M_{K^n}^2 F_{K^n} \tilde{f}_+^{(n)}(0)\right] - \left(\frac{m_s^2 - m_u^2}{4}\right) I_\pi F_\pi, \quad (4.9)$$

where $\tilde{f}_+^{(n)}(0)$ is the form factor defined by the matrix element $\langle K^n|Q^K|\pi\rangle$. Taking into account charge-conjugation invariance and the fact that the form factors $\tilde{f}_+^{(n)}$ and $f_+^{(n)}$ defined in (4.2) are first-order quantities, one can show to this order that $\tilde{f}_+^{(n)}(0) = -f_+^{(n)}(0)$. Therefore, using Eq. (3.2) and the fact that at zeroth order $M_{K^n}^2 = M_{\pi^n}^2$, Eq. (4.9) implies

$$M_K^2 F_K f_+(0) = \left(\frac{m_s+m_u}{2m_u}\right)\left[M_\pi^2 F_\pi + \sum_{n=1}^{\infty} M_{\pi^n}^2 F_{\pi^n} f_+^{(n)}(0)\right] + \left(\frac{m_s^2 - m_u^2}{4}\right) I_\pi F_\pi. \quad (4.10)$$

To second order in chiral-symmetry breaking, this is equivalent to Eq. (4.2) which was derived by the application of (generalized) pion PCAC.

V. COUPLINGS OF THE RADIAL EXCITATIONS

A. Coupling to vector and pseudoscalar mesons, VPP^n

We now turn to a consideration of matrix elements of the type

$$\langle 0|[Q_5^a, D_5^b]|V^c, \lambda=0\rangle \quad (a, b = 1, \ldots, 7), \quad (5.1)$$

where $|V, \lambda=0\rangle$ represents a vector meson in its longitudinal light-plane helicity state. We take into account the $[SU(3)\times SU(3)]_L$ transformation properties of the axial-vector divergences, saturate the matrix element by the sequence of radially excited pseudoscalar mesons, and use Eq. (3.7). If V is a ρ meson, one gets the two relations

$$M_\pi^2 F_\pi^2 f_{\rho\pi\pi} + \sum_{n+1}^{\infty} M_{\pi^n}^2 F_{\pi^n} F_\pi f_{\rho\pi^n\pi} + \sum_{n=1}^{\infty} M_\pi^2 F_\pi F_{\pi^n} f_{\rho\pi^n\pi} + \sum_{n,m=1}^{\infty} M_{\pi^n}^2 F_{\pi^n} F_{\pi^m} f_{\rho\pi^n\pi^m} = m_u^2 I_\rho F_\rho M_\rho, \quad (5.2a)$$

$$M_K^2 F_K^2 f_{\rho KK} + \sum_{n=1}^{\infty} M_{K^n}^2 F_{K^n} F_K f_{\rho K^n K} + \sum_{n=1}^{\infty} M_K^2 F_K F_{K^n} f_{\rho K^n K} + \sum_{n,m=1}^{\infty} M_{K^n}^2 F_{K^n} F_{K^m} f_{\rho K^n K^m} = \left(\frac{m_s+m_u}{2}\right)^2 I_\rho F_\rho M_\rho, \quad (5.2b)$$

where I_ρ is the analog of the quantities I_K, I_π defined above in Eqs. (4.3) and (4.4). The coupling constants are defined as

$$\langle V^a(k,\lambda)|j_{P^b}|P^c(p)\rangle = f_{abc}\, 2\epsilon^\lambda\cdot p f_{VPP}. \quad (5.3)$$

Note that because of its even charge-conjugation parity, the $[(\overline{3},3)+(3,\overline{3})]$ part of the commutator does not contribute to the above matrix elements, and the latter come out to be of second order in the quark masses.

We next analyze, up to second order, the quark-mass dependence of the left-hand sides of Eqs. (5.2).

(*i*) *The quadratic-solution case*. Let us first point out that if we had retained only the pion or kaon contribution (ordinary PCAC), we would have concluded that M_π^2 and M_K^2 are second-order quantities in the quark masses. Therefore, the PCAC hypothesis in its generalized form Eq. (3.7) maintains the viability of the quadratic solution. Recall that in this case both M_π^2 and F_{π^n} are of first order in m_u. The third and fourth terms of the left-hand side of Eq. (5.2a) are therefore of second order in the quark mass. Since the first term of the left-hand side is of first order in m_u, the second term must also be of first order so as to cancel it; therefore,

$$M_\pi^2 F_\pi f_{\rho\pi\pi} + \sum_{n=1}^{\infty} M_{\pi^n}^2 F_{\pi^n} f_{\rho\pi^n\pi} = O(m^2). \quad (5.4)$$

If we assume that the ρ meson couples only to nearby resonances then we can neglect in the summation above all but the first radially excited meson (the π'). To this approximation,

$$|f_{\rho\pi'\pi}/f_{\rho\pi\pi}| \simeq |M_\pi^2 F_\pi / M_{\pi'}^2 F_{\pi'}| = O(1). \quad (5.5)$$

A similar conclusion can also be deduced from (5.2b), as well as from analysis of the matrix element $\langle 0|[Q_5^K, D_5^K]|\omega\rangle$:

$$|f_{\rho K'K}/f_{\rho KK}| \simeq |f_{\omega K'K}/f_{\omega KK}| \simeq |M_K^2 F_K / M_{K'}^2 F_{K'}| = O(1). \quad (5.6)$$

(*ii*) *The linear-solution case*. Here the first and fourth terms of Eq. (5.2a) are of second order, while the third term is of third order (and so can be neglected). Therefore, the second term must also be of second order. With the same approximation as above, this implies

$$f_{\rho\pi'\pi} = O(m_u) \quad (5.7)$$

and similarly

$$f_{\rho K'K} \simeq f_{\omega K'K} = O\left(\frac{m_s+m_u}{2}\right). \quad (5.8)$$

Next we consider the matrix elements Eq. (5.1) corresponding to the K^* meson state. One gets[29] the two equations

$$M_K^2 F_K F_\pi f_{K^*K\pi} + \sum_{n=1}^{\infty} M_{K^n}^2 F_{K^n} F_\pi f_{K^*K^n\pi} + \sum_{n=1}^{\infty} M_K^2 F_K F_{\pi^n} f_{K^*K\pi^n} + \sum_{n,m=1}^{\infty} M_{K^n}^2 F_{K^n} F_{\pi^m} f_{K^*K^n\pi^m} = m_u \left(\frac{m_s+m_u}{2}\right) I_{K^*} F_{K^*} M_{K^*}, \quad (5.9a)$$

$$M_\pi^2 F_\pi F_K f_{K^*K\pi} + \sum_{n=1}^{\infty} M_{\pi^n}^2 F_{\pi^n} F_K f_{K^*K\pi^n} + \sum_{n=1}^{\infty} M_\pi^2 F_\pi F_{K^n} f_{K^*K^n\pi} + \sum_{n,m=1}^{\infty} M_{\pi^n}^2 F_{\pi^n} F_{K^m} f_{K^*K^m\pi^n} = m_u \left(\frac{m_s+m_u}{2}\right) I_{K^*} F_{K^*} M_{K^*}. \quad (5.9b)$$

(i) In the quadratic-solution case the conclusions are analogous to Eq. (5.4): The first and second terms in the left-hand side must cancel to first order, while the third and fourth terms have the same quark-mass dependence as the right-hand side. The approximation of retaining only the first radial excitation implies

$$|f_{K^*K'\pi}/f_{K^*K\pi}| \simeq |M_K^2 F_K/M_{K'}^2 F_{K'}| = O(1), \quad (5.10a)$$

$$|f_{K^*K\pi'}/f_{K^*K\pi}| \simeq |M_\pi^2 F_\pi/M_{\pi'}^2 F_{\pi'}| = O(1). \quad (5.10b)$$

(ii) In the linear-solution case, the third term in Eqs. (5.9) is of third order, while the fourth term is of second order and has the same type of quark mass dependence as the right-hand side. The first term is also of second order, but is proportional to m_u^2 (or $[(m_s+m_u)/2]^2$) which does not match the right-hand side. Consequently, the second term of the left-hand side not only has to be of second order, but also has to cancel the contribution of the first term.

Let us assume for simplicity that the quark-mass dependence of the second term is entirely of the same type as that of the first term. Thus, to lowest order,

$$M_K^2 F_K f_{K^*K\pi} + \sum_{n=1}^{\infty} M_{K^n}^2 F_{K^n} f_{K^*K^n\pi} = O(m^3), \quad (5.11a)$$

$$M_\pi^2 F_\pi f_{K^*K\pi} + \sum_{n=1}^{\infty} M_{\pi^n}^2 F_{\pi^n} f_{K^*K\pi^n} = O(m^3). \quad (5.11b)$$

Keeping the π' and K' contributions only, after using Eq. (3.3), we find

$$|f_{K^*K'\pi}/f_{K^*K\pi}| \simeq M_K/M_{K'}, \quad (5.12a)$$

$$|f_{K^*K\pi'}/f_{K^*K\pi}| \simeq M_\pi/M_{\pi'}. \quad (5.12b)$$

Similarly,

$$|f_{\rho K'K}/f_{\rho KK}| \simeq |f_{\omega K'K}/f_{\omega KK}| \simeq M_K/M_{K'}, \quad (5.13a)$$

$$|f_{\rho\pi'\pi}/f_{\rho\pi\pi}| \simeq M_\pi/M_{\pi'}. \quad (5.13b)$$

One can now estimate the partial decay widths of the π' and K' mesons (within 20–40%),

$$\begin{aligned} &\Gamma_L(\pi' \to \rho\pi) \simeq 10 \text{ MeV}, \\ &\Gamma_L(K' \to \rho K) \simeq 30 \text{ MeV}, \\ &\Gamma_L(K' \to \omega K) \simeq 10 \text{ MeV}, \\ &\Gamma_L(K' \to K^*\pi) \simeq 60 \text{ MeV}, \end{aligned} \quad (5.14)$$

where the subscript L indicates that these estimates refer to the linear solution. Recall now that the ratios $F_{\pi'}/F_\pi$ and $F_{K'}/F_K$ satisfy in the quadratic solution case the bounds of Eq. (3.6); this allows us to place the following lower bounds (which we trust to about 20%) on the partial decay widths for π' and K':

$$\begin{aligned} &\Gamma_Q(\pi' \to \rho\pi) \gtrsim 500 \text{ MeV}, \\ &\Gamma_Q(K' \to \rho K) \gtrsim 125 \text{ MeV}, \\ &\Gamma_Q(K' \to \omega K) \gtrsim 40 \text{ MeV}, \\ &\Gamma_Q(K' \to K^*\pi) \gtrsim 250 \text{ MeV}. \end{aligned} \quad (5.15)$$

The subscript Q indicates that these estimates refer to the quadratic solution.

In summary, analysis of the quark-mass dependence of the matrix elements Eq. (5.1) leads to predictions about the coupling constants of the radially excited pseudoscalar mesons to the low-lying vector and pseudoscalar mesons. The conclusions differ, however, in the quadratic- and linear-solution cases. In the former case, they are predicted to be zeroth-order quantities in chiral-symmetry breaking, with consequently rather large partial decay widths. In the latter case, they appear to be first-order quantities, thus leading to relatively small partial decay widths.

B. Couplings to scalar and pseudoscalar mesons, SPP^n

The preceding analysis can be extended to the couplings of scalar to pseudoscalar mesons. We consider the matrix elements

$$\langle 0|[Q_5^a, D_5^b]|S^c\rangle \quad (a, b = 1, \ldots, 7), \quad (5.16)$$

where $|S\rangle$ is a scalar meson state.

If S is the isoscalar ϵ meson, presumably made of nonstrange quarks, we find

$$M_\pi{}^2 F_\pi{}^2 \frac{f_{\epsilon\pi\pi}}{M_\epsilon^2 - M_\pi{}^2} + \sum_{n=1}^\infty M_\pi{}^{n\,2} F_\pi{}^n F_\pi \frac{f_{\epsilon\pi^n\pi}}{M_\epsilon^2 - M_\pi{}^{n\,2}} + \sum_{n=1}^\infty M_\pi{}^2 F_\pi F_\pi{}^n \frac{f_{\epsilon\pi^n\pi}}{M_\epsilon^2 - M_\pi{}^2} + \sum_{n,m=1}^\infty M_\pi{}^{n\,2} F_\pi{}^n F_\pi{}^m \frac{f_{\epsilon\pi^n\pi^m}}{M_\epsilon^2 - M_\pi{}^{n\,2}} = 2m_u G_\epsilon\,, \tag{5.17}$$

where G_a represents the coupling of the scalar meson S^b to the vacuum via the scalar density u^a:

$$\langle 0|u^a|S^b\rangle = G_a \delta_{ab} \qquad (a,b=1,\ldots,7)\,, \tag{5.18}$$

$$\langle S^a|j_{P^b}|P^c\rangle = [d_{abc} + (\tfrac{2}{3})^{1/2}\delta_{a0}\delta_{bc}] f_{SPP} \qquad (b,c=1,\ldots,7)\,. \tag{5.19}$$

Here, in contrast to the vector-meson case, it is the $[(\overline{3},3)+(3,\overline{3})]$ part of the commutator which provides the unique contribution to the matrix element. In general, the coupling constant G_a is expected to be a zeroth-order quantity in chiral-symmetry breaking and therefore the right-hand side of the equation is of first order. The quark-mass-dependence analysis is now straightforward and proceeds as in Sec. V A.

(i) For the quadratic-solution case, the first and second terms of the left-hand side are of first order; there are no results of importance for the coupling constants $f_{\epsilon\pi^n\pi}$. Presumably they are zeroth-order quantities.

(ii) For the linear-solution case, only the second term of the left-hand side can be a first-order quantity, and therefore it must balance (in leading order) the right-hand side of the equation:

$$\sum_{n=1}^\infty M_\pi{}^{n\,2} F_\pi{}^n F_\pi \frac{f_{\epsilon\pi^n\pi}}{M_\epsilon^2 - M_\pi{}^{n\,2}} = 2m_u G_\epsilon + O(m^2)\,. \tag{5.20}$$

After retaining in the series the π' only, one obtains the estimate

$$f_{\epsilon\pi'\pi} \simeq \frac{2m_u G_\epsilon (M_\epsilon{}^2 - M_{\pi'}{}^2)}{M_{\pi'}{}^2 F_{\pi'} F_\pi}\,. \tag{5.21}$$

The coupling constants G_a are not measurable quantities; however, they can be evaluated on theoretical grounds. In Ref. 20, the following relation was derived between the κ-meson parameters and that of K and π:

$$M_\kappa F_\kappa = M_K F_K - M_\pi F_\pi\,. \tag{5.22}$$

Remembering that

$$i\langle 0|D^K|\kappa\rangle = M_\kappa{}^2 F_\kappa = (m_s - m_u)\langle 0|u^K|\kappa\rangle = (m_s - m_u) G_\kappa \tag{5.23}$$

and using the quark mass formulas obtained in Ref. 20,

$$m_u \simeq M_\pi F_\pi/\sqrt{6}F_\rho\,, \quad (m_s + m_u)/2 \simeq M_K F_K/\sqrt{6}\,F_{K^*}\,, \tag{5.24}$$

we find

$$G_\kappa \simeq (\tfrac{3}{2})^{1/2} M_\kappa F_{K^*}\,, \tag{5.25}$$

which can be generalized to the other scalar mesons

$$G_\epsilon = (\tfrac{3}{2})^{1/2} M_\epsilon F_\omega\,, \quad G_\delta \simeq (\tfrac{3}{2})^{1/2} M_\delta F_\rho\,. \tag{5.26}$$

(5.21) then reduces to

$$|f_{\epsilon\pi'\pi}| \simeq M_\epsilon (M_{\pi'}{}^2 - M_\epsilon{}^2)/F_\pi M_{\pi'}\,. \tag{5.27a}$$

Similar expressions can also be obtained for the coupling constants $f_{\epsilon K'K}$, $f_{\kappa K'\pi}$, $f_{\kappa K\pi'}$ by choosing the appropriate SU(3) indices in the matrix elements of (5.16),

$$|f_{\epsilon K'K}| \simeq M_\epsilon (M_{K'}{}^2 - M_\epsilon{}^2)/F_K M_{K'}\,, \tag{5.27b}$$

$$|f_{\kappa K'\pi}| \simeq M_\kappa (M_{K'}{}^2 - M_\kappa{}^2)/F_K M_{K'}\,, \tag{5.27c}$$

$$|f_{\kappa K\pi'}| \simeq M_\kappa (M_{\pi'}{}^2 - M_\kappa{}^2)/F_\pi M_\pi\,. \tag{5.27d}$$

These formulas allow us to calculate the partial decay widths of π' and K' into scalar and pseudoscalar mesons. The only difficulty lies in the lack of knowledge of the masses of the scalar mesons. These are not accurately known; we adopt here a purely phenomenological attitude by assigning them "effective" values. We choose $M_\epsilon \approx 700$ MeV and $M_\kappa \approx 1100-1300$ MeV. We then get the partial decay widths

$$\Gamma_L(\pi' \to \epsilon\pi) \simeq 400 \text{ MeV}\,, \quad \Gamma_L(K' \to \epsilon K) \simeq 100 \text{ MeV}\,, \quad \Gamma_L(K' \to \kappa\pi) \lesssim 100 \text{ MeV}\,. \tag{5.28}$$

Let us suppose that π' and K' decay entirely via the channels just discussed; furthermore, it is reasonable to suppose that for the quadratic solution the $P' \to SP$ rates are at least as great as for the linear solution. Then the total widths satisfy

$$\Gamma_L^{\text{tot}}(\pi') \sim 400 \text{ MeV}, \quad \Gamma_Q^{\text{tot}}(\pi') > 900 \text{ MeV}, \quad 200 \text{ MeV} \lesssim \Gamma_L^{\text{tot}}(K') \lesssim 300 \text{ MeV}, \quad \Gamma_Q^{\text{tot}}(K') > 600 \text{ MeV}\,. \tag{5.29}$$

Qualitatively, then, the linear solution gives a K' total as well as partial width consistent with experiment,[21] while the quadratic solution gives the corresponding widths too large.[30] The π' total width is so large in both cases as to make its detection quite difficult.

C. Couplings to two pseudoscalar mesons, PPV^n

As we analyzed the couplings of radially excited pseudoscalar mesons, we may similarly treat their vector-meson partners. If we assume that the ρ' (1.55-GeV) meson is one of the SU(6) partners of π' and K', then we get the equalities[31]

$$\langle\pi'|Q_5^\pi|\rho\rangle=\langle\rho'|Q_5^\pi|\pi\rangle, \qquad (5.30a)$$

$$\langle K'|Q_5^K|\rho\rangle=\langle\rho'|Q_5^K|K\rangle. \qquad (5.30b)$$

We can now apply generalized PCAC, Eq. (3.7), to get

$$\left(F_\pi f_{\rho\pi'\pi}+\sum_{n=1}^{\infty}F_{\pi^n}f_{\rho\pi'\pi^n}\right)\Big/M_\rho = \left(F_\pi f_{\rho'\pi\pi}+\sum_{n=1}^{\infty}F_{\pi^n}f_{\rho'\pi\pi^n}\right)\Big/M_{\rho'}, \qquad (5.31a)$$

$$\left(F_K f_{\rho K'K}+\sum_{n=1}^{\infty}F_{K^n}f_{\rho K'K^n}\right)\Big/M_\rho = \left(F_K f_{\rho'KK}+\sum_{n=1}^{\infty}F_{K^n}f_{\rho'KK^n}\right)\Big/M_{\rho'}. \qquad (5.31b)$$

(i) For the quadratic-solution case the first term of the left-hand side of Eqs. (5.31) is of zeroth order in chiral-symmetry breaking, while the second terms of the left-hand and right-hand sides are of first order. Therefore the first term of the right-hand sides must also be of zeroth order, and must satisfy the equalities

$$f_{\rho\pi'\pi}/M_\rho\simeq f_{\rho'\pi\pi}/M_{\rho'}\simeq\left|M_\pi^2F_\pi f_{\rho\pi\pi}/M_{\pi'}^2F_{\pi'}M_\rho\right|, \qquad (5.32a)$$

$$f_{\rho K'K}/M_\rho\simeq f_{\rho'KK}/M_{\rho'}\simeq\left|M_K^2F_K f_{\rho KK}/M_{K'}^2F_{K'}M_\rho\right|. \qquad (5.32b)$$

The second equalities come from Eqs. (5.5) and (5.6). These relations imply that the coupling constants $f_{\rho'\pi\pi}$ and $f_{\rho'KK}$ are zeroth-order quantities and the decay widths $\Gamma(\rho'\to\pi\pi)$, $\Gamma(\rho'\to KK)$ [evaluated by using the estimates Eq. (3.6)] are of the order of a few hundred MeV.

(ii) For the linear-solution case all the terms of the left-hand sides of Eqs. (5.31) are of first order in chiral-symmetry breaking, which implies that the first term of the right-hand sides is also of first order. Therefore the coupling constants $f_{\rho'\pi\pi}$ and $f_{\rho'KK}$ are expected to be small. To get a quantitative estimate of their values, let us suppose that the equalities Eq. (5.31) hold also (approximately) term by term, that is

$$f_{\rho\pi'\pi}/M_\rho\approx f_{\rho'\pi\pi}/M_{\rho'}\approx f_{\rho\pi\pi}M_\pi/M_\rho M_{\pi'}, \qquad (5.33a)$$

$$f_{\rho K'K}/M_\rho\approx f_{\rho'KK}/M_{\rho'}\approx f_{\rho KK}M_K/M_\rho M_{K'}. \qquad (5.33b)$$

The second equalities come from Eqs. (5.13). One then gets the predictions

$$\Gamma_L(\rho'\to\pi\pi)\approx 10\text{ MeV}, \quad \Gamma_L(\rho'\to KK)\approx 30\text{ MeV}. \qquad (5.34)$$

which seem to be in agreement with the experimental data.[32] This indicates qualitatively that the decay modes of the ρ' into two pseudoscalar mesons are completely negligible, since $\Gamma_{\rho'}$ is on the order of several hundred MeV.

There are experimental indications[33] for the existence of a ρ resonance at 1250 MeV, but the overall evidence is inconclusive at present. In the conventional quark-model framework, with a harmonic-oscillator spectrum, only even daughters of Regge trajectories appear; therefore, the first radial excitation of the ρ is found near 1.6 GeV. If one were forced to include odd daughter trajectories [as a $\rho(1250)$ would imply], then the conventional framework would have to be drastically modified. Similar remarks hold, *mutatis mutandis*, for daughter trajectories of the pions.

VI. SUMMARY AND CONCLUDING REMARKS

The idea of an approximate hadronic symmetry such as SU(2), SU(3), or SU(3) × SU(3) has proved to be extremely useful, in spite of our lack of knowledge of the forces which break these symmetries. The key to success lies in our ability to recognize an appropriate "symmetric" world as a reference, from which we may proceed in a perturbative sense if the symmetry breaking is small. We have been concerned in this paper with the problem of SU(3) × SU(3) symmetry in general, and with the breaking of this symmetry via PCAC in particular. The quark–vector-gluon model suggests that chiral symmetry is only broken by the quark mass term; we have adopted this viewpoint, with the further assumption that the quark masses may be treated as small parameters in a perturbative expansion about the "massless" limit. Note that the quark masses enter not as inertial masses of free quarks, but as symmetry-breaking parameters. In particular, the mass $m_u=m_d$ measures the breaking of SU(2) × SU(2), while m_s-m_u describes the breaking of SU(3).

We have stressed the necessity of generalizing the notion of PCAC by assuming the dominance of the axial-vector divergence by the sequence of all pseudoscalar mesons appearing in the model. The ground state of this sequence is the usual almost-Goldstone boson, while the remaining particles are its radial excitations, whose masses do not vanish in the chiral-symmetry limit. Such a generalization provides a simple and systematic way of treating corrections to ordinary PCAC.

We have studied vacuum-to-one-particle matrix elements of commutators of light-plane charges and axial-vector divergences, expanded up to second order in the quark masses. It was shown that the quadratic [$M_\pi^2=0(m_u)$] and linear [$M_\pi=O(m_u)$] solutions lead to consistent results, at least formally. However, the respective predictions frequently differ from each other, notably with respect to the decay modes of the radially excited pseudoscalar and vector mesons. It appears that the linear solution provides a natural explanation for the smallness of some decay amplitudes which are predicted to be first-order chiral-symmetry-breaking quantities.

One may also study low-energy theorems, such as those involving σ terms in πN scattering, using the approach we have developed in this paper. However, one will encounter technical difficulties here which arise from the mass-shell extrapolation. One may adopt one of the several prescriptions that have been proposed for extrapolating onto the mass shell. Then one saturates the pseudoscalar sector of the absorptive part of the relevant amplitude by the entire family of pseudoscalar mesons and radially excited partners, and not only by the lowest-lying state. Therefore, the corrections to PCAC and the corrections arising from the mass-shell extrapolation may be treated separately and the effects added.

It is clear that a determination (or even an estimate) of the corrections to ordinary PCAC, such as we have given, leads to a firmer theoretical basis for certain sum rules and low-energy theorems. If done carefully, a perturbative approach to chiral SU(3) symmetry breaking can be consistent.

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*Laboratory associated with C.N.R.S.

[1]Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); 124, 246 (1961); Y. Nambu and D. Lurié, *ibid.* 125, 1429 (1962).

[2]R. Dashen, Phys. Rev. 183, 1245 (1969); R. Dashen and M. Weinstein, *ibid.* 183, 1261 (1969); 188, 2330 (1969).

[3]For a recent review, with an extensive bibliography, see H. Pagels, Phys. Rep. 16C, 219 (1974).

[4]M. Gell-Mann and M. Lévy, Nuovo Cimento 26, 53 (1960).

[5]J. L. Rosner, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974,* edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. II-171; S. Meshkov, *ibid.* p. II-101; A. J. G. Hey, *ibid.* p. II-120; A. J. G. Hey, Southampton University report, 1976 (unpublished).

[6]F. J. Gilman, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974,* edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p.IV-149.

[7]H. Fritzsch and M. Gell-Mann, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 2, p. 135; H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47B, 365 (1973).

[8]M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[9]S. L. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968).

[10]H. Leutwyler, Nucl. Phys. B76, 413 (1974).

[11]M. D. Scadron and H. F. Jones, Phys. Rev. D 10, 967 (1974).

[12]H. Sazdjian and J. Stern, Nucl. Phys. B94, 163 (1975); H. Sazdjian, *ibid.* B129, 319 (1977).

[13]N. H. Fuchs, Phys. Rev. D 14, 1709 (1976).

[14]P. Sinha and V. P. Gautam, Indian J. Phys. 50, 981 (1976); Indian J. Phys. (to be published).

[15]D. G. Caldi and H. Pagels, Phys. Rev. D 14, 809 (1976); 15, 2668 (1977).

[16]For earlier attempts along these lines, see C. A. Dominguez, Phys. Rev. D 7, 1252 (1973); 15, 1350 (1977); S. D. Drell, *ibid.* 7, 2190 (1973); J. L. Newmeyer and S. D. Drell, *ibid.* 8, 4070 (1973); I. Bars and M. B. Halpern, *ibid.* 7, 3043 (A73).

[17]See, for example, S. B. Treiman, R. Jackiw, and D. J. Gross, *Lectures on Current Algebra and Its Applications* (Princeton University Press, Princeton, New Jersey, 1972).

[18]H. Pagels, Phys. Rev. 179, 1337 (1968); H. Pagels and A. Zepeda, Phys. Rev. D 5, 3262 (1972); H. F. Jones and M. D. Scadron, *ibid.* 11, 174 (1975).

[19]J. F. Gunion, P. C. McNamee, and M. D. Scadron, Nucl. Phys. B123, 445 (1977).

[20]H. Sazdjian, Nucl. Phys. B129, 319 (1977).

[21]G. W. Brandenberg *et al.*, Phys. Rev. Lett. 36, 1239 (1976).

[22]These bounds are obtained by the use of duality and finite-energy sum rules (FESR) as in Ref. 20 and in H. Fritzsch and H. Leutwyler, Phys. Rev. D 10, 1624 (1974). As suggested by these authors, the pion (kaon) may have a long asymptotic tail in the free two-point spectral function, which in the chiral-symmetry limit would behave as $1/m_u$, thus implying $M_\pi^2 \sim m_u M_0$. Hence, the FESR applied to the radially excited pseudoscalar mesons yield upper bounds on the decay constants, since part of the higher-mass intervals receive contributions from the pion (kaon). For quark masses, we use the estimates of Ref. 10, since we are concerned with the quadratic solution at this point.

[23]A related analysis was given in Ref. 20, where it was shown that one could estimate, starting from the experimental value of F_π and using Eq. (3.2), the corrective effects in kaon PCAC, as applied to the ΛNK coupling. It was found that in the linear solution, these effects (which were of the order of 25%) reproduced rather well the experimental value of the ΛNK coupling constant, $g_{\Lambda NK}$. On the other hand, the quadratic solution disagreed with the observed value of $g_{\Lambda NK}$ by roughly a factor of 2.

[24]H. Leutwyler, in *Proceedings of the Adriatic Summer*

Meeting on Particle Physics, Rovinj, Yugoslavia, 1973, edited by M. Martinis, S. Pallua, and N. Zovko (North-Holland, Amsterdam, 1974).

[25]S. Deser, W. Gilbert, and E. C. G. Sudarshan, Phys. Rev. 115, 731 (1959).

[26]H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Caltech Report No. CALT-68-456 (unpublished).

[27]H. Sazdjian, Nucl. Phys. B62, 286 (1973).

[28]N. H. Fuchs, Phys. Rev. D 16, 1535 (1977).

[29]We have neglected in the right-hand side of Eqs. (5.9) a third-order quantity transforming as (1, 8) + (8, 1). Also recall that the K^* does not have a definite charge-conjugation parity, so the $(3,\bar{3})+(\bar{3},3)$ part of the commutator might be expected to have a nonzero contribution [first order in SU (3) breaking]. The spin condition, $\langle 0|D^K|K^*\rangle = 0$, allows this term to be expressed in terms of the (1, 8) + (8, 1) representation contributions.

[30]Experimentally $\Gamma_{K'} \simeq 250$ MeV, the K' decaying predominantly via the scalar mode ϵK, the vector modes ρK and $K^*\pi$ being less important.

[31]The equalities (5.30) are valid in the presence of Melosh transformation, provided the effects of quark-pair creation are negligible; see e.g. Ref. 10 and S. P. De Alwis and J. Stern, Nucl. Phys. B77, 509 (1974). In that case the axial charge Q_5^π (Q_5^K) belongs, in the constituent-quark basis, to a mixture of $[(8,1)-(1,8)]$ and $[(\bar{3},3)-(3,\bar{3})]$ representations.

[32]Particle Data Group, Rev. Mod. Phys. 48, S 1 (1976).

[33]S. Bartalucci *et al.*, DESY Report No. 77/59 (unpublished); G. Bassompierre *et al.*, Phys. Lett. 65B, 397 (1976).